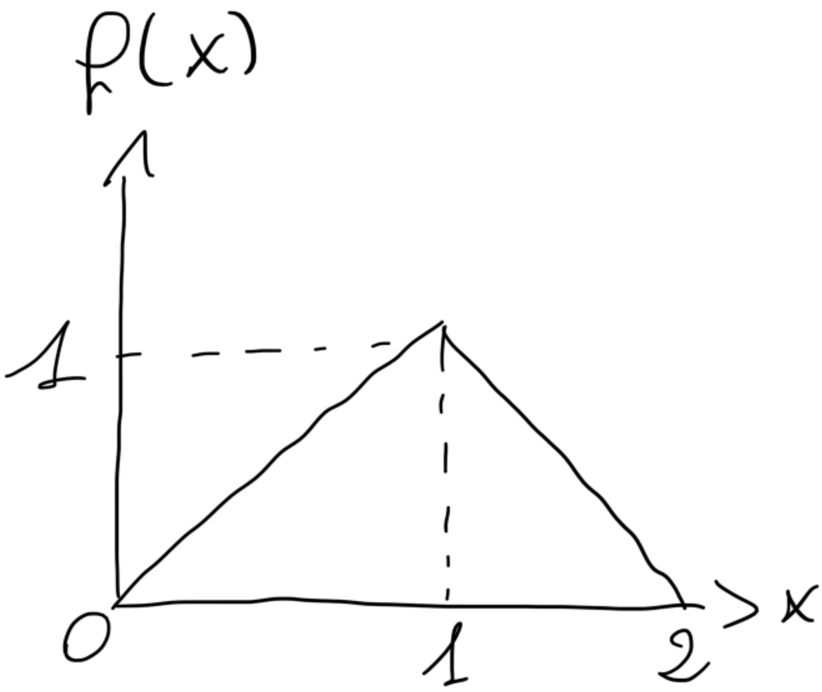
14 novembre 2025, 15:47

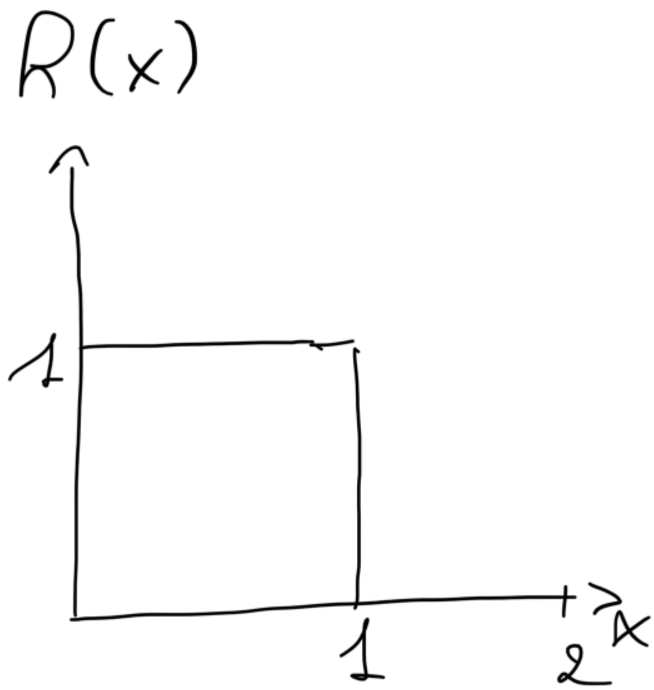
KERNEL EXERCISE

$$g(x) = f(x) * R(x)$$

$$= \int_{-\infty}^{+\infty} f(\tau) R(x-\tau)\, d\tau$$

GIVEN:





Let's consider τ:

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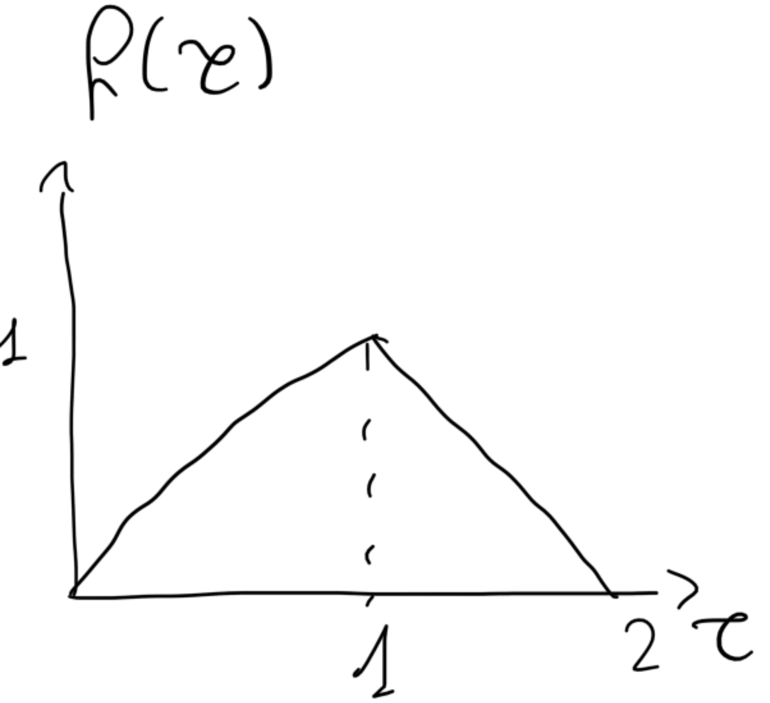
