Linguaggi di Programmazione

Roberta Gori

Esercitazione #5

HOFL, inferenza di tipi
e semantica operazionale

[Es 1.]

Determinare il tipo del termine HOFL

$$t \stackrel{\text{def}}{=} \mathbf{rec}\ x.\ \big((\lambda y.\ \mathbf{if}\ y\ \mathbf{then}\ 0\ \mathbf{else}\ 0)\ x\big).$$

Quindi, calcolare la sua forma canonica (in valutazione *lazy*).

$$t \triangleq \mathbf{rec}\ x.\ (\ (\ \lambda y.\ \mathbf{if}\ y\ \mathbf{then}\ 0\ \mathbf{else}\ 0\ )\ x\ )$$

Es. 1, inferenza di tipi

$$t \triangleq \underbrace{\mathbf{rec}\ \underbrace{x}_{int}.\ \underbrace{(\ \underbrace{(\ \lambda \underbrace{y}_{int}.\ \underbrace{\mathbf{if}\ \underbrace{y}_{int}\ \mathbf{then}\ \underbrace{0}_{int}\ \mathbf{else}\ \underbrace{0}_{int}}_{int}\ )}_{int \to int}\ \underbrace{x}_{int}\ )}_{int}}_{int} \quad : int$$

Es. 1, forma canonica?

$$t \triangleq \mathbf{rec}\ x.\ (\ (\ \lambda y.\ \mathbf{if}\ y\ \mathbf{then}\ 0\ \mathbf{else}\ 0\ )\ x\ )$$

$$\frac{t[^{\mathbf{rec}\ x.\ t}/_x] \to c}{\mathbf{rec}\ x.\ t \to c}$$

$$t \to c \ \nwarrow\ (\ (\ \lambda y.\ \mathbf{if}\ y\ \mathbf{then}\ 0\ \mathbf{else}\ 0\ )\ x\ )[^t/_x] \to c$$
$$= (\ (\ \lambda y.\ \mathbf{if}\ y\ \mathbf{then}\ 0\ \mathbf{else}\ 0\ )\ t\ ) \to c$$

$$\frac{t_1 \to \lambda x.\ t_1' \quad t_1'[^{t_0}/_x] \to c}{(t_1\ t_0) \to c}$$

$$\nwarrow\ \lambda y.\ \mathbf{if}\ y\ \mathbf{then}\ 0\ \mathbf{else}\ 0 \to \lambda x'.\ t'\ ,\ t'[^t/_{x'}] \to c$$

$$\nwarrow_{x'=y, t'=\mathbf{if}\ldots}\ (\ \mathbf{if}\ y\ \mathbf{then}\ 0\ \mathbf{else}\ 0\ )[^t/_y] \to c$$
$$= (\ \mathbf{if}\ t\ \mathbf{then}\ 0\ \mathbf{else}\ 0\ ) \to c$$

$$\nwarrow\ t \to n,\ 0 \to c$$ (non importa se $n = 0$)

Stesso goal di quello da cui siamo partiti:
non esiste canonical form

[Es 2.]

Determinare il tipo del termine HOFL

$$map \stackrel{\text{def}}{=} \lambda f.\, \lambda x.\, \big((f\ \mathbf{fst}(x)),\ (f\ \mathbf{snd}(x))\big).$$

Quindi, calcolare le forme canoniche (in semantica *lazy*) dei termini

$$t_1 \stackrel{\text{def}}{=} map\ (\lambda z.\, 2 \times z)\ (1, 2) \qquad\qquad t_2 \stackrel{\text{def}}{=} \mathbf{fst}\big(map\ (\lambda z.\, 2 \times z)\ (1, 2)\big).$$

$$map \triangleq \lambda f \ . \ \lambda x \ . \ (\ (\ f \ \mathbf{fst}(x) \ ) \ , \ (\ f \ \mathbf{snd}(x) \ ) \ )$$

