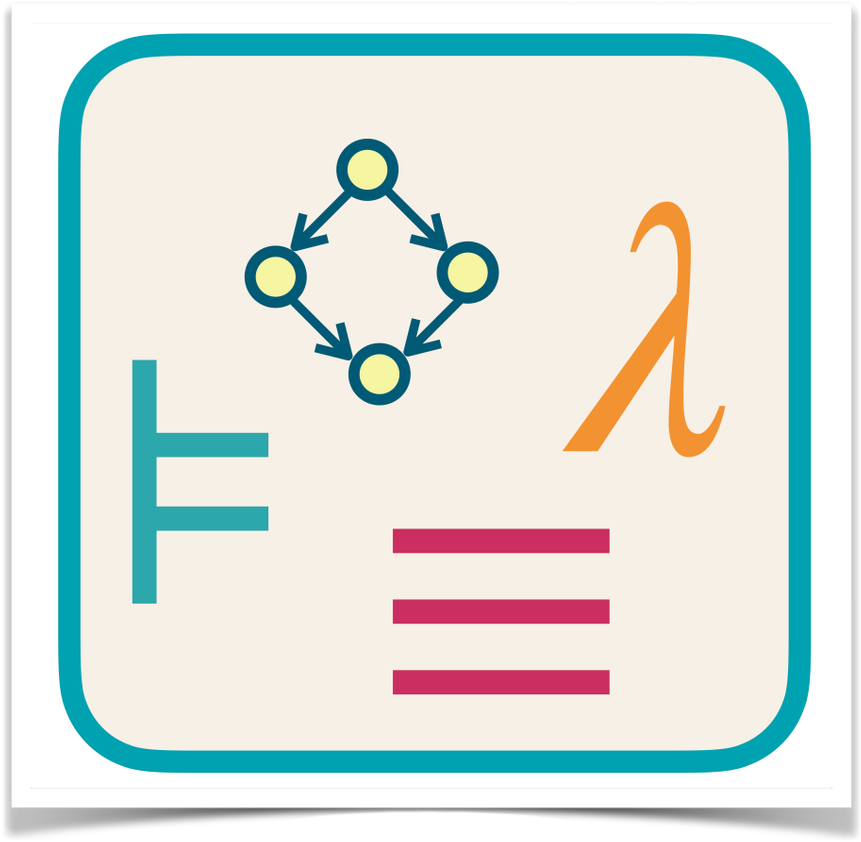


Linguaggi di Programmazione

Roberta Gori

CCS semantica astratta-11.4

CCS

isomorfismo tra grafi

CCS sintassi

$$
\begin{array}{lcll}
p, q & ::= & \mathbf{nil} & \text{processo inattivo} \\
& | & x & \text{variabile di processo (per la ricorsione)} \\
& | & \mu.p & \text{prefisso azione} \\
& | & p\backslash\alpha & \text{canale ristretto} \\
& | & p[\phi] & \text{rietichettatura del canale} \\
& | & p + q & \text{scelta nondeterministica (somma)} \\
& | & p|q & \text{compozione parallela} \\
& | & \mathbf{rec}\ x.\ p & \text{ricorsione}
\end{array}
$$

(gli operatori sono elencati in ordine di precedenza)

CCS semantica operazionale

Act) $$\frac{}{\mu.p \xrightarrow{\mu} p}$$

Res) $$\frac{p \xrightarrow{\mu} q \quad \mu \notin \{\alpha, \overline{\alpha}\}}{p\backslash\alpha \xrightarrow{\mu} q\backslash\alpha}$$

Rel) $$\frac{p \xrightarrow{\mu} q}{p[\phi] \xrightarrow{\phi(\mu)} q[\phi]}$$

SumL) $$\frac{p_1 \xrightarrow{\mu} q}{p_1 + p_2 \xrightarrow{\mu} q}$$

SumR) $$\frac{p_2 \xrightarrow{\mu} q}{p_1 + p_2 \xrightarrow{\mu} q}$$

ParL) $$\frac{p_1 \xrightarrow{\mu} q_1}{p_1 | p_2 \xrightarrow{\mu} q_1 | p_2}$$

Com) $$\frac{p_1 \xrightarrow{\lambda} q_1 \quad p_2 \xrightarrow{\overline{\lambda}} q_2}{p_1 | p_2 \xrightarrow{\tau} q_1 | q_2}$$

ParR) $$\frac{p_2 \xrightarrow{\mu} q_2}{p_1 | p_2 \xrightarrow{\mu} p_1 | q_2}$$

Rec) $$\frac{p\left[{}^{\mathbf{rec}\ x.\ p}/_{x}\right] \xrightarrow{\mu} q}{\mathbf{rec}\ x.\ p \xrightarrow{\mu} q}$$

Isomorfismo tra LTS

processi sintatticamente diversi
possono mostrare esattamente lo stesso comportamento

$\mathbf{nil}$ $\quad$ $\mathbf{nil}\backslash\alpha$ $\quad$ $\mathbf{nil}[\phi]$

p $\quad$ $p + \mathbf{nil}$ $\quad$ $p + p$ $\quad$ $p|\mathbf{nil}$

$p + q$ $\quad$ $q + p$

$p|q$ $\quad$ $q|p$

i loro LTS sono diversi (stati sintatticamente diversi)

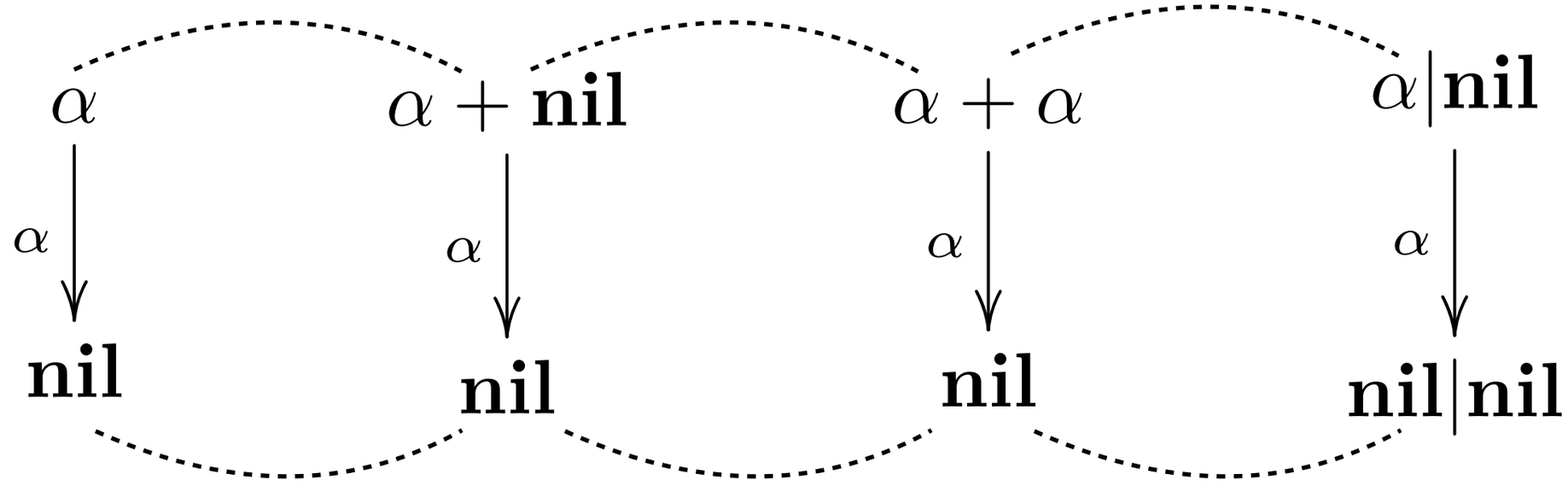
l'isomorfismo tra grafi astrae dagli stati

