





# Individual mobility laws



# Understanding the laws of individual human mobility

is there a typical traveling distance?

 can we profile individuals according to their mobility behavior?

to what extent are humans predictable?

# **Modelling individual human mobility**

• What determines the decision to start a trip?

What determines the choice of the destination?

 What determines the decision to come back home or to explore new locations?

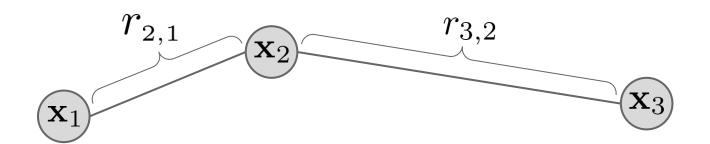
Can we generate realistic individual trajectories?

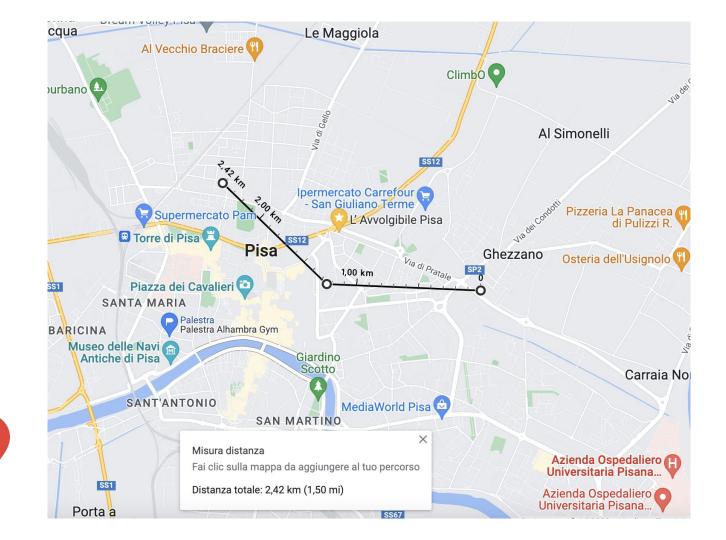
# Distances

#### **Travel distance (jump length)**

Distance between two consecutive locations visited by a moving object

$$r=|\mathbf{x}_2-\mathbf{x}_1|$$





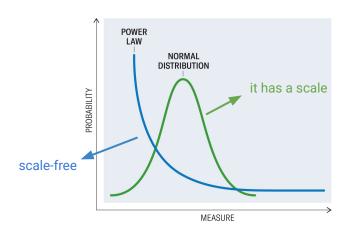
#### **Travel distance probability**

$$P(r)$$
 = probability of finding a trip of length  $\,r\,$ 

#### A Pareto Distribution vs. a Gaussian Curve

A normal distribution (i.e., a Gaussian curve) is bell-shaped, whereas a Pareto distribution (i.e., power law) is shaped like a hockey stick with long tails.

What's the shape of this distribution?









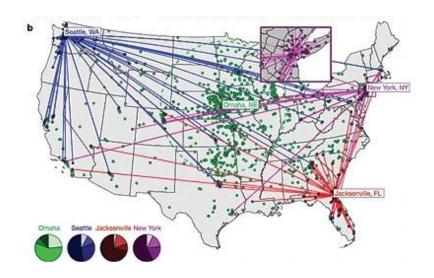
#### Brockmann et al., 2006:

Dollar bills: 464,670

• Records: 1,033,095

• Area: US

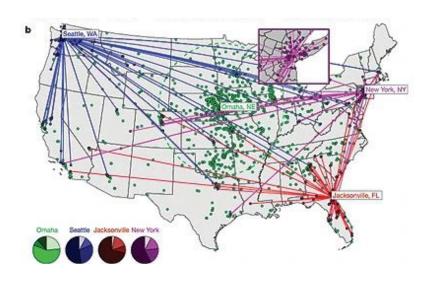
(excluding Alaska and Hawaii)



Trajectories of bank notes originating from four different places with travelling time T < 14 days.

