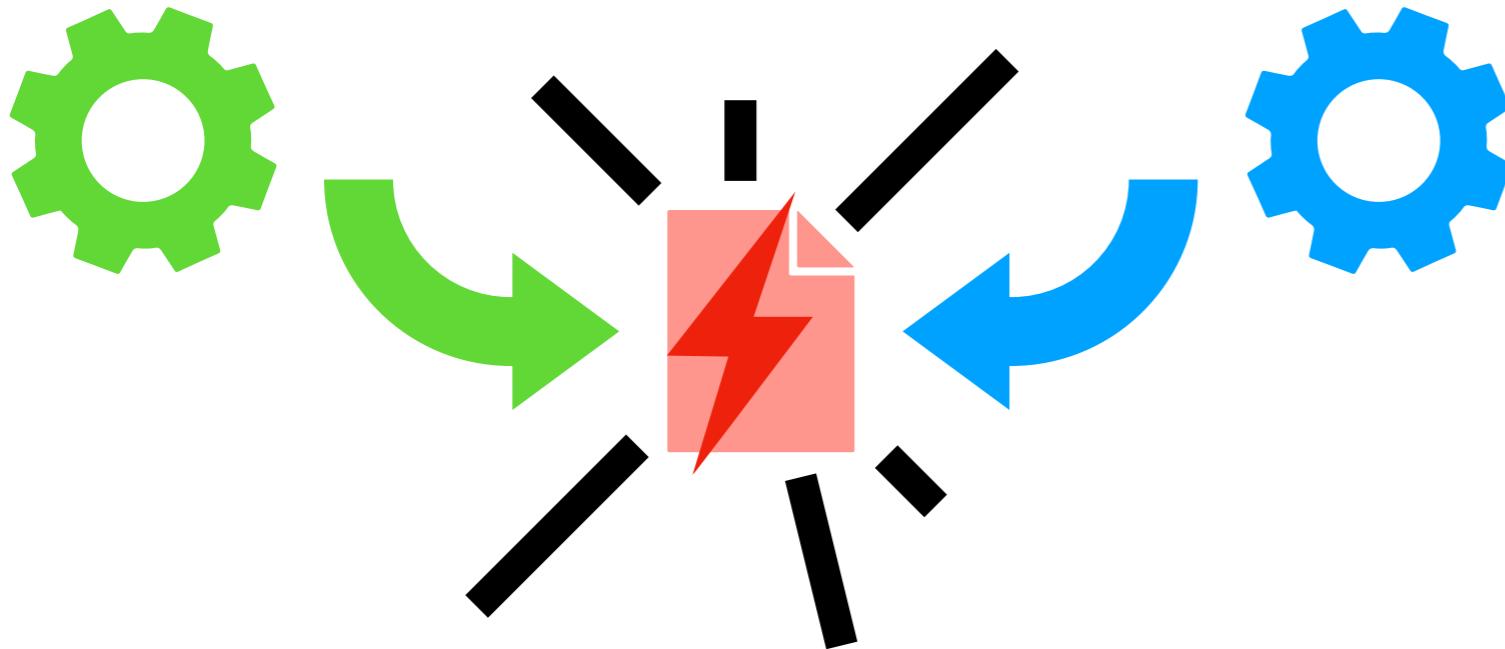


Linguaggi di Programmazione



Roberta Gori

Semantica operazionale di HOFL-cap 7.2

Attenzione

$$t ::= x \mid n \mid t_0 \text{ op } t_1 \mid \text{if } t \text{ then } t_0 \text{ else } t_1 \\ \mid (t_0, t_1) \mid \text{fst}(t) \mid \text{snd}(t) \\ \mid \lambda x. t \mid t_0 \ t_1 \\ \mid \text{rec } x. t$$
$$\tau ::= \text{int} \mid \tau_0 * \tau_1 \mid \tau_0 \rightarrow \tau_1$$

$$\frac{}{x : \widehat{x}} \quad \frac{}{n : \text{int}} \quad \frac{t_0 : \text{int} \quad t_1 : \text{int}}{t_0 \text{ op } t_1 : \text{int}} \quad \frac{t : \text{int} \quad t_0 : \tau \quad t_1 : \tau}{\text{if } t \text{ then } t_0 \text{ else } t_1 : \tau}$$

$$\frac{t_0 : \tau_0 \quad t_1 : \tau_1}{(t_0, t_1) : \tau_0 * \tau_1} \quad \frac{t : \tau_0 * \tau_1}{\text{fst}(t) : \tau_0} \quad \frac{t : \tau_0 * \tau_1}{\text{snd}(t) : \tau_1}$$

$$\frac{x : \tau_0 \quad t : \tau_1}{\lambda x. t : \tau_0 \rightarrow \tau_1} \quad \frac{t_1 : \tau_0 \rightarrow \tau_1 \quad t_0 : \tau_0}{t_1 \ t_0 : \tau_1}$$