Isomorfismo tra grafi

$$G = (V, E) \qquad G' = (V', E')$$

$$v \xrightarrow{\mu} w \quad \Leftrightarrow \quad f(v) \xrightarrow{\mu} f(w)$$

$f : V \to V'$ bigettiva



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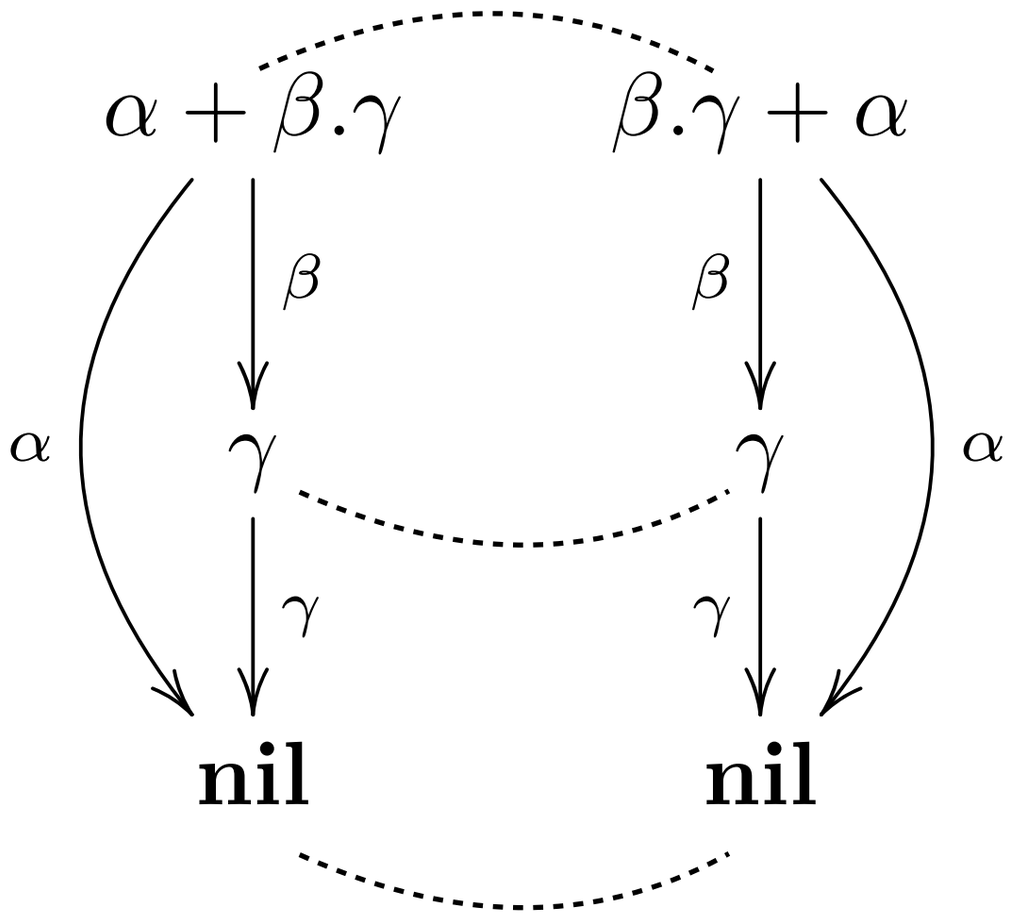

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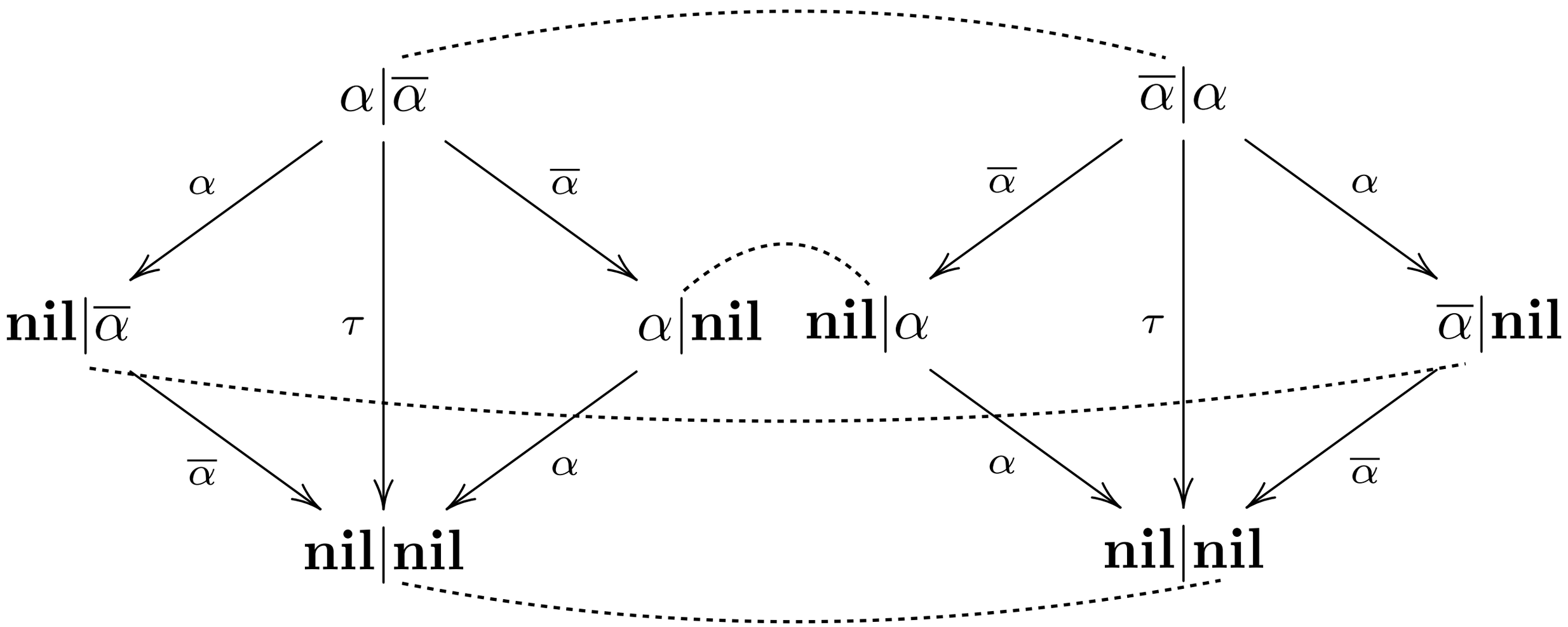