- Most bank notes are reported close the initial entry, r < 10km
  - Seattle 53%, NYC 58%, Jacksonville 71%

- A small but **considerable** fraction is reported at large distances, r>800km
  - Seattle 8%, NYC 7%,
     Jacksonville 3%

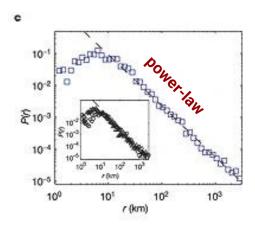


Trajectories of bank notes originating from four different places with travelling time T < 14 days.

 Probability of traversing a distance in 1-4 days (20,540 bills)

$$P(r) \sim r^{-(1+\beta)}$$

$$\beta = 0.59 \pm 0.02$$



Measured P(r) of traversing a distance in less than T = 4 days. The inset shows P(r) for metropolitan areas, cities of intermediate size, small towns.



#### **Mobile Phone Records**

#### González et al., 2008:

Dataset D1 (CDRs):

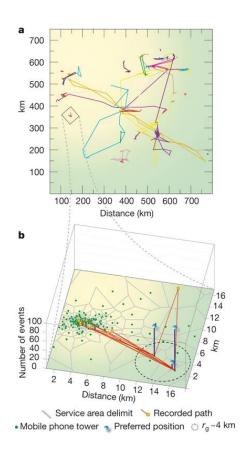
Users: 100,000

Records: 16,264,308

Dataset D2 (CPRs):

o Users: 206

Records: 10,407



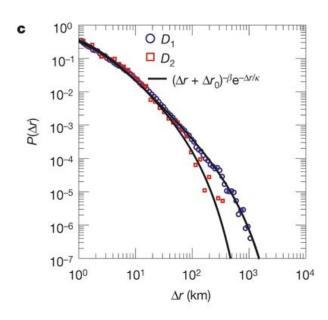
Week-long trajectory of 40 mobile phone users

Detailed trajectory of a single user

- 186 two-hourly reports
- 12 locations.

The circle represents the radius of gyration centred in the user's centre of mass.

#### **Mobile Phone Records**



$$P(r) = (r + r_0)^{-\beta} \exp(-r/\kappa)$$

$$\beta = 1.75 \pm 0.15$$

$$r_0 = 1.5km$$

$$\kappa_{D_1} = 400km$$

$$\kappa_{D_2} = 80km$$

### **Radius of gyration**

Characteristic distance of an individual

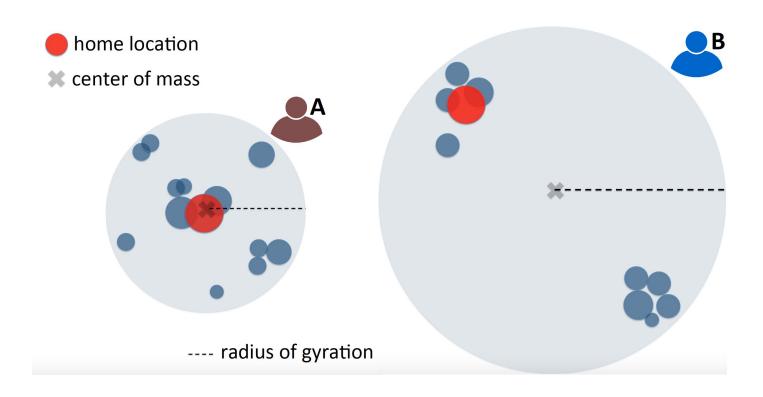
$$r_g(u) = \sqrt{\frac{1}{n_u} \sum_{i=1}^{n_u} (\mathbf{r}_i - \mathbf{r}_{cm})^2}$$

Center of mass

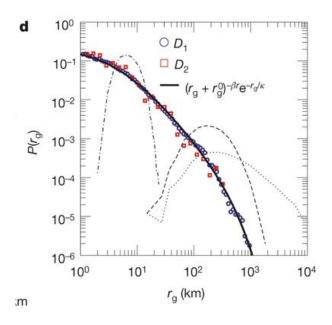
$$\mathbf{r}_{cm} = \frac{1}{n_u} \sum_{i=1}^{n} \mathbf{r}_i$$