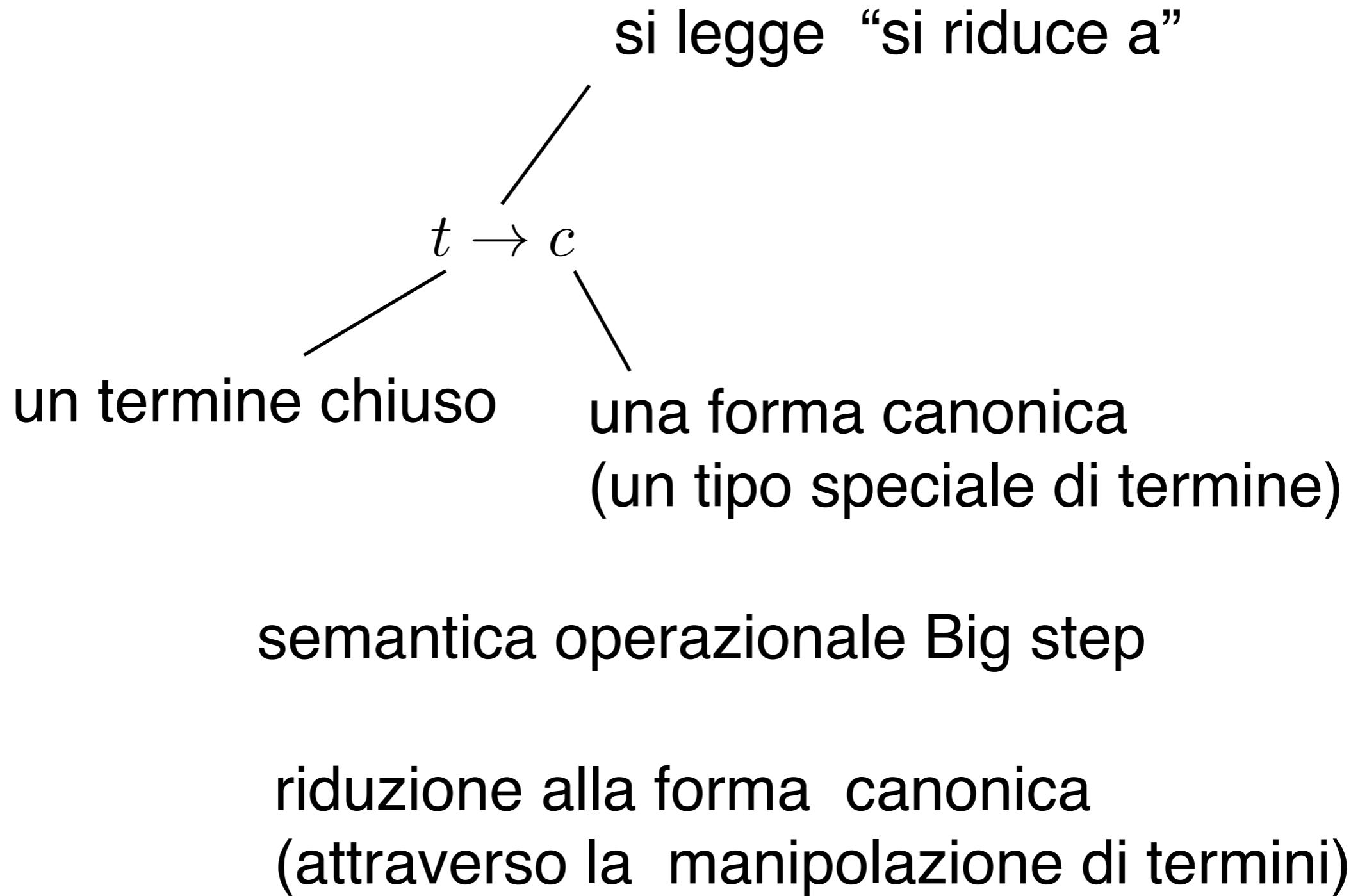
$$\frac{x : \tau \quad t : \tau}{\text{rec } x. t : \tau}$$

