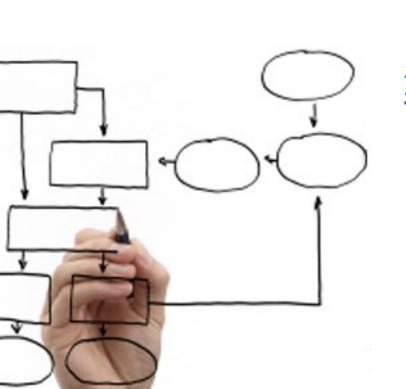
## Business Processes Modelling MPB (6 cfu, 295AA)



#### Roberto Bruni

http://www.di.unipi.it/~bruni

12 - Analysis of WF nets

#### Object



We study suitable soundness properties of Workflow nets

#### Bondedness, liveness

$$(P, T, F, M_0)$$

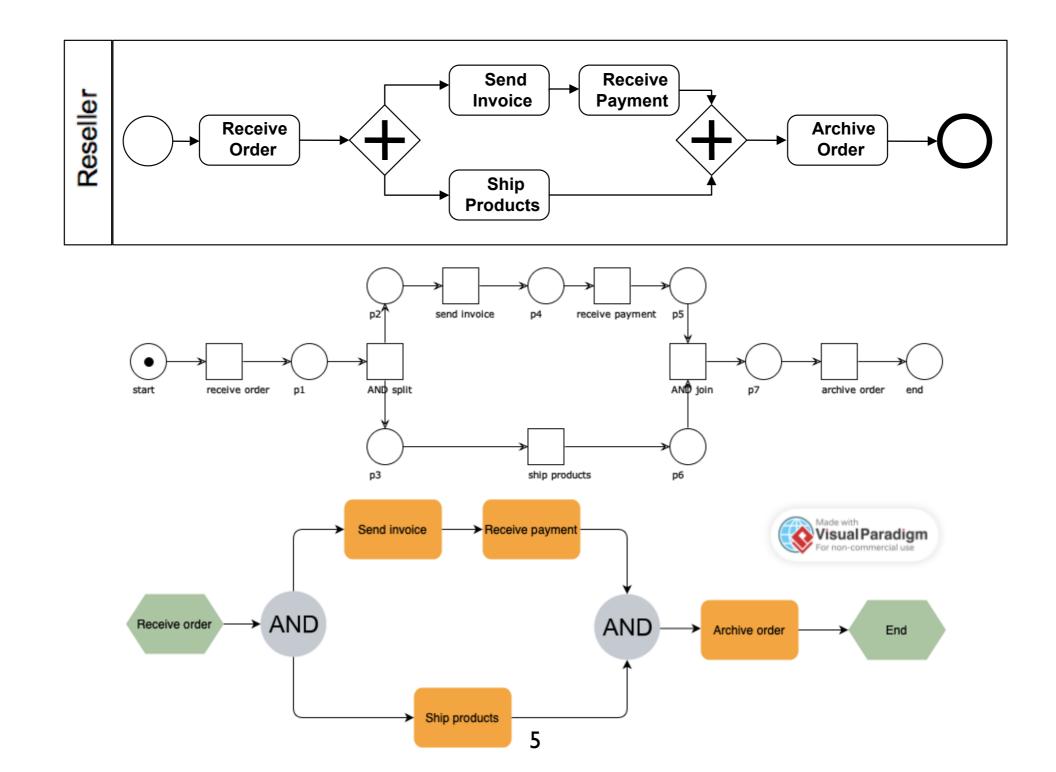
$$\exists k \in \mathbb{N}, \quad \forall p \in P, \quad \forall M \in [M_0), \quad M(p) \le k$$

Boundedness? Liveness?

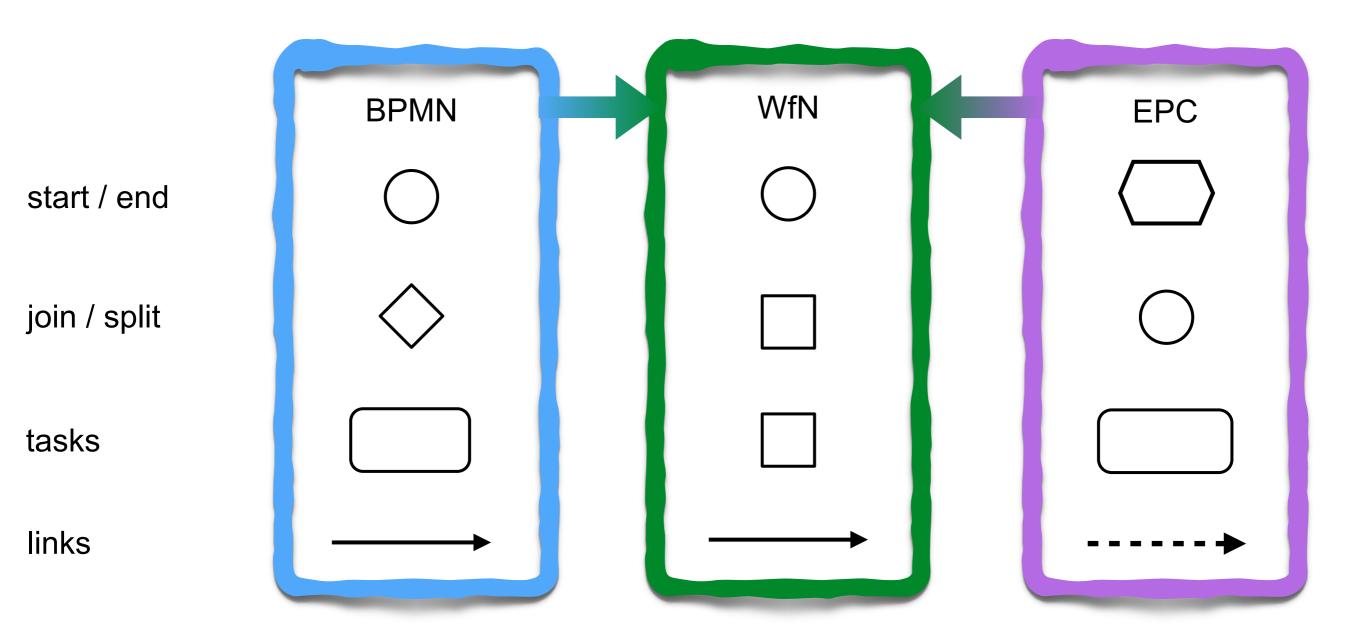
$$\forall t \in T, \quad \forall M \in [M_0], \quad \exists M' \in [M], \quad M' \stackrel{t}{\rightarrow}$$

## Soundness informally

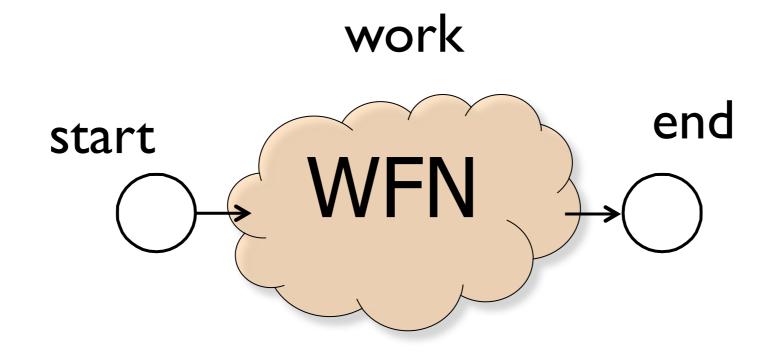
### Example: Reseller



### Diagram verification



#### Workflow net: idea



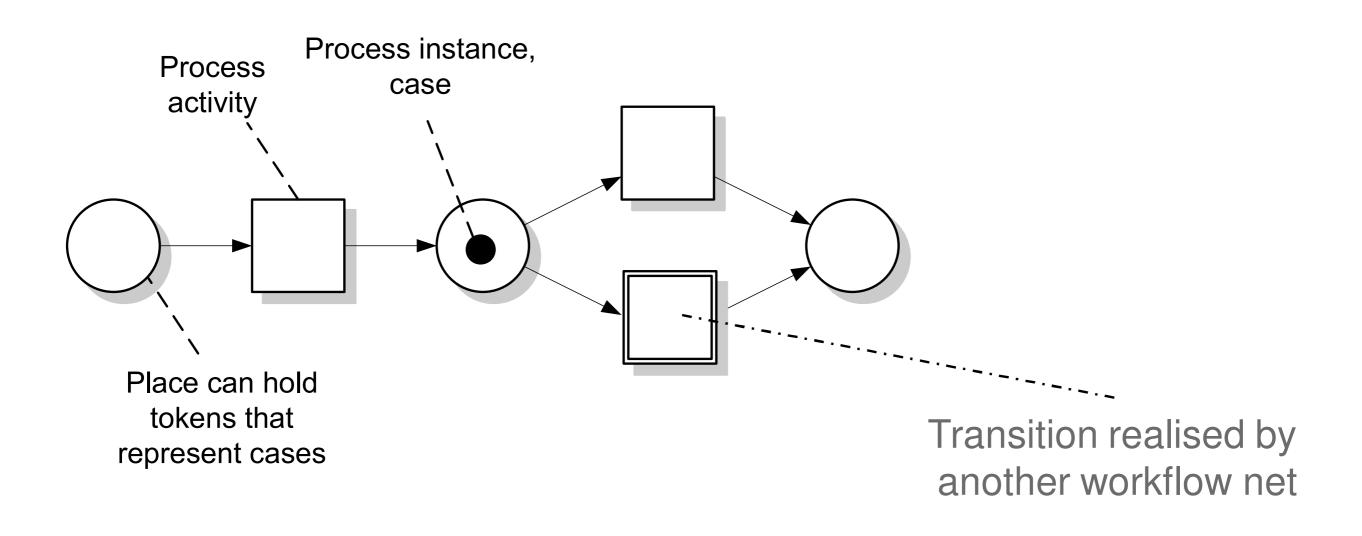
#### Workflow net

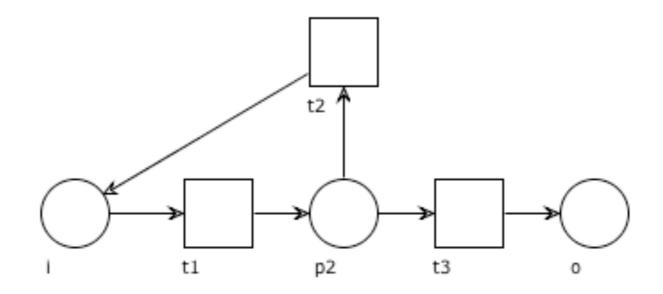
#### **Definition:**

A Petri net (P, T, F) is called **workflow net** if:

- 1. there is a distinguished *initial place*  $i \in P$  with  $\bullet i = \emptyset$
- 2. there is a distinguished final place  $o \in P$  with  $o \bullet = \emptyset$
- 3. every other place and transition belongs to a path from i to o

# WF nets as business processes



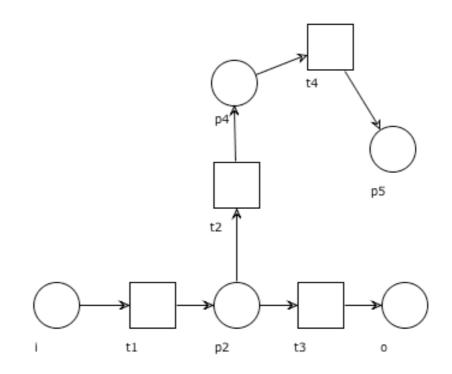


No distinguished entry / exit point

no entry: when should the case start?

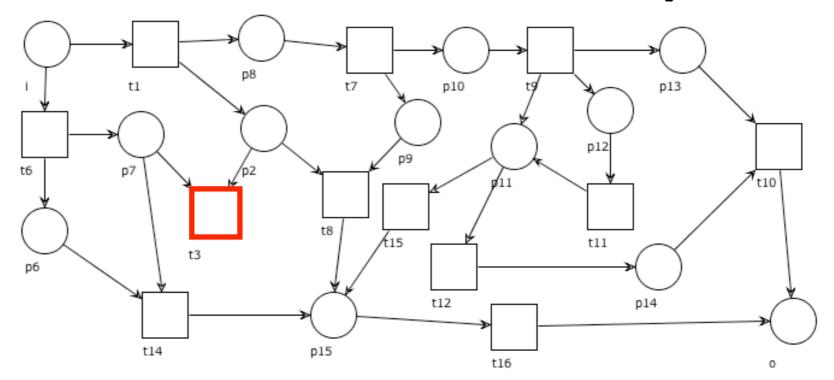
no exit: when should the case end?

not a workflow net!



Multiple entry / exit points

multiple entries: when should the case start?
multiple exit: when should the case end?
not a workflow net!



Tasks t without incoming and/or outgoing arcs

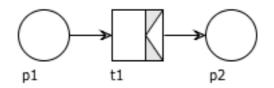
no input: when should t be carried out?

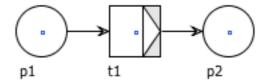
no output: t does not contribute to case completion

not a workflow net!

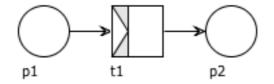
Wrong decorations of transitions

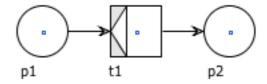
split with only one outgoing arc





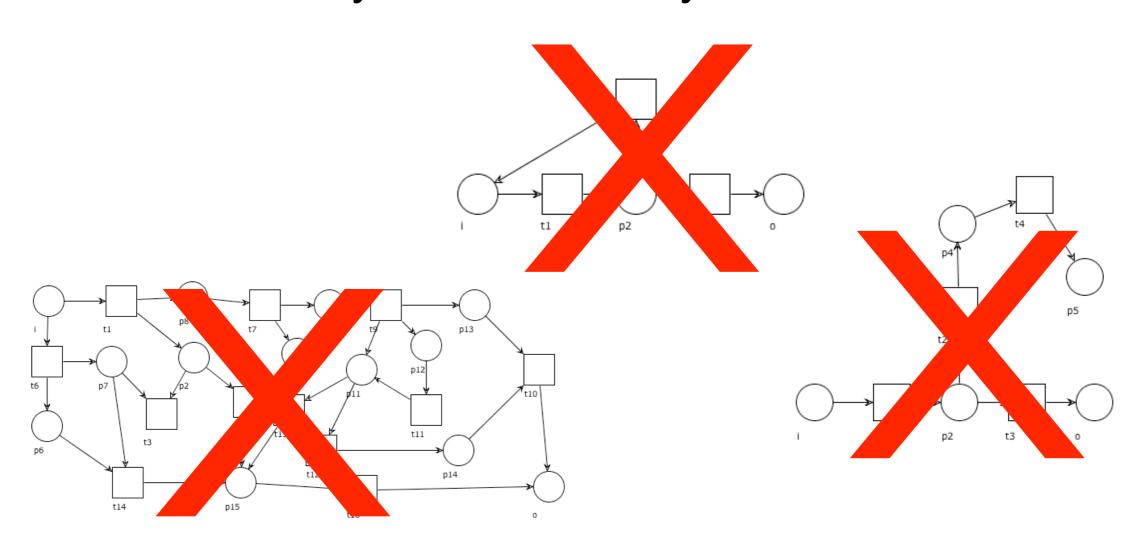
join with only one incoming arc





non-sense: left to designer responsibility

The definition of Workflow nets is purely structural but already rules out many erroneous models



#### Structural properties

All the properties we have seen so far are structural (or static)

(i.e., they depend on the shape of the graph, on its connectivity or topology, but NOT on the initial marking and enabled firings)

We also care about **behavioural** properties (e.g., how the system can evolve, which firing sequences will be possible, which markings will be reachable)

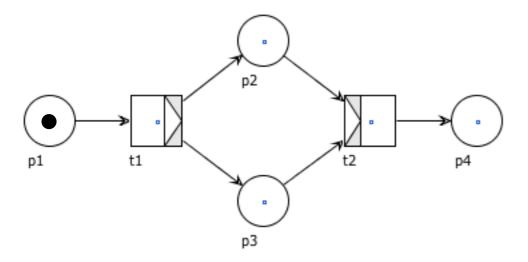
#### A matter of terminology

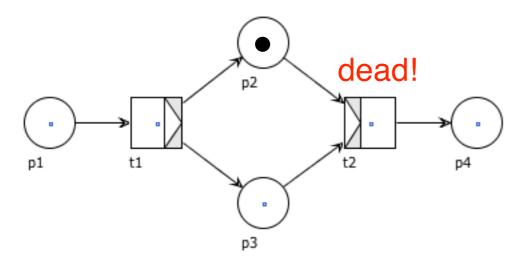
To better reflect the above distinction, it is frequent:

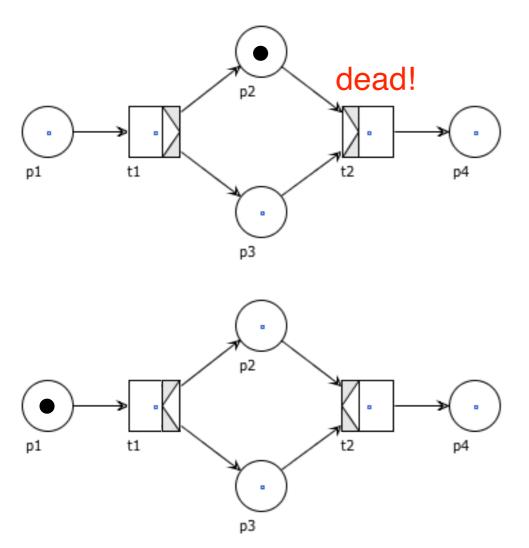
to use the term **net system** for denoting a Petri net **with** a given initial marking (we study behavioural properties of systems)

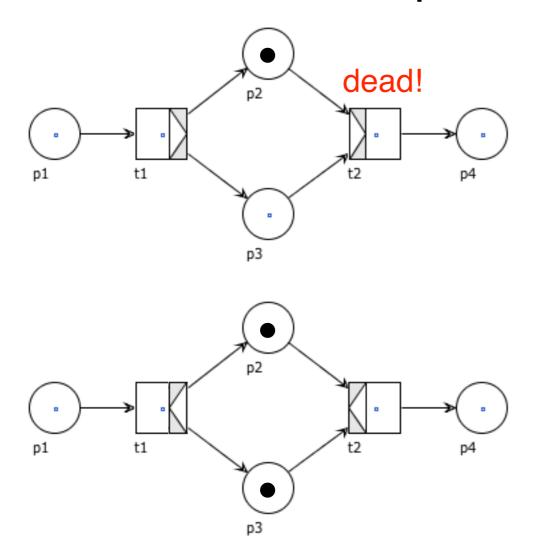
to use the term **net** for denoting a Petri net **without** specifying any initial marking (we study structural properties of nets)

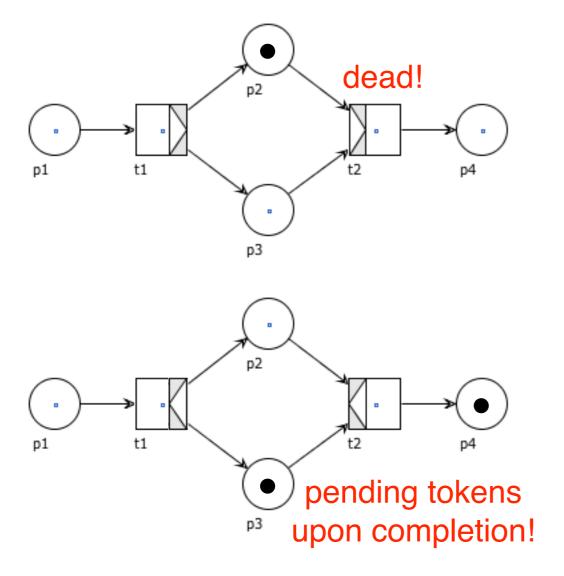
even if, in the case of workflow nets, the initial markings will consist of one token in the initial place