 $n_u$  number of records  $\mathbf{r}_i$  position

# **Radius of gyration**



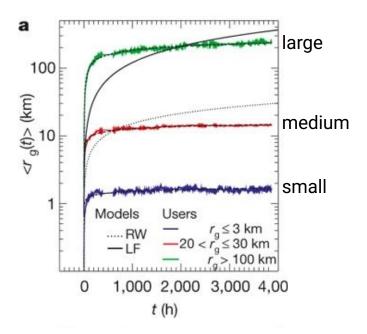
# **Radius of gyration**



$$P(r_g) = (r_g + r_g^0)^{-\beta_r} \exp(-r_g/\kappa)$$
$$r_g^0 = 5.8km$$
$$\beta_r = 1.65 \pm 0.15$$
$$\kappa = 350km$$

Measured P(r<sub>g</sub>) on datasets D1 and D2. The dotted, dashed and dot-dashed curves show P(rg) obtained from null models

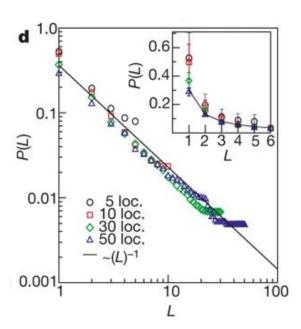
### **Radius of gyration - time evolution**



Radius of gyration versus time for users of three groups. The black curves correspond to the analytical predictions for the random walk models. The dashed curves corresponding to a logarithmic fit.

- Radius increases logarithmically with time
  - Indicating a saturation process

# **Location frequency**



Frequency of visiting locations for users observed to visit 5, 10, 30 and 50 locations. L is the rank of the location listed in the order of the visit frequency. 40% of the time individuals are found at their first two preferred locations

- Rank each location based on how many times an individual is recorded there
  - E.g., L=3 is the third-most-visited location for an individual

$$P(L) \sim 1/L$$

People devote most of their time to a few locations, spending their time to places with diminished regularity

### k-radius of gyration

Recurrent characteristic distance of an individual

$$r_g^{(k)} = \sqrt{\frac{1}{N_k} \sum_{i=1}^k w_i (\mathbf{r}_i - \mathbf{r}_{cm}^{(k)})^2}$$

k-center of mass

$$\mathbf{r}_{cm}^{(k)} = \frac{1}{N_k} \sum_{i=1}^{k} w_i \mathbf{r}$$

 $N_k$  number of records in location  $_k$ 

#### **Mobile Phone Records**

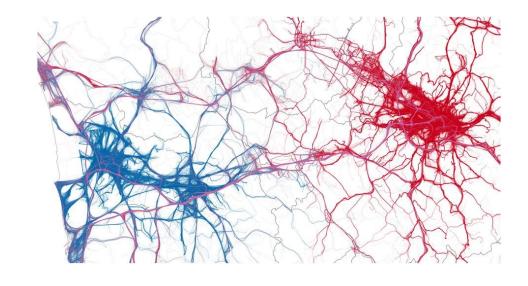
#### Pappalardo et al., 2015:

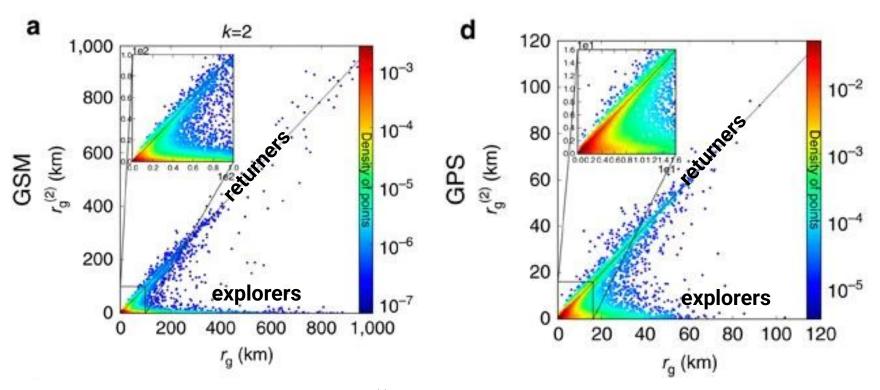
• CDRs:

o Users: 67,000

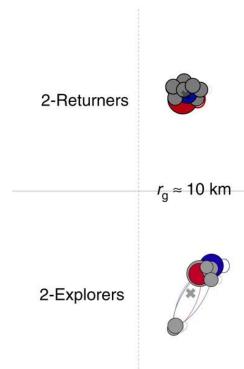
• GPS traces:

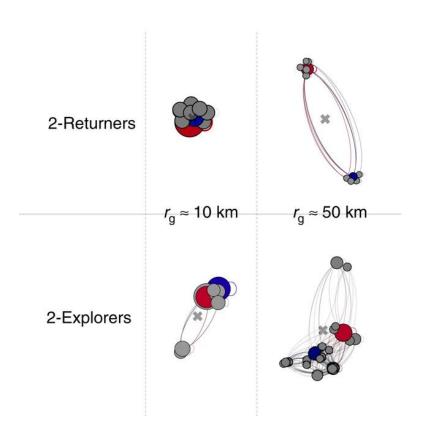
o Users: 46,000

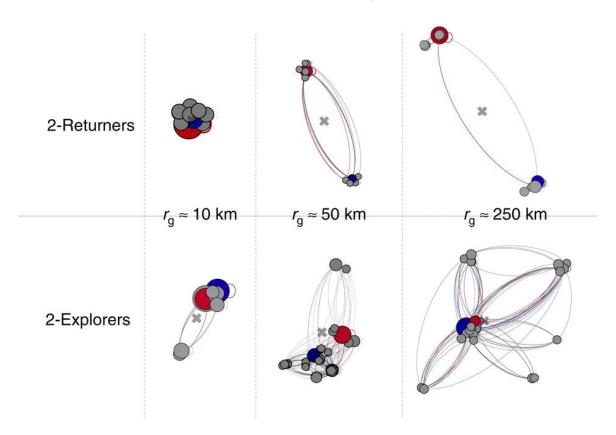




Correlation between total rg and  $rg^{(k)}$  for k=2, 4, 8 for CDRs and GPS traces. Each point is coloured from blue to red, indicating the density of points in the corresponding region.

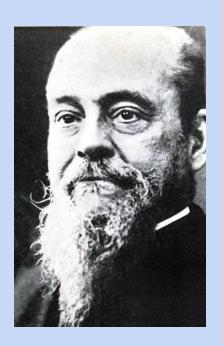






#### **INTERVALLO**

# Vilfredo Pareto and the 80/20 rule



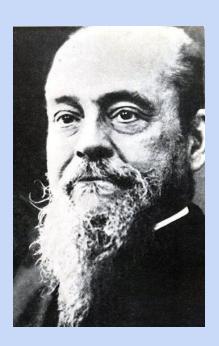
He noticed that in Italy a few wealthy individuals earned most of the money, while the majority of the population earned rather small amounts.

He connected this disparity to the observation that **incomes follow a power law**, representing the first known report of a power-law distribution.

**The 80/20 rule**: Roughly 80 percent of money is earned by only 20 percent of the population.

#### **INTERVALLO**

# Vilfredo Pareto and the 80/20 rule



The 80/20 rule emerges in many areas:

- 80% of profits are produced by 20% of the employees
- 80% of decisions are made during 20% of meeting time
- 80% of links on the Web point to only 15% of webpages
- 80% of citations go to only 38% of scientist
- 80% of links in Hollywood connected to 30% of actors

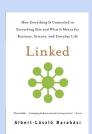
#### • The 1% phenomena:

- o In the US, 1% of the population earns 15% of the total income
- signature of income disparity, it is a consequence of the power-law nature of the income distribution

#### References

- [article] We Need to Let Go of the Bell Curve, Harvard Business Review, 2022
- [article] Visualizing power-law distributions, Capital as Power, 2019