assegniamo una
semantica
solo ai termini che sono:
ben formati e chiusi

$$t : \tau \quad \text{fv}(t) = \emptyset$$

Forme Canoniche

Forme canoniche



Forme canoniche

insieme di forme canoniche $C_\tau \subseteq T_\tau$
con tipo τ

(laziness)
non e' richiesto
siano in forma canonica

$$\frac{\begin{array}{c} / \quad \backslash \\ t_0 : \tau_0 \quad t_1 : \tau_1 \quad t_0, t_1 \text{ closed} \end{array}}{(t_0, t_1) \in C_{\tau_0 * \tau_1}}$$

$$\frac{\lambda x. t : \tau_0 \rightarrow \tau_1 \quad \lambda x. t \text{ closed}}{\lambda x. t \in C_{\tau_0 \rightarrow \tau_1}}$$

Forme Canoniche?

$$\frac{\frac{t_0 : \tau_0 \quad t_1 : \tau_1 \quad t_0, t_1 \text{ closed}}{(t_0, t_1) \in C_{\tau_0 * \tau_1}} \quad \frac{\lambda x. \, t : \tau_0 \rightarrow \tau_1 \quad \lambda x. \, t \text{ closed}}{\lambda x. \, t \in C_{\tau_0 \rightarrow \tau_1}}}{n \in C_{int}}$$

$1 + 2 \times 3$ 

$\text{if } 0 \text{ then } 0 \text{ else } 0$ 

$(1, 2)$ 

$\lambda x. \, 1$ 

$(1 + 2, 2 - 1)$ 

$\lambda x. \, 1 + 2 \times 3$ 

$\text{fst}(1, 2)$ 

$\lambda x. \, \text{fst}(1, 2)$ 

HOFL

Semantica operazionale Lazy

Assiomi della semantica Operazionale

$$\frac{c \in C_\tau}{c \rightarrow c}$$

Dobbiamo vedere i vari casi

$$\frac{\begin{array}{ccc} t_0 : \tau_0 & t_1 : \tau_1 & t_0, t_1 \text{ closed} \\ \hline n \rightarrow n \end{array}}{(t_0, t_1) \rightarrow (t_0, t_1)} \qquad \frac{\lambda x. t : \tau_0 \rightarrow \tau_1 \quad \lambda x. t \text{ closed}}{\lambda x. t \rightarrow \lambda x. t}$$

Gli interi, le coppie e le astrazioni sono già in forma canonica

Semantica op. Lazy

$$\frac{}{n \rightarrow n} \quad \frac{t_0 : \tau_0 \quad t_1 : \tau_1 \quad t_0, t_1 \text{ closed}}{(t_0, t_1) \rightarrow (t_0, t_1)} \quad \frac{\lambda x. \, t : \tau_0 \rightarrow \tau_1 \quad \lambda x. t \text{ closed}}{\lambda x. \, t \rightarrow \lambda x. \, t}$$

$$\frac{t_0 \rightarrow n_0 \quad t_1 \rightarrow n_1}{t_0 \text{ op } t_1 \rightarrow n_0 \text{ op } n_1} \quad \frac{t \rightarrow 0 \quad t_0 \rightarrow c_0}{\text{if } t \text{ then } t_0 \text{ else } t_1 \rightarrow c_0} \quad \frac{t \rightarrow (t_0, t_1) \quad t_0 \rightarrow c_0}{\mathbf{fst}(t) \rightarrow c_0}$$

$$\frac{t[\mathbf{rec} \ x. \ t/x] \rightarrow c}{\mathbf{rec} \ x. \ t \rightarrow c} \quad \frac{t \rightarrow n \quad n \neq 0 \quad t_1 \rightarrow c_1}{\text{if } t \text{ then } t_0 \text{ else } t_1 \rightarrow c_1} \quad \frac{t \rightarrow (t_0, t_1) \quad t_1 \rightarrow c_1}{\mathbf{snd}(t) \rightarrow c_1}$$

$$\frac{t_1 \rightarrow \lambda x. \, t'_1 \quad t'_1[t^0/x] \rightarrow c}{(t_1 \ t_0) \rightarrow c} \quad (\text{lazy})$$