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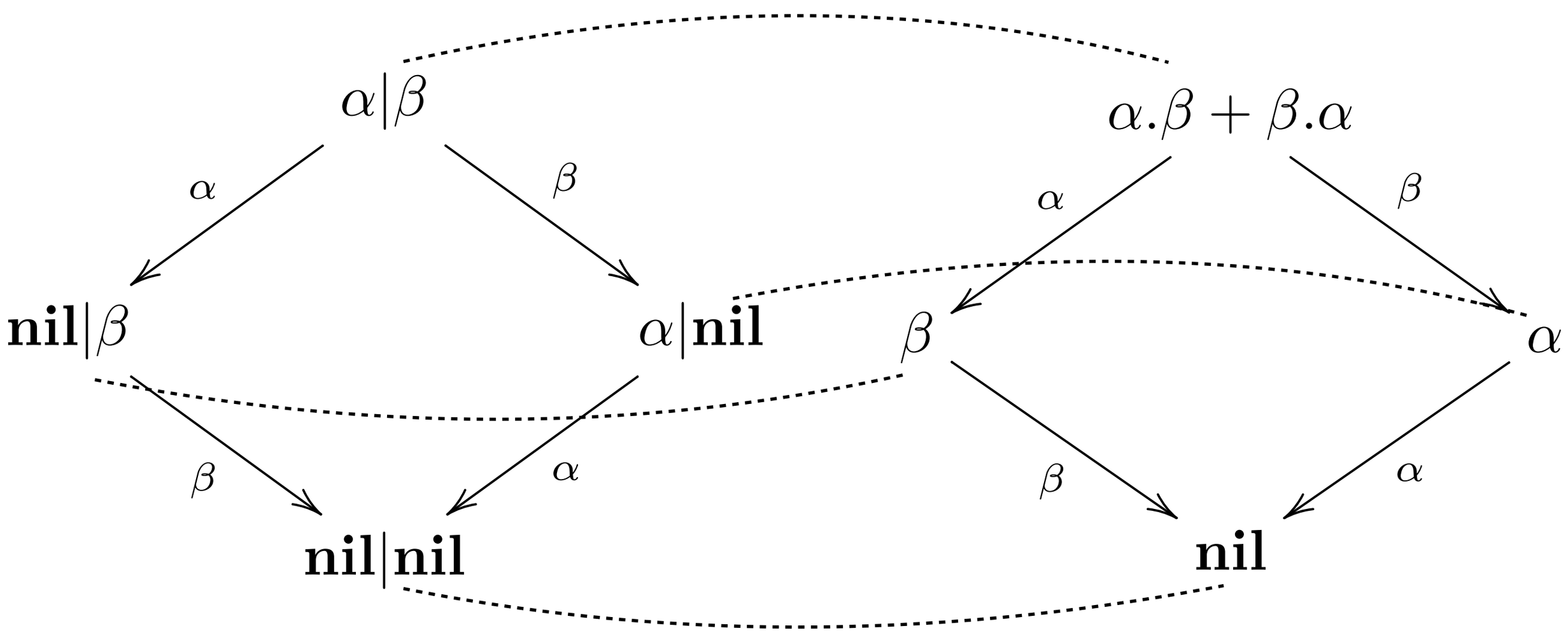
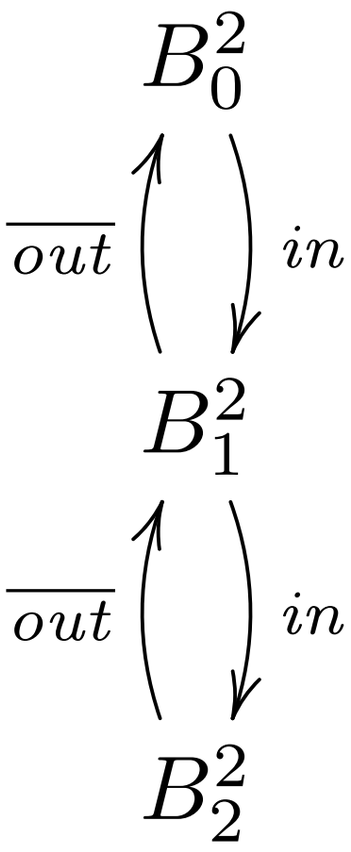
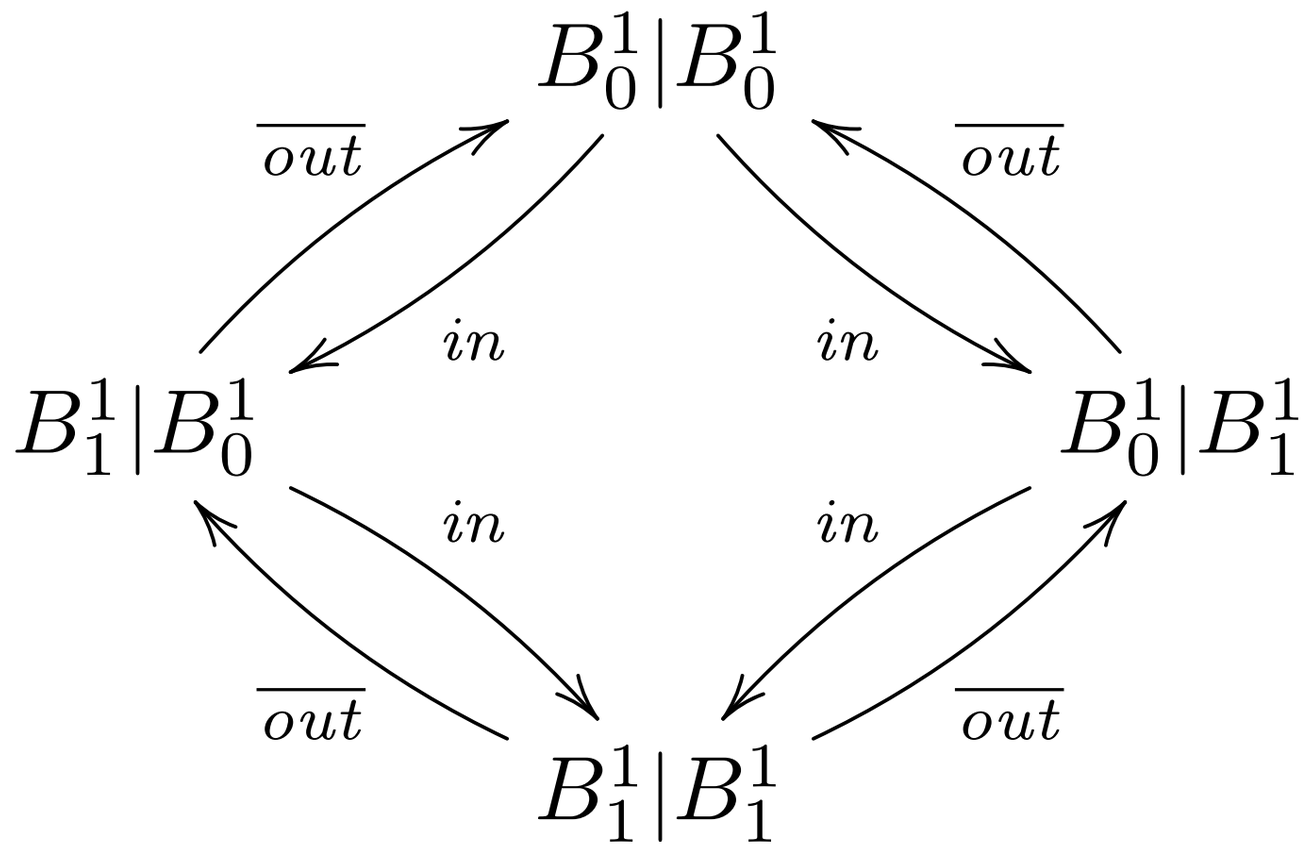


