

Principles for software composition 2024/25

02 - Structural induction and rule induction

[Ex. 1] Complete the proof of termination of boolean expressions by structural induction.

[Ex. 2] Extend the syntax of arithmetic expressions with the operator $a_0 \sqcap a_1$ whose big-step operational semantics is given by the rules:

$$\frac{\langle a_0, \sigma \rangle \longrightarrow n \quad \langle a_1, \sigma \rangle \longrightarrow n}{\langle a_0 \sqcap a_1, \sigma \rangle \longrightarrow n}$$

1. Prove termination or exhibit a counterexample.
2. Prove determinacy or exhibit a counterexample.

[Ex. 3] Extend the syntax of arithmetic expressions with the operator $a_0 \sqcup a_1$ whose big-step operational semantics is given by the rule:

$$\frac{\langle a_0, \sigma \rangle \longrightarrow n_0}{\langle a_0 \sqcup a_1, \sigma \rangle \longrightarrow n_0} \quad \frac{\langle a_1, \sigma \rangle \longrightarrow n_1}{\langle a_0 \sqcup a_1, \sigma \rangle \longrightarrow n_1}$$

1. Prove termination or exhibit a counterexample.
2. Prove determinacy or exhibit a counterexample.

[Ex. 4] Consider the command

$$w \stackrel{\text{def}}{=} \mathbf{while} \ x > y \ \mathbf{do} \ (x := x + 1 ; y := y - 1)$$

Find out the set S of memories σ such that $\langle w, \sigma \rangle \not\rightarrow$ and prove that this is the case by using the inference rule for divergence.

[Ex. 5] Prove determinacy of boolean expressions by rule induction.

[Ex. 6] Let b be a boolean expression and c a command. Consider the command

$$w \stackrel{\text{def}}{=} \mathbf{while} \ b \ \mathbf{do} \ c$$

Prove by rule induction that:

$$\forall \sigma, \sigma'. \langle w, \sigma \rangle \longrightarrow \sigma' \Rightarrow \langle b, \sigma' \rangle \longrightarrow \text{false}$$