Ricordiamo il sistema di tipi

$$\frac{}{x : \widehat{x}} \quad \frac{}{n : int} \quad \frac{t_0 : int \quad t_1 : int}{t_0 \text{ op } t_1 : int} \quad \frac{t : int \quad t_0 : \tau \quad t_1 : \tau}{\mathbf{if } \ t \ \mathbf{then } \ t_0 \ \mathbf{else } \ t_1 : \tau}$$

$$\frac{t_0 : \tau_0 \quad t_1 : \tau_1}{(t_0, t_1) : \tau_0 * \tau_1} \quad \frac{t : \tau_0 * \tau_1}{\mathbf{fst}(t) : \tau_0} \quad \frac{t : \tau_0 * \tau_1}{\mathbf{snd}(t) : \tau_1}$$

$$\frac{x : \tau_0 \quad t : \tau_1}{\lambda x. \ t : \tau_0 \rightarrow \tau_1} \quad \frac{t_1 : \tau_0 \rightarrow \tau_1 \quad t_0 : \tau_0}{t_1 \ t_0 : \tau_1}$$

$$\frac{x : \tau \quad t : \tau}{\mathbf{rec } \ x. \ t : \tau}$$

Esempio

$$t \triangleq \lambda x. \frac{\underline{x}}{int} + \frac{\underline{1}}{int} : int \rightarrow int$$

$$\frac{\underline{\quad\quad\quad int}}{\underline{\quad\quad\quad int}}$$

$$\frac{\underline{\quad\quad\quad int}}{int \rightarrow int}$$

$$\lambda x. x + 1 \rightarrow c \xleftarrow{c = \lambda x. x + 1} \square$$

	$\frac{t_0 : \tau_0 \quad t_1 : \tau_1 \quad t_0, t_1 \text{ closed}}{(t_0, t_1) \rightarrow (t_0, t_1)}$	$\frac{\lambda x. t : \tau_0 \rightarrow \tau_1 \quad \lambda x. t \text{ closed}}{\lambda x. t \rightarrow \lambda x. t}$
$t_0 \rightarrow n_0 \quad t_1 \rightarrow n_1$	$\frac{t \rightarrow 0 \quad t_0 \rightarrow c_0}{\text{if } t \text{ then } t_0 \text{ else } t_1 \rightarrow c_0}$	$\frac{t \rightarrow (t_0, t_1) \quad t_0 \rightarrow c_0}{\mathbf{fst}(t) \rightarrow c_0}$
$t[\mathbf{rec} \ x. t/x] \rightarrow c$	$\frac{t \rightarrow n \quad n \neq 0 \quad t_1 \rightarrow c_1}{\text{if } t \text{ then } t_0 \text{ else } t_1 \rightarrow c_1}$	$\frac{t \rightarrow (t_0, t_1) \quad t_1 \rightarrow c_1}{\mathbf{snd}(t) \rightarrow c_1}$
	$\frac{t_1 \rightarrow \lambda x. t'_1 \quad t'_1[t_0/x] \rightarrow c}{(t_1 \ t_0) \rightarrow c}$	(lazy)

Esempio

$$t \triangleq \underbrace{\left(\lambda x. x + 1, \underbrace{1 + 2}_{\begin{array}{c} int \\ int \end{array}} \right)}_{\begin{array}{c} int \rightarrow int \\ int \end{array}} : (int \rightarrow int) * int$$

laziness:
non dobbiamo valutare $1+2$

$$(\lambda x. x + 1, 1 + 2) \rightarrow c \quad \xleftarrow{c = (\lambda x. x + 1, 1 + 2)} \square$$