Equivalenti o no?





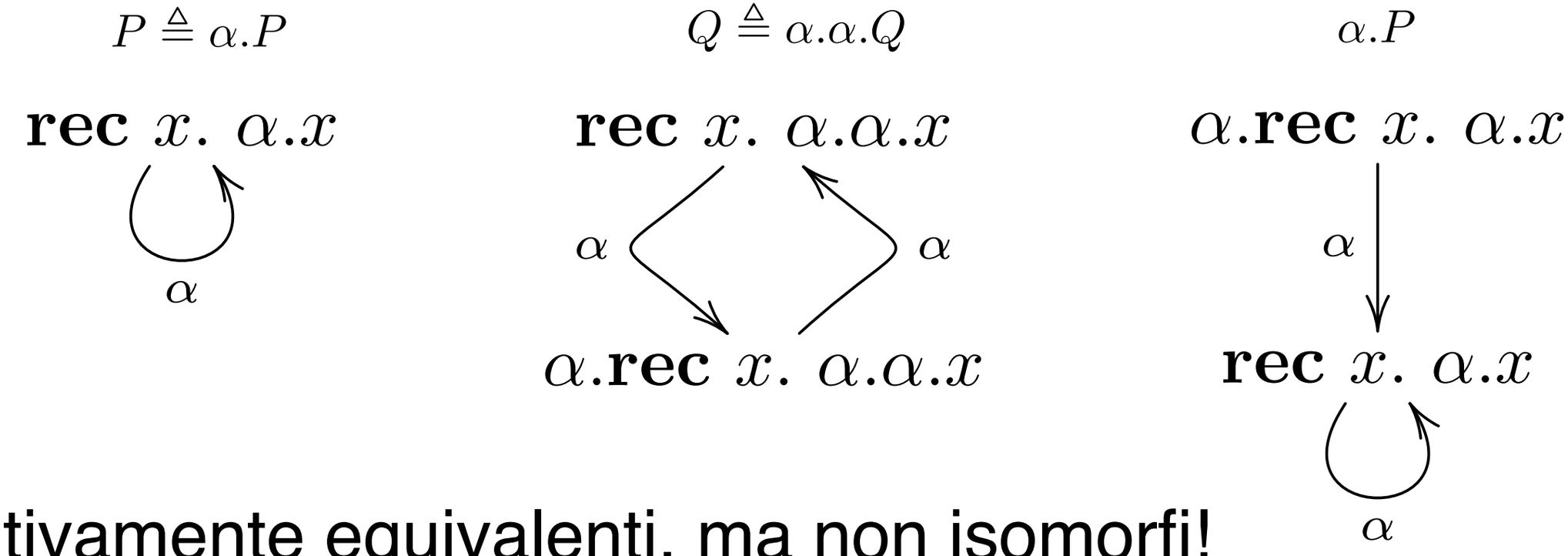
intuitivamente equivalenti, ma non isomorfi!

Iso e' troppo concreta

se due processi hanno LTS isomorfi
allora devono essere considerati equivalenti

L'isomorfismo tra grafi cattura alcune equivalenze interessanti

ma e' abbastanza?



intuitivamente equivalenti, ma non isomorfi!

CCS

equivalenza tra le tracce

Equivalenza per traccia

nella teoria degli automi: equivalenza tra linguaggi

nozione di tracce (finite) $p \xrightarrow{\mu_1 \mu_2 \cdots \mu_k} q$

$$p = p_0 \xrightarrow{\mu_1} p_1 \xrightarrow{\mu_2} \cdots \xrightarrow{\mu_k} p_k = q$$

semantica delle tracce (finite) di un processo

$$\mathcal{T}(p) = \{\mu_1 \mu_2 \cdots \mu_k \mid \exists q.\ p \xrightarrow{\mu_1 \mu_2 \cdots \mu_k} q\}$$

due processi sono equivalenti per traccia
se hanno la stessa semantica
delle tracce

$$p \equiv_{\mathsf{tr}} q \text{ iff } \mathcal{T}(p) = \mathcal{T}(q)$$

l'equivalenza per traccia è una buona nozione per sistemi concorrenti?

Equivalenza per traccia

isomorfismo tra grafi implica l'equivalenza per traccia

conserviamo tutte le equivalenze utili viste prima

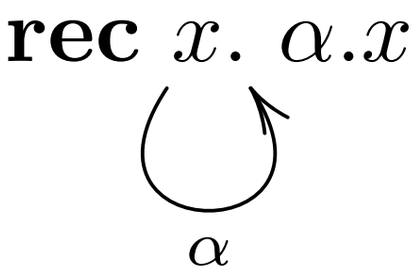
$$p \equiv_{\mathsf{tr}} p + \mathbf{nil} \equiv_{\mathsf{tr}} p + p \equiv_{\mathsf{tr}} p|\mathbf{nil}$$