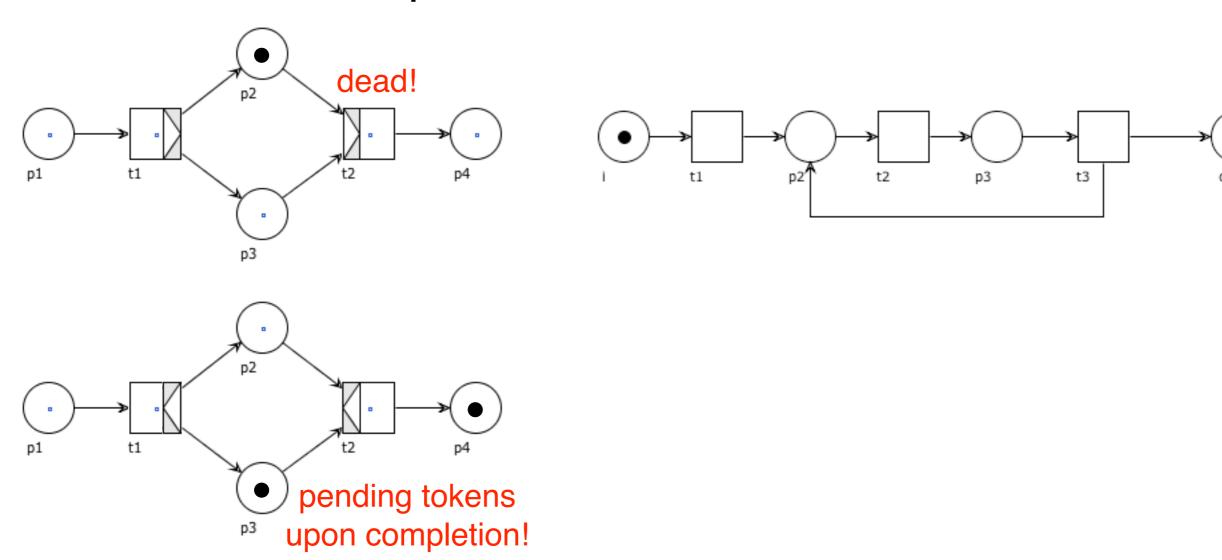


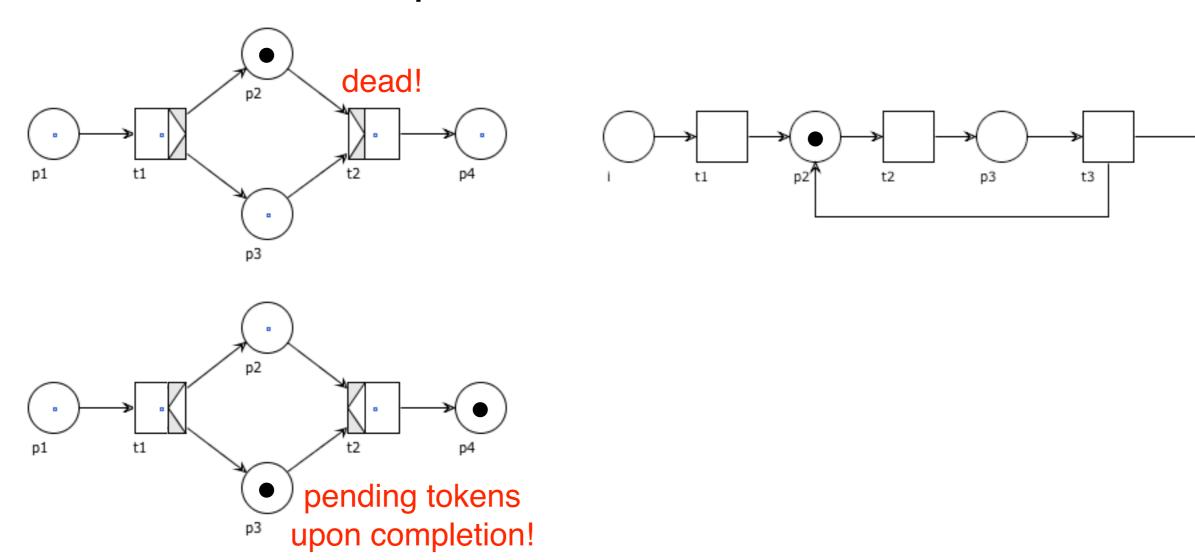


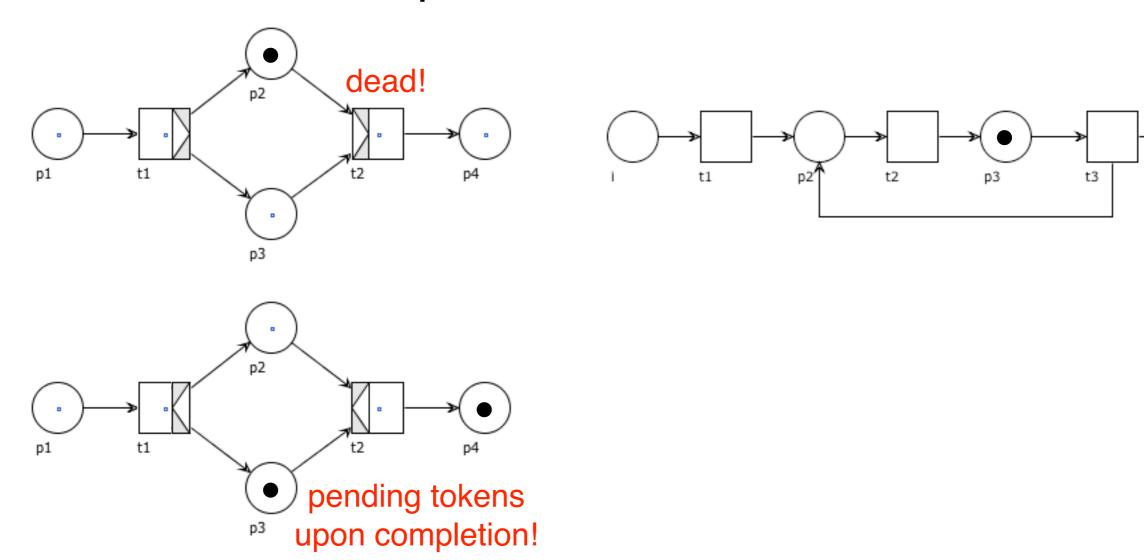




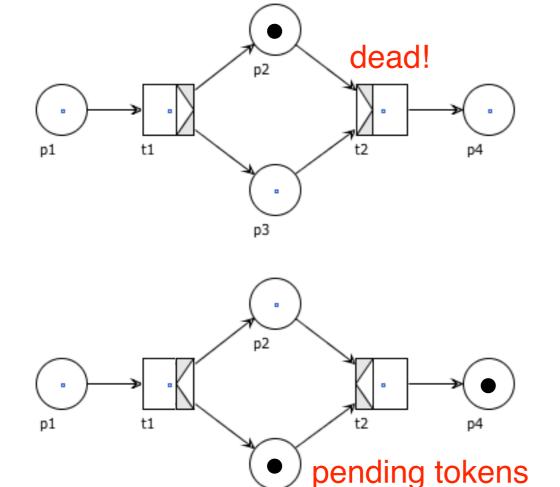




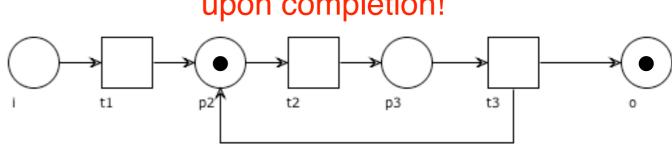


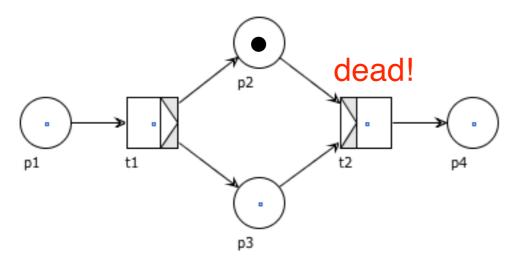


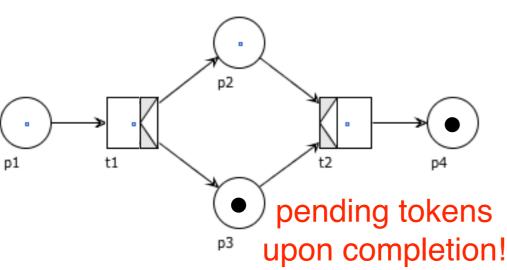
Structural correctness cannot rule out many other problematic issues...



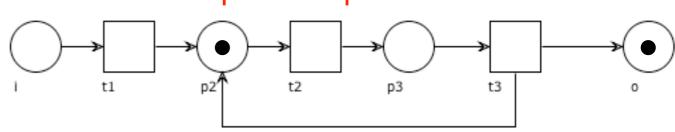
upon completion!

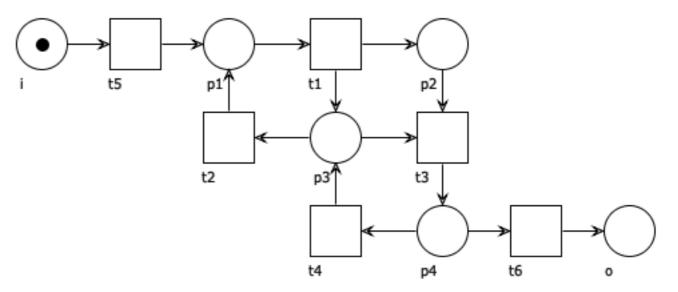




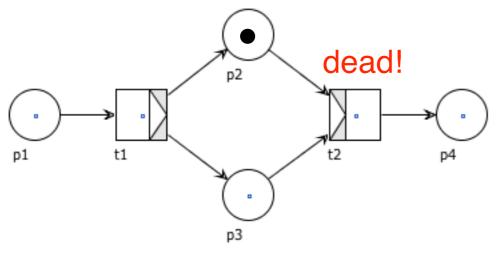


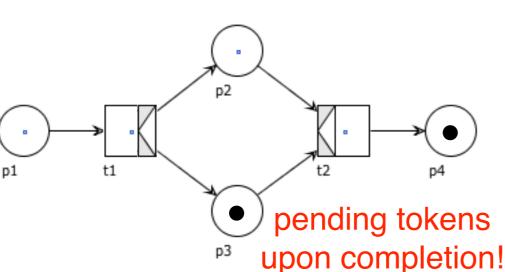


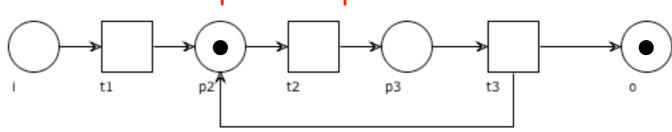


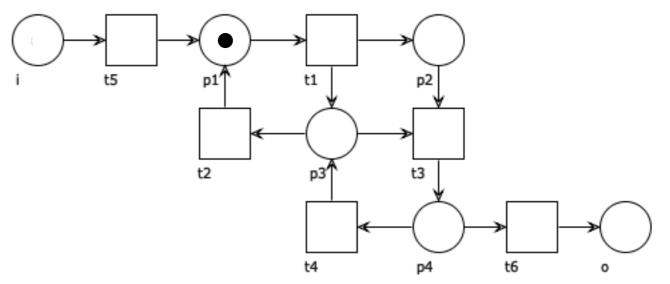


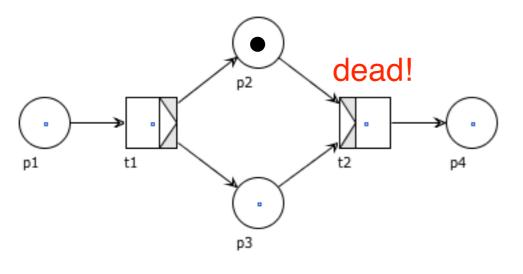
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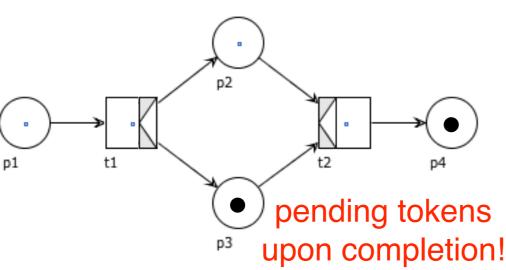




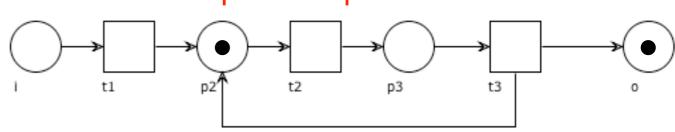


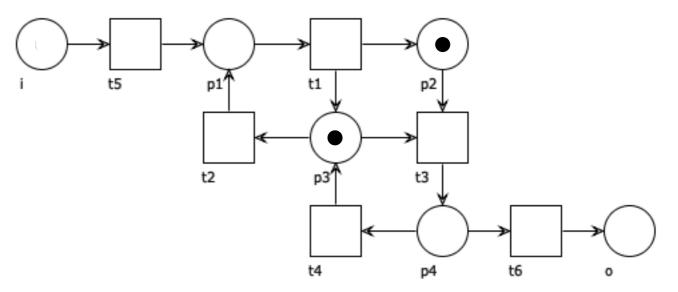


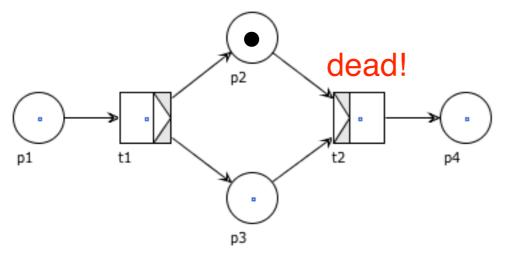


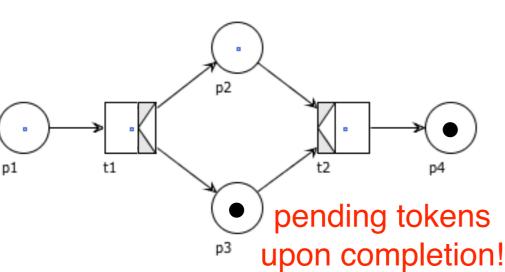




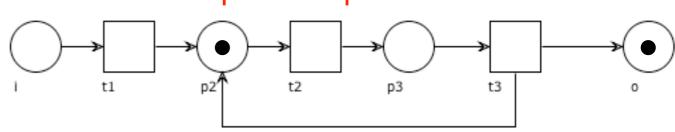


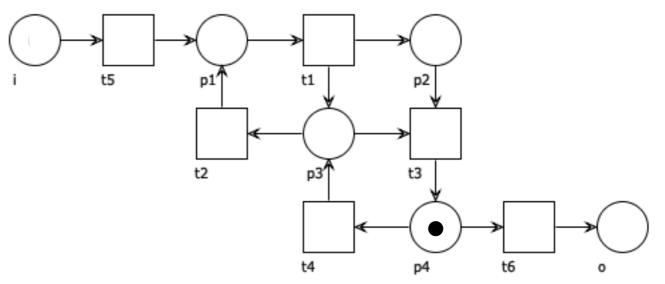




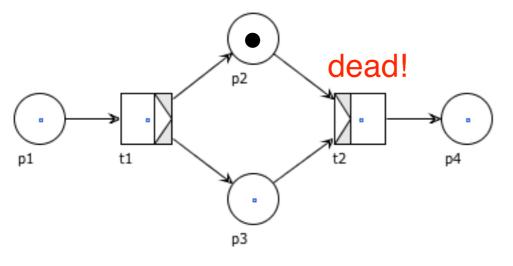


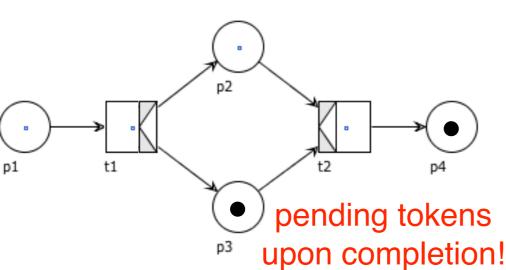


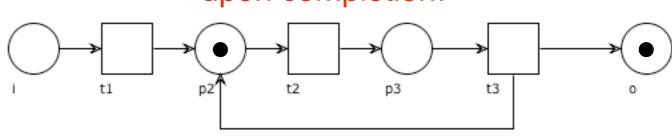


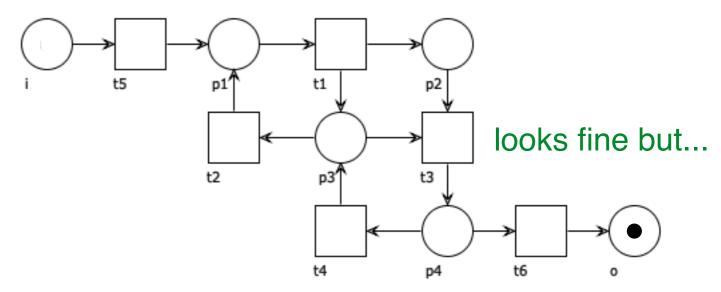


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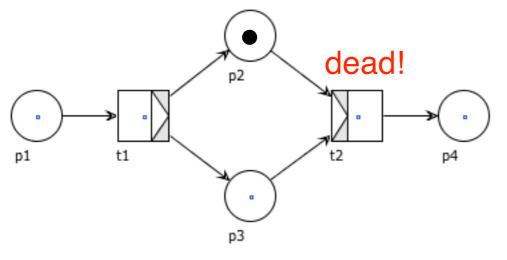


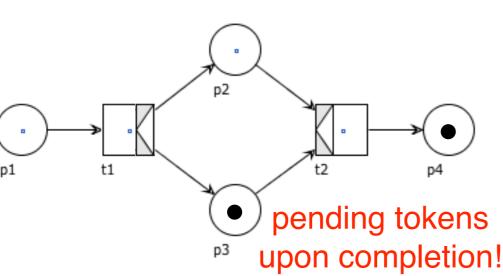


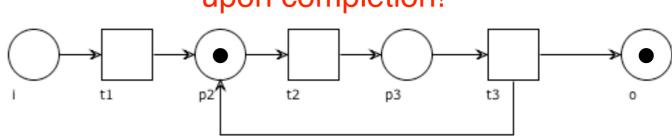


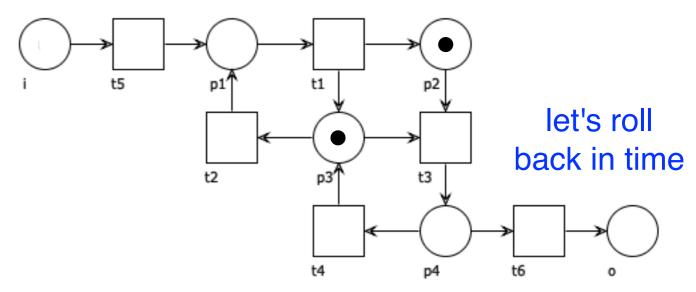


Structural correctness cannot rule out many other problematic issues...

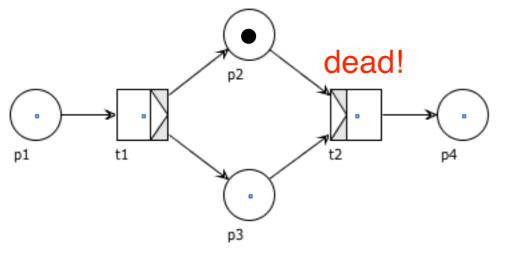


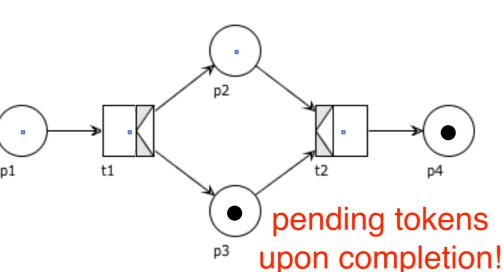


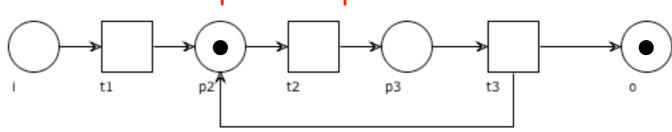


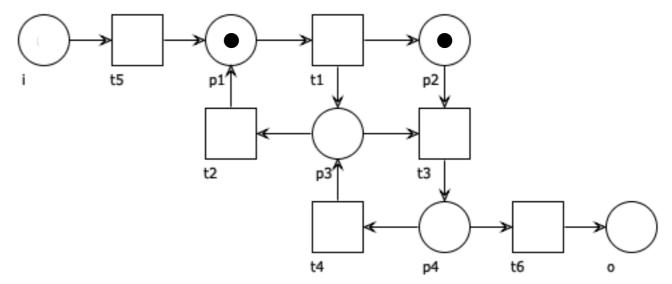


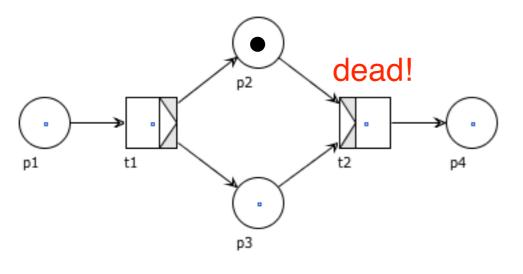
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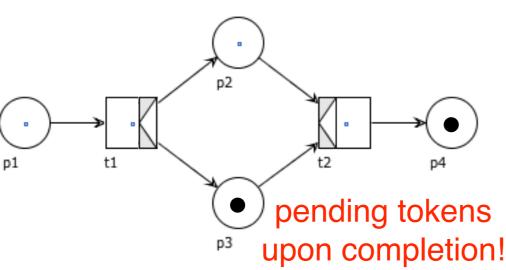




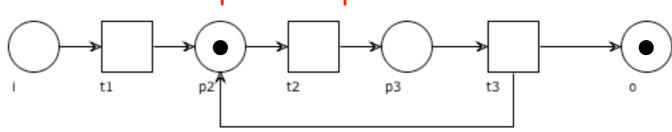


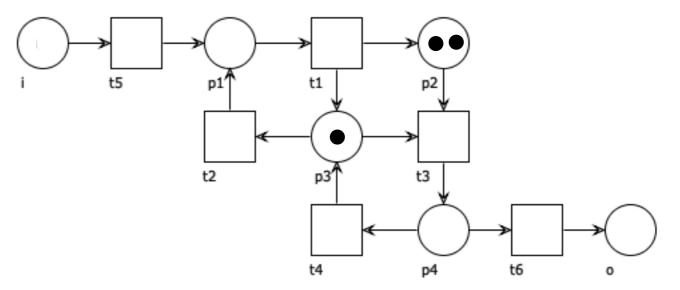


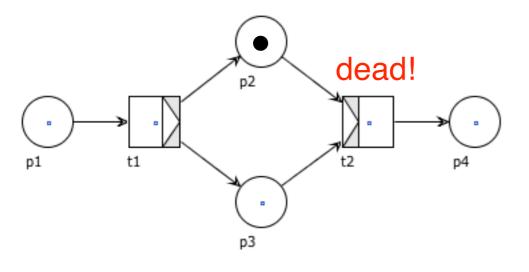


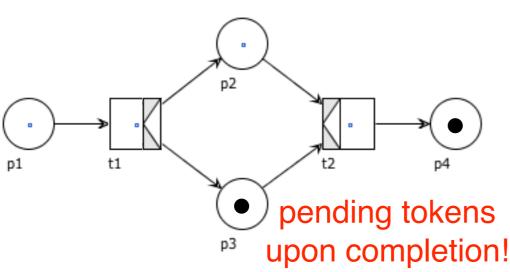




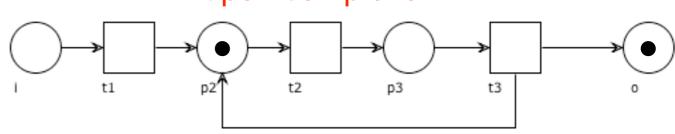


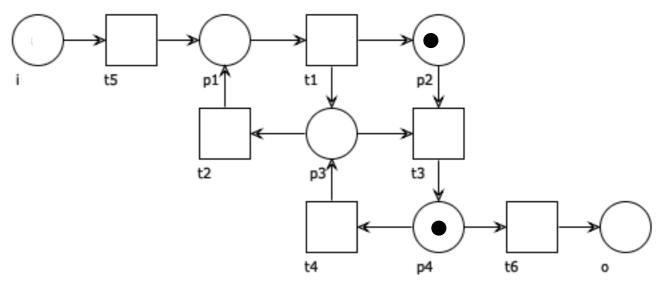




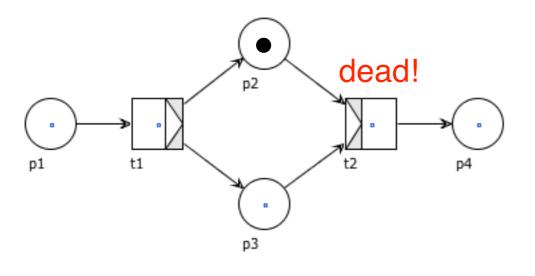


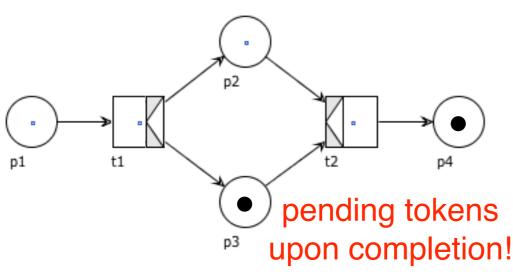


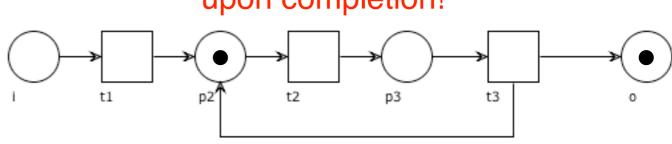


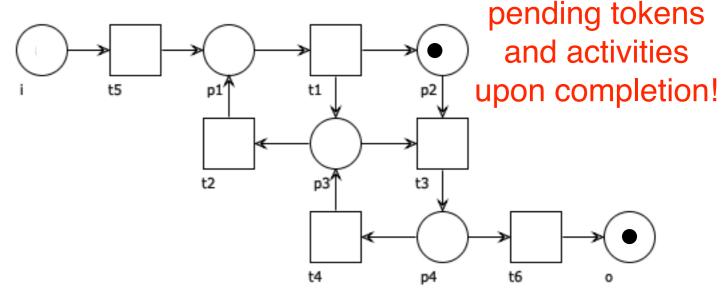


Structural correctness cannot rule out many other problematic issues...

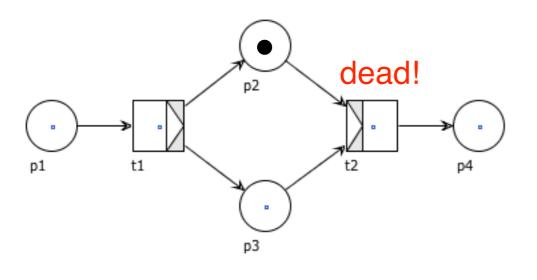


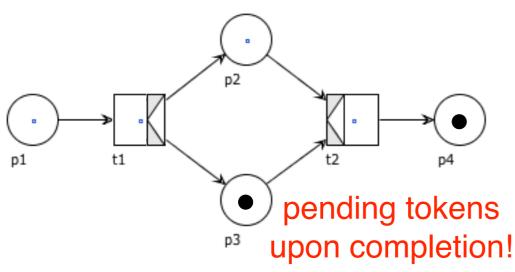


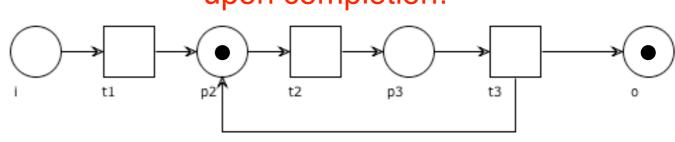


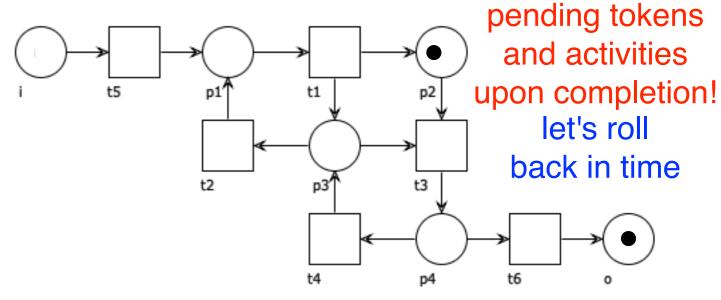


Structural correctness cannot rule out many other problematic issues...