	$t_0 : \tau_0 \quad t_1 : \tau_1 \quad t_0, t_1 \text{ closed}$	$\lambda x. t : \tau_0 \rightarrow \tau_1 \quad \lambda x. t \text{ closed}$
$n \rightarrow n$	$(t_0, t_1) \rightarrow (t_0, t_1)$	$\lambda x. t \rightarrow \lambda x. t$
$t_0 \rightarrow n_0 \quad t_1 \rightarrow n_1$	$t \rightarrow 0 \quad t_0 \rightarrow c_0$	$t \rightarrow (t_0, t_1) \quad t_0 \rightarrow c_0$
$t_0 \text{ op } t_1 \rightarrow n_0 \text{ op } n_1$	$\text{if } t \text{ then } t_0 \text{ else } t_1 \rightarrow c_0$	$\text{fst}(t) \rightarrow c_0$
$t[\text{rec } x. t / x] \rightarrow c$	$t \rightarrow n \quad n \neq 0 \quad t_1 \rightarrow c_1$	$t \rightarrow (t_0, t_1) \quad t_1 \rightarrow c_1$
$\text{rec } x. t \rightarrow c$	$\text{if } t \text{ then } t_0 \text{ else } t_1 \rightarrow c_1$	$\text{snd}(t) \rightarrow c_1$
	$t_1 \rightarrow \lambda x. t'_1 \quad t'_1[t_0 / x] \rightarrow c$	$(t_1 \ t_0) \rightarrow c \quad (\text{lazy})$

Esempio

$$t \triangleq \lambda x. \text{ if } \mathbf{fst}(x) \text{ then } 1 \text{ else } \mathbf{snd}(x) : (\mathit{int} * \mathit{int}) \rightarrow \mathit{int}$$
$$\frac{\frac{\frac{\frac{\mathit{int} * \mathit{int}}{\mathit{int}}} {\mathit{int} * \tau_1}} {\mathit{int}}} {\mathit{int}} \quad \frac{\frac{\frac{\mathit{int}}{\mathit{int} * \tau_1}} {\mathit{int}}} {\mathit{int} = \tau_1}$$
$$\frac{\mathit{int}}{(\mathit{int} * \mathit{int}) \rightarrow \mathit{int}}$$

Esempio (con.)

$$t \triangleq \lambda x. \text{if } \text{fst}(x) \text{ then } 1 \text{ else } \text{snd}(x)$$

$n \rightarrow n$	$t_0 : \tau_0 \quad t_1 : \tau_1 \quad t_0, t_1 \text{ closed}$	$\lambda x. t : \tau_0 \rightarrow \tau_1 \quad \lambda x. t \text{ closed}$
	$(t_0, t_1) \rightarrow (t_0, t_1)$	$\lambda x. t \rightarrow \lambda x. t$
$t_0 \rightarrow n_0 \quad t_1 \rightarrow n_1$	$t \rightarrow 0 \quad t_0 \rightarrow c_0$	$t \rightarrow (t_0, t_1) \quad t_0 \rightarrow c_0$
$t_0 \text{ op } t_1 \rightarrow n_0 \text{ op } n_1$	$\text{if } t \text{ then } t_0 \text{ else } t_1 \rightarrow c_0$	$\text{fst}(t) \rightarrow c_0$
$t^{[\text{rec } x. t / x]} \rightarrow c$	$t \rightarrow n \quad n \neq 0 \quad t_1 \rightarrow c_1$	$t \rightarrow (t_0, t_1) \quad t_1 \rightarrow c_1$
$\text{rec } x. t \rightarrow c$	$\text{if } t \text{ then } t_0 \text{ else } t_1 \rightarrow c_1$	$\text{snd}(t) \rightarrow c_1$
		$t_1 \rightarrow \lambda x. t'_1 \quad t'_1[t_0 / x] \rightarrow c$
		$(t_1 t_0) \rightarrow c \quad (\text{lazy})$

$$t (1, 2) \rightarrow c \leftarrow t \rightarrow \lambda x'. t' , \quad t'[(1, 2) / x'] \rightarrow c$$

$$\leftarrow_{x'=x, \quad t'=\text{if } \dots} (\text{if } \text{fst}(x) \text{ then } 1 \text{ else } \text{snd}(x))[(1, 2) / x] \rightarrow c$$

$$= \text{if } \text{fst}(1, 2) \text{ then } 1 \text{ else } \text{snd}(1, 2) \rightarrow c$$

$$\leftarrow \text{fst}(1, 2) \rightarrow n , \quad n \neq 0 , \quad \text{snd}(1, 2) \rightarrow c$$

$$\leftarrow (1, 2) \rightarrow (n_1, n_2) , \quad n_1 \rightarrow n , \quad n \neq 0 , \quad \text{snd}(1, 2) \rightarrow c$$