Es. 2, inferenza di tipi

$$map \triangleq \underbrace{\lambda \underbrace{f}_{\tau_1 \to \tau} \; . \; \underbrace{\lambda \underbrace{x}_{\tau_1 * \tau_1} \; . \; \underbrace{(\; \underbrace{(\; \underbrace{f}_{\tau_1 \to \tau} \; \underbrace{\mathbf{fst}(\underbrace{x}_{\tau_1 * \tau_2})}_{\tau_1} \;)}_{\tau} \; , \; \underbrace{(\; \underbrace{f}_{\tau_1 \to \tau} \; \underbrace{\mathbf{snd}(\underbrace{x}_{\tau_1 * \tau_2})}_{\tau_2 = \tau_1} \;)}_{\tau} \;)}_{\tau * \tau}}_{\tau_1 * \tau_1 \to \tau * \tau}}_{(\tau_1 \to \tau) \to \tau_1 * \tau_1 \to \tau * \tau}$$

$$map \; : \; (\tau_1 \to \tau) \to \tau_1 * \tau_1 \to \tau * \tau$$

Es. 2a, forma canonica

$$map \triangleq \lambda f\ .\ \lambda x\ .\ (\ (\ f\ \mathbf{fst}(x)\ )\ ,\ (\ f\ \mathbf{snd}(x)\ )\ )$$

$$t_1 \triangleq map\ (\lambda z.\ 2 \times z)\ (1,2)$$

$$\frac{t_1 \to \lambda x.\, t_1' \quad t_1'[^{t_0}/_x] \to c}{(t_1\ t_0) \to c}$$

$$t_1 \to c\ \nwarrow\ (map\ (\lambda z.\ 2 \times z)) \to \lambda x'.\ t'\ ,\ t'[^{(1,2)}/_{x'}] \to c$$

$$\nwarrow\ map \to \lambda f'.\ t''\ ,\ t''[^{\lambda z.\ 2\times z}/_{f'}] \to \lambda x'.\ t'\ ,\ t'[^{(1,2)}/_{x'}] \to c$$

$$\nwarrow_{f'=f,t''=\lambda x\ldots}\ (\lambda x.\ ((f\ \mathbf{fst}(x)),(f\ \mathbf{snd}(x))))[^{\lambda z.\ 2\times z}/_f] \to \lambda x'.\ t'\ ,\ t'[^{(1,2)}/_{x'}] \to c$$

$$= (\lambda x.\ (((\lambda z.\ 2 \times z)\ \mathbf{fst}(x)),((\lambda z.\ 2 \times z)\ \mathbf{snd}(x)))) \to \lambda x'.\ t'\ ,\ t'[^{(1,2)}/_{x'}] \to c$$

$$\nwarrow_{x'=x,t'=(\ldots,\ldots)}\ (((\lambda z.\ 2 \times z)\ \mathbf{fst}(x)),((\lambda z.\ 2 \times z)\ \mathbf{snd}(x)))[^{(1,2)}/_x] \to c$$

$$= (((\lambda z.\ 2 \times z)\ \mathbf{fst}(1,2)),((\lambda z.\ 2 \times z)\ \mathbf{snd}(1,2))) \to c$$

$$\nwarrow_{c=(((\lambda z.\ 2\times z)\ \mathbf{fst}(1,2)),((\lambda z.\ 2\times z)\ \mathbf{snd}(1,2)))}\ \square$$

Es. 2b, forma canonica

$$t_1 \to (\ ((\lambda z.\ 2 \times z)\ \mathbf{fst}(1,2))\ ,\ ((\lambda z.\ 2 \times z)\ \mathbf{snd}(1,2))\ )$$

$$\mathbf{fst}(t_1) \to c\ \nwarrow\ \ t_1 \to (t_1', t_2')\ ,\ t_1' \to c$$

$$\nwarrow^{*}_{t_1'=(\lambda z.\ 2\times z)\ \mathbf{fst}(1,2)\ ,\ t_2'=(\lambda z.\ 2\times z)\ \mathbf{snd}(1,2)}\ (\lambda z.\ 2 \times z)\ \mathbf{fst}(1,2) \to c$$