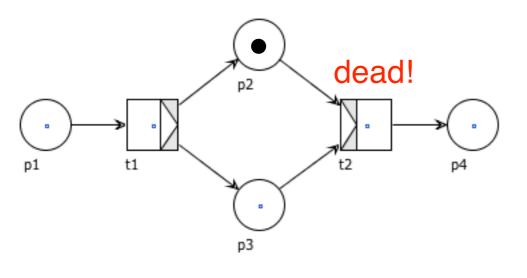


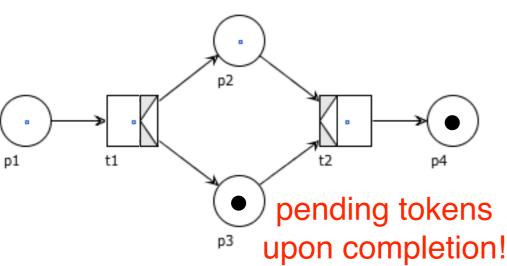




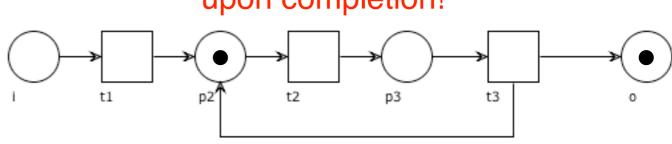


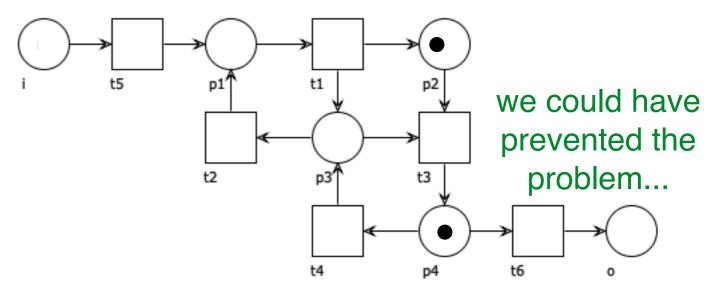
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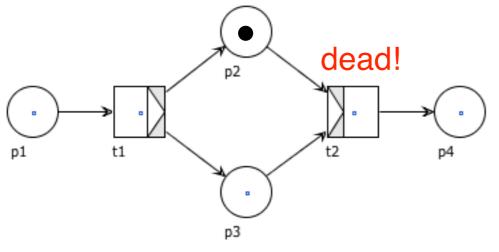


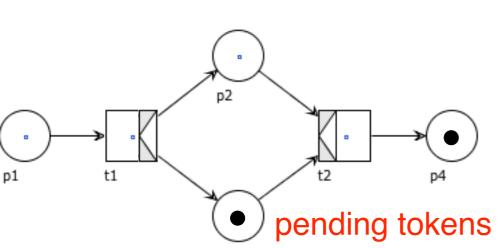






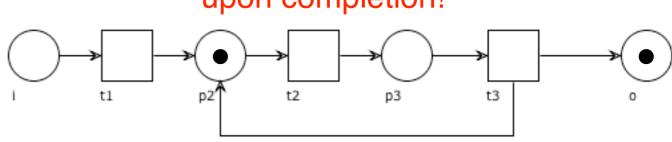
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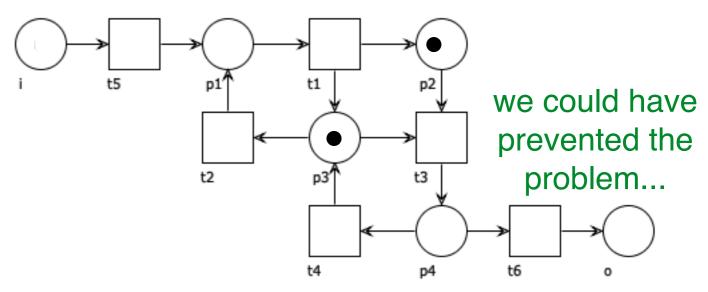




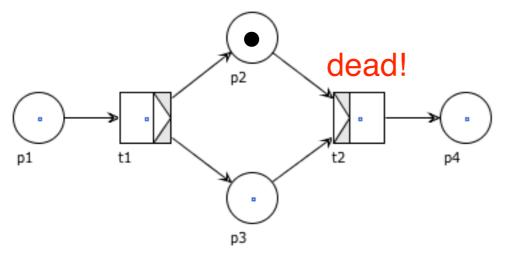
upon completion!

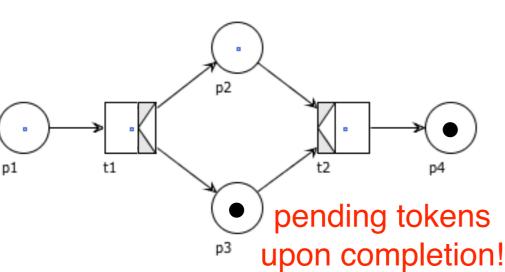
pending tokens and activities upon completion!



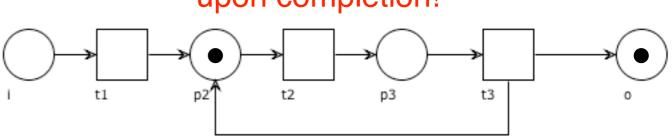


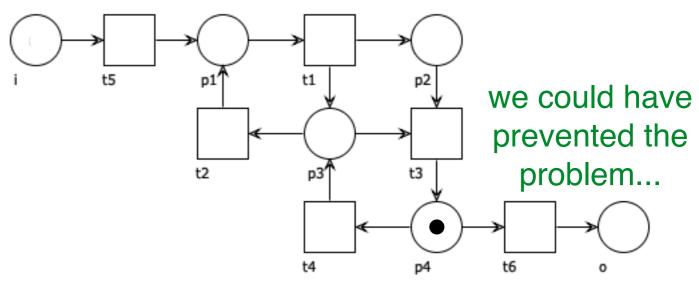
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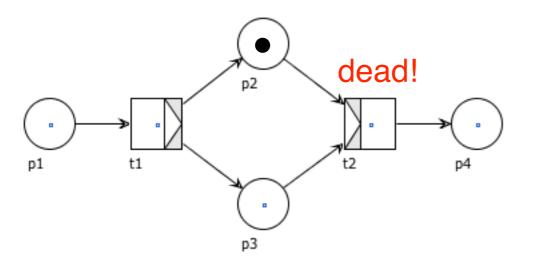


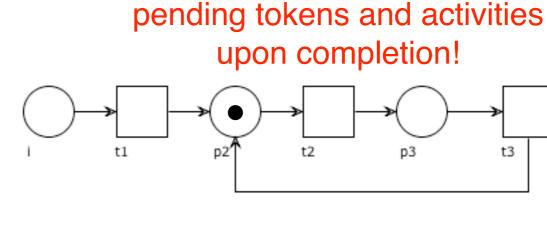
pending tokens and activities upon completion!

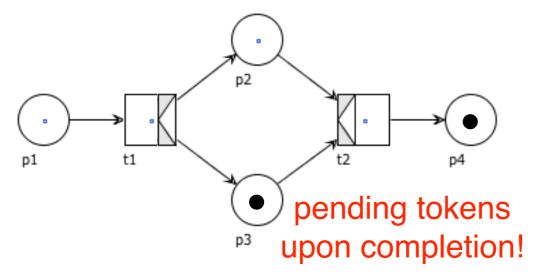


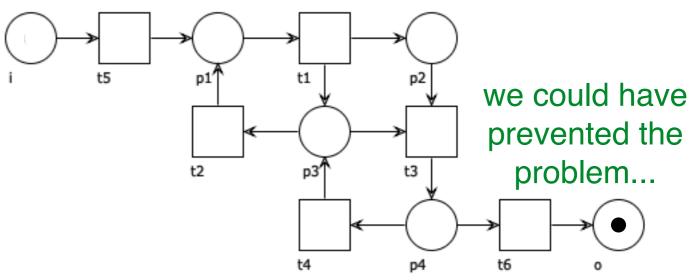


Structural correctness cannot rule out many other problematic issues...

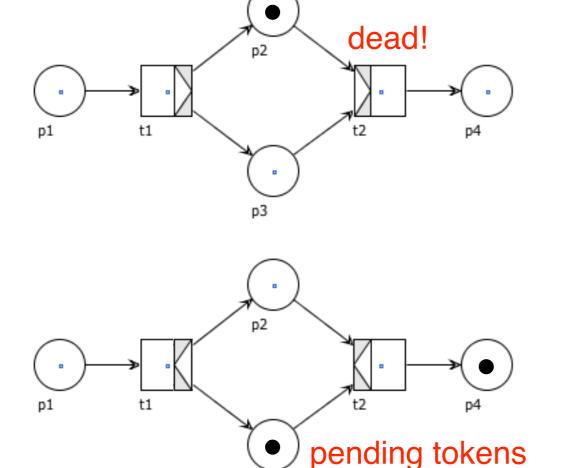




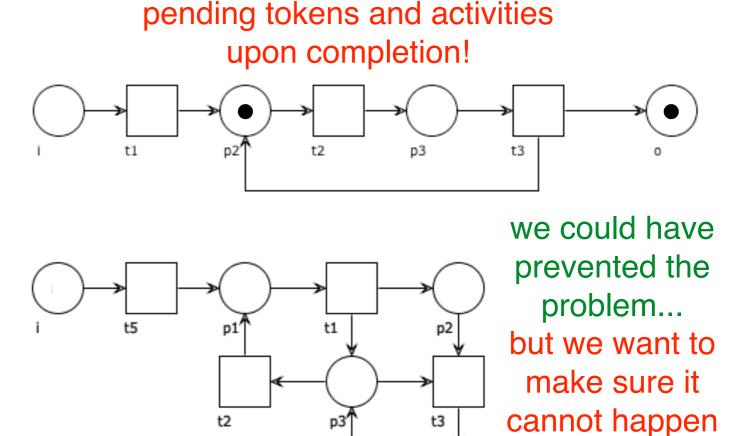




Structural correctness cannot rule out many other problematic issues...

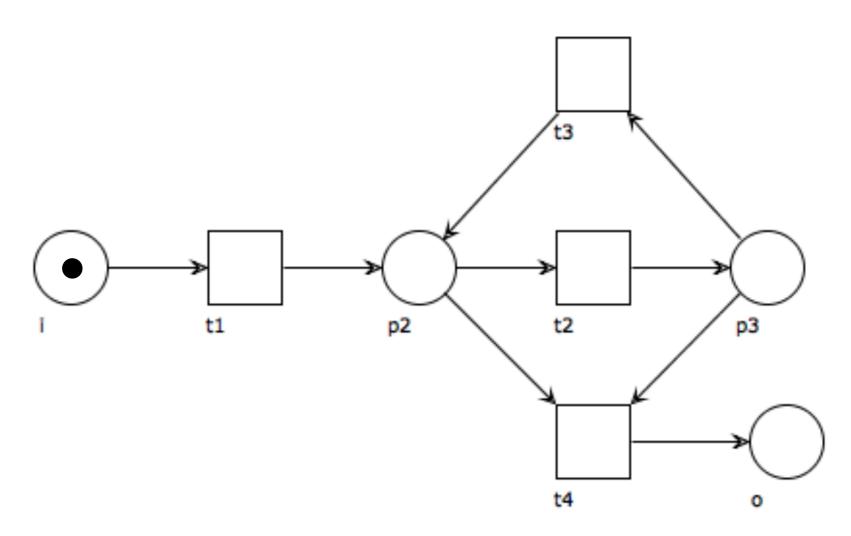


upon completion!



#### Livelock

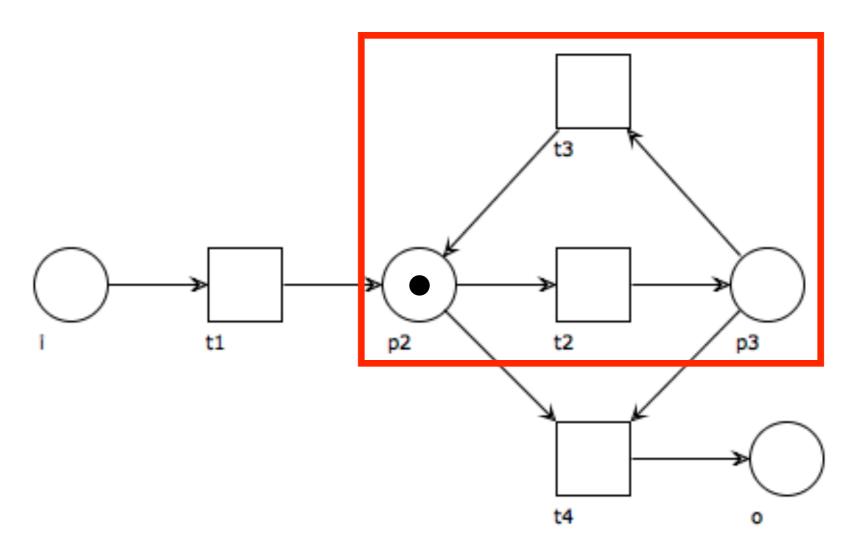
A case is trapped in a cycle with no opportunity to end



can arise in workflow nets

#### Livelock

A case is trapped in a cycle with no opportunity to end



can arise in workflow nets

#### Remark

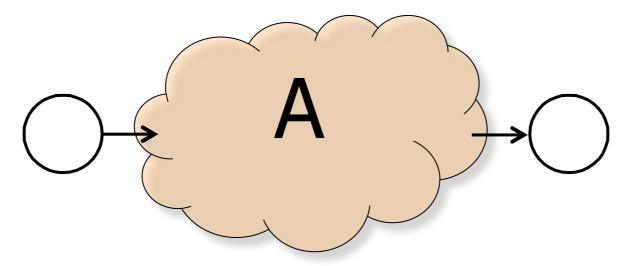
All the previous flaws are typical errors that can be detected without any knowledge about the actual goal of the Business Process

# System validation and verification

Validation is concerned with the relation between the model and the reality How does a model fit log files? Which model does fit better?

Verification aims to answer qualitative questions
Is there a deadlock possible?
Is it possible to successfully handle a specific case?
Will all cases terminate eventually?
Is it possible to execute a certain task?

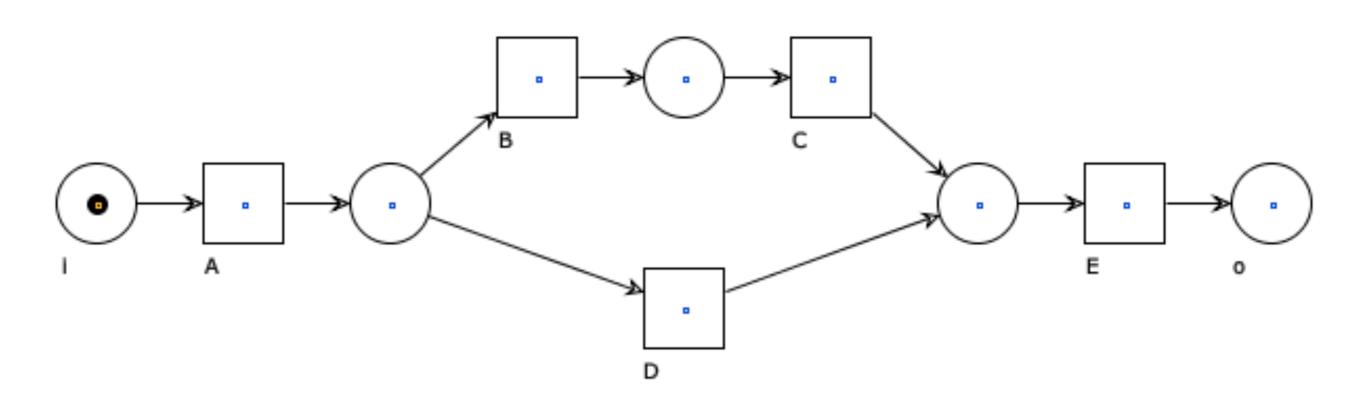
## Language of a workflow net



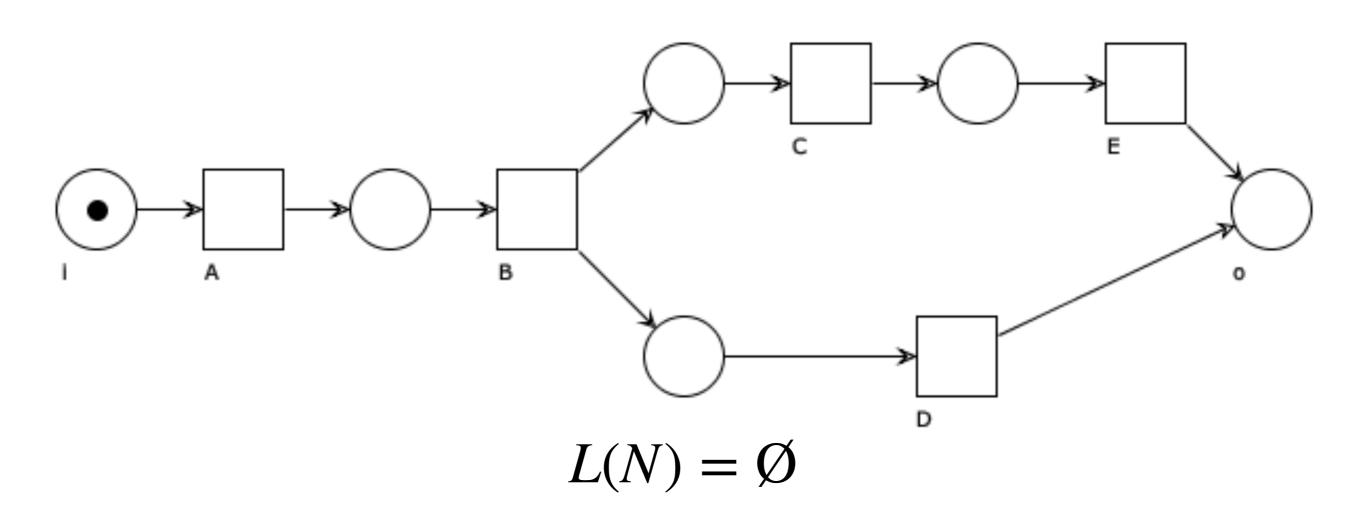
The language of a workflow net is the set of firing sequences that lead from marking i to marking o

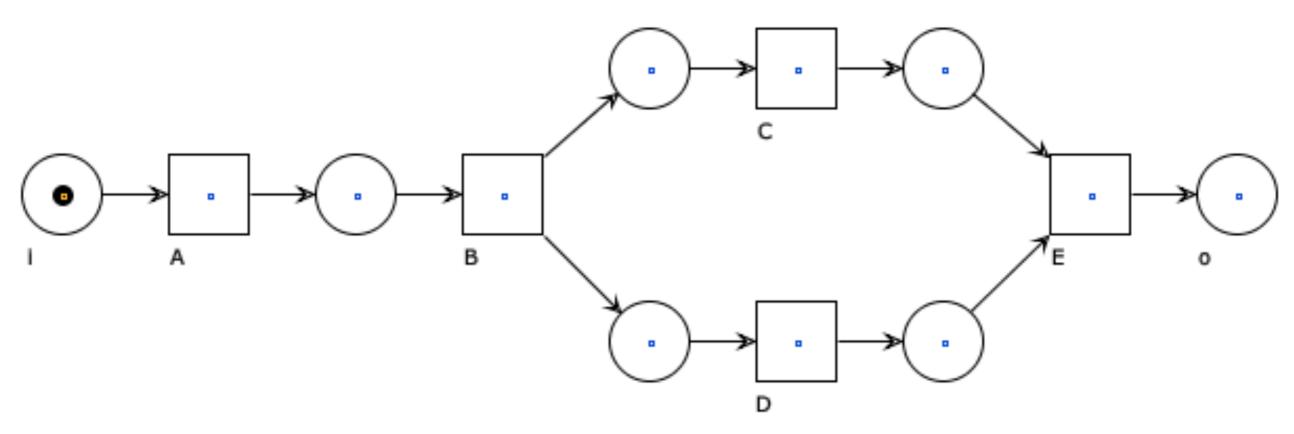
$$L(N) = \{ \sigma \mid i \xrightarrow{\sigma} o \}$$

L(N) defines all the admissible traces of the workflow

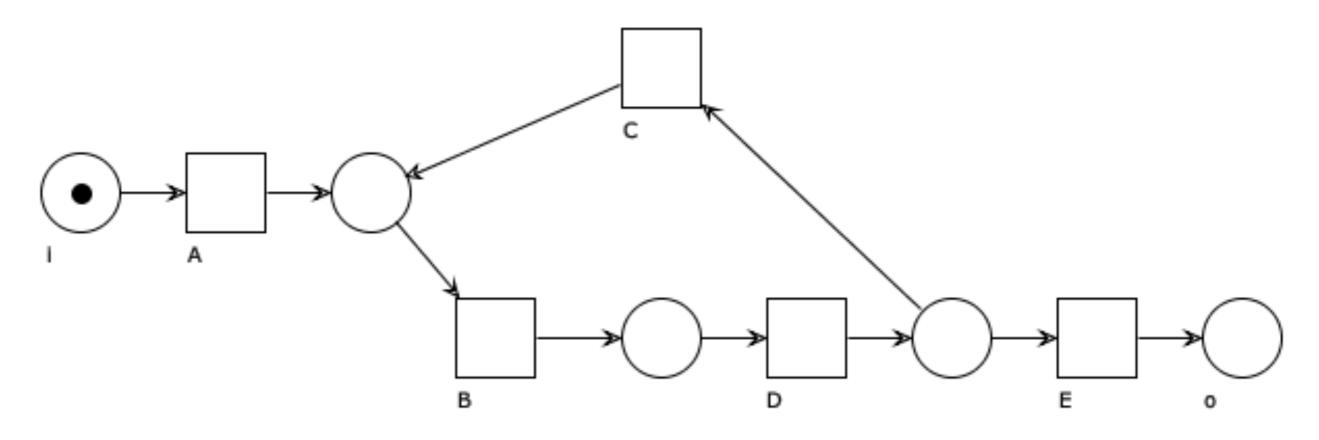


$$L(N) = \{ ABCE, ADE \}$$

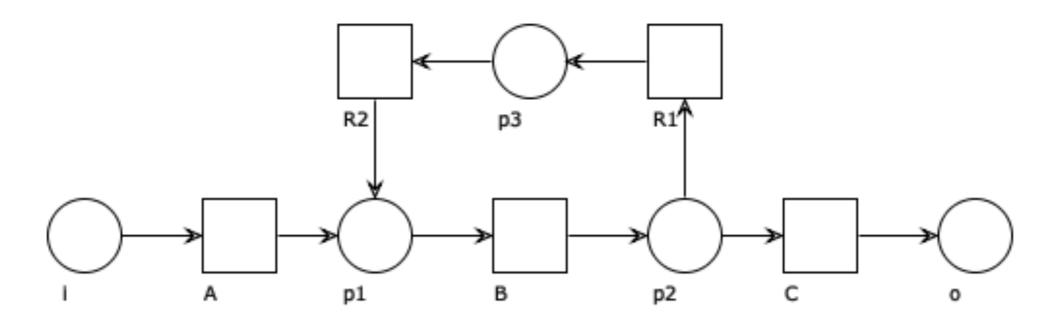




 $L(N) = \{ ABCDE, ABDCE \}$ 



 $L(N) = \{ \text{ ABDE , ABDCBDE , ABDCBDCBDE ,}$   $\text{ABDCBDCBDCBDE , ABDCBDCBDCBDE , ... } \}$   $L(N) = \{ \text{ ABD(CBD)}^k \text{E } \mid k \geq 0 \}$ 



$$L(N) \stackrel{?}{=} \{ A (B R1 R2)^k C | k \ge 0 \}$$

No

$$L(N) \stackrel{?}{=} \{ A (B R1 R2 B)^k C | k \ge 0 \}$$

No

$$L(N) \stackrel{?}{=} \{ A B (R1 R2 B)^k C | k \ge 0 \}$$

Yes

$$L(N) \stackrel{?}{=} \{ A (B R1 R2)^k B C | k \ge 0 \}$$

Yes

## Question time

Consider the workflow net below

How many times can A be executed?