$$\leftarrow_{n_1=1, n_2=2, n=1}^* \text{snd}(1, 2) \rightarrow c$$

$$\leftarrow (1, 2) \rightarrow (n_3, n_4) , \quad n_4 \rightarrow c$$

$$\leftarrow_{n_3=1, n_4=2, c=2}^* \square$$

$$t (1, 2) \rightarrow 2$$

Esempio

$$t \triangleq \mathbf{rec} \underbrace{x.}_{\tau} \underbrace{x}_{\tau} : \tau$$

$$\mathbf{rec} \ x. \ x \rightarrow c \quad \leftarrow \quad x[x[\mathbf{rec} \ x. \ x / x] \rightarrow c]$$

$$= \mathbf{rec} \ x. \ x \rightarrow c$$

stesso goal da cui siamo partiti
nessun'altra opzione da esplorare:
divergenza!

Esempio

$fact \triangleq \text{rec } f. \lambda x. \text{ if } x \text{ then } 1 \text{ else } x \times (f(x - 1))$

$$\begin{aligned}
 fact \rightarrow c &\leftarrow (\lambda x. \text{ if } x \text{ then } 1 \text{ else } x \times (f(x - 1)))^{[fact/f]} \rightarrow c \\
 &= \lambda x. \text{ if } x \text{ then } 1 \text{ else } x \times (\overbrace{(\text{rec } f. \dots)}^{fact}(x - 1)) \rightarrow c \\
 &\quad \swarrow_{c = \lambda x. \text{ if } x \text{ then } 1 \text{ else } x \times (fact(x - 1))} \square
 \end{aligned}$$

$\frac{}{n \rightarrow n}$	$\frac{t_0 : \tau_0 \quad t_1 : \tau_1 \quad t_0, t_1 \text{ closed}}{(t_0, t_1) \rightarrow (t_0, t_1)}$	$\frac{\lambda x. t : \tau_0 \rightarrow \tau_1 \quad \lambda x. t \text{ closed}}{\lambda x. t \rightarrow \lambda x. t}$
$t_0 \rightarrow n_0 \quad t_1 \rightarrow n_1$	$\frac{t \rightarrow 0 \quad t_0 \rightarrow c_0}{\text{if } t \text{ then } t_0 \text{ else } t_1 \rightarrow c_0}$	$\frac{t \rightarrow (t_0, t_1) \quad t_0 \rightarrow c_0}{\text{fst}(t) \rightarrow c_0}$
$\text{rec } x. t \rightarrow c$	$\frac{t \rightarrow n \quad n \neq 0 \quad t_1 \rightarrow c_1}{\text{if } t \text{ then } t_0 \text{ else } t_1 \rightarrow c_1}$	$\frac{t \rightarrow (t_0, t_1) \quad t_1 \rightarrow c_1}{\text{snd}(t) \rightarrow c_1}$
		$\frac{t_1 \rightarrow \lambda x. t'_1 \quad t'_1[t_0/x] \rightarrow c}{(t_1 t_0) \rightarrow c} \text{ (lazy)}$

Esempio

	$\frac{t_0 : \tau_0 \quad t_1 : \tau_1 \quad t_0, t_1 \text{ closed}}{(t_0, t_1) \rightarrow (t_0, t_1)}$	$\frac{\lambda x. \ t : \tau_0 \rightarrow \tau_1 \quad \lambda x. t \text{ closed}}{\lambda x. \ t \rightarrow \lambda x. \ t}$
$t_0 \rightarrow n_0 \quad t_1 \rightarrow n_1$	$\frac{t \rightarrow 0 \quad t_0 \rightarrow c_0}{\text{if } t \text{ then } t_0 \text{ else } t_1 \rightarrow c_0}$	$\frac{t \rightarrow (t_0, t_1) \quad t_0 \rightarrow c_0}{\mathbf{fst}(t) \rightarrow c_0}$
$t[\mathbf{rec} \ x. \ t / x] \rightarrow c$	$\frac{t \rightarrow n \quad n \neq 0 \quad t_1 \rightarrow c_1}{\text{if } t \text{ then } t_0 \text{ else } t_1 \rightarrow c_1}$	$\frac{t \rightarrow (t_0, t_1) \quad t_1 \rightarrow c_1}{\mathbf{snd}(t) \rightarrow c_1}$
	$\frac{t_1 \rightarrow \lambda x. \ t'_1 \quad t'_1[t_0/x] \rightarrow c}{(t_1 \ t_0) \rightarrow c}$	(lazy)

