Social Network Analysis

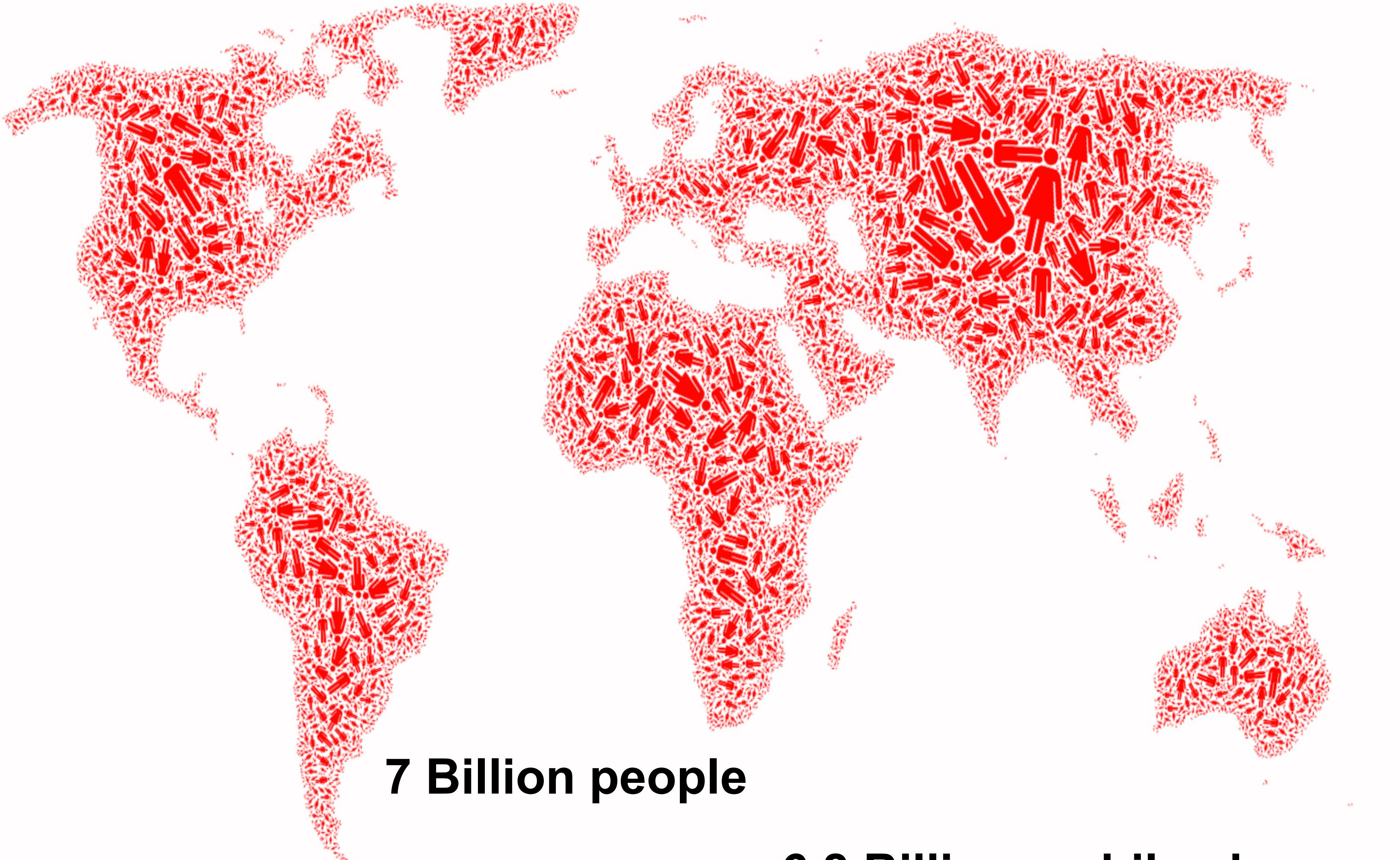
A crash course @ UPF

Dino Pedreschi

ISTI-CNR & Università di Pisa

http://kdd.isti.cnr.it


2005
Luca Bruno / AP
2013
NBC NEWS
Michael Sohn / AP



7 Billion people

