Models of computation (MOD) 2014/15
Exam – July 22, 2015

[Ex. 1] Add to IMP the arithmetic expression

$$\textbf{eval}\ a\ \textbf{after}\ c$$

that returns the value of a in the store obtained by the execution of c (if it terminates).

1. Define the operational semantics for the new expression.
2. Show a simple counterexample to the property
$$\forall a \in Aexp.\ \forall \sigma \in \Sigma.\ \exists n \in \mathbb{N}.\ \langle a, \sigma \rangle \to n$$
3. Redesign the denotational semantics of IMP expressions and commands to take into account the fact that the evaluation of expressions may not terminate: illustrate all needed changes.

[Ex. 2] Let $\mathcal{D} = (D, \sqsubseteq_D)$ be a CPO$_\perp$. Let us consider the function $\mathsf{min} : \mathcal{D} \times \mathcal{D} \to \mathcal{D}$ defined by letting

$$\mathsf{min}(x, y) = \begin{cases} x & \text{if } x \sqsubseteq_D y \\ y & \text{if } y \sqsubseteq_D x \\ \perp_D & \text{if } x \not\sqsubseteq_D y \text{ and } y \not\sqsubseteq_D x \end{cases}$$

1. Exhibit a CPO$_\perp$ $\mathcal{D}$ such that min is not monotone.[1]
2. Prove that if $\sqsubseteq_D$ is a total order then min is monotone.
3. Prove that if $\sqsubseteq_D$ is a total order then min is continuous.

[Ex. 3] Let us consider the HOFL term

$$t \stackrel{\text{def}}{=} \textbf{rec}\ f.\ \lambda x.\ \textbf{if}\ \textbf{fst}(x) - \textbf{snd}(x)\ \textbf{then}\ \textbf{snd}(x)\ \textbf{else}\ f((\textbf{fst}(x) - \textbf{snd}(x), \textbf{snd}(x)))$$

1. Find the principal type of t.
2. Compute the (lazy) canonical form of $(t\ (6,3))$.

[Ex. 4] Consider the CCS processes

$$\begin{aligned} p &\stackrel{\text{def}}{=} \textbf{rec}\ X.\ (\tau.X\ +\ \alpha.\textbf{nil}) \\ q &\stackrel{\text{def}}{=} \textbf{rec}\ Y.\ \tau.(Y\ +\ \alpha.\textbf{nil}) \\ r &\stackrel{\text{def}}{=} \textbf{rec}\ Z.\ (\tau.(Z\ +\ \alpha.\textbf{nil})\ +\ \alpha.\textbf{nil}) \end{aligned}$$

1. Draw the labelled transition systems for p, q and r.
2. Which of the above processes are strong bisimilar?
3. Which of the above processes are weak bisimilar?

---

[1] Assume the usual order over the product domain $\mathcal{D} \times \mathcal{D}$.