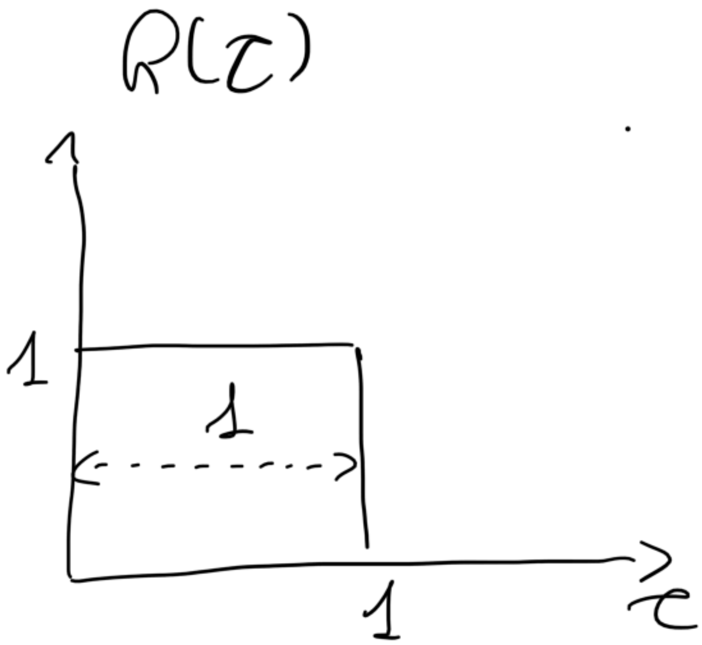
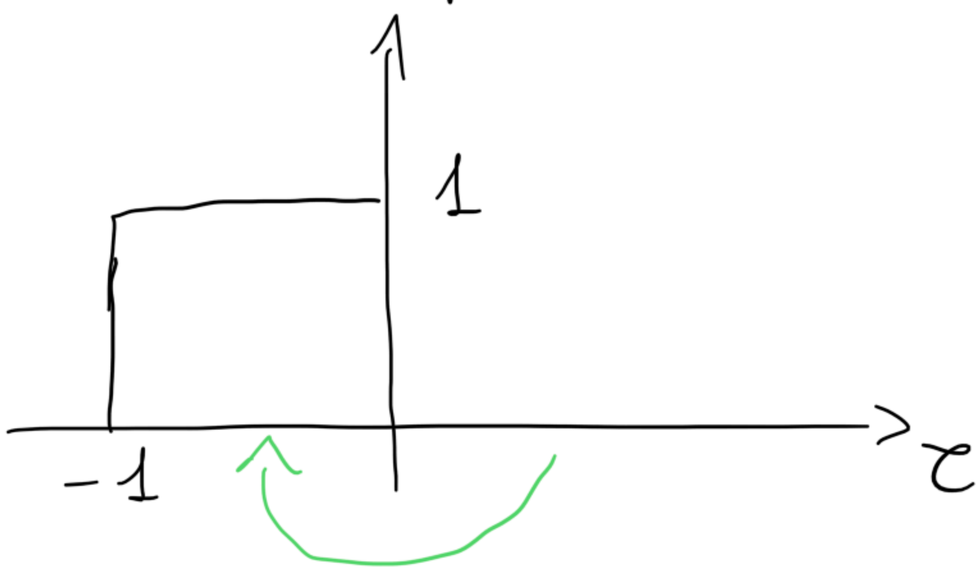
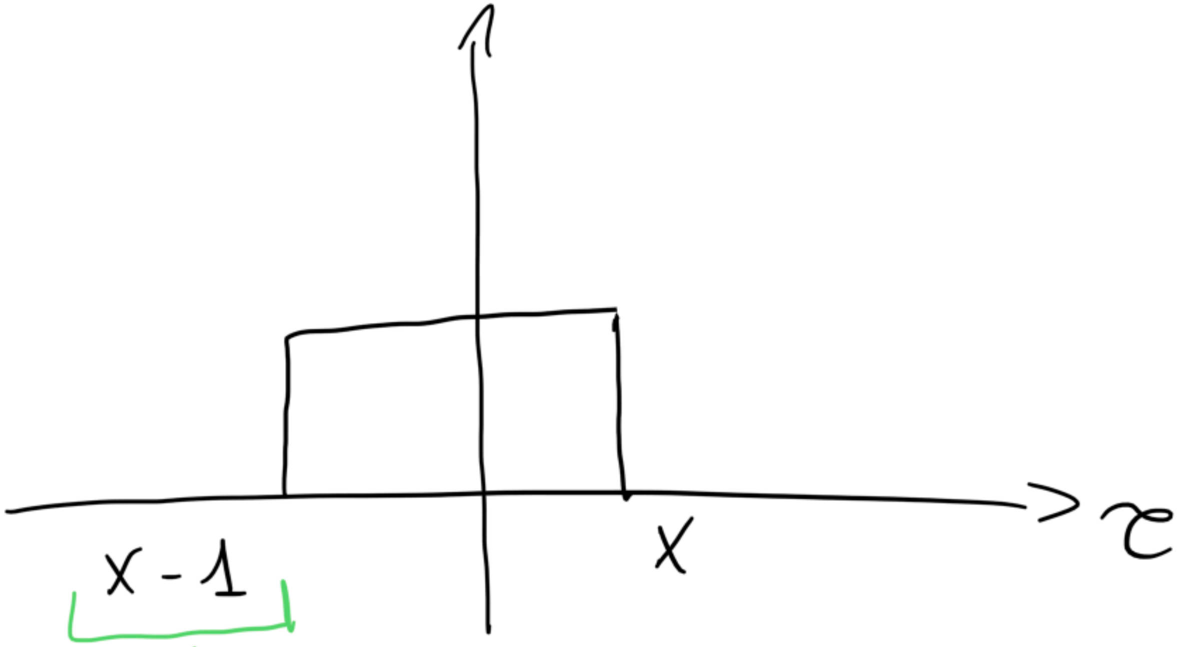