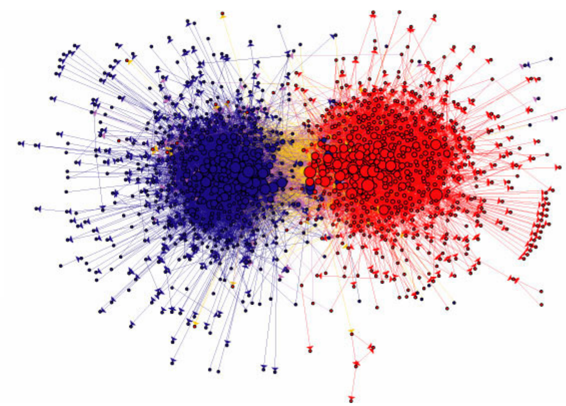
6.8 Billion mobile phones

Siamo tutti Pollicini digitali
Tots som Pollicini digitals

Big data proxies of social life

Shopping patterns & lyfestyle

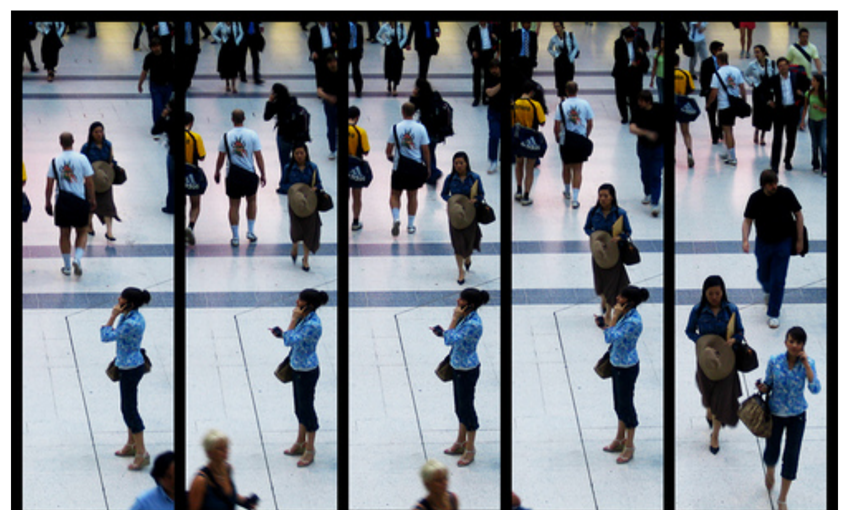
RELATIONSHIPS & SOCIAL TIES