$$fact \triangleq \mathbf{rec} \ f. \ \lambda x. \ \mathbf{if} \ x \mathbf{then} \ 1 \mathbf{else} \ x \times (f \ (x - 1))$$

$$\begin{aligned}
 (fact \ 1) \rightarrow c &\leftarrow fact \rightarrow \lambda x'. \ t' , \ t'[1/x'] \rightarrow c \\
 &\nwarrow_{x'=x, t'=\mathbf{if}\dots}^* (\mathbf{if} \ x \mathbf{then} \ 1 \mathbf{else} \ x \times (fact \ (x - 1))) [1/x] \rightarrow c \\
 &= \mathbf{if} \ 1 \mathbf{then} \ 1 \mathbf{else} \ 1 \times (fact \ (1 - 1)) \rightarrow c \\
 &\nwarrow 1 \rightarrow n , \ n \neq 0 , \ 1 \times (fact \ (1 - 1)) \rightarrow c \\
 &\nwarrow_{n=1, c=n_1 \times n_2}^* 1 \rightarrow n_1 , \ (fact \ (1 - 1)) \rightarrow n_2
 \end{aligned}$$

Esempio

	$\frac{t_0 : \tau_0 \quad t_1 : \tau_1 \quad t_0, t_1 \text{ closed}}{(t_0, t_1) \rightarrow (t_0, t_1)}$	$\frac{\lambda x. \ t : \tau_0 \rightarrow \tau_1 \quad \lambda x. t \text{ closed}}{\lambda x. \ t \rightarrow \lambda x. \ t}$
$t_0 \rightarrow n_0 \quad t_1 \rightarrow n_1$	$\frac{t \rightarrow 0 \quad t_0 \rightarrow c_0}{\text{if } t \text{ then } t_0 \text{ else } t_1 \rightarrow c_0}$	$\frac{t \rightarrow (t_0, t_1) \quad t_0 \rightarrow c_0}{\mathbf{fst}(t) \rightarrow c_0}$
$t[\mathbf{rec} \ x. \ t / x] \rightarrow c$	$\frac{t \rightarrow n \quad n \neq 0 \quad t_1 \rightarrow c_1}{\text{if } t \text{ then } t_0 \text{ else } t_1 \rightarrow c_1}$	$\frac{t \rightarrow (t_0, t_1) \quad t_1 \rightarrow c_1}{\mathbf{snd}(t) \rightarrow c_1}$
	$\frac{t_1 \rightarrow \lambda x. \ t'_1 \quad t'_1[t_0 / x] \rightarrow c}{(t_1 \ t_0) \rightarrow c}$	(lazy)

- $\nwarrow_{n=1, c=n_1 \times n_2}^* 1 \rightarrow n_1, (fact(1 - 1)) \rightarrow n_2$ laziness
 $\nwarrow_{n_1=1} fact \rightarrow \lambda x''. \ t'', t''[1^{-1} / x''] \rightarrow n_2$ si vede qui
 $\nwarrow_{x''=x, t''=\text{if...}}^* (\text{if } x \text{ then } 1 \text{ else } x \times (fact(x - 1)))[1^{-1} / x] \rightarrow n_2$
 $= \text{if } 1 - 1 \text{ then } 1 \text{ else } (1 - 1) \times (fact((1 - 1) - 1)) \rightarrow n_2$
 $\nwarrow 1 - 1 \rightarrow 0, 1 \rightarrow n_2$
 $\nwarrow_{n_2=1}^* \square$ $c = n_1 \underline{\times} n_2 = 1 \underline{\times} 1 = 1$

HOFL

Semantica operazionale Eager

Lazy vs Eager

$$\frac{t_1 \rightarrow \lambda x. t'_1 \quad t'_1[t_0/x] \rightarrow c}{(t_1\ t_0) \rightarrow c} \quad (\text{lazy})$$

$$\frac{t_1 \rightarrow \lambda x. t'_1 \quad t_0 \rightarrow c_0 \quad t'_1[c_0/x] \rightarrow c}{(t_1\ t_0) \rightarrow c} \quad (\text{eager})$$