How many times can B be executed?

Can a firing sequence contain two As in a row?

Can a firing sequence contain two Bs in a row?

Can a firing sequence contain more Bs than As?

## Question time

Consider the workflow net below

How many times can A be executed? 1 or more
How many times can B be executed? 0 or more
Can a firing sequence contain two As in a row? yes
Can a firing sequence contain two Bs in a row? no
Can a firing sequence contain more Bs than As? no

#### Simulation

#### **Test analysis**

Try and see which firing sequences are possible

Using WoPeD:

Play (forward and backward) with net tokens Record certain runs (to replay or explain) Randomly select alternatives

Problem: how to make sure that all possible runs have been examined?

## Reachability analysis

All possible runs of a workflow net are represented in its Reachability / Coverability Graph

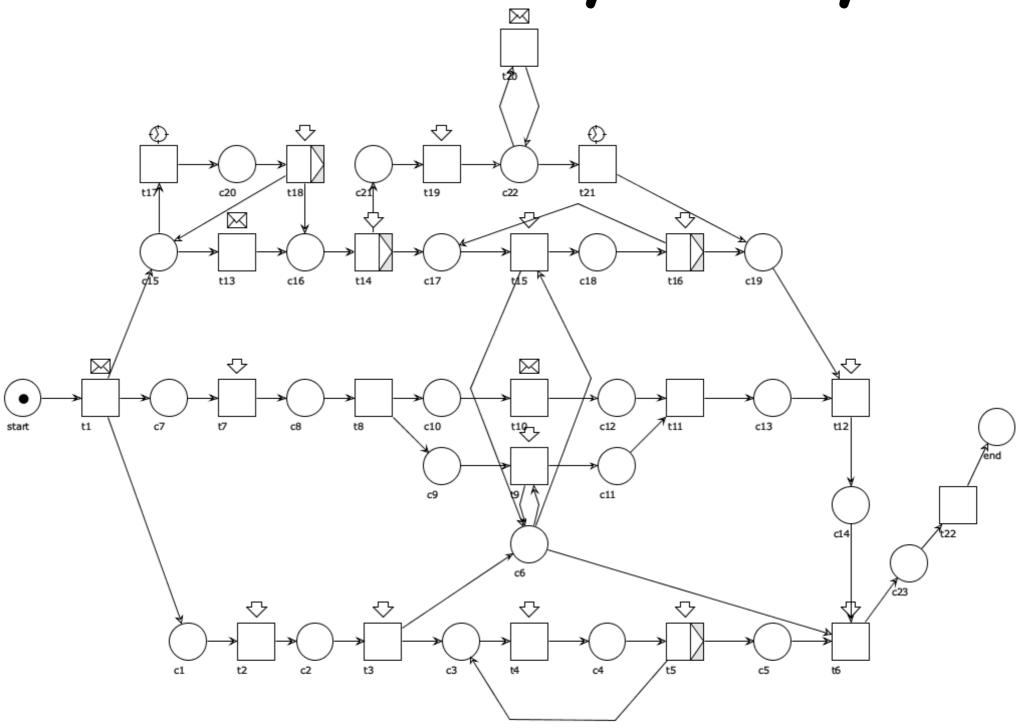
Using WoPeD:

all reachable states are shown
(a single run does not necessarily visit all nodes)
End states are evident (no outgoing arc)

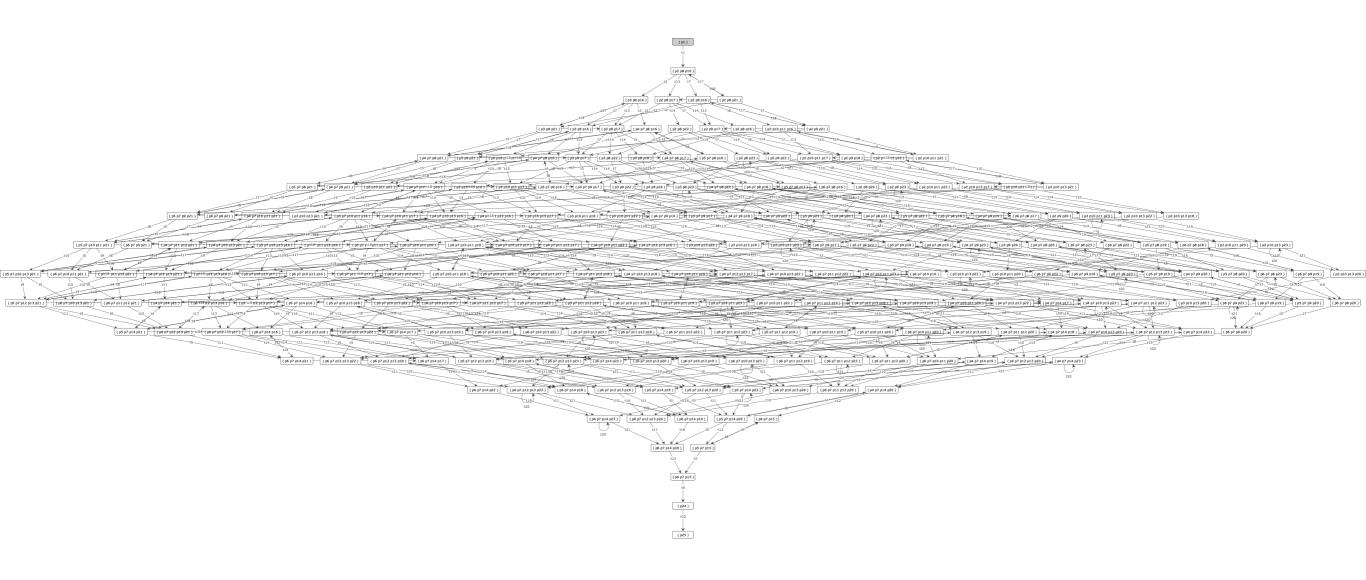
Useful to check if dangerous or undesired states can arise (e.g. the green-green state in the two-traffic-lights)

Problem: state explosion

## Reachability analysis

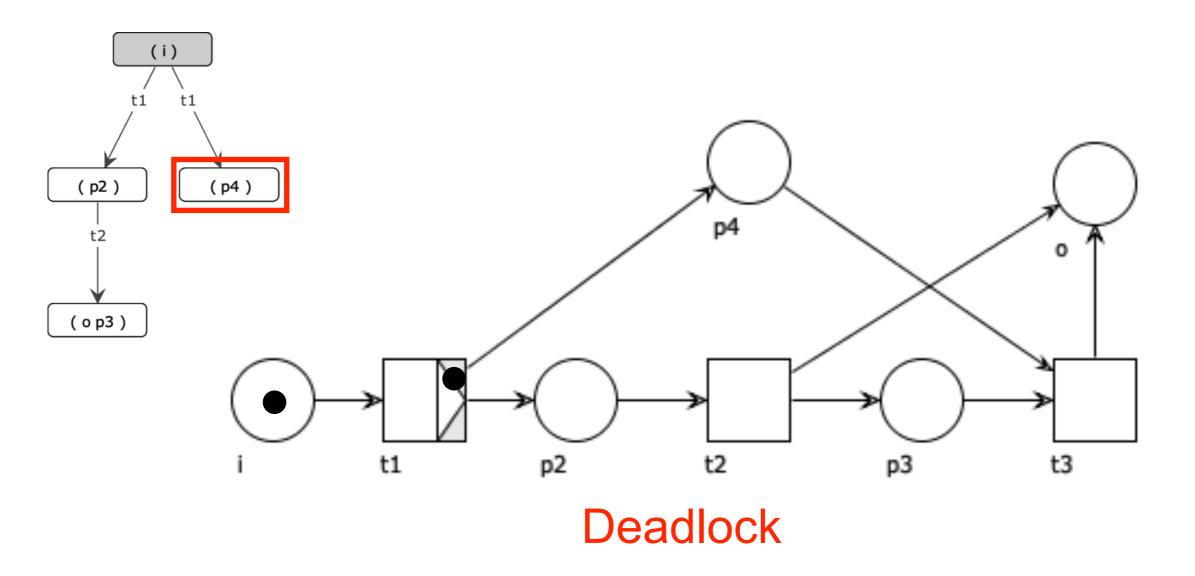


## Reachability analysis

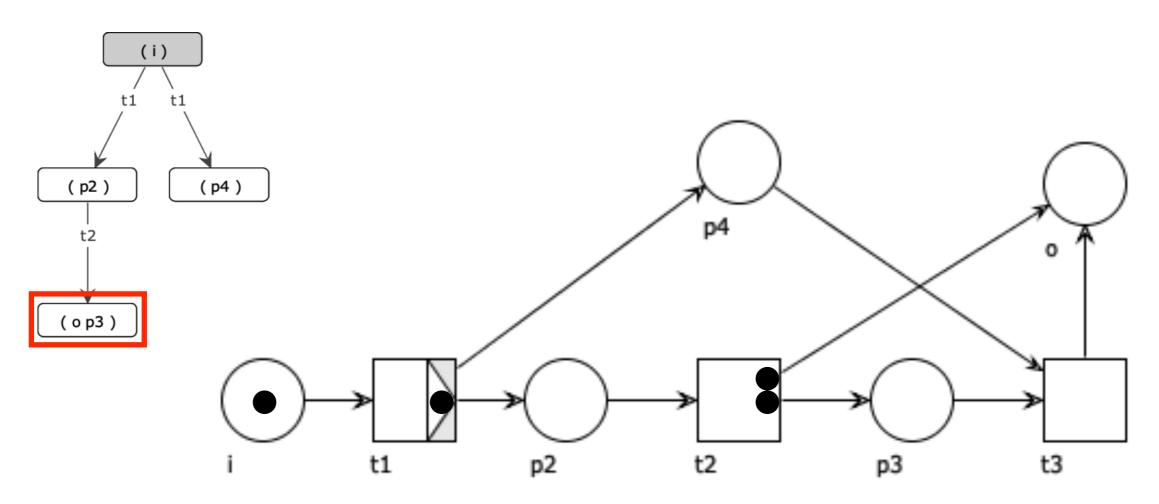


Problem: state explosion

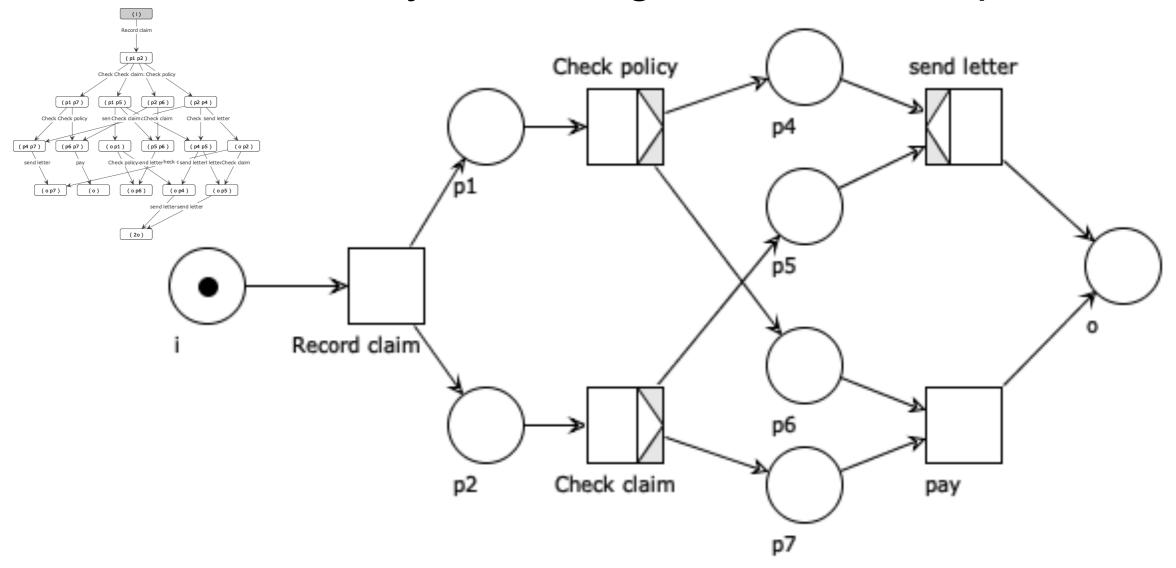
Do you see any problem in the workflow net below?

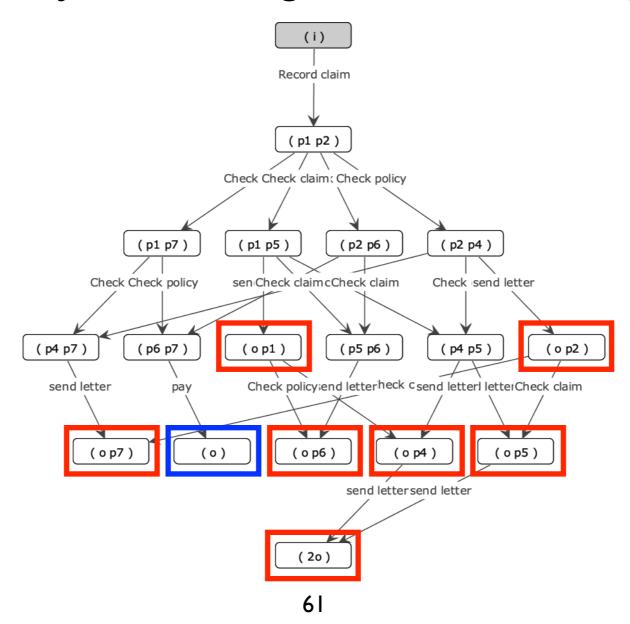


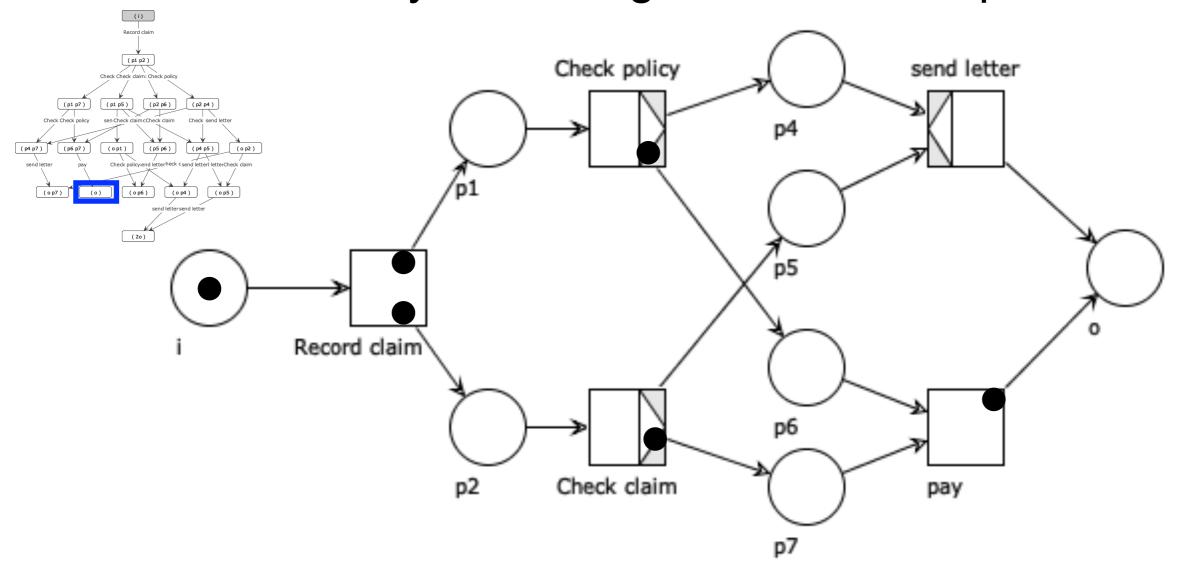
Do you see any problem in the workflow net below?



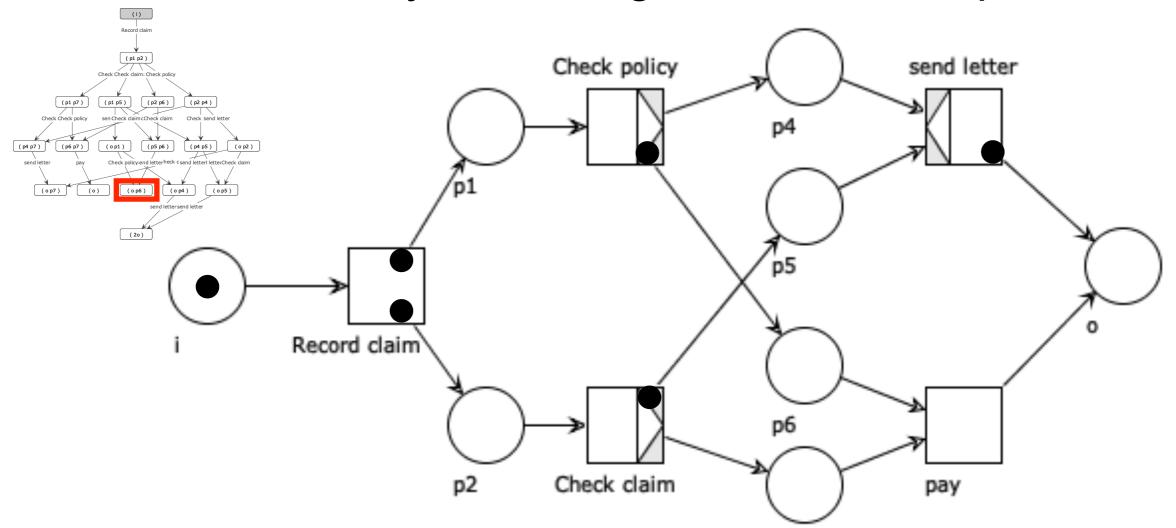
Some tokens left in the net after case completion





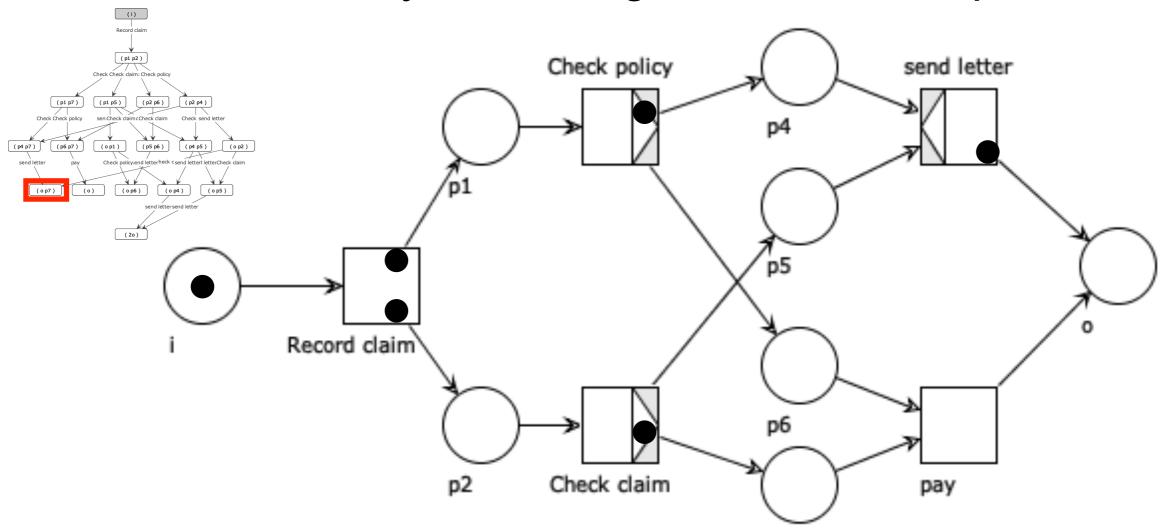


Which problem(s) in the workflow net below? How would you redesign the business process?



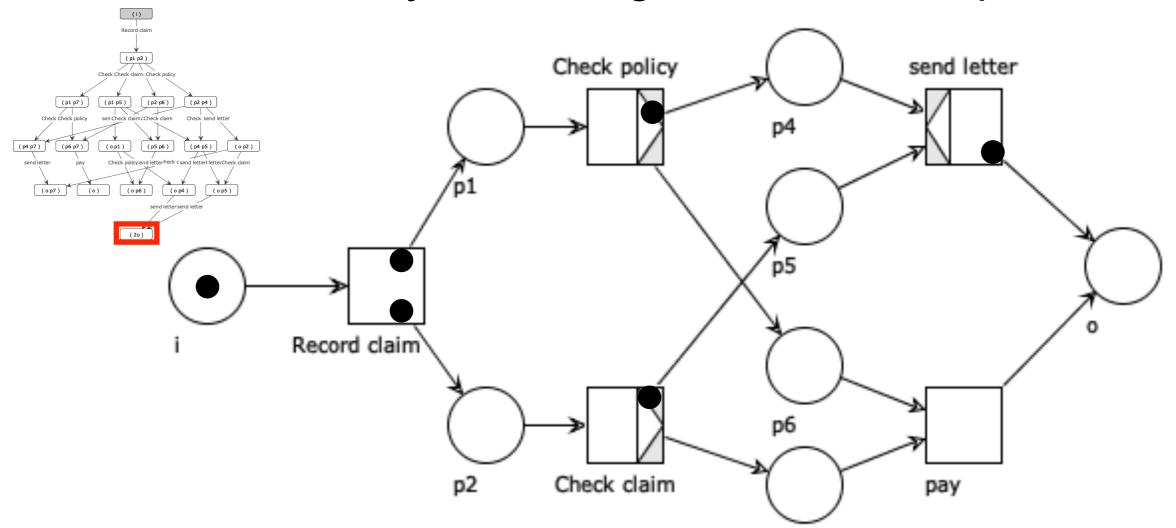
Some tokens left in the net after case completion

Which problem(s) in the workflow net below? How would you redesign the business process?