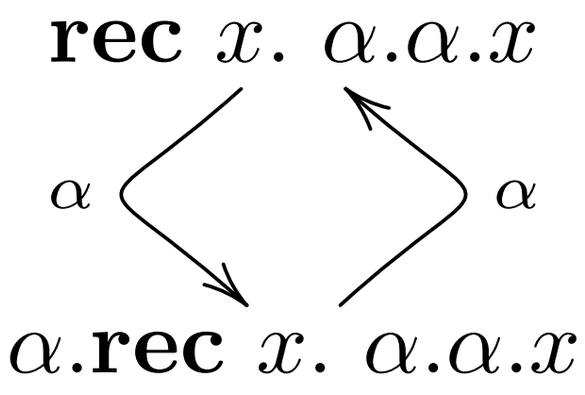
$$p + q \equiv_{\mathsf{tr}} q + p$$

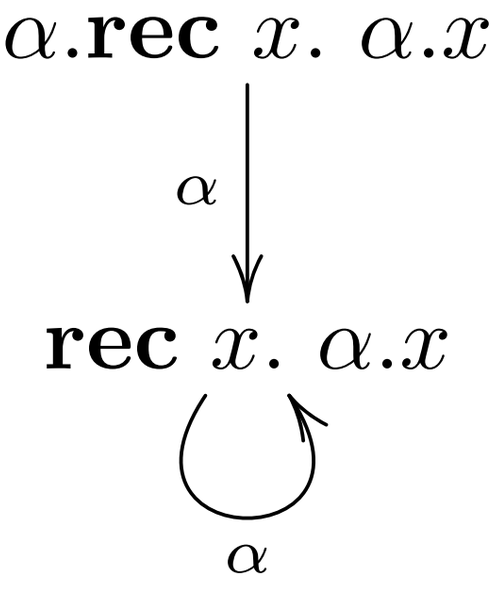
$$p|q \equiv_{\mathsf{tr}} q|p$$

la semantica delle tracce è chiusa per prefissi
(se una traccia è presente, sono presenti anche tutti i suoi prefissi)

Equivalenza per traccia





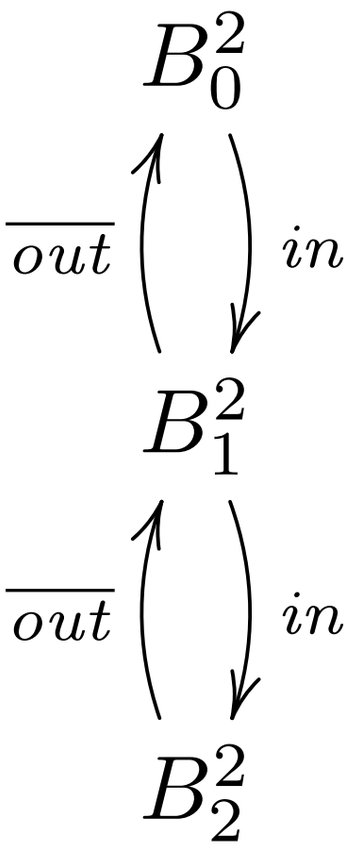


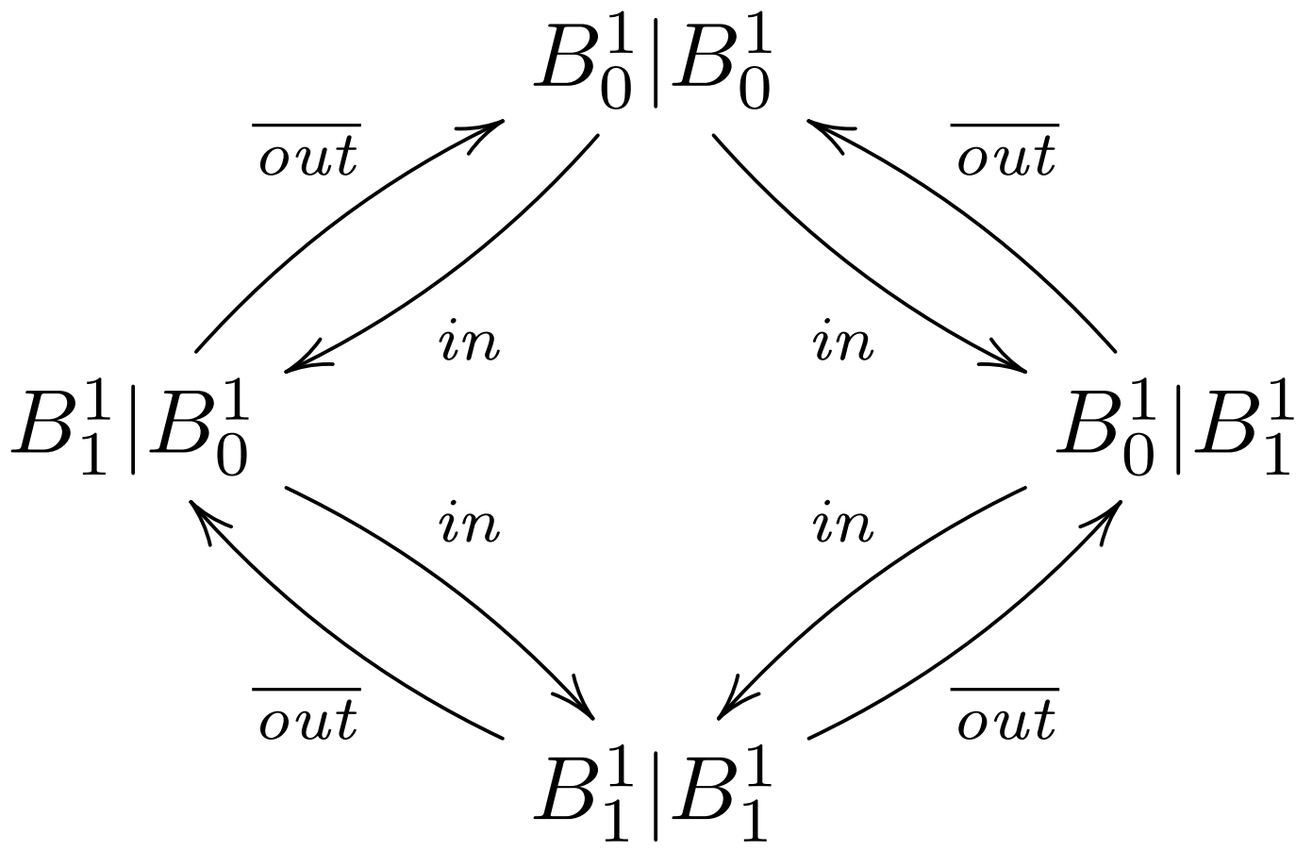
$$\mathcal{T}(\textbf{rec } x.\ \alpha.x) = \{\alpha^n \mid n \in \mathbb{N}\}$$

$$\mathcal{T}(\textbf{rec } x.\ \alpha.\alpha.x) = \{\alpha^n \mid n \in \mathbb{N}\}$$

$$\mathcal{T}(\alpha.\textbf{rec } x.\ \alpha.x) = \{\alpha^n \mid n \in \mathbb{N}\}$$