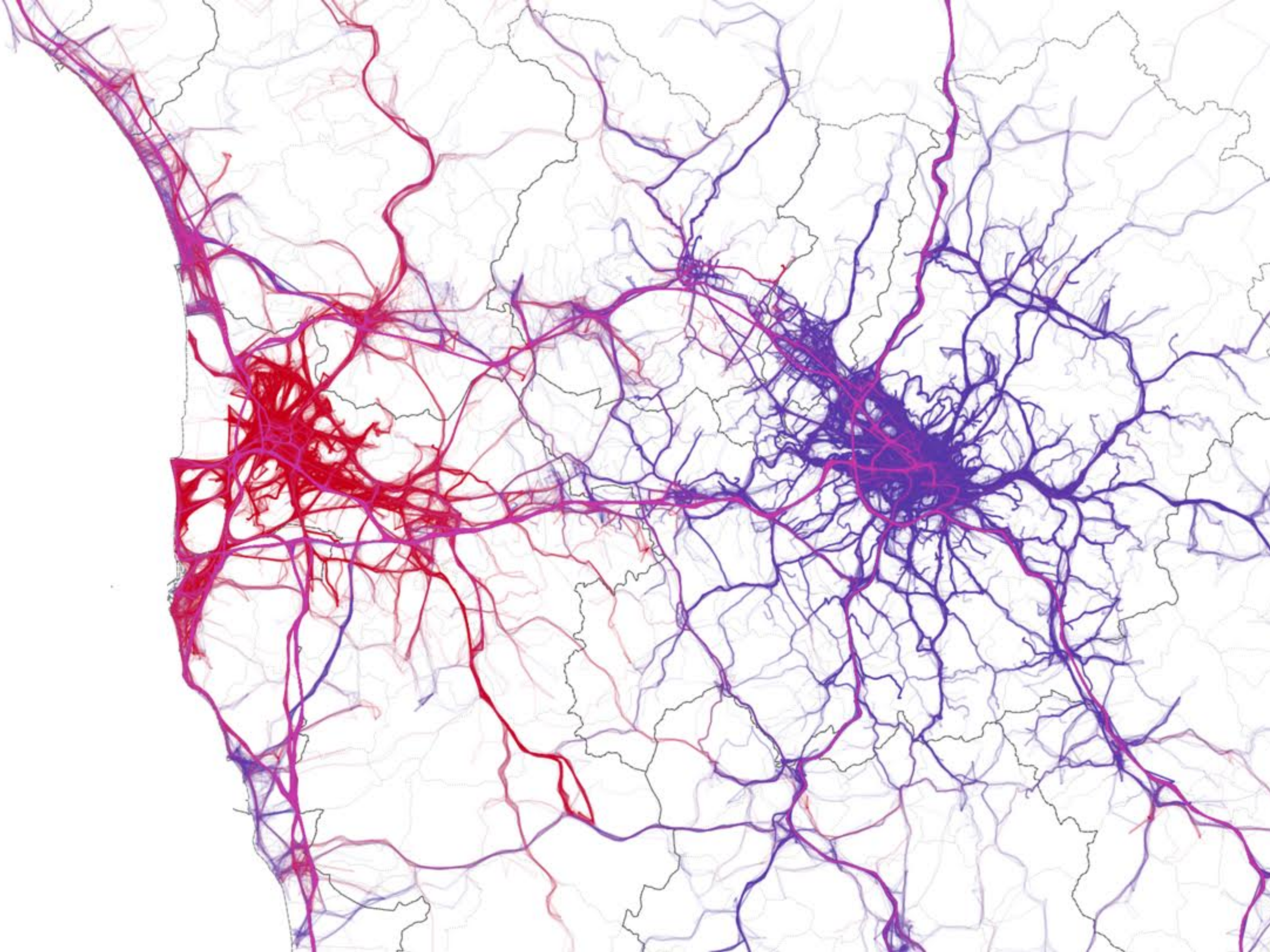


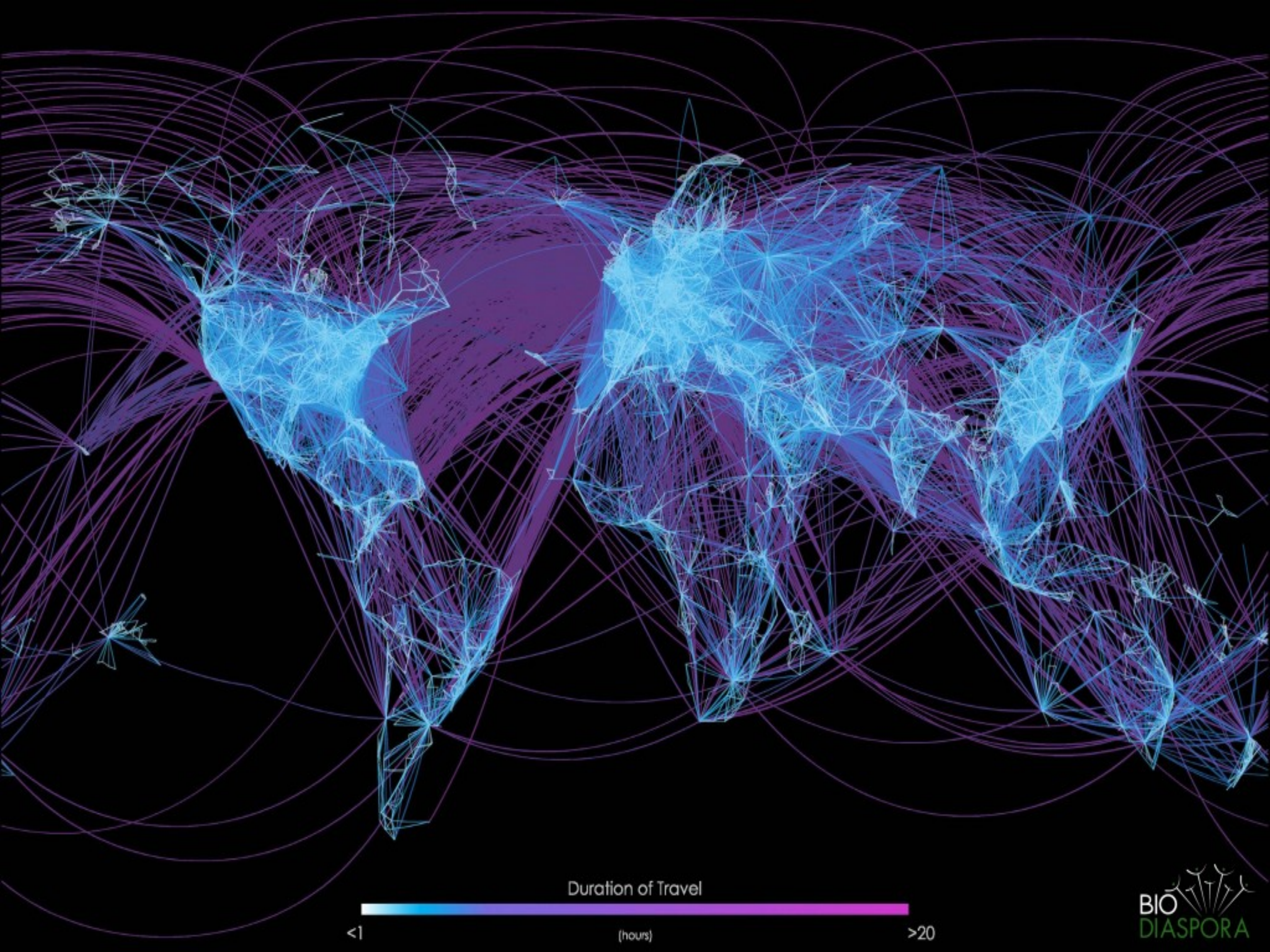
DESIRES, OPINIONS, SENTIMENTs

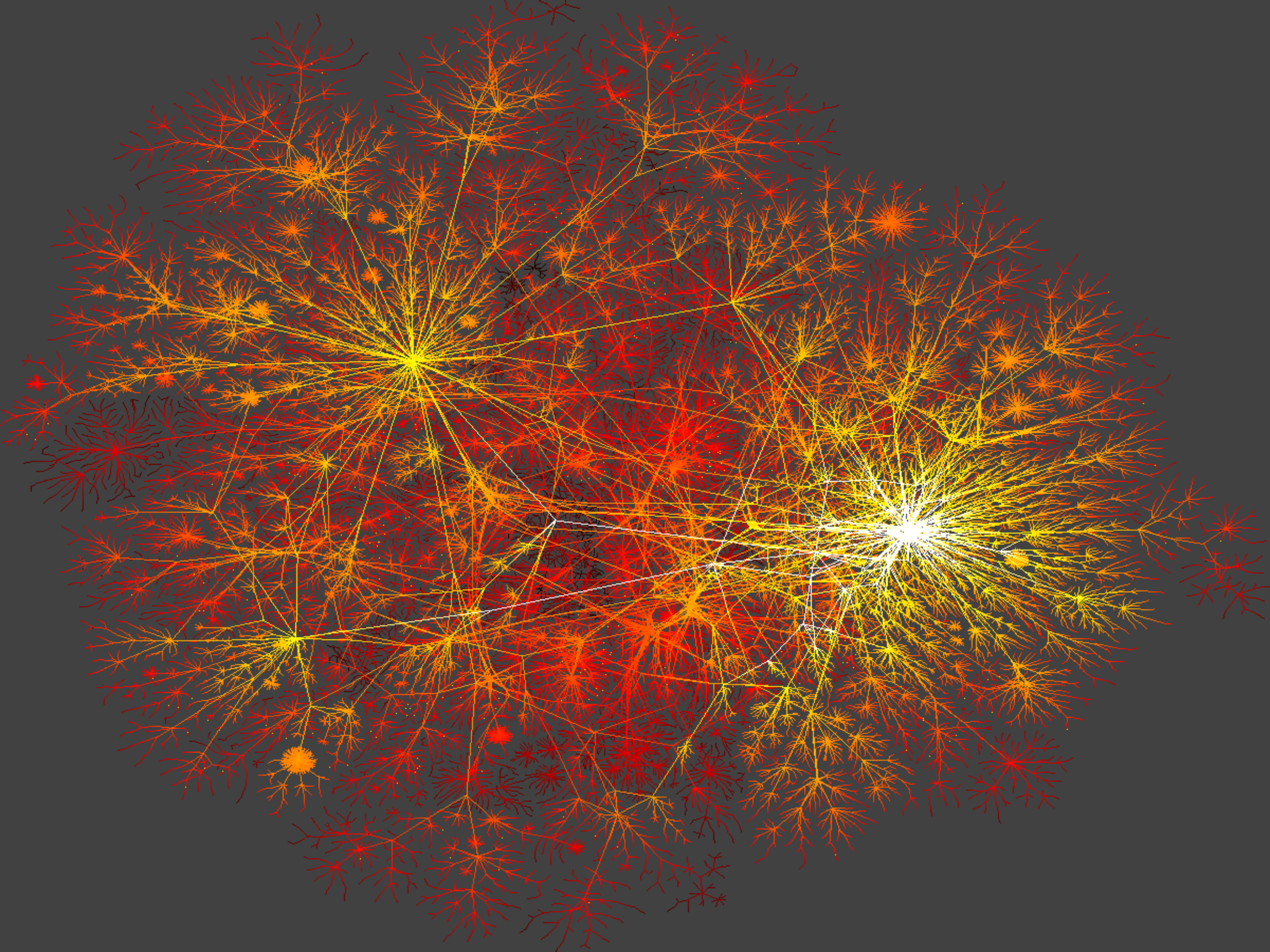
WIKIPEDIA
The Free Encyclopedia

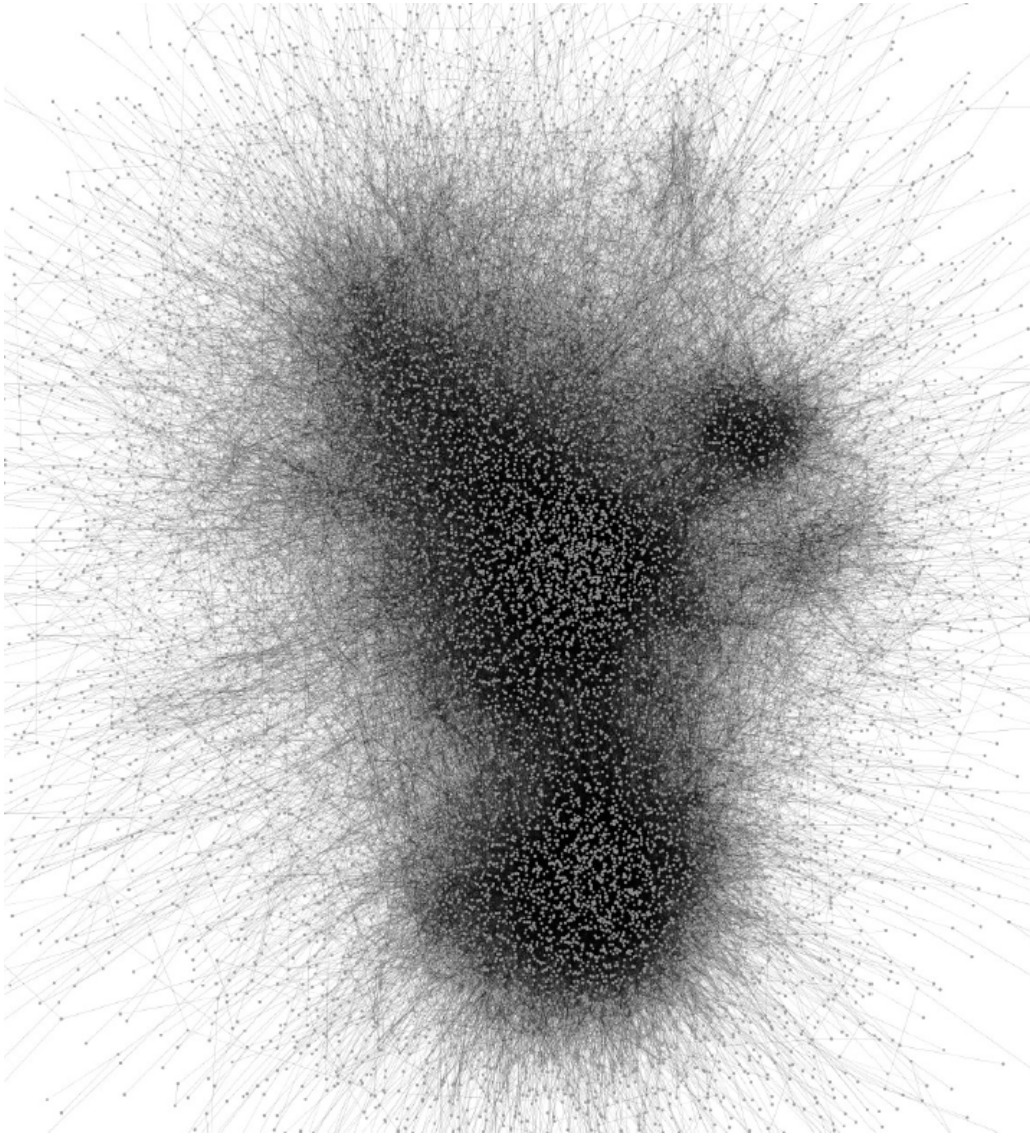
MOVEMENTS