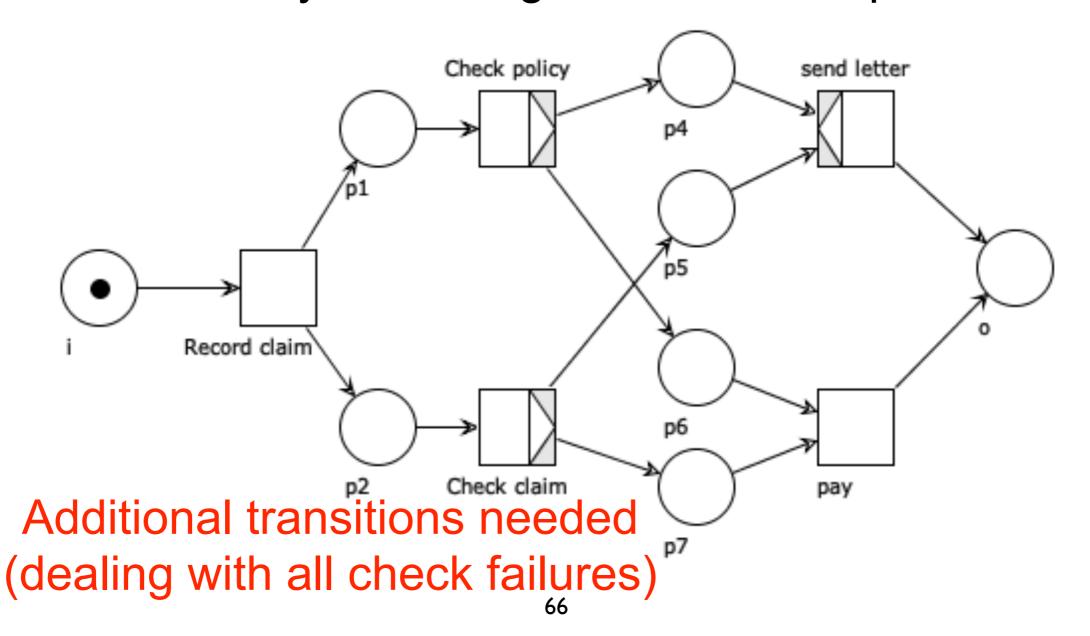


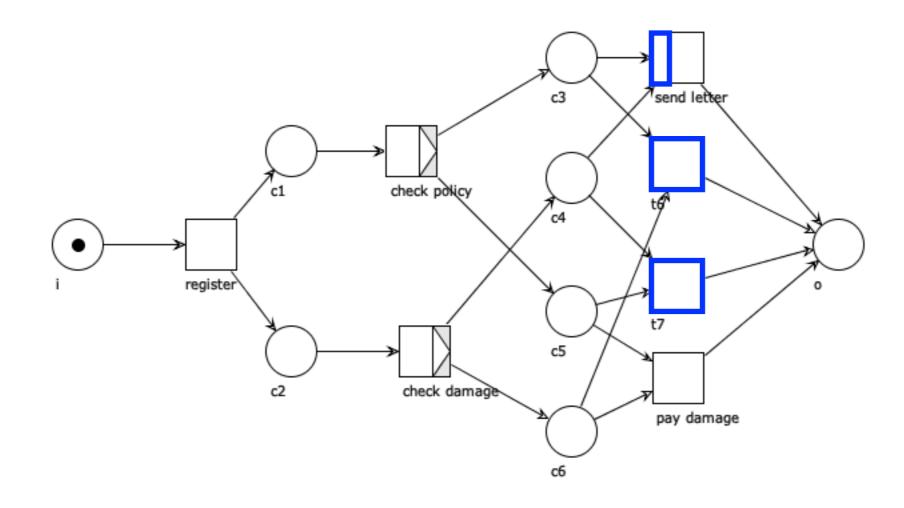
Some tokens left in the net after case completion

Which problem(s) in the workflow net below? How would you redesign the business process?



Some activities take place after case completion





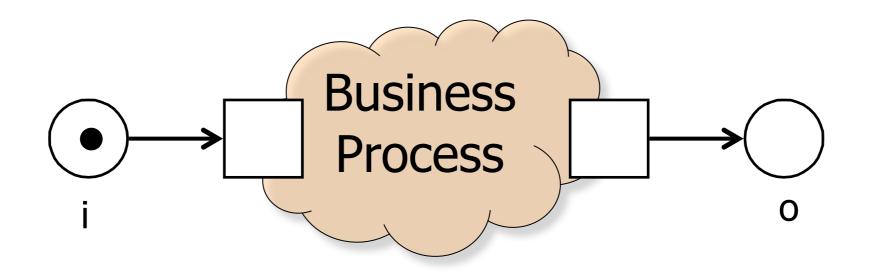
### Soundness

# Soundness of Business Processes

#### A process is called **sound** if

- 1. it contains no unnecessary tasks
- 2. every case, once started, can always be completed in full
- 3. no pending items are left upon case completion

# Soundness of Business Processes



# Soundness of Workflow nets

A workflow net is called **sound** if

- 1. for each transition t, there is a marking M (reachable from i) that enables t
- 2. for each token put in place i, one token eventually appears in the place o
- 3. when a token is in place o, all other places are empty

## Fairness assumption

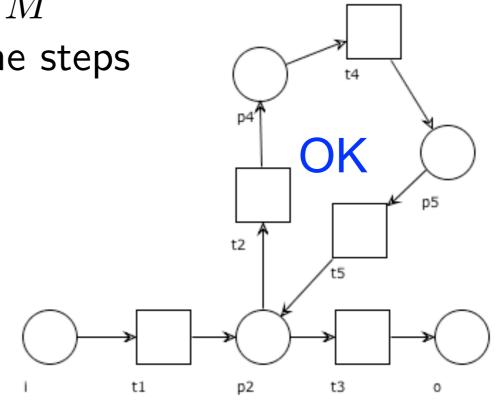
#### Remark:

Condition 2 does not mean that iteration must be forbidden or bound

It says that from any reachable marking  ${\cal M}$  there must be possible to reach o in some steps

#### Fairness assumption:

A task cannot be postponed indefinitely



# Soundness, Formally

A workflow net is called **sound** if

no dead task no transition is dead

$$\forall t \in T. \; \exists M \in [i\rangle. \; M \xrightarrow{t}$$

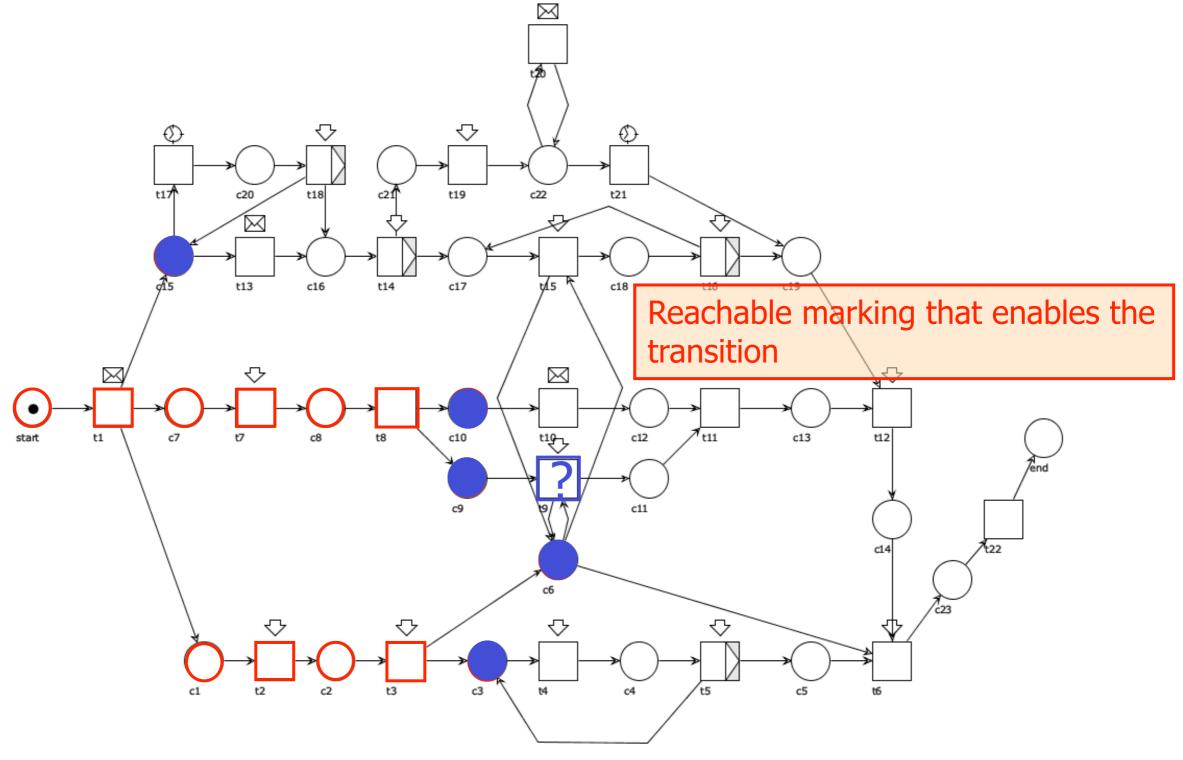
option to complete place o is eventually marked

$$\forall M \in [i \rangle. \exists M' \in [M \rangle. M'(o) \ge 1$$

proper completion when o is marked, no other token is left

$$\forall M \in [i] . M(o) \ge 1 \Rightarrow M = o$$

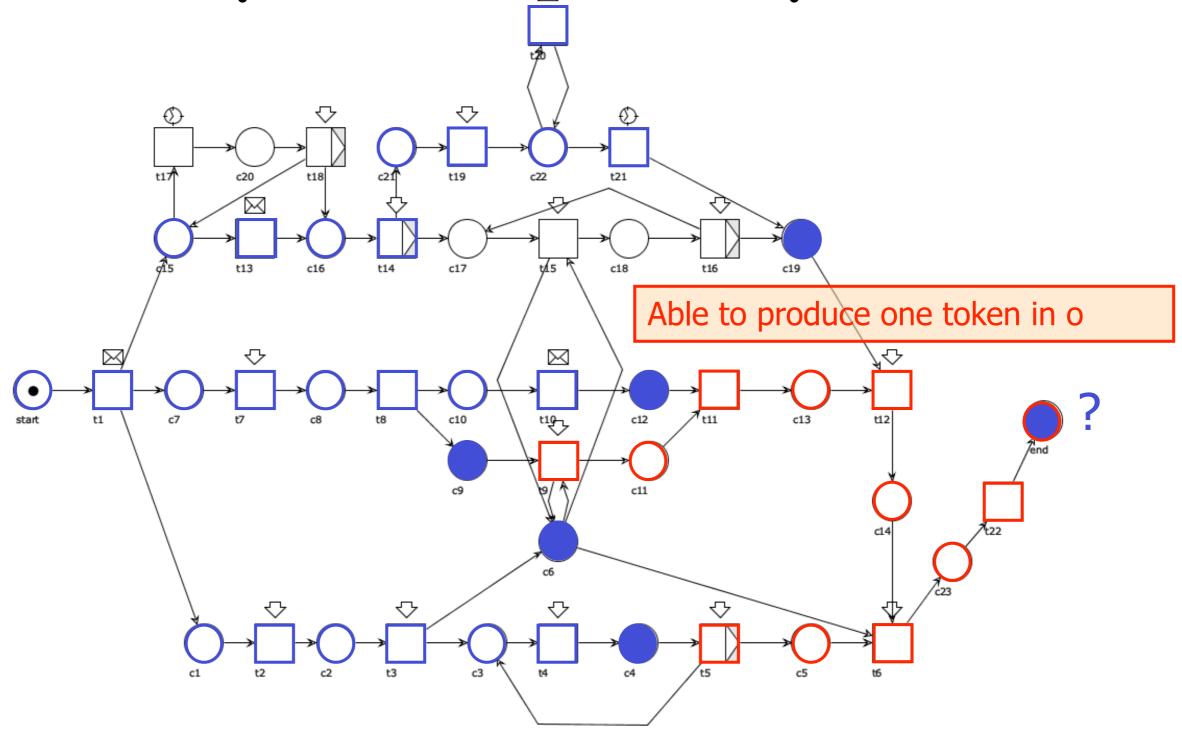
### 1: no dead tasks



### 1: no dead tasks

The check must be repeated for each task

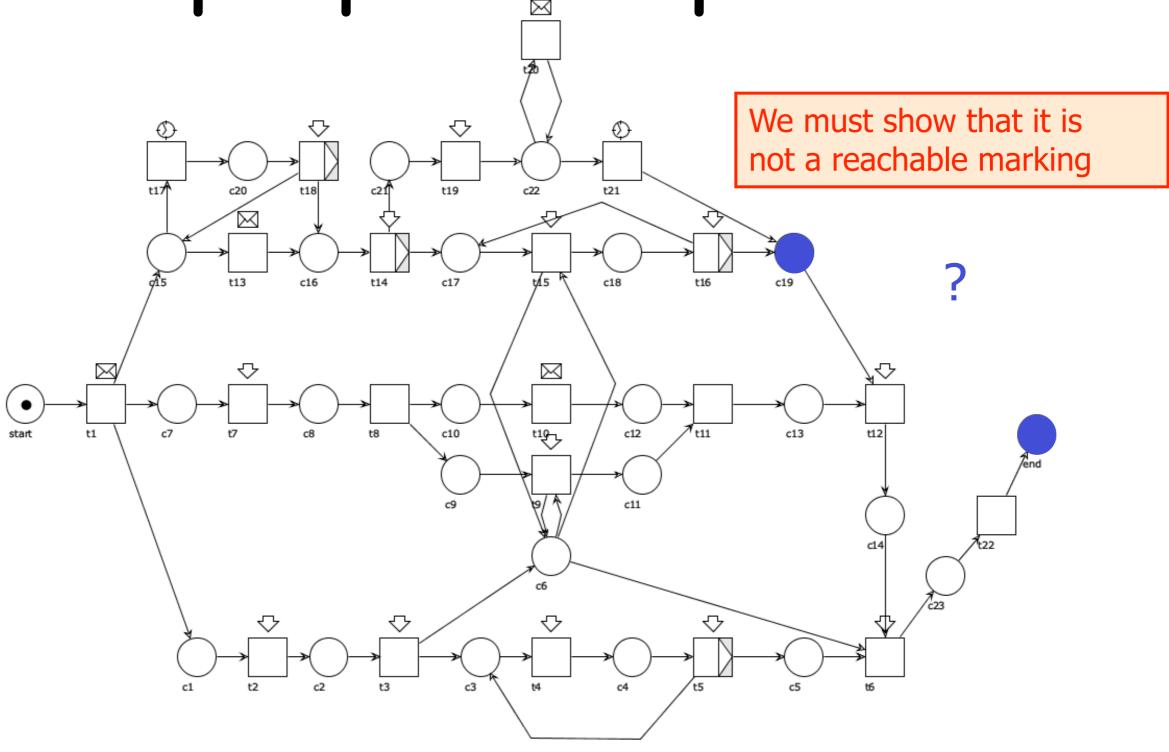
2: option to complete



# 2: option to complete

The check must be repeated for each reachable marking

# 3: proper completion



# 3: proper completion

The check must be repeated for each marking M such that M>o

# Brute-force analysis

First, check if the Petri net is a workflow net easy "structural" check

Second, check if it is sound (more difficult):
 build the Reachability Graph

to check 1: for each transition t there must be an arc in the
 RG that is labelled with t

to check 2&3: the RG must have only one final state (sink),
 that consists of one token in o
 and is reachable from any other state,
 and no other marking has a token in o

# Some Pragmatic Considerations

All checks can better be done automatically (computer aided)

but nevertheless RG construction...

- 1. can be computationally expensive for large nets (because of state explosion)
- 2. provides little support in repairing unsound processes
- 3. can be infinite (CG can be used, but it is not exact)

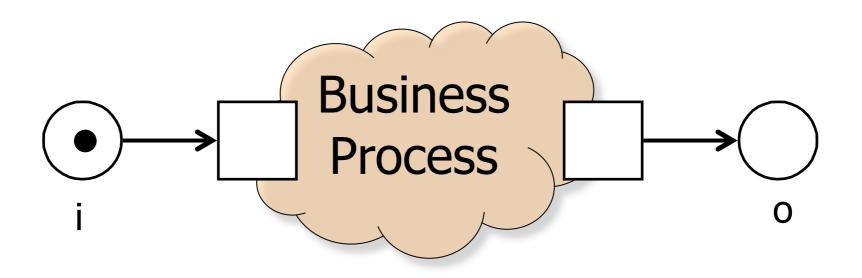
# Advanced support

Translate soundness to other well-known properties that can be checked more efficiently:

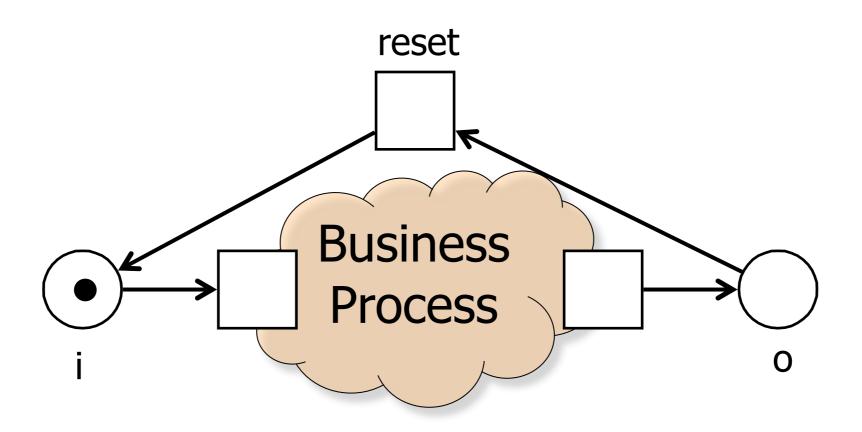
boundedness and liveness



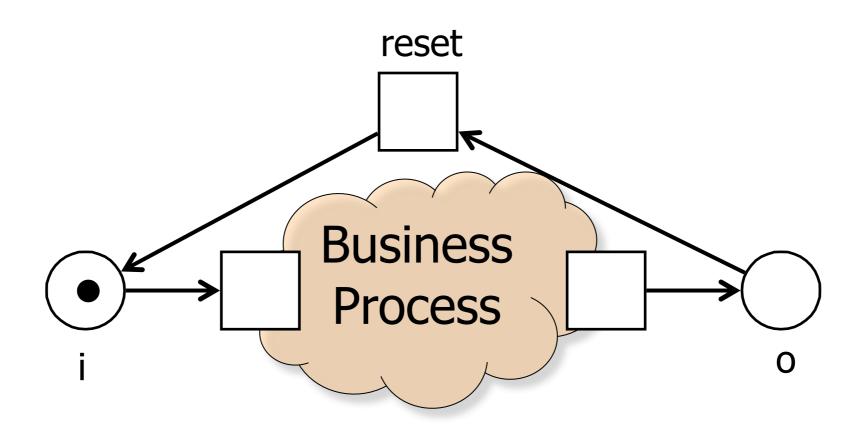
# Play once

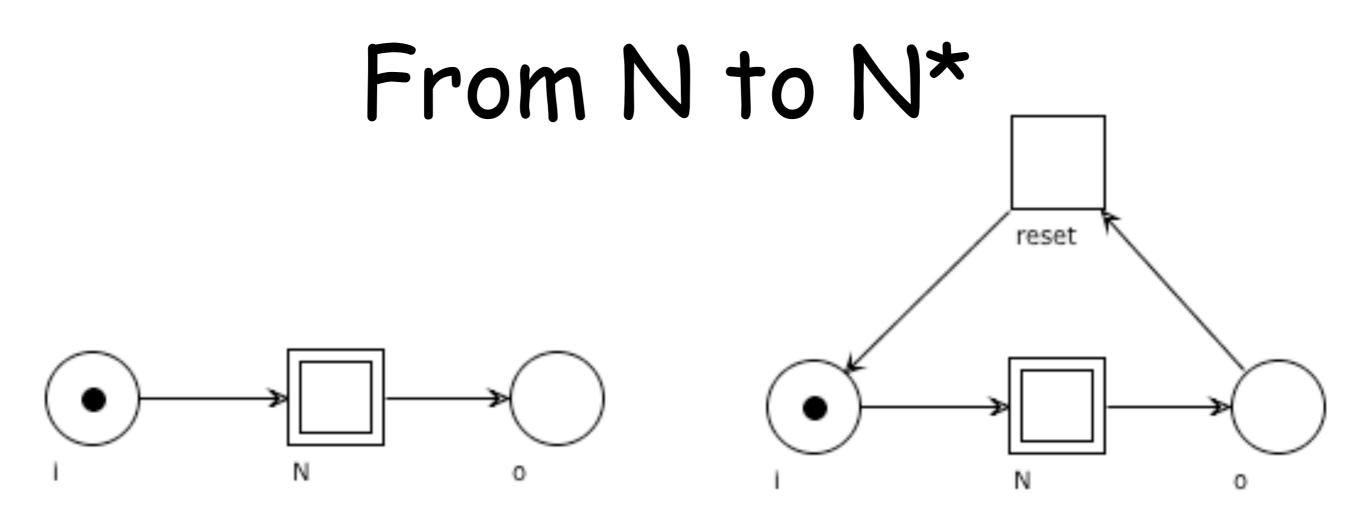


# Play twice



# Play any number of times





Let us denote by  $N:i\to o$  a workflow net with entry place i and exit place o.

Let  $N^*$  be the net obtained by adding the "reset" transition to N  $reset: o \rightarrow i$ .