$$\nwarrow\ \lambda z.\ 2 \times z \to \lambda z'.\ t'\ ,\ t'[^{\mathbf{fst}(1,2)}/_{z'}] \to c$$

$$\nwarrow_{z'=z, t'=2\times z}\ (2 \times z)[^{\mathbf{fst}(1,2)}/_{z}] \to c$$

$$= (2 \times \mathbf{fst}(1,2)) \to c$$

$$\nwarrow_{c=n_1 \underline{\times} n_2}\ 2 \to n_1\ ,\ \mathbf{fst}(1,2) \to n_2$$

$$\nwarrow^{*}_{n_1=2}\ (1,2) \to (t_1'', t_2'')\ ,\ t_1'' \to n_2$$

$$\nwarrow_{t_1''=1, t_2''=2}\ 1 \to n_2$$

$$\nwarrow_{n_2=1}\ \square$$

$$c = n_1 \underline{\times} n_2 = 2 \underline{\times} 1 = 2$$

Teoria dei domini

[Es.3] Sia $(D, \sqsubseteq_D)$ un CPO e $f : D \to D$ una funzione continua.
Dimostrare che l'insieme dei punti fissi di f è lui stesso un CPO (ordinato con $\sqsubseteq_D$).

ES. 3, CPO dei punti fissi

$(D, \sqsubseteq_D)$ CPO $\qquad f : D \to D$ continua

$\mathsf{FP}_f \triangleq \{ \ d \ \mid \ d = f(d) \ \}$ l'insieme di tutti i punti fissi di f

$(\mathsf{FP}_f, \sqsubseteq) \qquad \sqsubseteq \triangleq \sqsubseteq_D \cap (\mathsf{FP}_f \times \mathsf{FP}_f)$ CPO?

e' un PO (perche' $\mathsf{FP}_f \subseteq D$)

proviamo che e' completo prendiamo una catena $\{d_i\}_{i \in \mathbb{N}} \subseteq \mathsf{FP}_f$

mostriamo $\bigsqcup_{i \in \mathbb{N}} d_i$ come calcolato in D e' un puntofisso di f

$$f\left(\bigsqcup_{i \in \mathbb{N}} d_i\right) = \bigsqcup_{i \in \mathbb{N}} f(d_i) \qquad \text{by continuita'}$$

$$= \bigsqcup_{i \in \mathbb{N}} d_i \qquad \text{ogni } d_i \text{ e' un puntofisso}$$

HOFL semantica denotazionale

[Es.4] [Test di convergenza] Vogliamo modificare la semantica denotazionale di HOFL assegnando al costrutto

$$\textbf{if } t \textbf{ then } t_0 \textbf{ else } t_1$$

- la semantica di t_1 se la semantica di t è $\perp_{\mathbb{Z}_\perp}$, e
- la semantica di t_0 altrimenti.

È possibile? In caso contrario, perché?

Es. 4, test di convergenza

$$[\![\textbf{if } t \textbf{ then } t_0 \textbf{ else } t_1]\!]\rho \triangleq \mathsf{Cond}^{\perp}_{\tau}(\ [\![t]\!]\rho\ ,\ [\![t_0]\!]\rho\ ,\ [\![t_1]\!]\rho\ )$$

$$\mathsf{Cond}^{\perp}_{\tau}(v, d_0, d_1) \triangleq \begin{cases} d_0 & \text{se } v = \lfloor n \rfloor \text{ per qualche } n \\ d_1 & \text{altrimenti} \end{cases}$$

nessun problema?

$\mathsf{Cond}^{\perp}_{\tau}$ non e' monotona in *v* !