From the formula, we need $h(-\tau)$
To obtain it, we flip $h(\tau)$

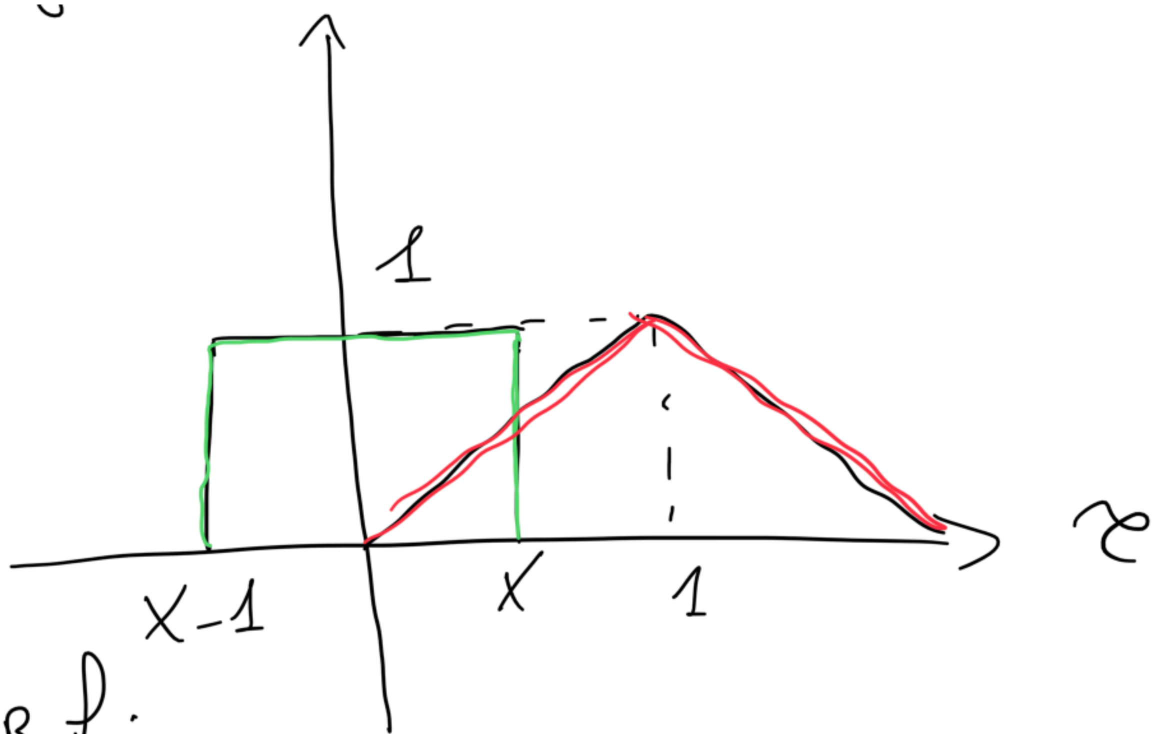


Now we need to compute $R(x-\tau)$



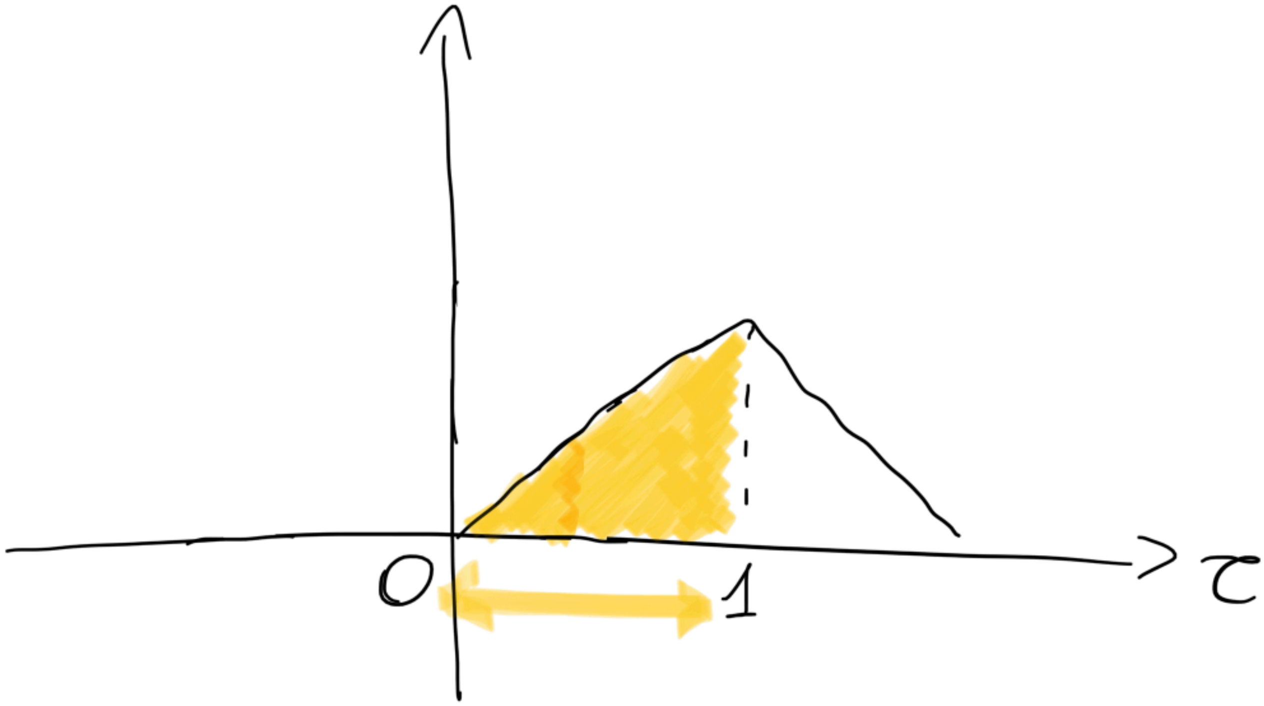
- 1 since the dimension of h is 1 (as presented before)

At this point, we can compute the convolution, passing the kernel (h) on all the function (f).
The integral is over τ.