### MAIN THEOREM

### Theorem:

N is sound iff  $N^{st}$  is live and bounded

### MAIN THEOREM

### Theorem:

N is sound iff  $N^{st}$  is live and bounded

1 no dead tasks 2 option to complete  $\Leftarrow$  1 3 proper completion

- $3 \Rightarrow$  at any reachable marking, every transition can fire in the future and
- $^2 \Rightarrow$  for some k, every place will contain less than k tokens

# Proof of MAIN THEOREM (1)

 $N^*$  live and bounded implies N sound:

 $(\Leftarrow)$ 

Since  $N^*$  is **live**: for each  $t \in T$  there is  $M \in [i]$ .  $M \stackrel{t}{\rightarrow}$ 

Take any  $M \in [i]$  enabling  $reset: o \rightarrow i$ , hence  $M \supseteq o$ 

Let  $M \stackrel{reset}{\longrightarrow} M'$ . Then  $M' \in [i]$  and  $M' \supseteq i$ 

Since  $N^*$  is bound, it must be M'=i (and M=o) Otherwise all places marked by M'-i=M-o would be unbounded

Hence  $N^*$  just allows multiple runs of N:

"option to complete" and "proper completion" hold (see above)

"no dead task" holds because  $N^{st}$  is live

### A technical lemma

#### Lemma:

If N is sound, M is reachable in N iff M is reachable in  $N^*$ 

 $\Rightarrow$ ) straightforward

$$\Leftarrow$$
) Let  $i \xrightarrow{\sigma} M$  in  $N^*$  for  $\sigma = t_1 t_2 ... t_n$ 

We proceed by induction on the number r of instances of reset in  $\sigma$  If r=0, then reset does not occur in  $\sigma$  and M is reachable in N If r>0, let k be the least index such that  $t_k=reset$ 

Let 
$$\sigma = \sigma' t_k \sigma''$$
 with  $\sigma' = t_1 t_2 ... t_{k-1}$  fireable in  $N$ 

Since 
$$N$$
 is sound:  $i \xrightarrow{\sigma'} o$  and  $i \xrightarrow{\sigma''} M$ 

Since  $\sigma''$  contains r-1 instances of reset:

by inductive hypothesis  ${\cal M}$  is reachable in  ${\cal N}$ 

# Proof of MAIN THEOREM (2)

N sound implies  $N^*$  bounded :

We proceed by contradiction, assuming  $N^{st}$  is unbounded

Since  $N^*$  is unbounded:

 $\exists M, M'$  such that  $i \to^* M \to^* M'$  with  $M \subset M'$ 

Let  $L = M' - M \neq \emptyset$ 

Since N is sound:

 $(\Rightarrow)$ 

 $\exists \sigma \in T^* \text{ such that } M \xrightarrow{\sigma} o$ 

By the monotonicity Lemma:  $M' \stackrel{\sigma}{\to} o + L$  and thus  $o + L \in [i]$  Which is absurd, because N is sound

# Proof of MAIN (←) THEOREM (3)

#### N sound implies $N^*$ live:

Take any transition t and let M be a marking reachable in  $N^{\ast}$  By the technical lemma, M is reachable in N

Since N is sound:  $\exists \sigma \in T^*$  with  $M \xrightarrow{\sigma} o$ 

Since N is sound:  $\exists \sigma' \in T^*$  with  $i \xrightarrow{\sigma'} M'$  and  $M' \xrightarrow{t}$ 

Let 
$$\sigma'' = \sigma \ reset \ \sigma'$$
, then:  $M \xrightarrow{\sigma''} M'$  in  $N^*$  and  $M' \xrightarrow{t}$ 

# A theorem on strong connectedness (whose proof we omit)

### Paths and circuits

(P,T,F)

A path is a non-empty sequence of nodes  $x_1x_2...x_k$  such that

$$(x_i, x_{i+1}) \in F$$
 for every  $1 \le i \le k$ 

and we say it leads from  $x_1$  to  $x_k$ 

A path  $x_1x_2...x_k$  is called a **circuit** if

all its nodes are distinct and  $(x_k, x_1) \in F$ 

since there is no node x with  $(x,x) \in F$ , any circuit has at least two nodes

### Paths and circuits

An undirected path is a non-empty sequence of nodes  $x_1x_2...x_k$  s.t.

$$(x_i, x_{i+1}) \in (F \cup F^{-1})$$
 for every  $1 \le i \le k$ 

(denotes the inverse of a binary relation)

$$F^{-1} = \{ (y, x) \mid (x, y) \in F \}$$

(a path where we disregard the orientation of arcs)

### Connectedness

A net (P,T,F) is **weakly connected** if there is an undirected path between any two distinct nodes

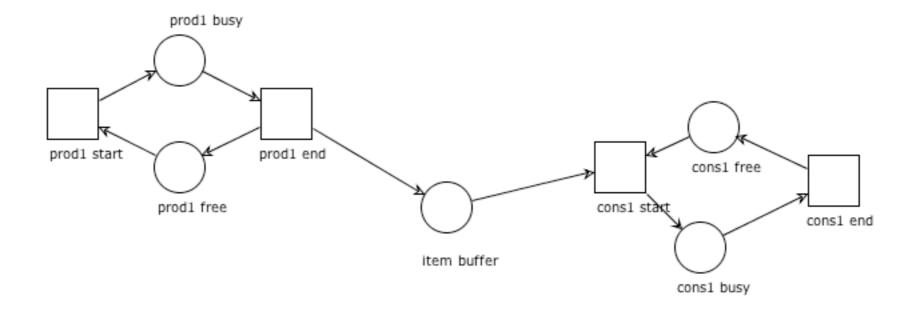
A net (P, T, F) is **strongly connected** if there is a path between any two distinct nodes

# Connectedness, again

A net (P,T,F) is **weakly connected**iff
it cannot be splitted in separated components

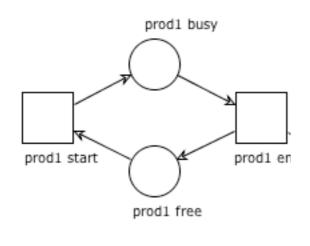
A weakly connected net is **strongly connected**iff
for every arc (x,y) there is a path from y to x

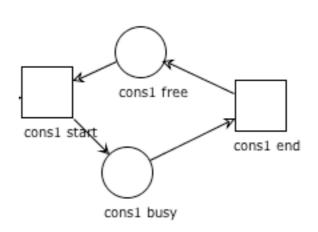
# Examples



weakly connected not strongly connected

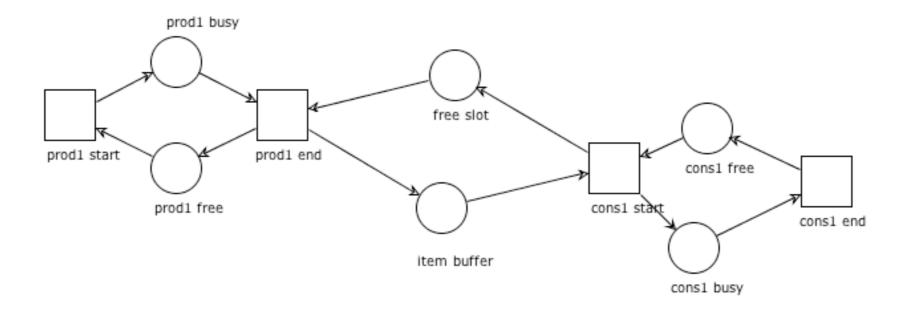
# Examples





not weakly connected not strongly connected

# Examples



weakly connected strongly connected

### Question time



not weakly connected strongly connected

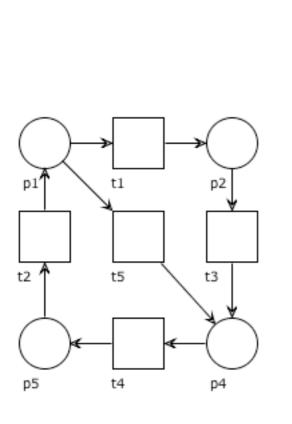
### A note

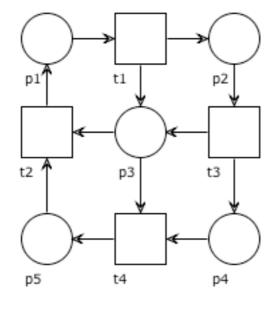
In the following we will consider (implicitly) weakly connected nets only

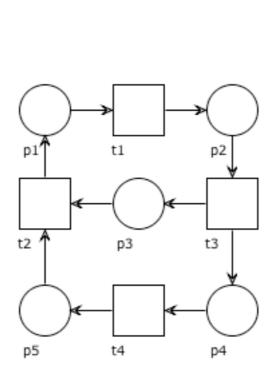
(if they are not, then we can study each of their subsystems separately)

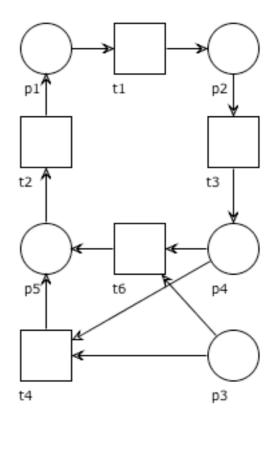
# Question time

Is the net strongly connected?



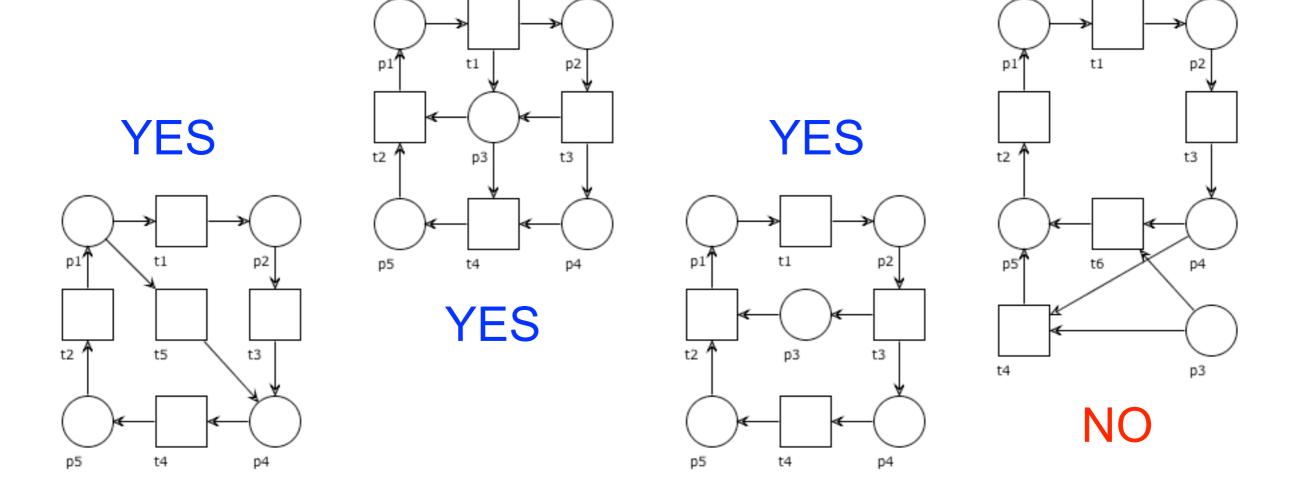






# Question time

Is the net strongly connected?



# Strong connectedness theorem

Theorem: If a weakly connected system is live and bounded then it is strongly connected

# Consequences

If a (weakly-connected) net is not strongly connected

then

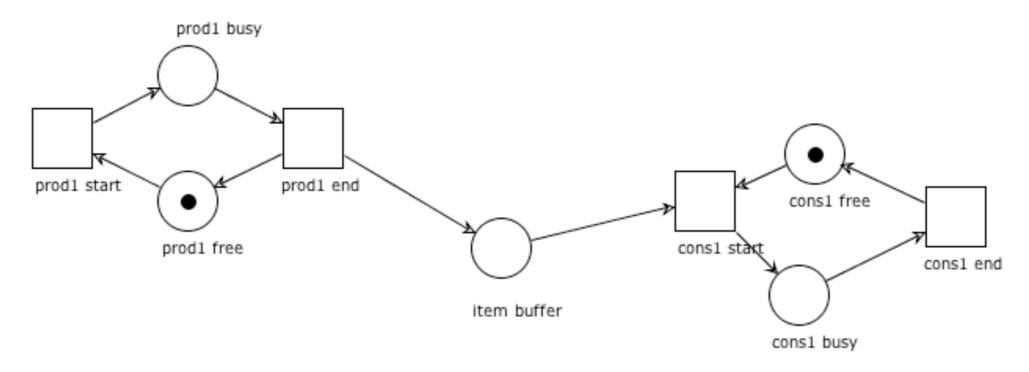
It is not "live and bounded"

If it is live, it is not bounded

If it is bounded, it is not live

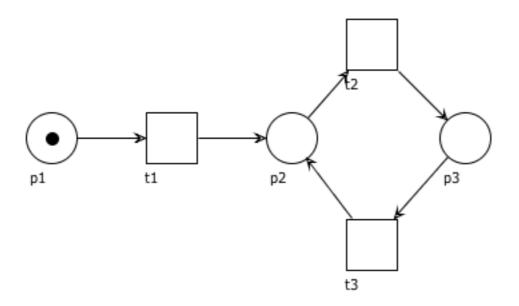
# Example

It is now immediate to see that this system (weakly connected, not strongly connected) cannot be live and bounded (it is live but not bounded)



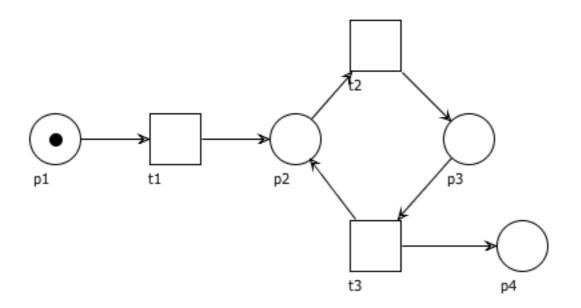
### Example

It is now immediate to see that this system (weakly connected, not strongly connected) cannot be live and bounded (it is bounded but not live)



## Example

It is now immediate to see that this system (weakly connected, not strongly connected) cannot be live and bounded (it is neither bounded nor live)



## Strong connectedness of N\*

#### **Proposition**:

 $N^{*}$  is strongly connected.

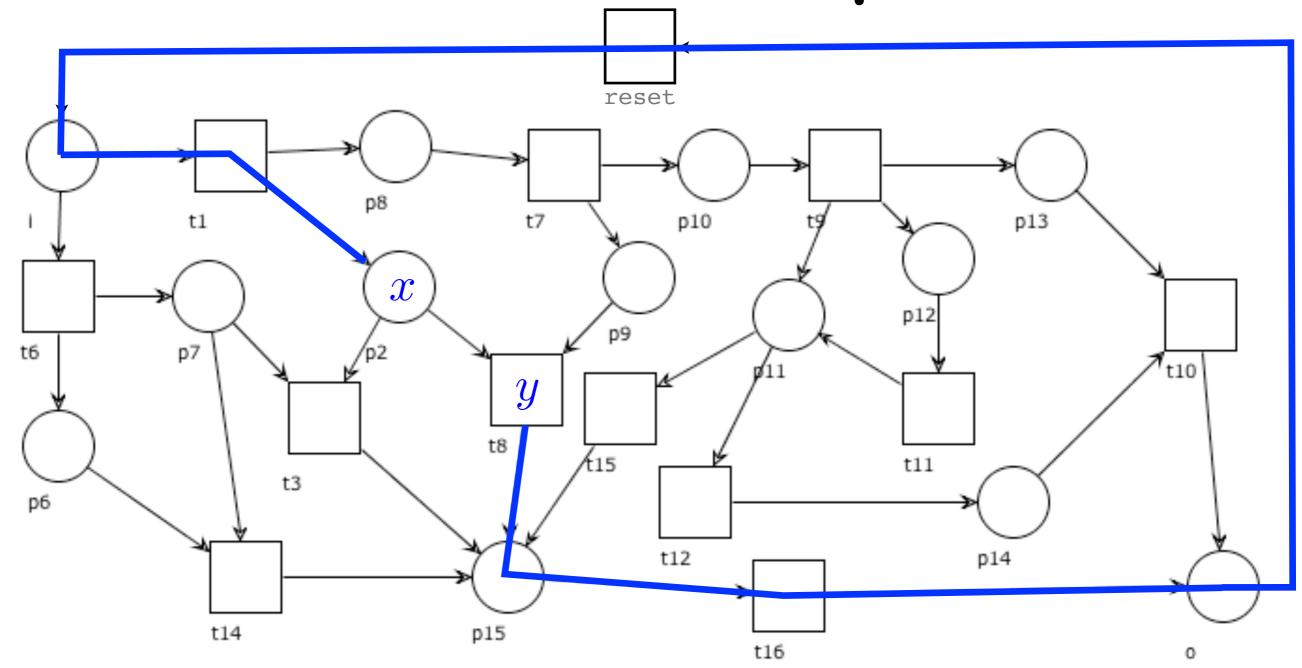
Take two nodes of  $(x,y) \in F_{N^*}$ , we want to build a path from y to x

If  $x,y \neq reset$ , then y lies on a path  $i \to^* y \to^* o$ , because N is a workflow net, x lies on a path  $i \to^* x \to^* o$ , because N is a workflow net, we combine the paths  $y \to^* o \to reset \to i \to^* x$ 

If x=o,y=reset, then we take any path  $i\to^* o$ , we build the path  $reset\to i\to^* o$ 

If x=reset, y=i, then take any path  $i\to^* o$ , we build the path  $i\to^* o\to reset$ 

# Strong connectedness of N\*: example

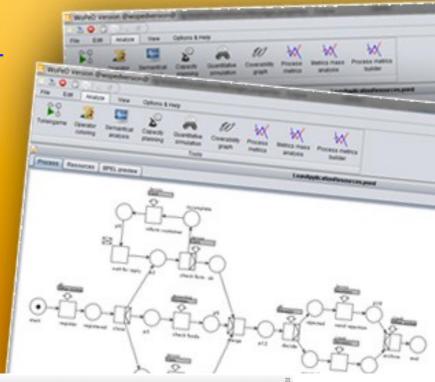


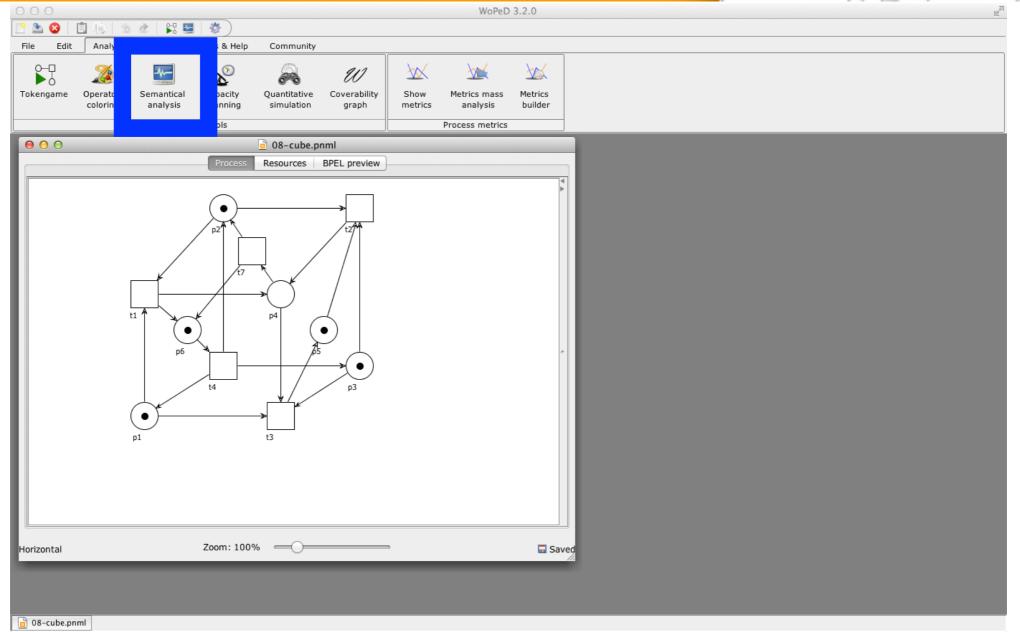
http://woped.dhbw-karlsruhe.de/woped/

### WoPeD

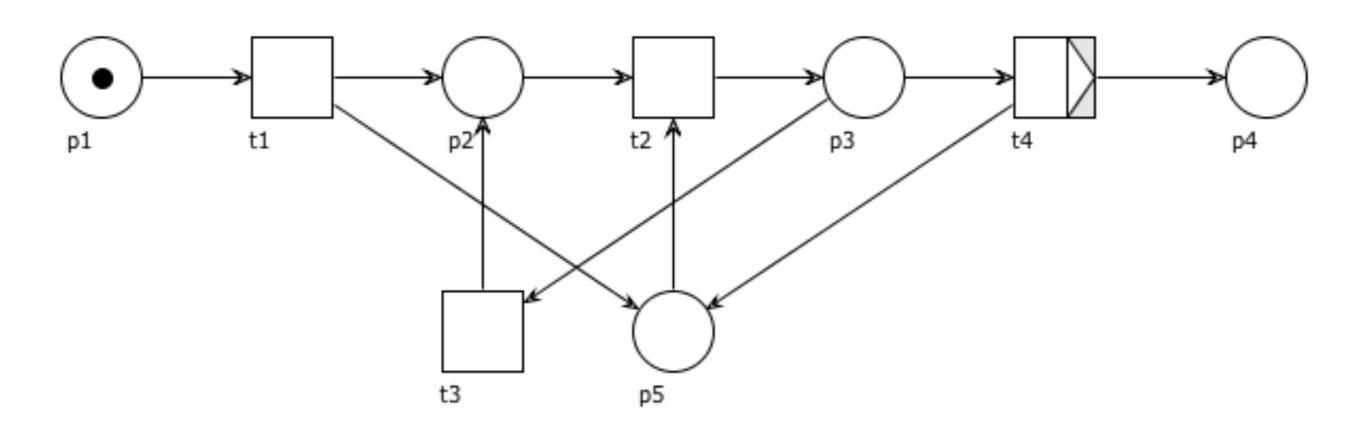
#### Workflow Petri Net Designer

Download WoPeD at sourceforge!