Equivalenza per traccia





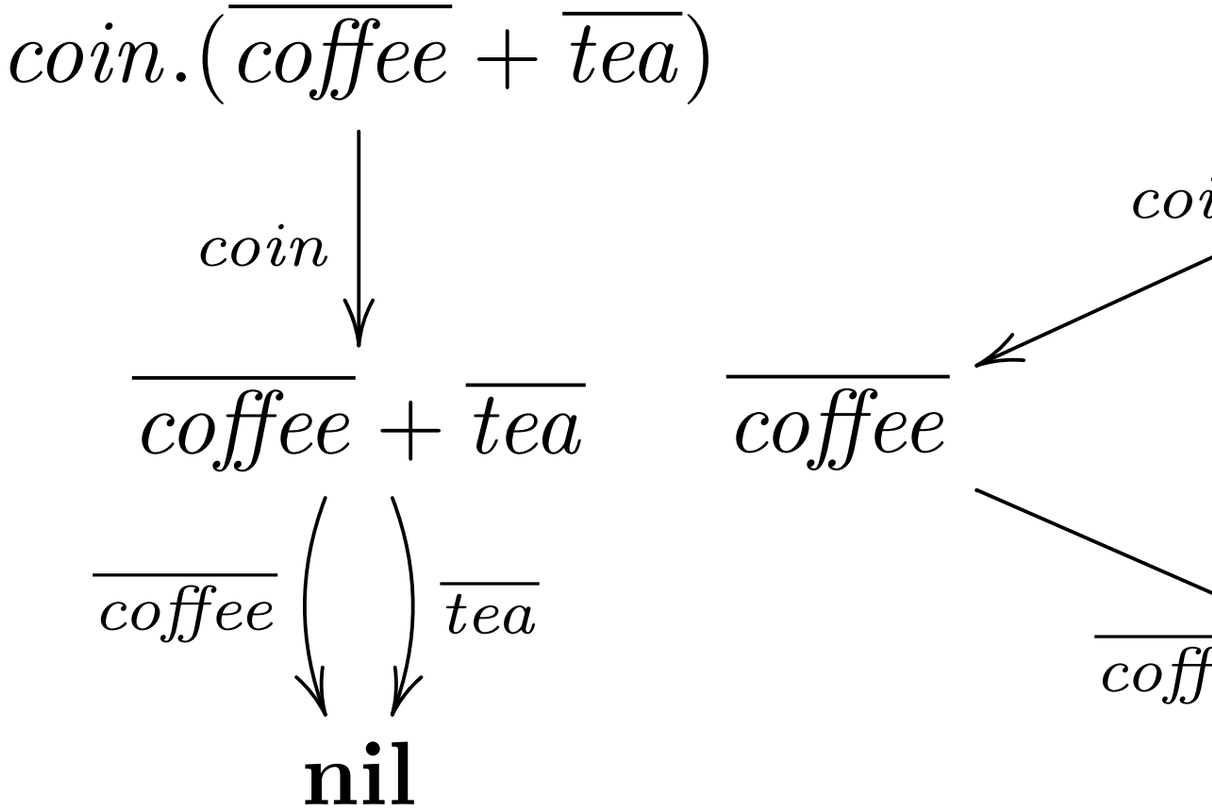
$$\mathcal{T}(B_0^2) = \mathcal{T}(B_0^1|B_0^1)$$

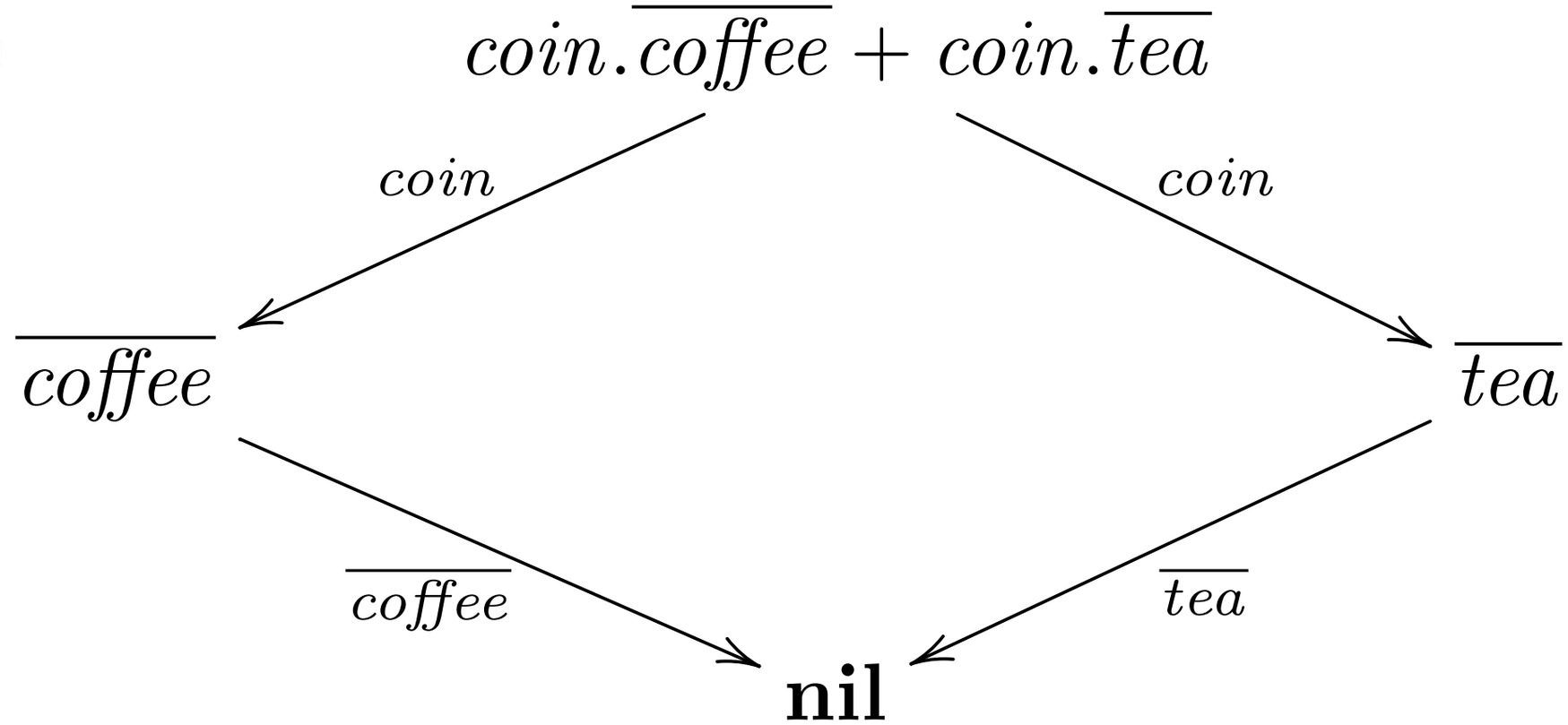
tentativo di descrizione:
$0 \leq \#in - \#out \leq 2$ (per ogni prefisso)