Contro esempio $\perp_{\mathbb{Z}_\perp} \sqsubseteq_{\mathbb{Z}_\perp} \lfloor 1 \rfloor$ Prendiamo $d_1 \not\sqsubseteq_{D_\tau} d_0$

$$(\perp_{\mathbb{Z}_\perp}, d_0, d_1) \sqsubseteq_{\mathbb{Z}_\perp \times D_\tau \times D_\tau} (\lfloor 1 \rfloor, d_0, d_1)$$

$$\mathsf{Cond}^{\perp}_{\tau}(\perp_{\mathbb{Z}_\perp}, d_0, d_1) = d_1 \not\sqsubseteq_{D_\tau} d_0 = \mathsf{Cond}^{\perp}_{\tau}(\lfloor 1 \rfloor, d_0, d_1)$$

Es. 4, test di convergenza

Per esempio prendiamo $d_0 = \lfloor 0 \rfloor$ $\quad d_1 = \lfloor 1 \rfloor$

$$\llbracket \textbf{if rec } x.\ x \textbf{ then } 0 \textbf{ else } 1 \rrbracket \rho = \lfloor 1 \rfloor$$

$$\not\sqsubseteq_{\mathbb{Z}_\perp}$$

$$\llbracket \textbf{if } 1 \textbf{ then } 0 \textbf{ else } 1 \rrbracket \rho = \lfloor 0 \rfloor$$

come conseguenza

$$t \triangleq \lambda x.\ \textbf{if } x \textbf{ then } 0 \textbf{ else } 1 \ :\ int \to int$$

non puo' avere una semantica in $D_{int \to int} = [\mathbb{Z}_\perp \to \mathbb{Z}_\perp]_\perp$

perche' $\llbracket t \rrbracket \rho$ non e' continua (non e' monotona)

[Es.5]

[Condizionale stict] Modificare la semantica operazionale di HOFL adottando le seguenti regole per i condizionali:

$$\frac{t \to 0 \quad t_0 \to c_0 \quad t_1 \to c_1}{\textbf{if } t \textbf{ then } t_0 \textbf{ else } t_1 \ \to \ c_0} \qquad \frac{t \to n \quad n \neq 0 \quad t_0 \to c_0 \quad t_1 \to c_1}{\textbf{if } t \textbf{ then } t_0 \textbf{ else } t_1 \ \to \ c_1}.$$

Senza modificare la semantica denotazionale, dimostrare che:

1. per ogni termine t e forma canonica c, vale

$$t \to c \ \Rightarrow \ \forall \rho.\ [[t]]\rho = [[c]]\rho;$$

2. in generale $t\Downarrow \not\Rightarrow \ t\downarrow$ (fornire un controesempio).

Es. 5.1, correttezza

$$\frac{t \to 0 \quad t_0 \to c_0 \quad t_1 \to c_1}{\textbf{if } t \textbf{ then } t_0 \textbf{ else } t_1 \ \to \ c_0} \qquad \frac{t \to n \quad n \neq 0 \quad t_0 \to c_0 \ \ t_1 \to c_1}{\textbf{if } t \textbf{ then } t_0 \textbf{ else } t_1 \ \to \ c_1}.$$

$$t \to c \ \Rightarrow \ \forall \rho. \ [\![t]\!] \, \rho = [\![c]\!] \, \rho$$

$$P(t \to c) \triangleq \forall \rho. \ [\![t]\!]\rho = [\![c]\!]\rho$$

estendiamo la prova di corretteza (per induzione sulle regole) per considerare le nuove regole

Es. 5.1, correttezza

$$P(t \to c) \triangleq \forall \rho.\ [\![t]\!] \rho = [\![c]\!] \rho$$

$$\frac{t \to 0 \quad t_0 \to c_0 \quad t_1 \to c_1}{\mathbf{if}\ t\ \mathbf{then}\ t_0\ \mathbf{else}\ t_1 \ \to\ c_0}$$

assumiamo

$$P(t \to 0) \triangleq \forall \rho.\ [\![t]\!] \rho = [\![0]\!] \rho = \lfloor 0 \rfloor$$