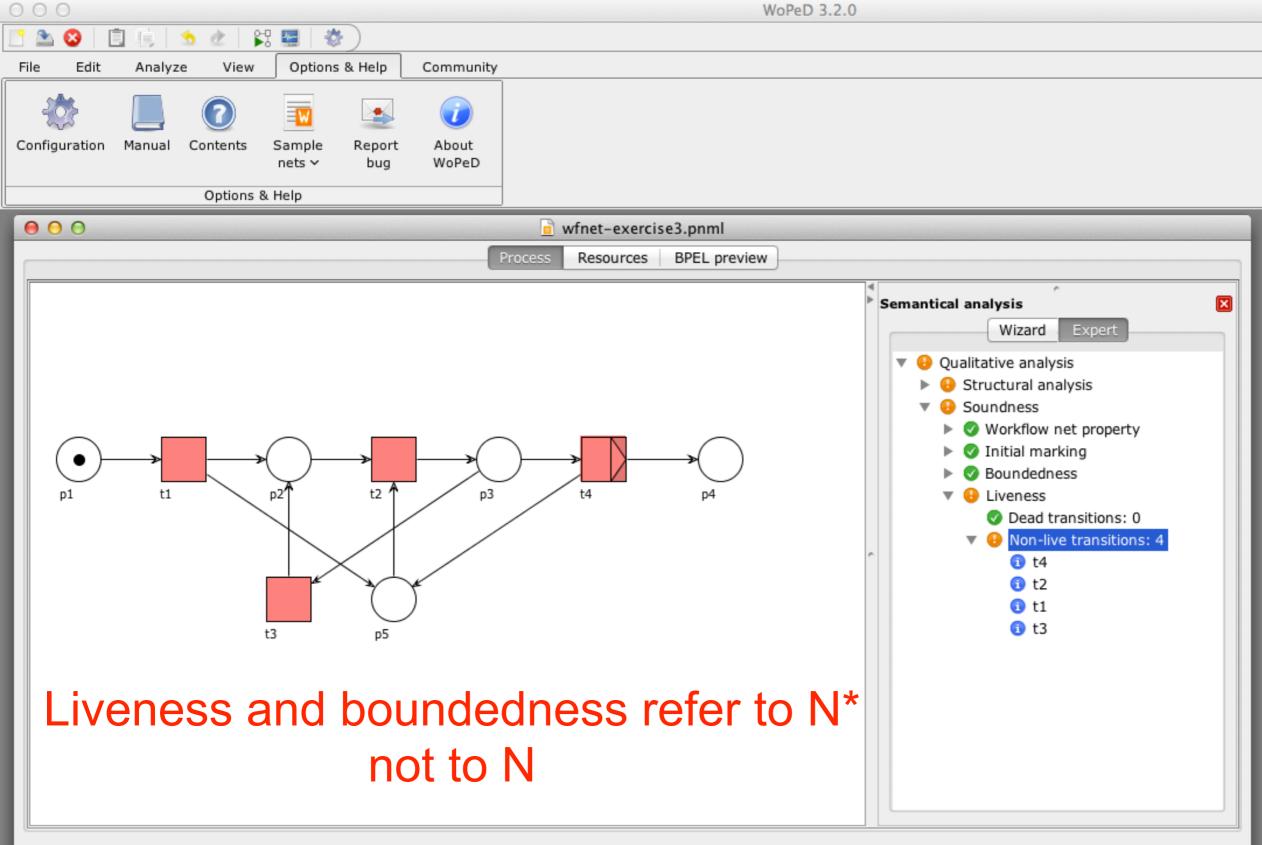




Use some tools to check if the net below is a sound workflow net or not



■ Not saved

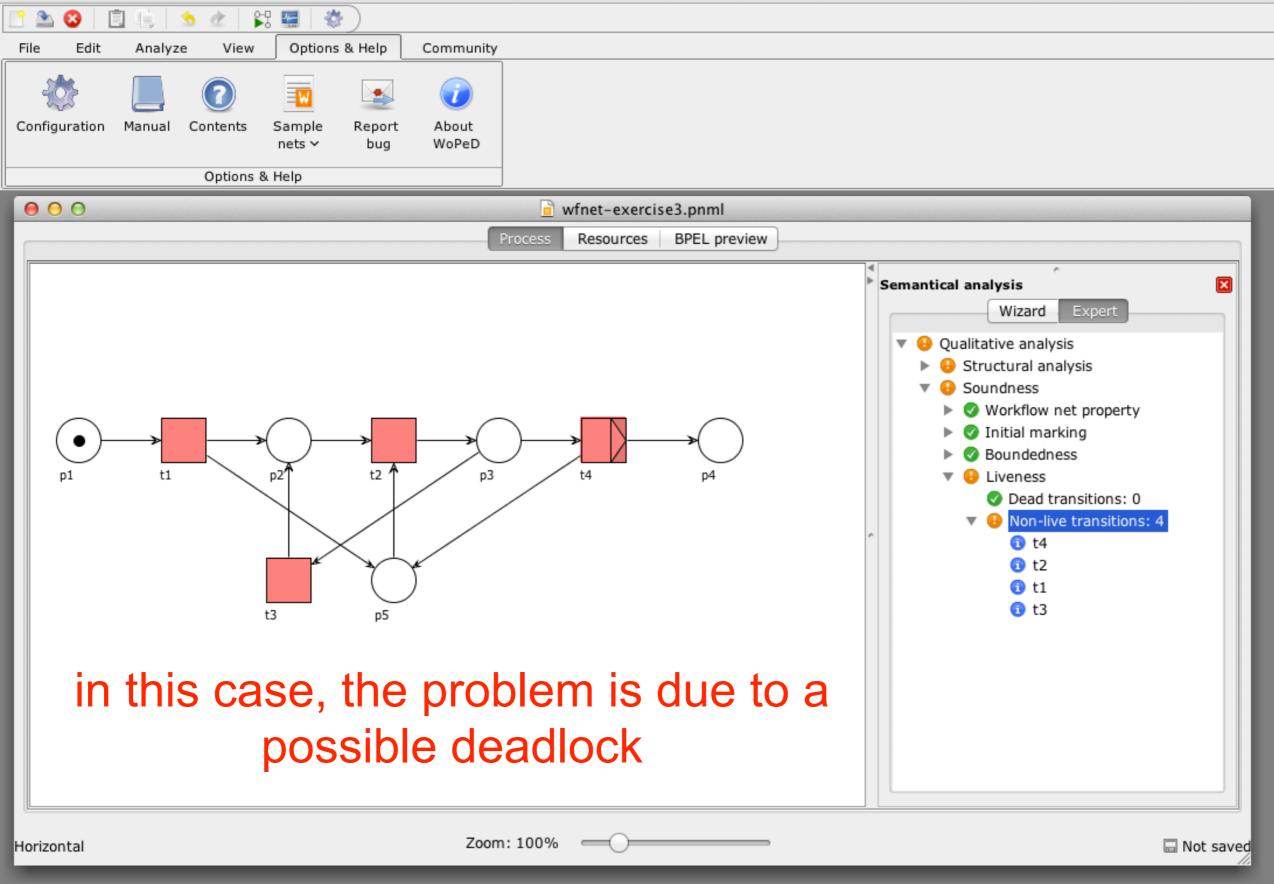


Zoom: 100%

wfnet-exercise3.pnml

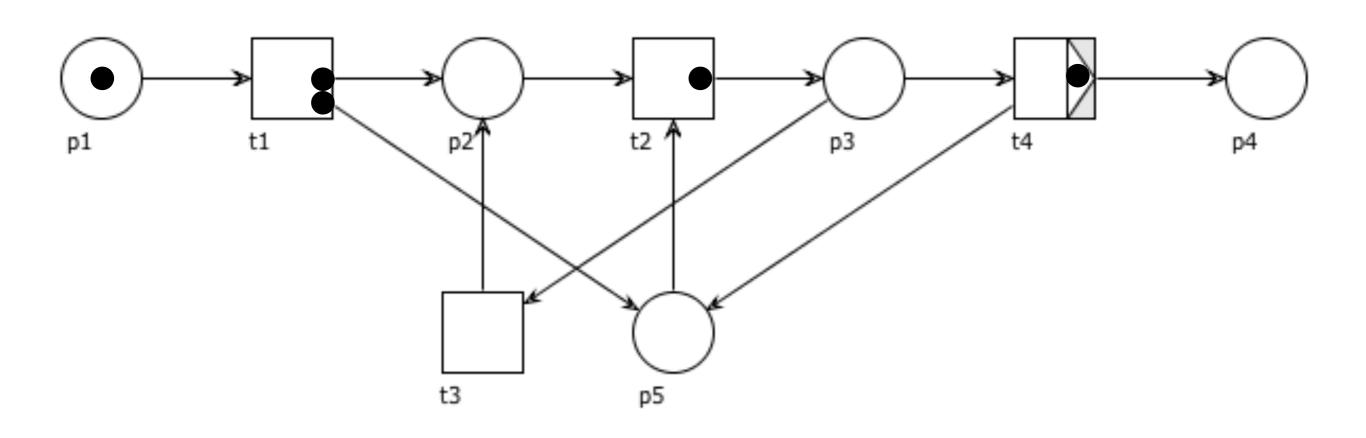
Horizontal



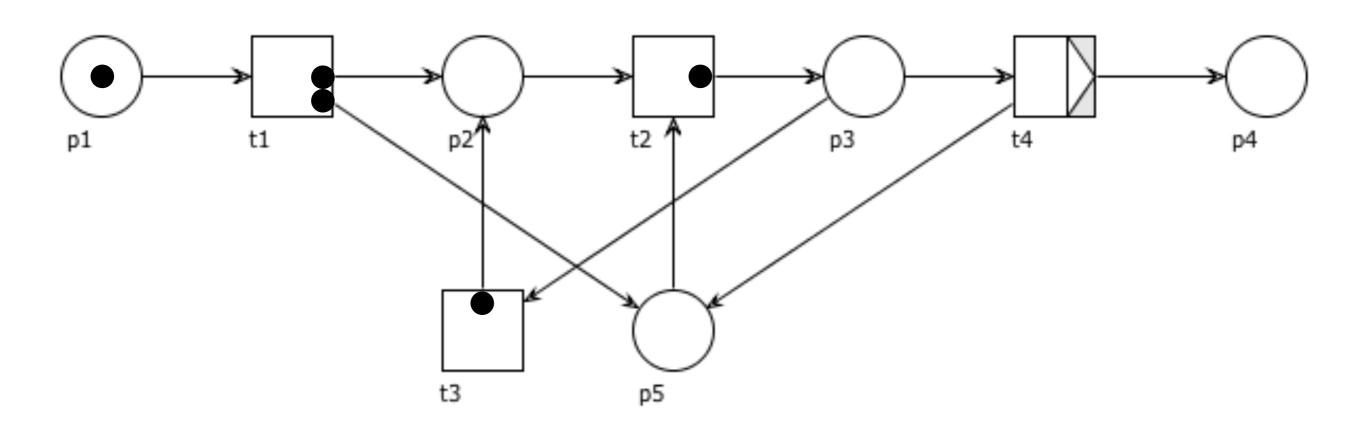


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Use some tools to check if the net below is a sound workflow net or not

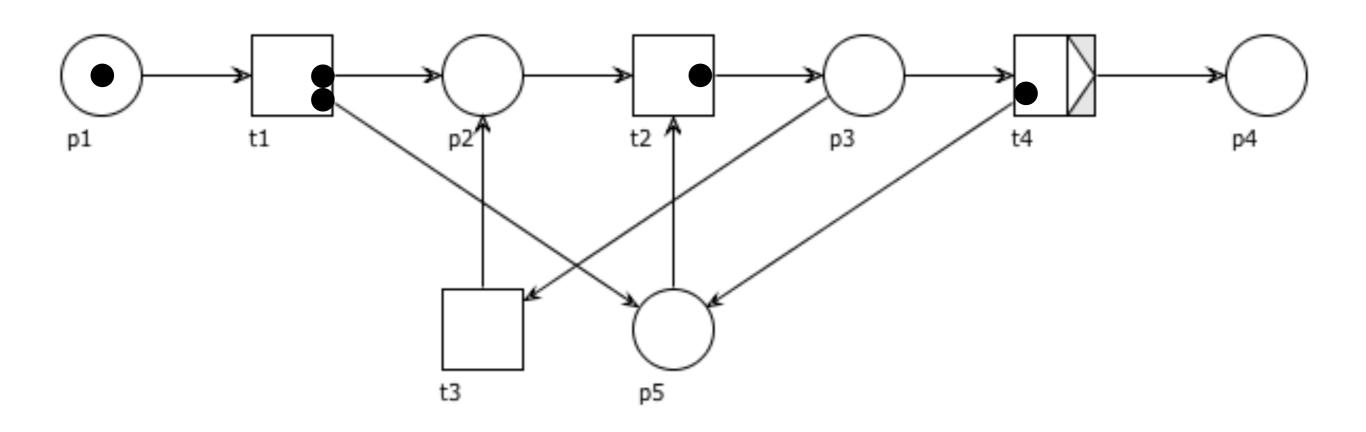


Use some tools to check if the net below is a sound workflow net or not



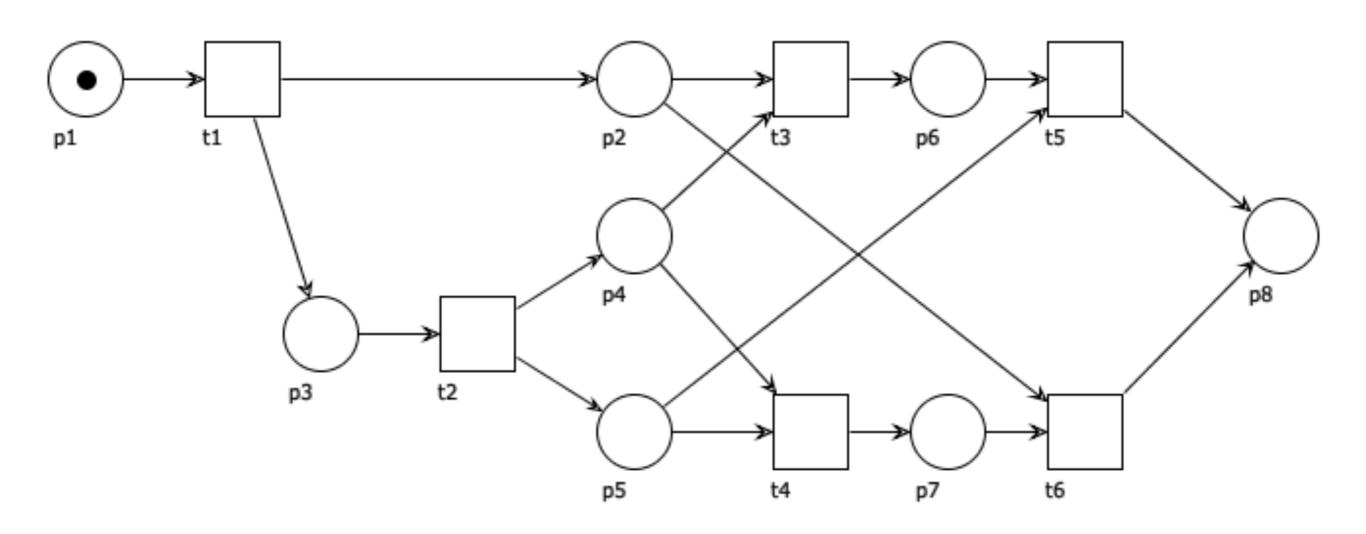
Deadlock

Use some tools to check if the net below is a sound workflow net or not



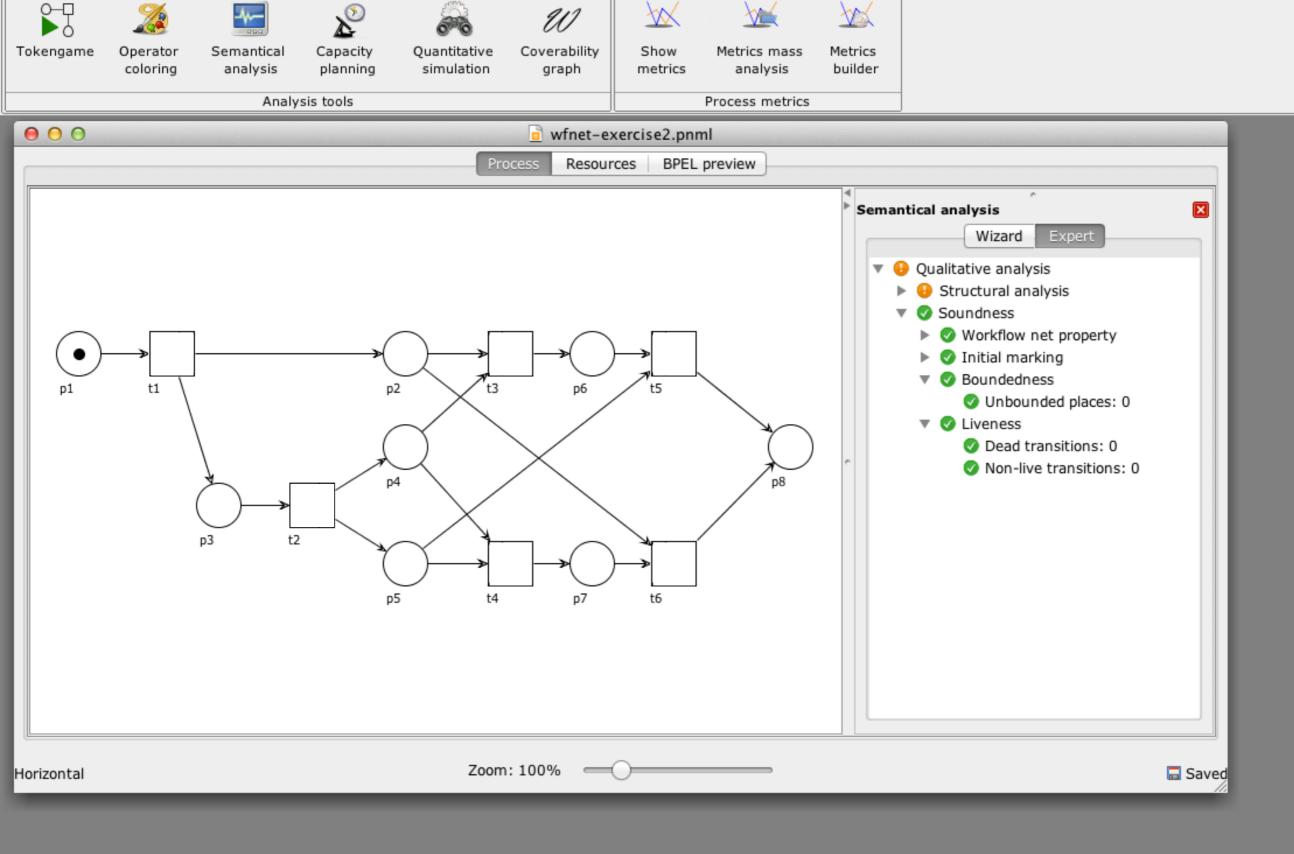
Deadlock

Use some tools to check if the net below is a sound workflow net or not



WoPeD 3.2.0

Community



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File

Edit

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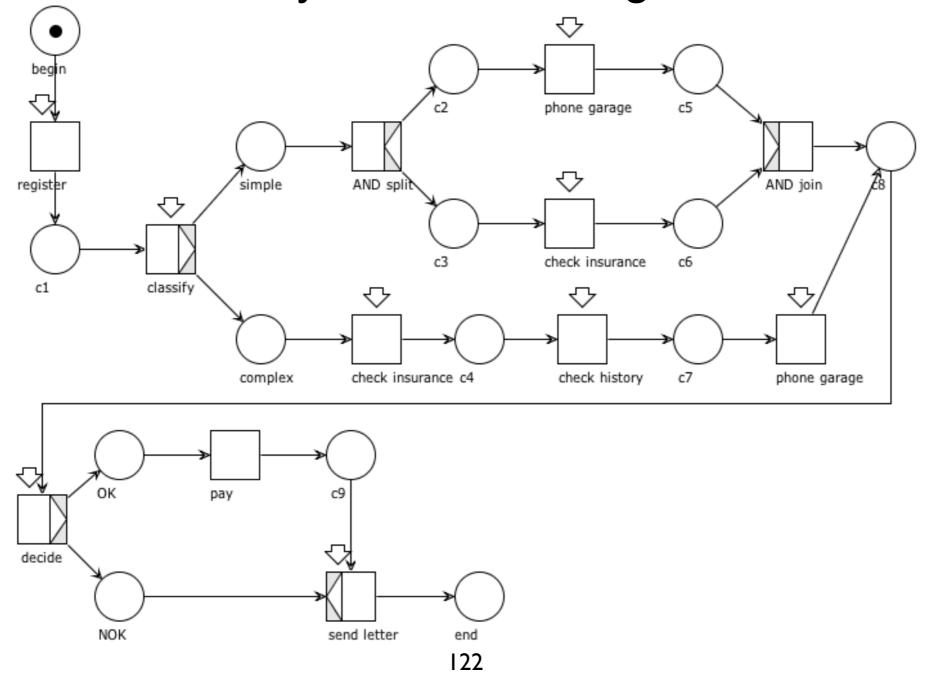
Analyze

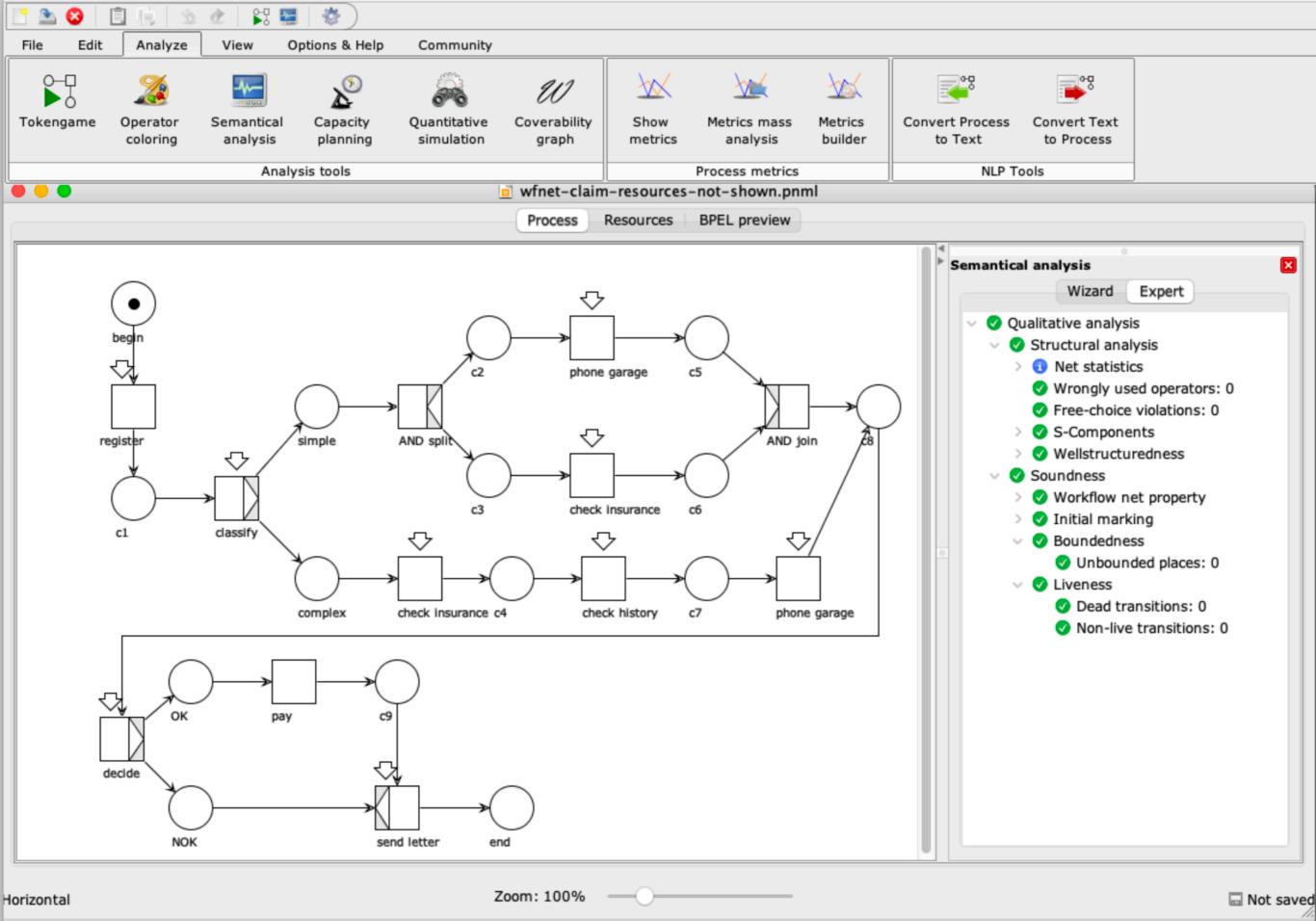
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View