2 distributori automatici

$$coin.(\overline{\overline{coffee} + \overline{tea}})$$

$$coin.\overline{\overline{coffee}} + coin.\overline{tea}$$

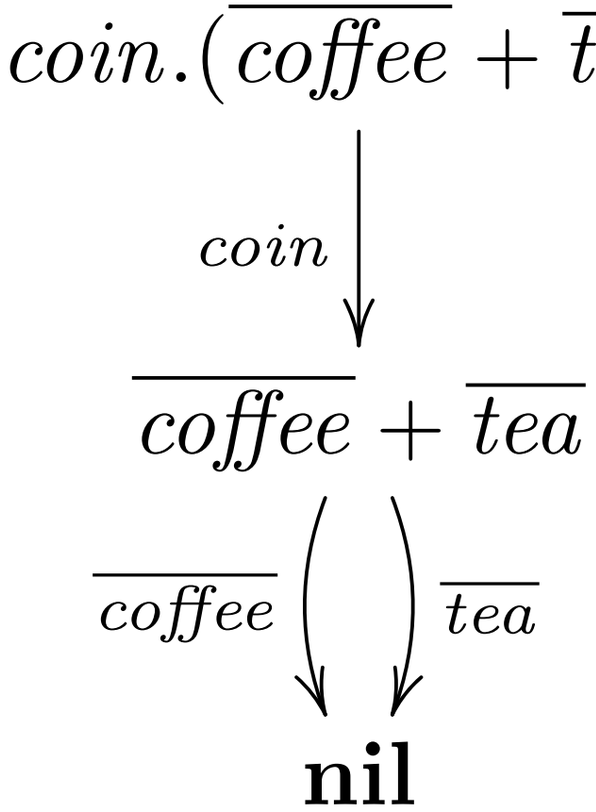




non isomorfi, ma equivalenti per traccia

La visione del cliente

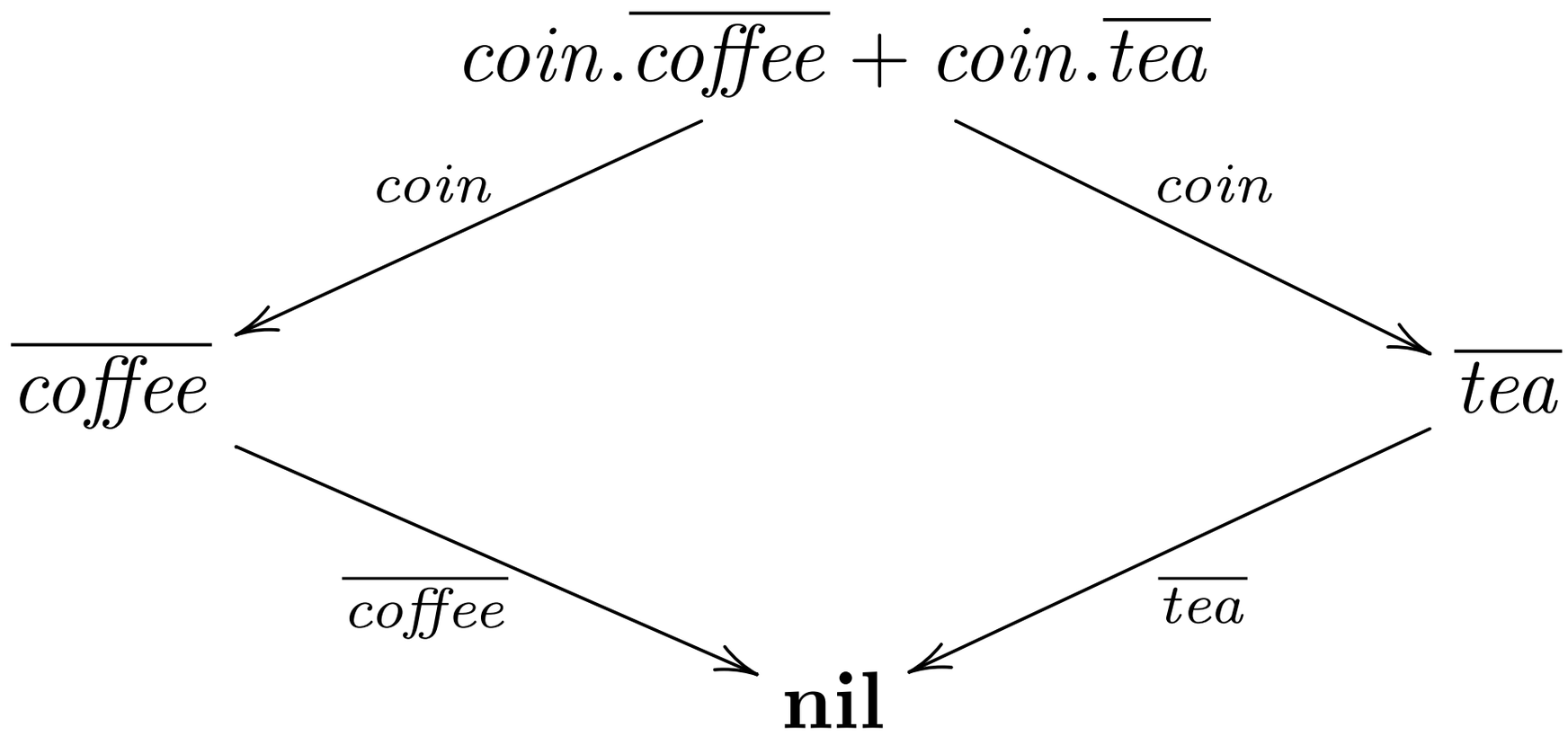
quale distributore automatico preferiresti?



inserisci una moneta, poi scegli la bevanda

La visione del cliente

quale distributore automatico preferiresti?