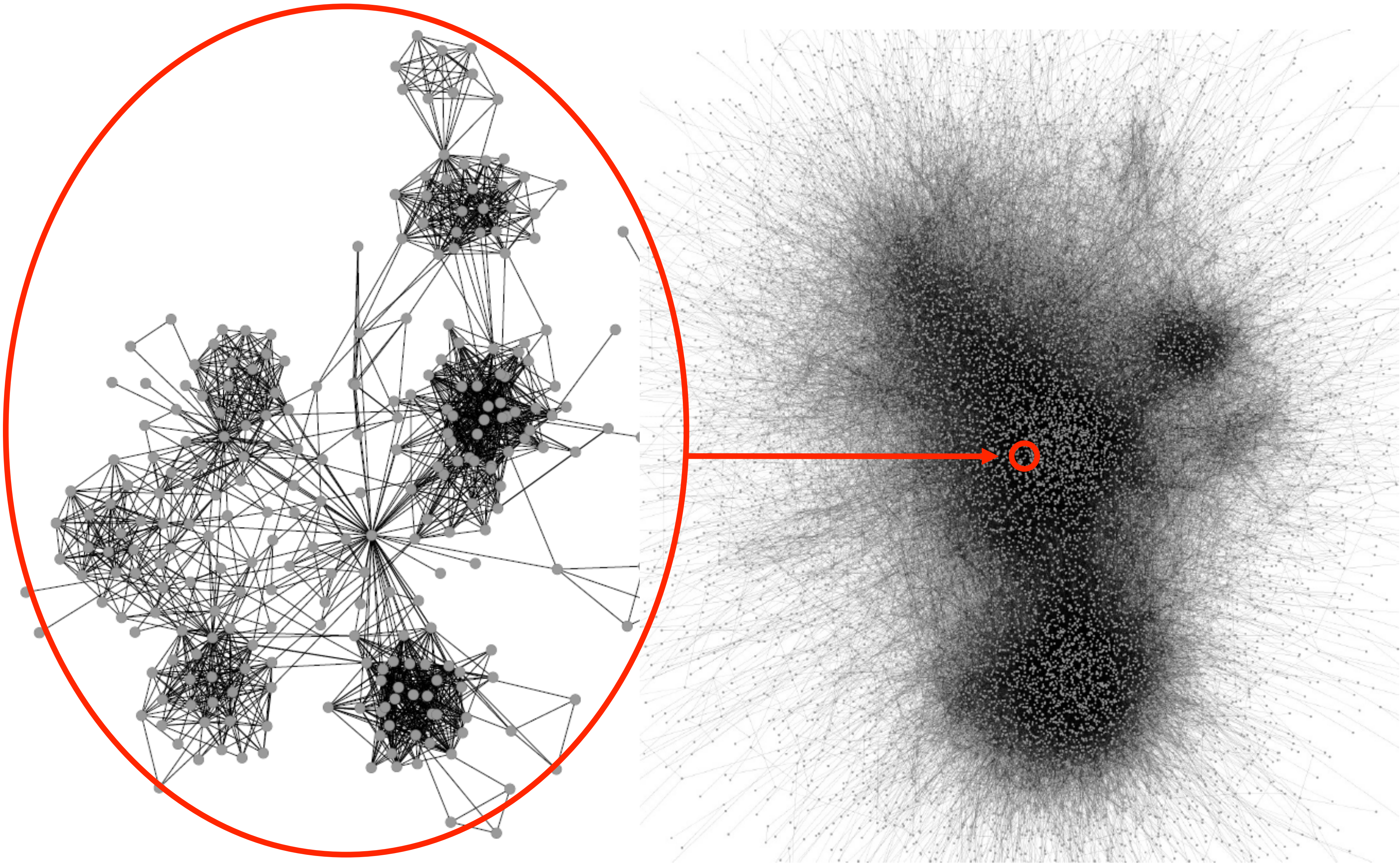





Duration of Travel
<1
(hours)
>20
BIO
DIASPORA







Complex (Social) Networks

- Big graph data and social, information, biological and technological networks
- The architecture of complexity and how real networks differ from random networks:
  - node degree and long tails,
  - social distance and small worlds,
  - clustering and triadic closure.
- Comparing real networks and random graphs.
- The main models of network science: small world and preferential attachment.