[book] Linked: the New Science of Networks, A.-L. Barabasi



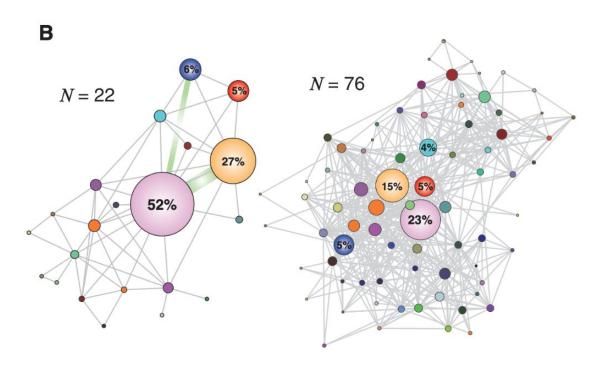
[book] Chi troppo chi niente, E. Ferragina



# Predictability

#### **Individual Mobility Network**

A network where nodes are an individual's visited locations and edges movements between locations



#### The role of randomness

1. What is the role of randomness in human mobility?

2. To what degree are our movements predictable?

# **Entropy**

Random entropy

$$S^{rand} = \log_2 / N$$

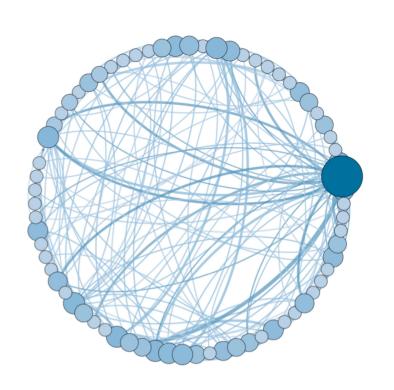
Uncorrelated entropy

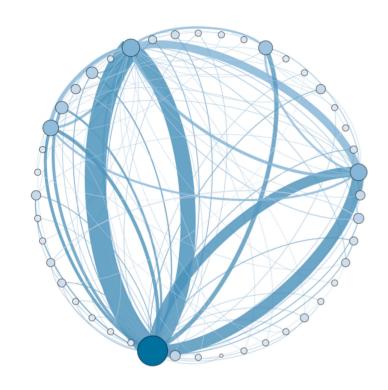
$$S^{unc} = -\sum_{i=1}^{n} p_i \log_2 p_i$$

Real entropy

$$S = -\sum_{T_i' \subset T_i} p_{T_i'} \log_2 p_{T_i'}$$

# Who's the most predictable?



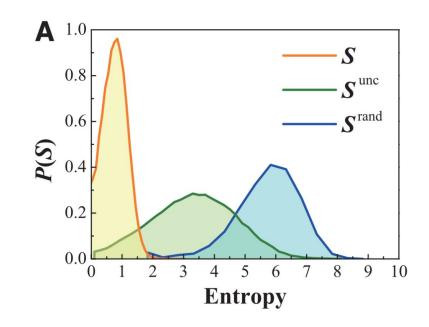


#### **Entropy**

#### Song et al., 2010:

- 50,000 users (CDRs)
- S peaks at 0.8

$$2^{0.8} = 1.74$$

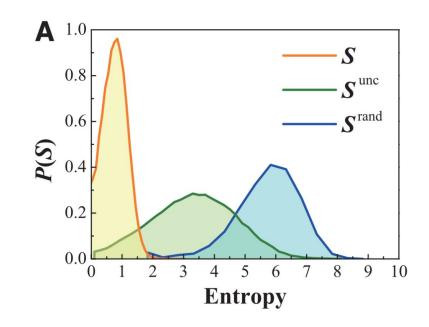


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#### References

- [paper] Human Mobility: Models and Applications, Barbosa et al., Physics Report, 2018, Section 3.1.1
- [paper] The scaling laws of human travel, Brockmann et al., Nature, 2006
- [paper] Understanding individual human mobility patterns, Gonzalez et al., Nature, 2008
- [paper] Returners and Explorers Dichotomy in Human Mobility,
   Pappalardo et al., Nature Communications, 2015
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