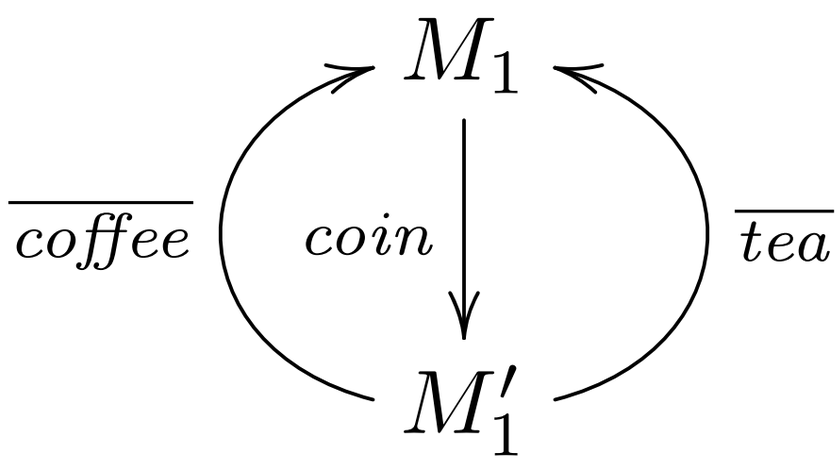


inserire una moneta, poi ottenere la bevanda scelta dalla macchina

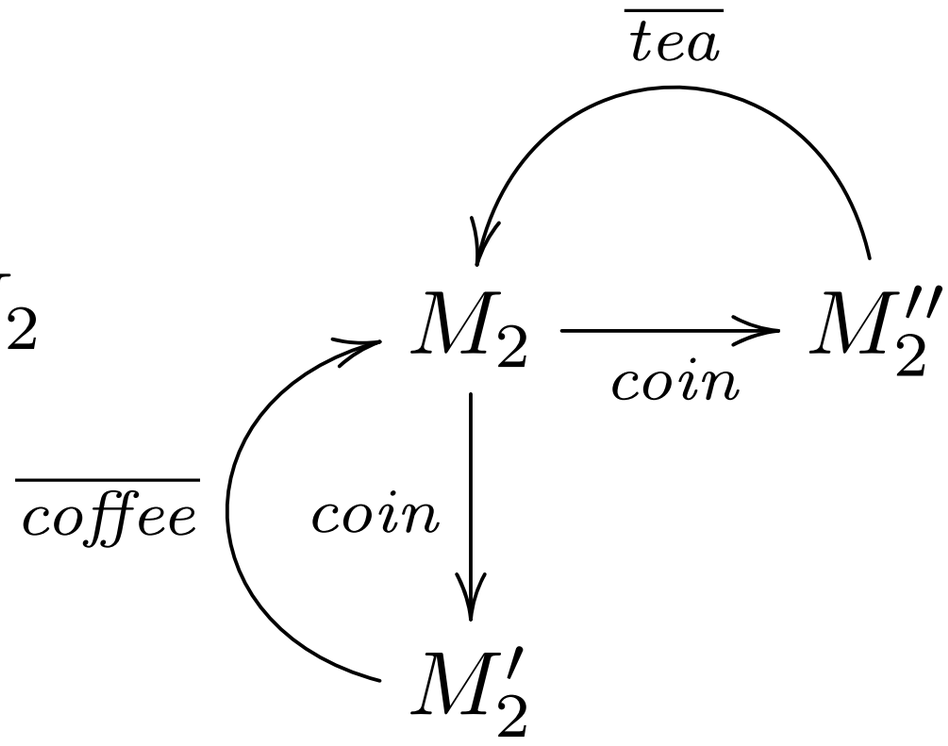
Distributori ricorsivi

$$M_2 \triangleq coin.\overbrace{\overline{coffee}.M_2}^{M_2'} + coin.\overbrace{\overline{tea}.M_2}^{M_2''}$$

$$M_1 \triangleq coin.\overbrace{(\overline{coffee}.M_1 + \overline{tea}.M_1)}^{M_1'}$$



$$M_1 \equiv_{\mathsf{tr}} M_2$$



la visione del sistema

$$M_1 \triangleq coin.\overbrace{(\overline{coffee}.M_1 + \overline{tea}.M_1)}^{M_1'}$$

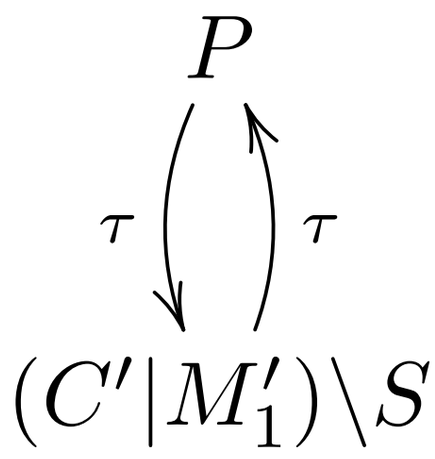
$$M_1 \equiv_{\mathsf{tr}} M_2$$

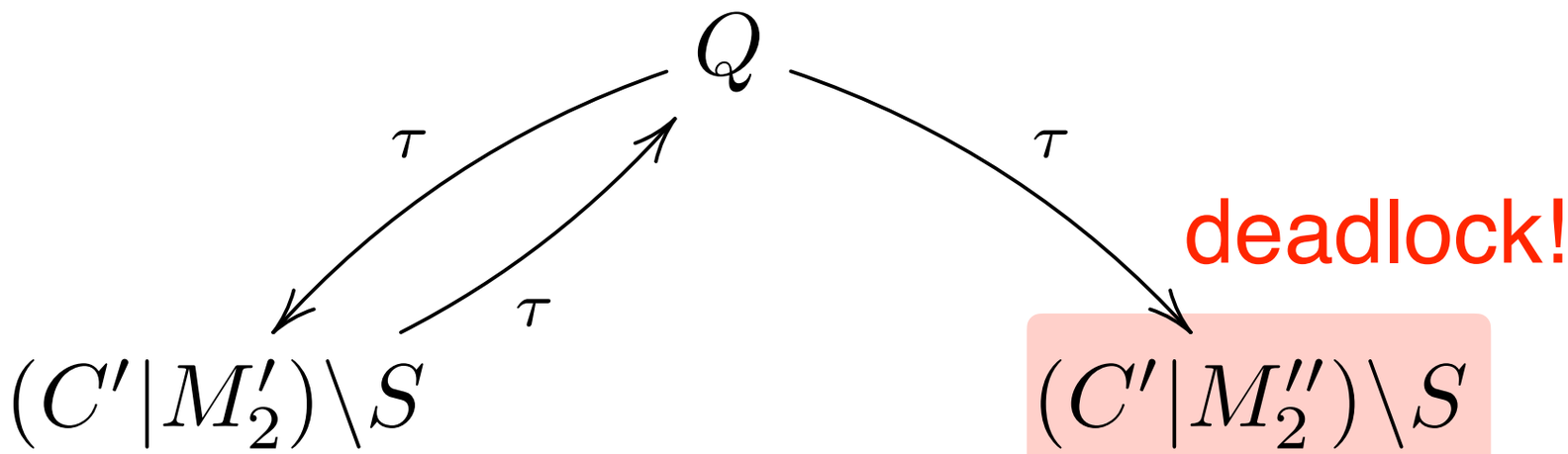
$$M_2 \triangleq coin.\overbrace{\overline{coffee}.M_2}^{M_2'} + coin.\overbrace{\overline{tea}.M_2}^{M_2''}$$

$$C \triangleq \overline{coin}.\overbrace{coffee.C}^{C'}$$

$$S \triangleq \{coin, coffee, tea\}$$

$$P \triangleq (C|M_1)\backslash S \qquad P \equiv_{\mathsf{tr}} Q \qquad Q \triangleq (C|M_2)\backslash S$$





prossimamente: bisimilarità

l'isomorfismo tra grafi distingue troppi processi

l'equivalenza delle tracce identifica troppi processi

abbiamo bisogno di qualche nozione di equivalenza tra i due

introduciamo la nozione di *strong bisimilarity*

come un gioco

come un punto fisso

come un equivalenza logica

da tenere a mente: due processi sono equivalenti a meno che abbiamo delle buone ragioni per distinguerli