Lazy vs Eager

$$t \triangleq (\lambda x. 1) (\mathbf{rec} \ y. \ y) : int$$

$x : \tau$
 $y : \tau$

$$t \rightarrow c \quad \nwarrow \quad \lambda x. 1 \rightarrow \lambda x'. t' , \ t'[\mathbf{rec} \ y. \ y/x'] \rightarrow c$$

lazy

$$\begin{aligned} &\nwarrow_{x'=x, \ t'=1} \quad 1[\mathbf{rec} \ y. \ y/x] \rightarrow c \\ &= 1 \rightarrow c \end{aligned}$$

$\nwarrow_{c=1} \square$

$$t \rightarrow c \quad \nwarrow \quad \lambda x. 1 \rightarrow \lambda x'. t' , \ \mathbf{rec} \ y. \ y \rightarrow c' , \ t'[c'/x'] \rightarrow c$$

eager

$$\begin{aligned} &\nwarrow_{x'=x, \ t'=1} \quad \mathbf{rec} \ y. \ y \rightarrow c' , \ 1[c'/x] \rightarrow c \end{aligned}$$

$$\begin{aligned} &\nwarrow \quad \mathbf{rec} \ y. \ y \rightarrow c' , \ 1[c'/x] \rightarrow c \end{aligned}$$

divergenza!

Lazy vs Eager

$$t \triangleq (\lambda x. x + x) (1 \times 2) : int$$

$x : int$

$$t \rightarrow c \leftarrow \lambda x. x + x \rightarrow \lambda x'. t' , t'[1 \times 2/x'] \rightarrow c$$

lazy

$$\begin{aligned} & \xleftarrow{x'=x, t'=x+x} (x + x)[1 \times 2/x] \rightarrow c \\ & = (1 \times 2) + (1 \times 2) \rightarrow c \end{aligned}$$

valutata
due volte

$$\xleftarrow{c=c_1 \pm c_2} \boxed{(1 \times 2) \rightarrow c_1 , (1 \times 2) \rightarrow c_2}$$

$$\xleftarrow{c_1=2, c_2=2} \square \quad c = c_1 \pm c_2 = 2 \pm 2 = 4$$

$$t \rightarrow c \leftarrow \lambda x. x + x \rightarrow \lambda x'. t' , 1 \times 2 \rightarrow c' , t'[c'/x'] \rightarrow c$$

eager

$$\begin{aligned} & \xleftarrow{x'=x, t'=x+x} 1 \times 2 \rightarrow c' , (x + x)[c'/x] \rightarrow c \\ & \xleftarrow{c'=2} (x + x)[2/x] \rightarrow c \\ & = 2 + 2 \rightarrow c \\ & \xleftarrow{c=4} \square \end{aligned}$$

HOFL

Proprieta' della semantica operazionale

Terminazione

termina?

$\forall t. \exists c. t \rightarrow c?$ 

rec $x.$ x

Determinismo?

determinismo? $\forall t. \forall c_1, c_2. t \rightarrow c_1 \wedge t \rightarrow c_2 \Rightarrow c_1 = c_2$? 

$$P(t \rightarrow c) \triangleq \forall c_1. t \rightarrow c_1 \Rightarrow c_1 = c$$

per ind. strutturale (provateci!)

Subject reduction

i tipi assegnati staticamente non cambiano in fase di esecuzione

subject reduction? $\forall t. \forall c. \forall \tau. t \rightarrow c \wedge t : \tau \Rightarrow c : \tau ?$ 

$$P(t \rightarrow c) \triangleq \forall \tau. t : \tau \Rightarrow c : \tau$$

per ind. strutturale (provateci!)

Congruenza?

$t_1 \equiv_{\text{op}} t_2$ iff $\forall c. (t_1 \rightarrow c \Leftrightarrow t_2 \rightarrow c)$

e' una congruenza? 

$$2 \equiv_{\text{op}} 1 + 1$$

$$\lambda x. 2 \not\equiv_{\text{op}} \lambda x. 1 + 1$$

$$\lambda x. 2 , \lambda x. 1 + 1 \in C_{\tau \rightarrow \text{int}}$$

$$\lambda x. 2 \rightarrow \lambda x. 2$$

$$\lambda x. 1 + 1 \rightarrow \lambda x. 1 + 1$$