$$P(t_0 \to c_0) \triangleq \forall \rho.\ [\![t_0]\!] \rho = [\![c_0]\!] \rho$$

$$P(t_1 \to c_1) \triangleq \forall \rho.\ [\![t_1]\!] \rho = [\![c_1]\!] \rho$$

vogliamo dimostrare

$$P(\mathbf{if}\ t\ \mathbf{then}\ t_0\ \mathbf{else}\ t_1 \to c_0) \triangleq \forall \rho.\ [\![\mathbf{if}\ t\ \mathbf{then}\ t_0\ \mathbf{else}\ t_1]\!] \rho = [\![c_0]\!] \rho$$

$$\begin{aligned} [\![\mathbf{if}\ t\ \mathbf{then}\ t_0\ \mathbf{else}\ t_1]\!] \rho &= \mathsf{Cond}_\tau([\![t]\!]\rho, [\![t_0]\!]\rho, [\![t_1]\!]\rho) && \text{per def} \\ &= \mathsf{Cond}_\tau(\lfloor 0 \rfloor, [\![c_0]\!]\rho, [\![c_1]\!]\rho) && \text{per ip.ind.} \\ &= [\![c_0]\!]\rho && \text{per Cond} \end{aligned}$$

Ex. 5.1, correttezza

$$P(t \to c) \triangleq \forall \rho.\ [\![t]\!]\rho = [\![c]\!]\rho$$

$$\frac{t \to n \quad n \neq 0 \quad t_0 \to c_0 \quad t_1 \to c_1}{\textbf{if } t \textbf{ then } t_0 \textbf{ else } t_1 \ \to \ c_1}$$

assumiamo $P(t \to n) \triangleq \forall \rho.\ [\![t]\!]\rho = [\![n]\!]\rho = \lfloor n \rfloor \quad n \neq 0$

$$P(t_0 \to c_0) \triangleq \forall \rho.\ [\![t_0]\!]\rho = [\![c_0]\!]\rho$$

$$P(t_1 \to c_1) \triangleq \forall \rho.\ [\![t_1]\!]\rho = [\![c_1]\!]\rho$$

vogliamo dimostrare

$$P(\textbf{if } t \textbf{ then } t_0 \textbf{ else } t_1 \to c_1) \triangleq \forall \rho.\ [\![\textbf{if } t \textbf{ then } t_0 \textbf{ else } t_1]\!]\rho = [\![c_1]\!]\rho$$

$$\begin{aligned}
[\![\textbf{if } t \textbf{ then } t_0 \textbf{ else } t_1]\!]\rho &= \mathsf{Cond}_\tau([\![t]\!]\rho, [\![t_0]\!]\rho, [\![t_1]\!]\rho) && \text{per def} \\
&= \mathsf{Cond}_\tau(\lfloor n \rfloor, [\![c_0]\!]\rho, [\![c_1]\!]\rho) && \text{per ip.ind.} \\
&= [\![c_1]\!]\rho && \text{per Cond}
\end{aligned}$$

Ex. 5.2, inconsistenza

vogliamo trovare un termine t tale che $t \Downarrow$ $t \uparrow$

consideriamo $t \triangleq \textbf{if } 0 \textbf{ then } 1 \textbf{ else rec } x.\ x \ : \ int$

$$[\![t]\!]\rho = \mathsf{Cond}_{int}([\![0]\!]\rho, [\![1]\!]\rho, [\![\textbf{rec } x.\ x]\!]\rho)$$

$$= \mathsf{Cond}_{int}(\lfloor 0 \rfloor, \lfloor 1 \rfloor, \bot_{\mathbb{Z}_\bot}) \quad = \lfloor 1 \rfloor \qquad t \Downarrow$$

$$t \to c \ \nwarrow \ 0 \to 0\ ,\ 1 \to c\ ,\ \textbf{rec } x.\ x \to c_1$$

$$\nwarrow^*_{c=1} \ \textbf{rec } x.\ x \to c_1$$

$$\nwarrow \ x[^{\textbf{rec } x.\ x}/_x] \to c_1$$

$$= \textbf{rec } x.\ x \to c_1 \qquad t \uparrow$$

[Es.6]

Determinare il tipo del termine HOFL

$$t \stackrel{\text{def}}{=} \mathbf{rec}\ f.\ (\ \lambda x.\, 1\ ,\ \mathbf{fst}(f)\ 0\ ).$$

Quindi, calcolare la semantica denotazionale (in valutazione *lazy*) di t.