Complex (Social) Networks

- Strong and weak ties, community structure and long-range bridges.
- Robustness of networks to failures and attacks.
- Cascades and spreading. Network models for diffusion and epidemics. The strength of weak ties for the diffusion of information. The strength of strong ties for the diffusion of innovation.
- Practical network analytics with Cytoscape and Gephi.
- Simulation of network processes with NetLogo.

Complex (Social) Networks

- Textbooks
  - Albert-Laszlo Barabasi. *Network Science* (2016)
  - http://barabasi.com/book/network-science
  - David Easley, Jon Kleinberg: *Networks, Crowds, and Markets* (2010)
  - http://www.cs.cornell.edu/home/kleinber/networks-book/
- Network Analytics Software (open):
  - Cytoscape: http://www.cytoscape.org/
  - Gephi: http://gephi.github.io/
- Network Data Repository
  - http://networkrepository.com/
- Simulation of network models: NetLogo

Complex

[adj., v. kuh m-pleks, kom-pleks; n. kom-pleks]

–adjective

1\. **composed of many interconnected parts**; compound; composite: a complex highway system.

2\. characterized by a very complicated or involved arrangement of parts, units, etc.: complex machinery.

3\. so complicated or intricate as to be hard to understand or deal with: a complex problem.

Source: Dictionary.com

Complexity, a **scientific theory** which asserts that some systems display behavioral phenomena that are completely inexplicable by any conventional analysis of the systems' constituent parts. These phenomena, commonly referred to as **emergent behaviour**, seem to occur in many complex systems involving living organisms, such as a stock market or the human brain.

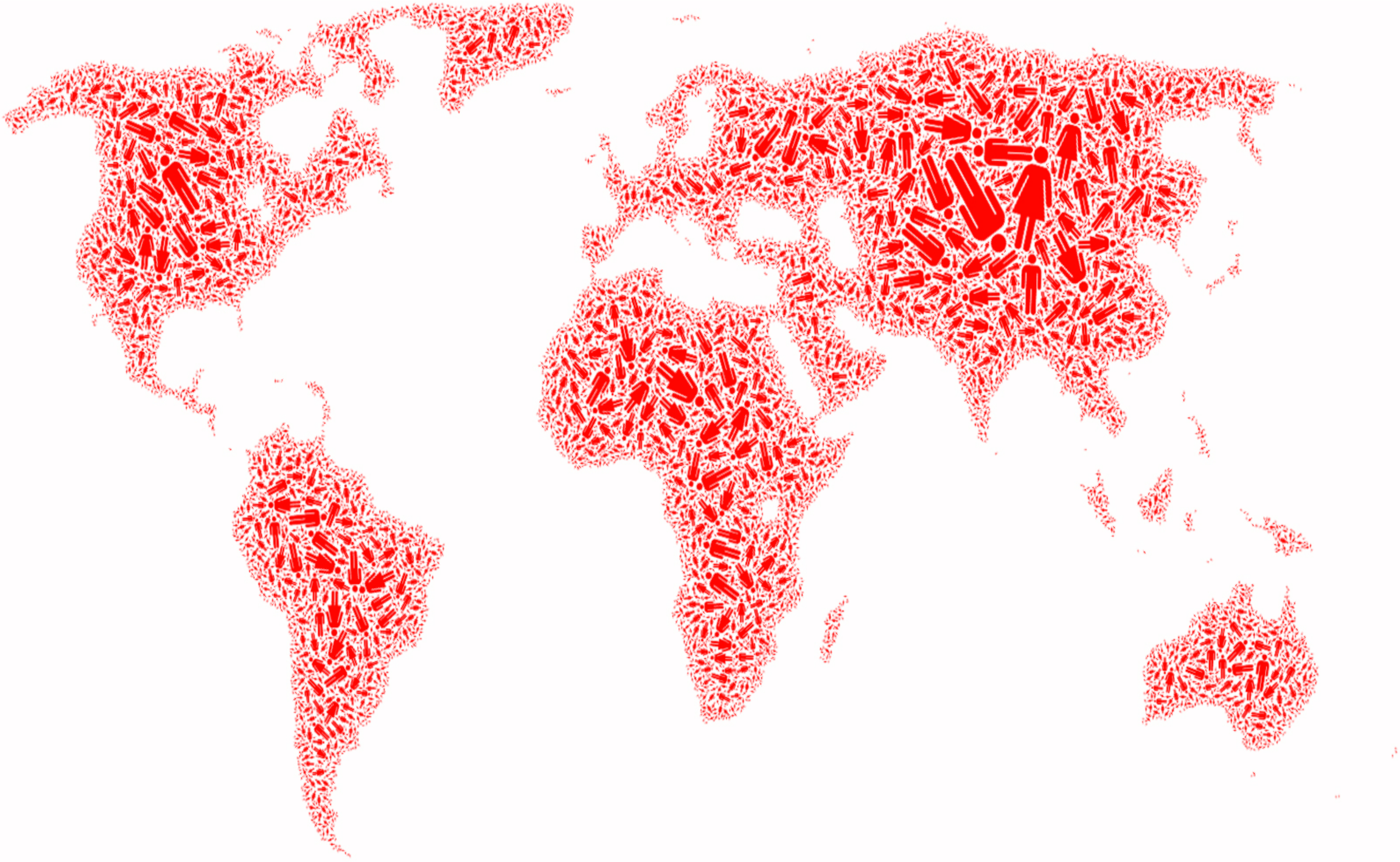
Source: John L. Casti, Encyclopædia Britannica