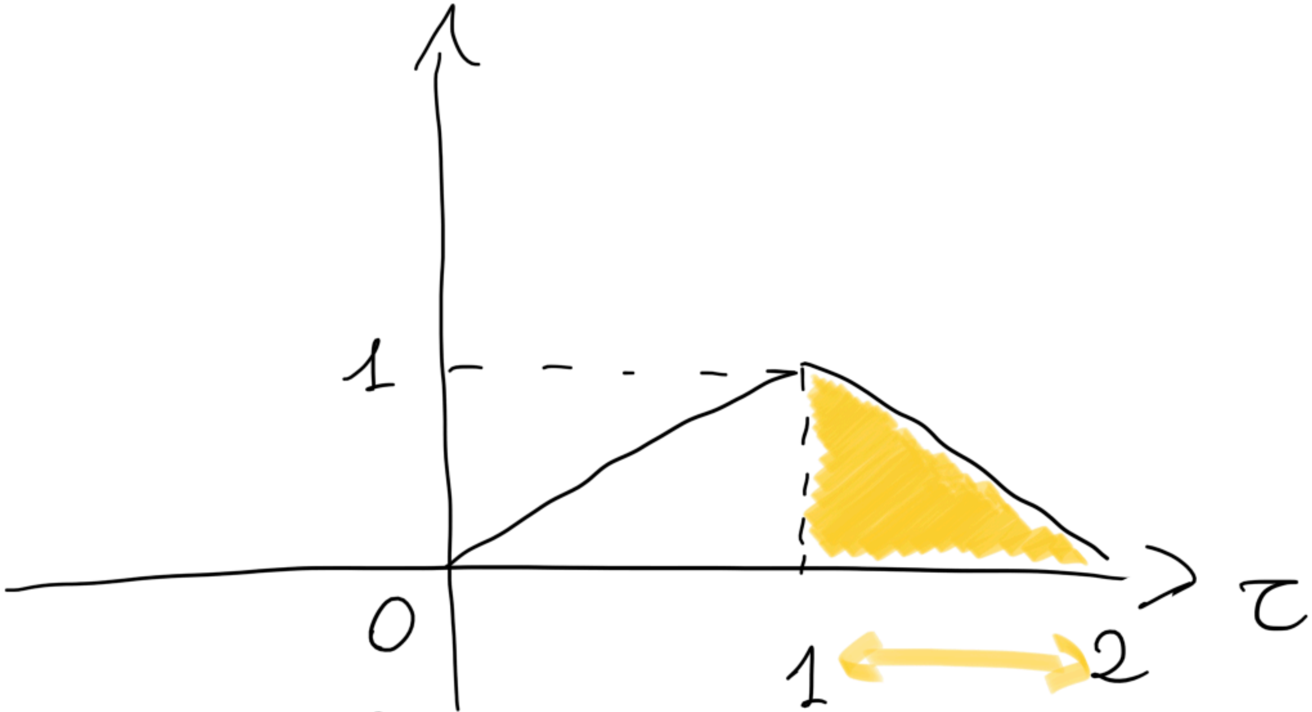


CASES FOR f:

if $0 < \tau < 1$: $f(\tau) = \tau$



if $1 < \tau < 2$: $f(\tau) = 2 - \tau$



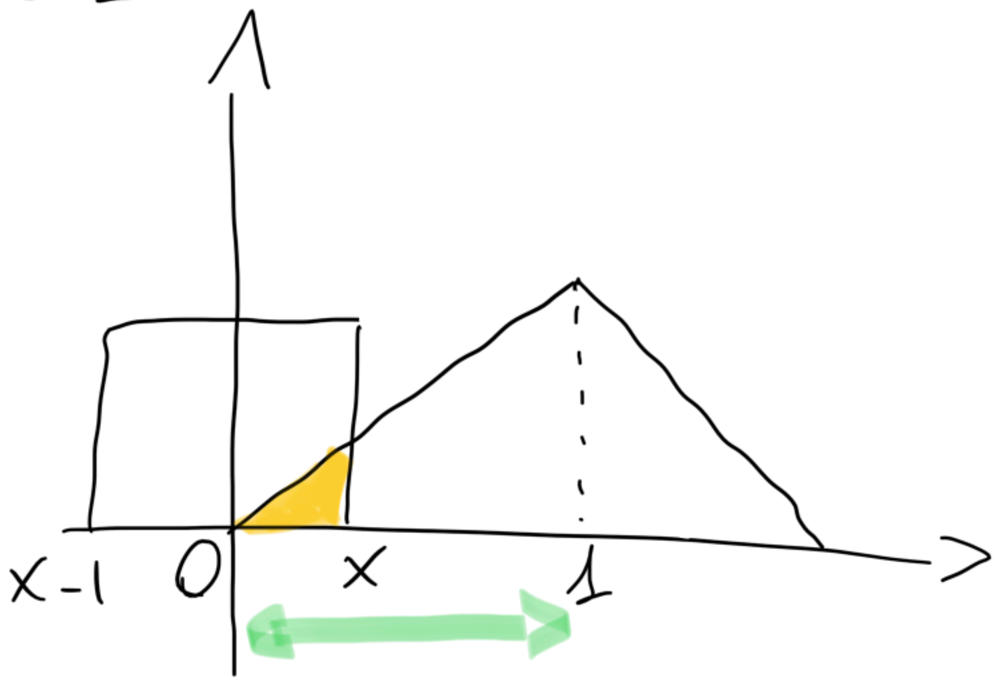
All the rest : $f(\tau) = 0$

HENCE:
OVERLAPS (for computing the integral)

if $x < 0$ → NO OVERLAPS

$f(x) = 0$

$0 < x < 1$

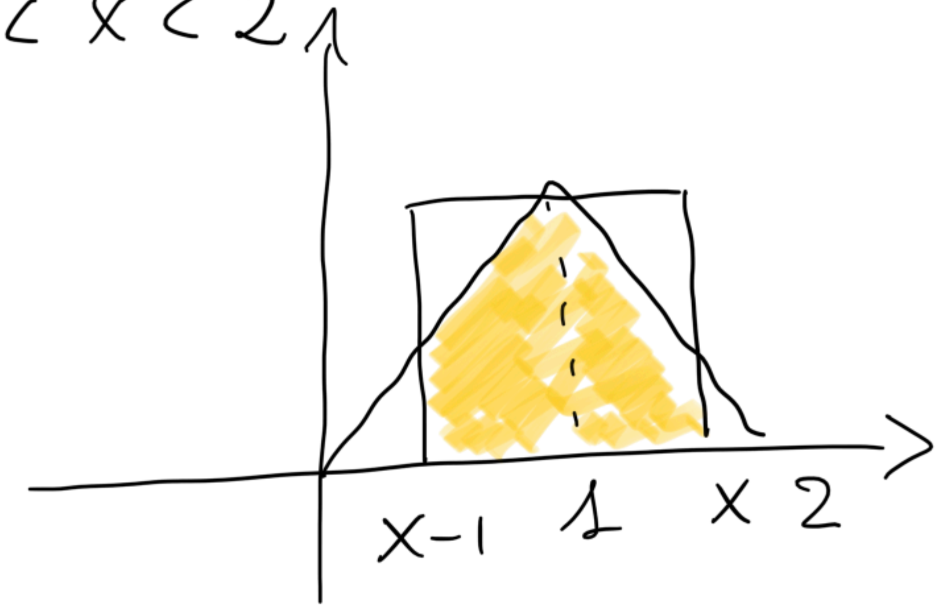


$0 < x < 1$

$$\int_0^x f(\tau)\,h(x-\tau)\,d\tau$$

$$= \int_0^x \tau(1)\,d\tau = \frac{\tau^2}{2}\Big|_0^x = \frac{x^2}{2} - 0$$

$\rightarrow \quad 1 < x < 2$



$$\int_{x-1}^{1} \tau(1)\,d\tau + \int_{1}^{x} (2-\tau)(1)\,d\tau$$

$$\frac{\tau^2}{2}\Big|_{x-1}^{1} + 2\tau - \frac{\tau^2}{2}\Big|_{1}^{x}$$

$$= \frac{1}{2} - \frac{(x-1)^2}{2} + \left(2x - \frac{x^2}{2} - 2 + \frac{1}{2}\right)$$