Es. 6, inferenza di tipi

$$t \triangleq \underbrace{\mathbf{rec}\ \underbrace{f}_{(int\to\tau_1)*\tau_2}.\ \underbrace{(\ \underbrace{\lambda \underbrace{x}_{\tau}.\ \underbrace{1}_{int}}_{\tau\to int}\ ,\ \underbrace{(\underbrace{\mathbf{fst}(\ \underbrace{f}_{(int\to\tau_1)*\tau_2}\ )}_{int\to\tau_1}\ \underbrace{0}_{int}\ )}_{\tau_1}\ )}_{(\tau\to int)*\tau_1}}_{(int\to\tau_1)*\tau_2 = (\tau\to int)*\tau_1} \quad : (int \to int) * int$$

$$\begin{cases} int = \tau \\ \tau_1 = int \\ \tau_2 = \tau_1 \end{cases}$$

$$\tau = \tau_1 = \tau_2 = int$$

Es. 6, sem. denotazionale

$$t \triangleq \textbf{rec}\ f.\ (\ \lambda x.\ 1\ ,\ (\textbf{fst}(\ f\ )\ 0\ )\ )\quad : (int \to int) * int$$

$$[\![\textbf{rec}\ x.\ t]\!]\rho \triangleq \mathit{fix}\ \lambda d.\ [\![t]\!]\rho[^d/_x]$$

$$[\![t]\!]\rho = \mathit{fix}\ \lambda d_f.\ [\![(\lambda x.\ 1, \textbf{fst}(f)\ 0)]\!]\rho[^{d_f}/_f]$$

$$[\![(t_1\ ,\ t_2)]\!]\rho \triangleq \lfloor\ (\ [\![t_1]\!]\rho\ ,\ [\![t_2]\!]\rho\ )\ \rfloor$$

$$= \mathit{fix}\ \lambda d_f.\ \lfloor\ (\ [\![\lambda x.\ 1]\!]\rho[^{d_f}/_f]\ ,\ [\![\textbf{fst}(f)\ 0]\!]\rho[^{d_f}/_f]\ )\ \rfloor$$

$$\rho' = \rho[^{d_f}/_f]$$

$$[\![\lambda x.\ t]\!]\rho \triangleq \lfloor\ \lambda d.\ [\![t]\!]\rho[^d/_x]\ \rfloor \qquad [\![\ t\ t_0\ ]\!]\rho \triangleq \textbf{let}\ \varphi \Leftarrow [\![t]\!]\rho.\ \varphi([\![t_0]\!]\rho)$$

$$= \mathit{fix}\ \lambda d_f.\ \lfloor\ (\ \lfloor\ \lambda d_x.\ [\![1]\!]\rho'[^{d_x}/_x]\ \rfloor\ ,\ (\textbf{let}\ \varphi \Leftarrow [\![\textbf{fst}(f)]\!]\rho'.\ \varphi([\![0]\!]\rho'))\ )\ \rfloor$$

Es. 6, sem. denotazionale

$$= \mathit{fix}\ \lambda d_f .\ \lfloor\ (\ \lfloor\ \lambda d_x .\ [\![1]\!]\rho'[^{d_x}/_x]\ \rfloor\ ,\ (\mathbf{let}\ \varphi \Leftarrow [\![\mathbf{fst}(f)]\!]\rho' .\ \varphi([\![0]\!]\rho'))\ )\ \rfloor$$

$$\rho' = \rho[^{d_f}/_f]$$

$$[\![n]\!]\rho \triangleq \lfloor n \rfloor \qquad [\![\mathbf{fst}(\ t\ )]\!]\rho \triangleq \pi_1^*\ (\ [\![t]\!]\rho\ )$$