Complexity

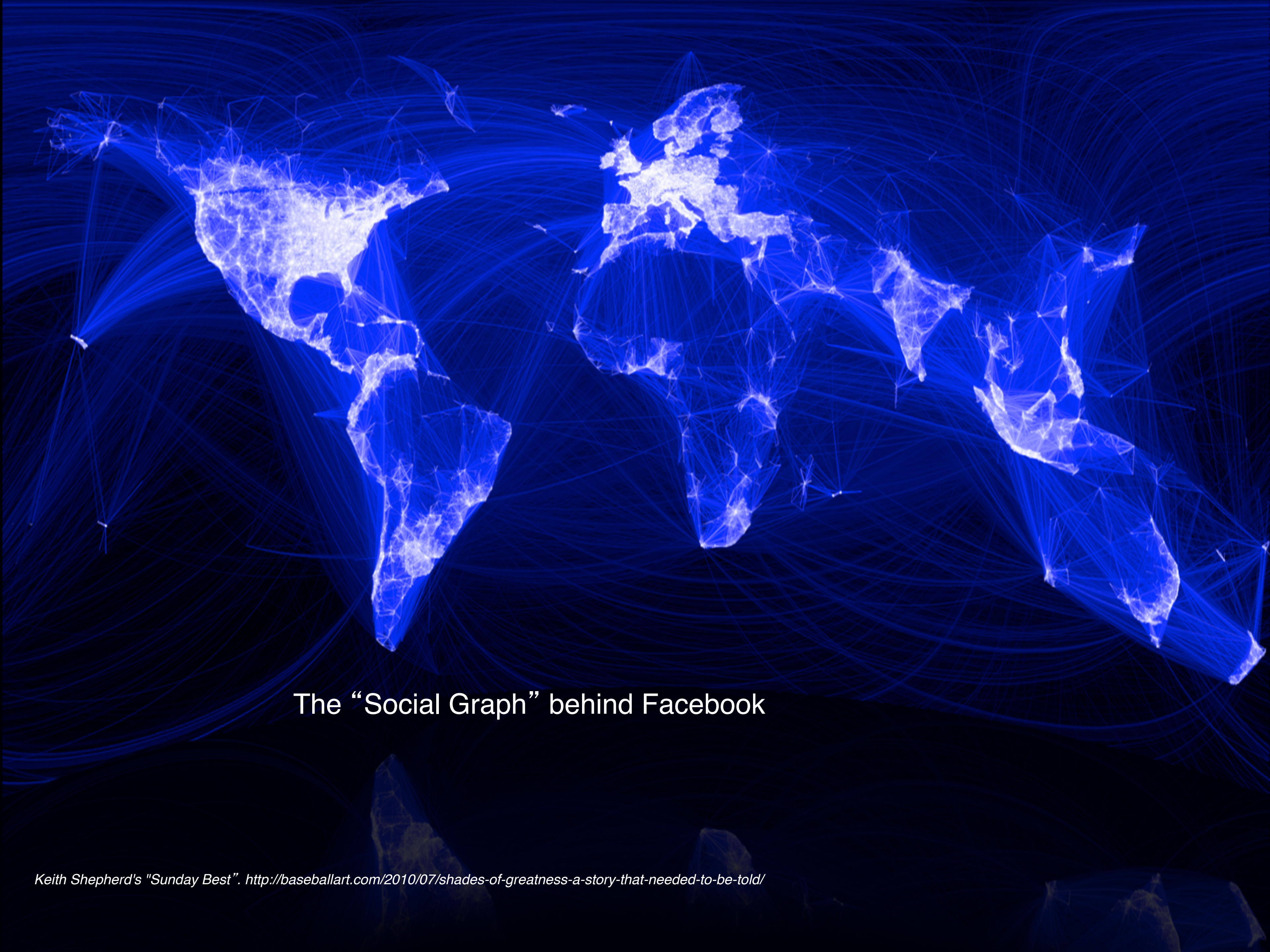
Emergent behavior: segregation

Behind each complex system there is a **network**, that defines the interactions between the components.