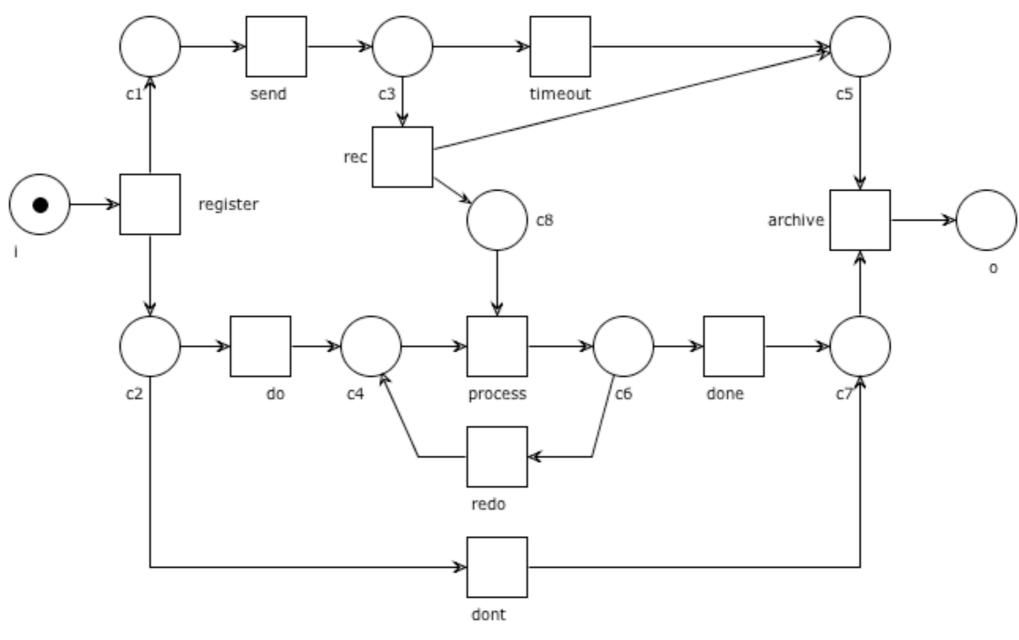
Options & Help

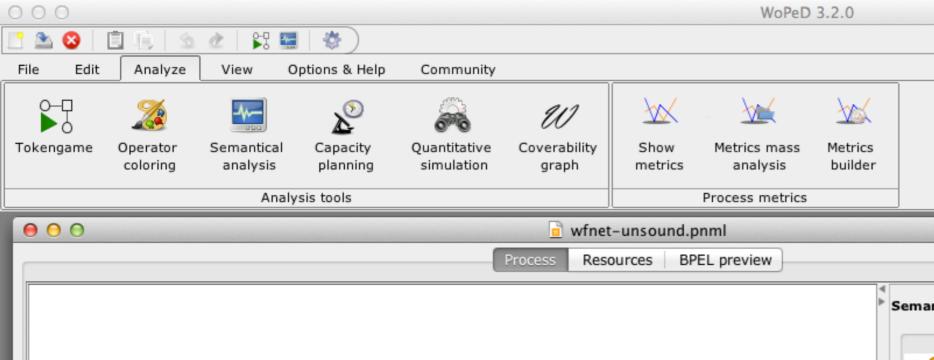
#### Analyse the following net

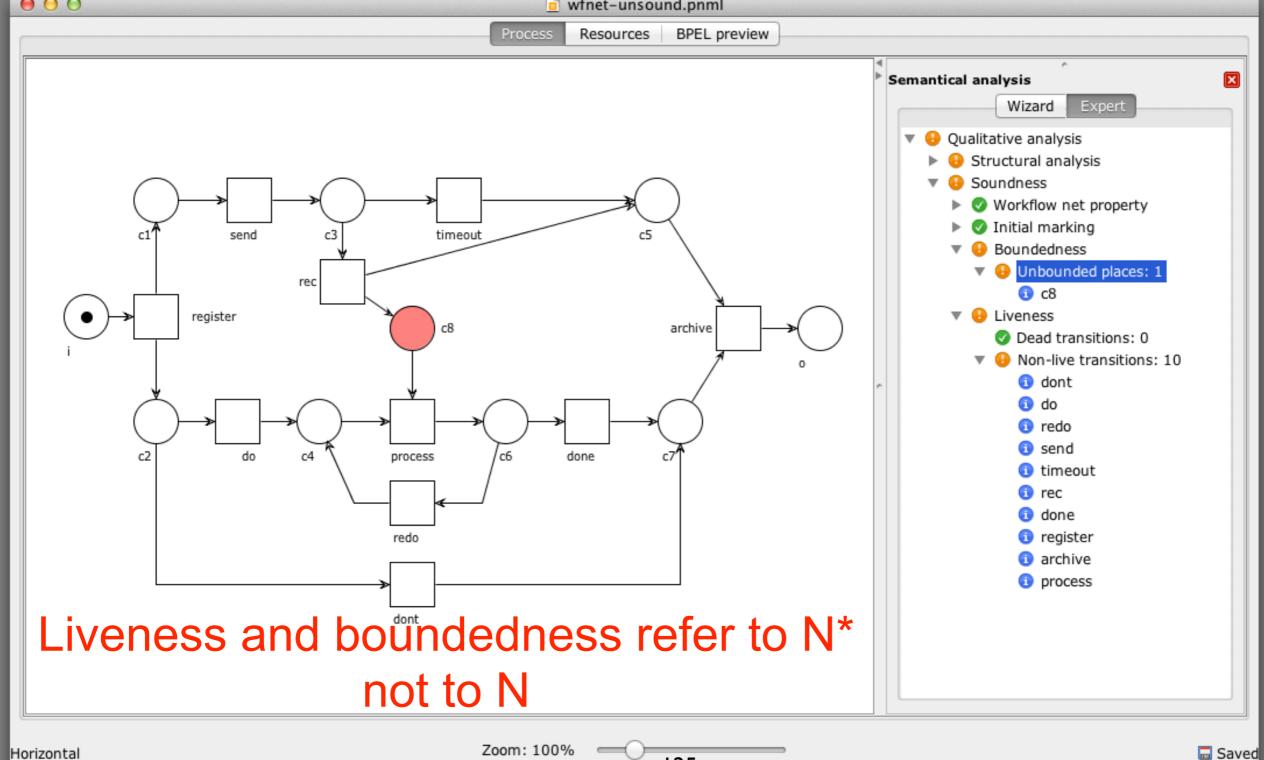




#### Analyse the following net

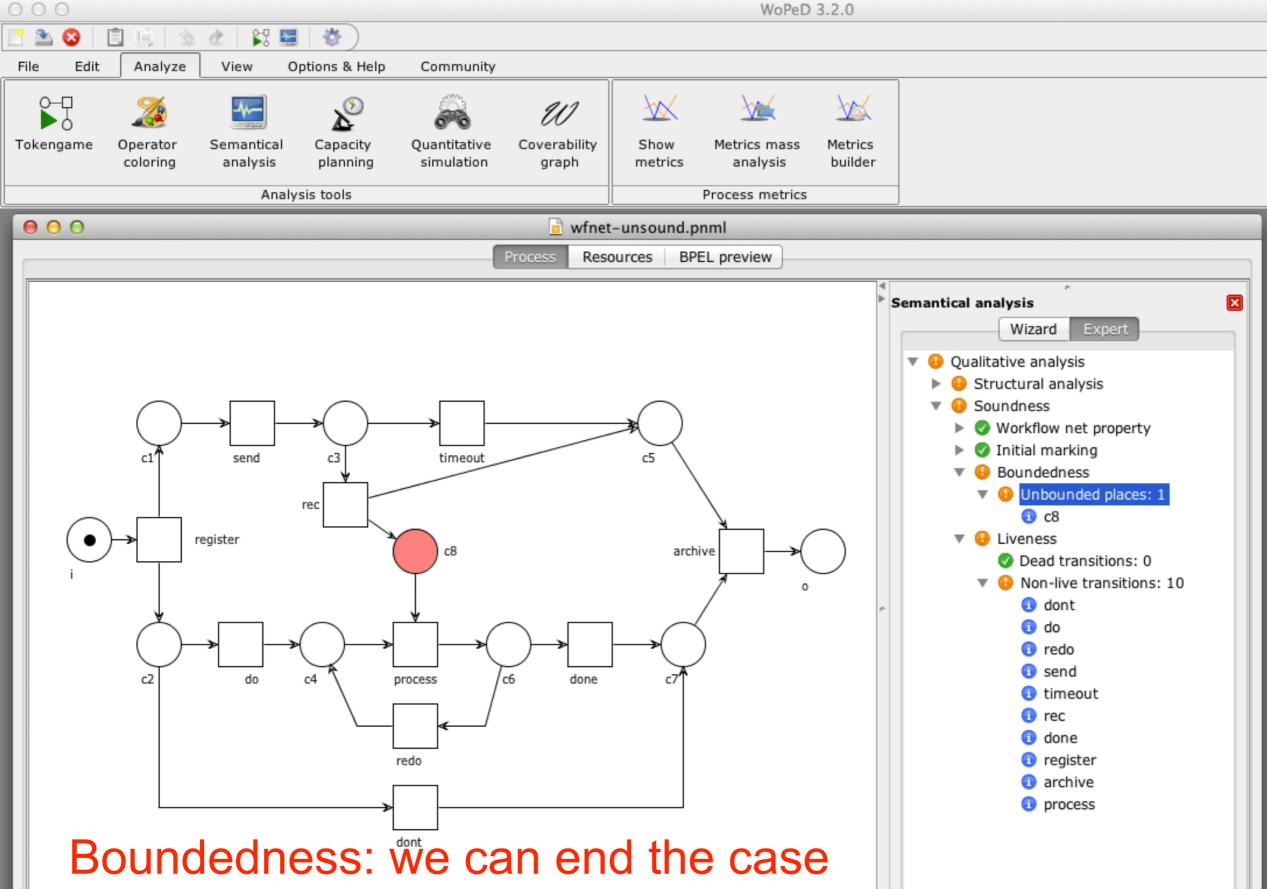






Horizontal

mfnet-unsound.pnml



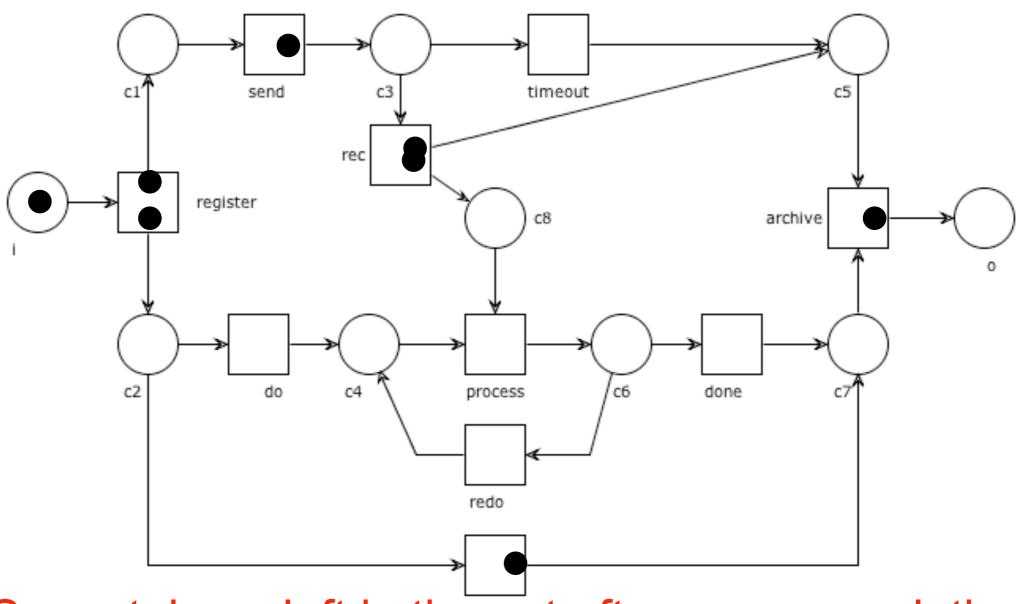
126

leaving a token in c8

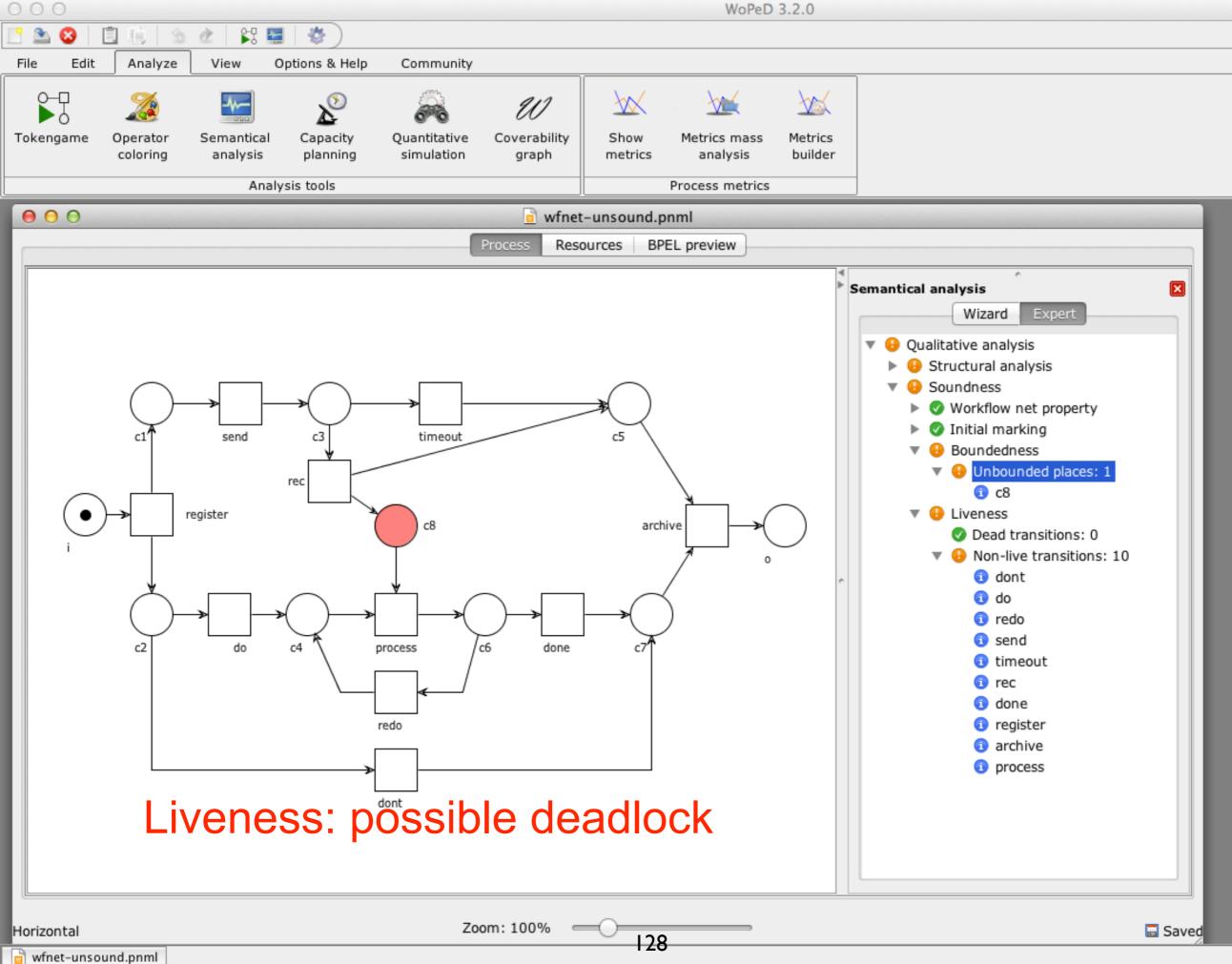
Zoom: 100%

Horizontal

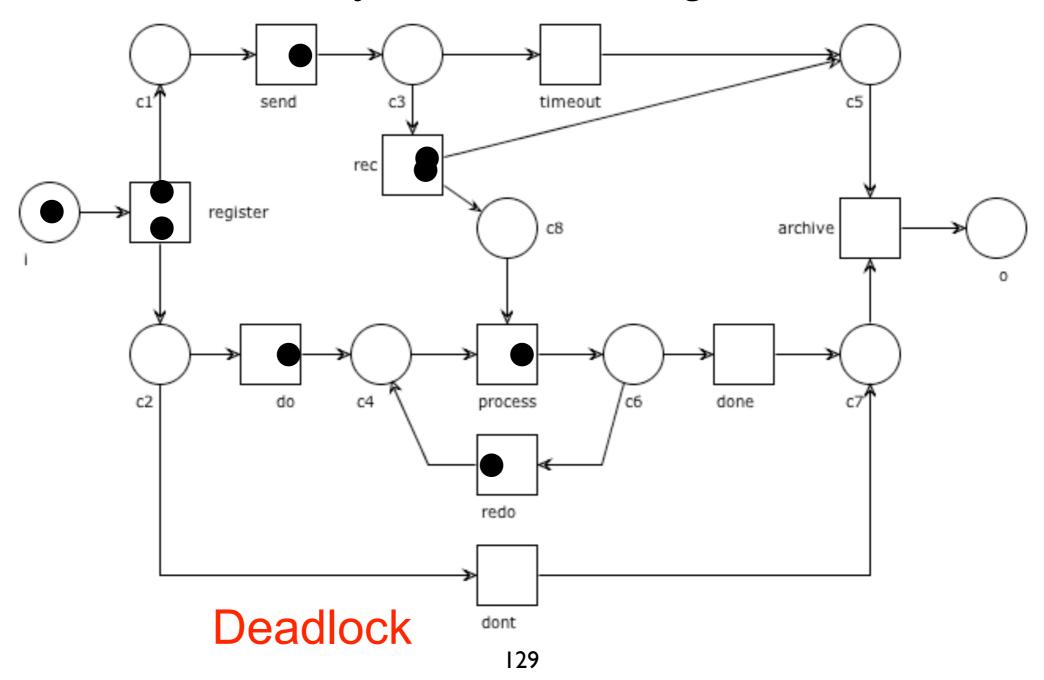
#### Analyse the following net



Some tokens left in the net after case completion



#### Analyse the following net



The workflow of a computer repair service (CRS) can be described as follows.

A customer brings in a defective computer and the CRS checks the defect and hands out a repair cost calculation back.

If the customer decides that the costs are acceptable, the process continues, otherwise she takes her computer home unrepaired.

The ongoing repair consists of two activities, which are executed sequentially but in an arbitrary order.

One activity is to check and repair the hardware,

whereas the other activity is to check and configure the software.

After both activities are completed, the proper system functionality is tested.

If an error is detected the repair procedure is repeated,

otherwise the repair is finished and the computer is returned.

Model the described workflow as a sound workflow net.

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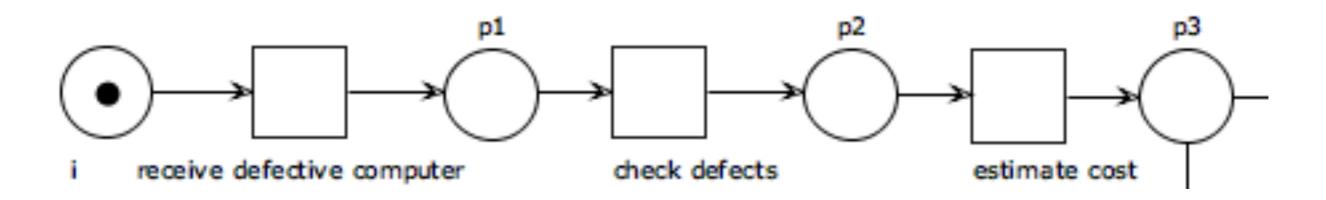
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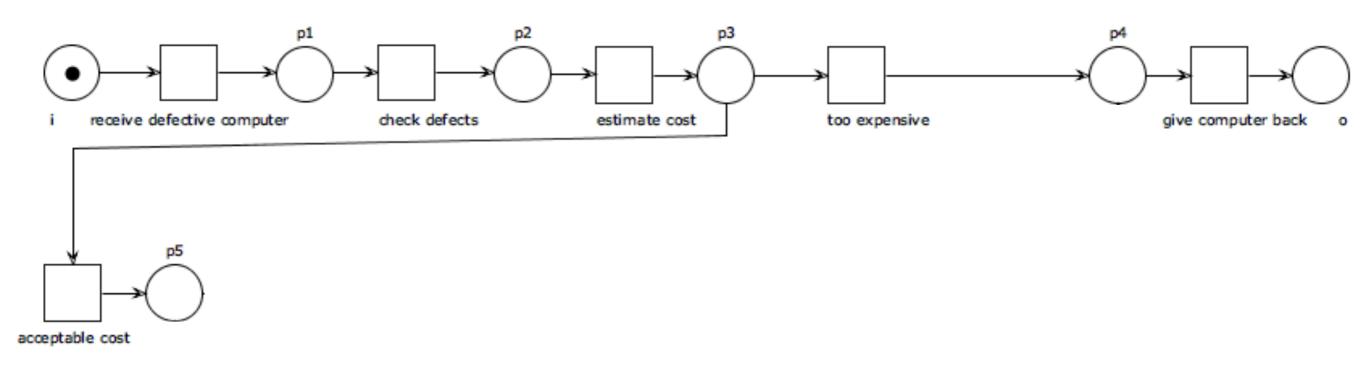
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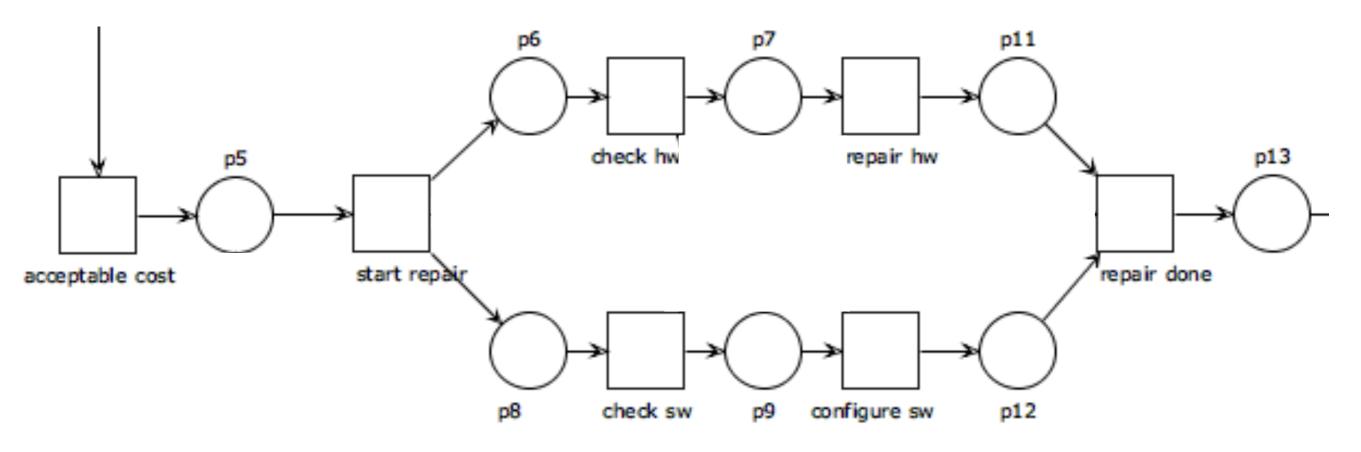


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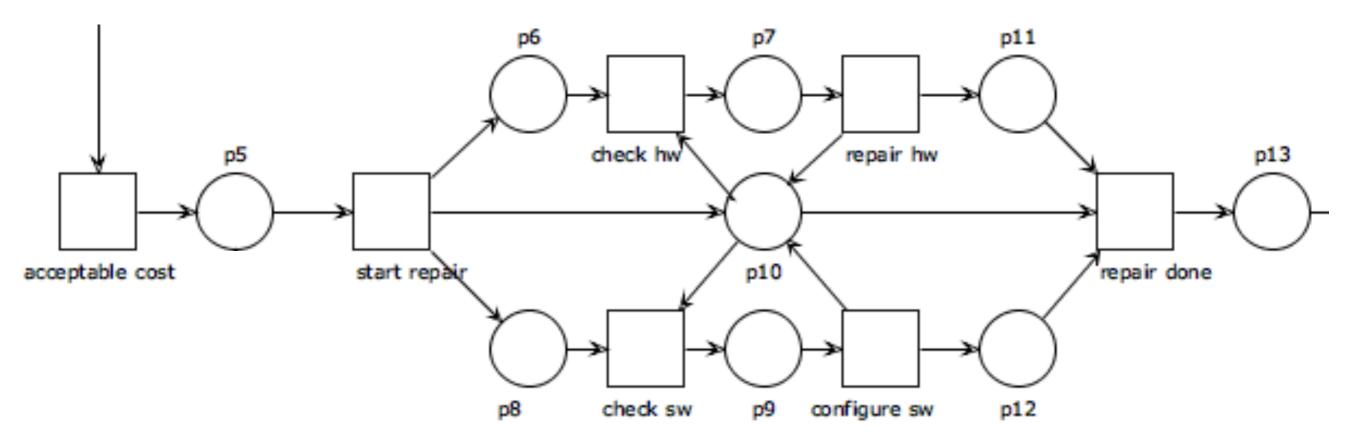
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One activity is to **check** and **repair** the **hardware**, whereas the other activity is to **check** and **configure** the **software**.

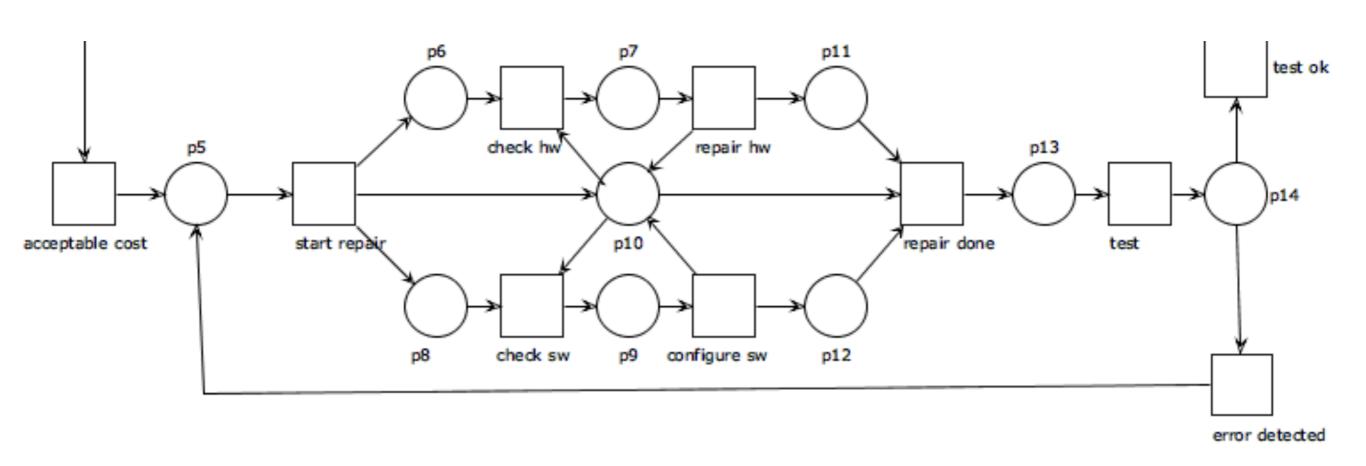


The ongoing repair consists of two activities, which are executed **sequentially but in an arbitrary order**.

One activity is to **check** and **repair** the **hardware**, whereas the other activity is to **check** and **configure** the **software**.



After both activities are completed, the proper system functionality is tested. If an error is detected the repair procedure is repeated,



otherwise the repair is finished and the computer is returned.