$$= \mathit{fix}\ \lambda d_f .\ \lfloor\ (\ \lfloor\ \lambda d_x .\ \lfloor 1 \rfloor\ \rfloor\ ,\ (\mathbf{let}\ \varphi \Leftarrow \pi_1^*([\![f]\!]\rho') .\ \varphi\ \lfloor 0 \rfloor)\ )\ \rfloor$$

$$= \mathit{fix}\ \lambda d_f .\ \lfloor\ (\ \lfloor\ \lambda d_x .\ \lfloor 1 \rfloor\ \rfloor\ ,\ (\mathbf{let}\ \varphi \Leftarrow \pi_1^*\ d_f .\ \varphi\ \lfloor 0 \rfloor)\ )\ \rfloor$$

Es. 6, sem. denonazionale

$$[\![t]\!]\rho = \mathit{fix}\ \lambda d_f.\ \lfloor\ (\ \lfloor\ \lambda d_x.\ \lfloor 1 \rfloor\ \rfloor\ ,\ (\mathbf{let}\ \varphi \Leftarrow \pi_1^*\ d_f.\ \varphi\ \lfloor 0 \rfloor)\ )\ \rfloor$$

$$f_0 = \perp_{D_{(int \to int) * int}}$$

$$f_1 = \lfloor\ (\ \lfloor\ \lambda d_x.\ \lfloor 1 \rfloor\ \rfloor\ ,\ (\mathbf{let}\ \varphi \Leftarrow \pi_1^*\ f_0.\ \varphi\ \lfloor 0 \rfloor)\ )\ \rfloor$$

$$= \lfloor\ (\ \lfloor\ \lambda d_x.\ \lfloor 1 \rfloor\ \rfloor\ ,\ \perp_{D_{int}}\ )\ \rfloor$$

$$f_2 = \lfloor\ (\ \lfloor\ \lambda d_x.\ \lfloor 1 \rfloor\ \rfloor\ ,\ (\mathbf{let}\ \varphi \Leftarrow \pi_1^*\ f_1.\ \varphi\ \lfloor 0 \rfloor)\ )\ \rfloor$$

$$= \lfloor\ (\ \lfloor\ \lambda d_x.\ \lfloor 1 \rfloor\ \rfloor\ ,\ (\mathbf{let}\ \varphi \Leftarrow \lfloor\ \lambda d_x.\ \lfloor 1 \rfloor\ \rfloor.\ \varphi\ \lfloor 0 \rfloor)\ )\ \rfloor$$

$$= \lfloor\ (\ \lfloor\ \lambda d_x.\ \lfloor 1 \rfloor\ \rfloor\ ,\ (\lambda d_x.\ \lfloor 1 \rfloor)\ \lfloor 0 \rfloor\ )\ \rfloor$$

$$= \lfloor\ (\ \lfloor\ \lambda d_x.\ \lfloor 1 \rfloor\ \rfloor\ ,\ \lfloor 1 \rfloor\ )\ \rfloor$$ elemento massimo!

Ex. 6, semantica den

$$t \triangleq \mathbf{rec}\ f.\ (\ \lambda x.\ 1\ ,\ (\mathbf{fst}(\ f\ )\ 0\ )\ )\quad : (int \to int) * int$$

$$[\![t]\!]\rho = \mathit{fix}\ \lambda d_f.\ \lfloor\ (\ \lfloor\ \lambda d_x.\ \lfloor 1 \rfloor\ \rfloor\ ,\ (\mathbf{let}\ \varphi \Leftarrow \pi_1^*\ d_f.\ \varphi\ \lfloor 0 \rfloor)\ )\ \rfloor$$

$$[\![t]\!]\rho = \lfloor\ (\ \lfloor\ \lambda d_x.\ \lfloor 1 \rfloor\ \rfloor\ ,\ \lfloor 1 \rfloor\ )\ \rfloor$$