Social, informational, technological, biological networks

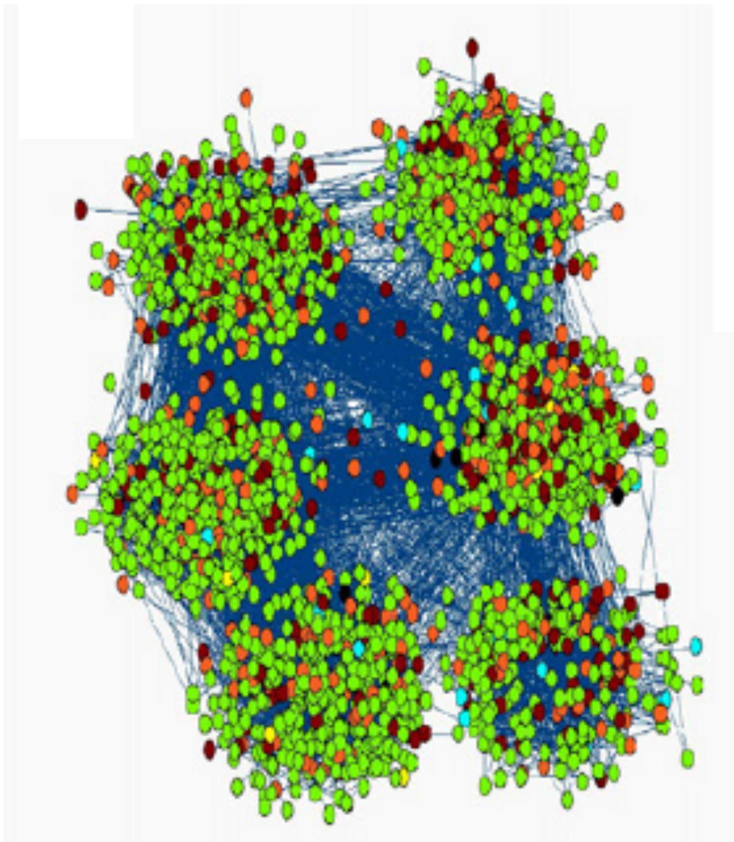
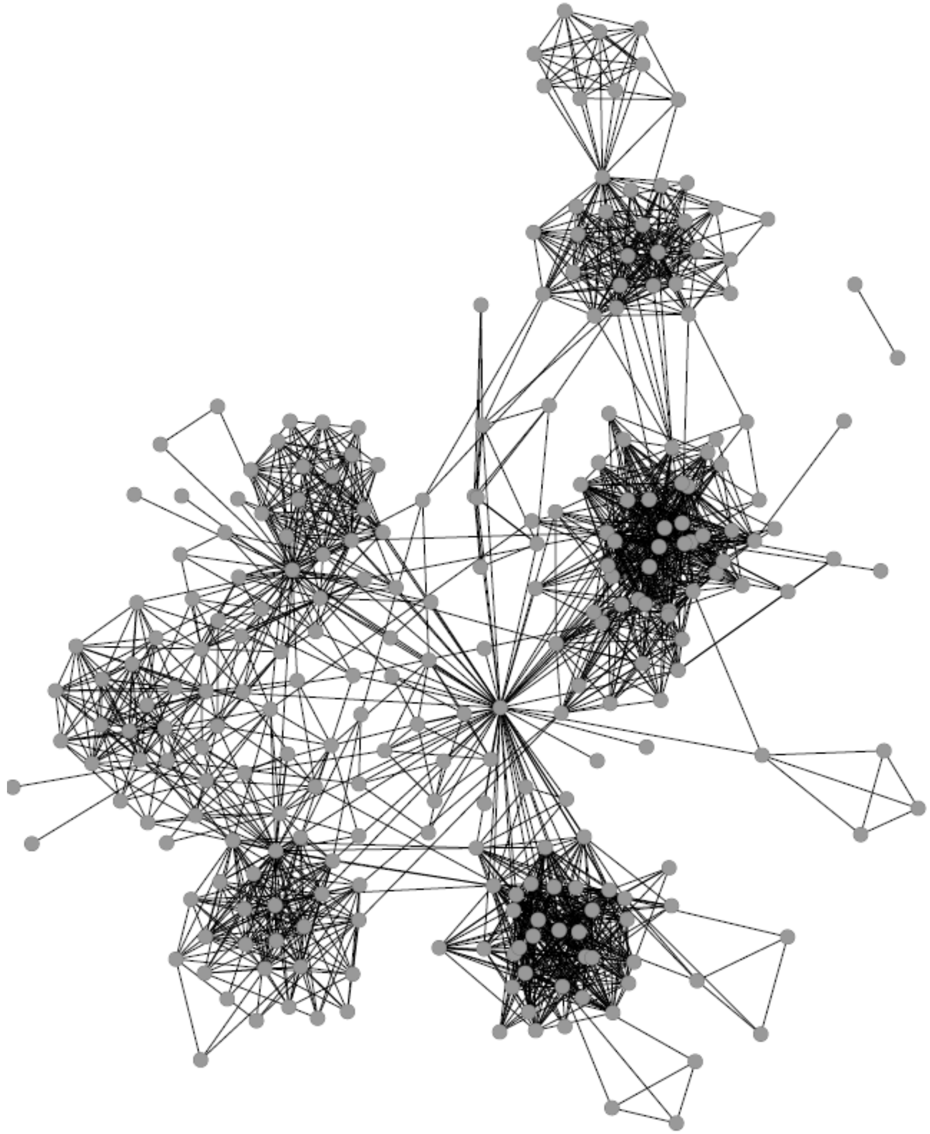


The "Day of 7 Billion“ has been in October 2011

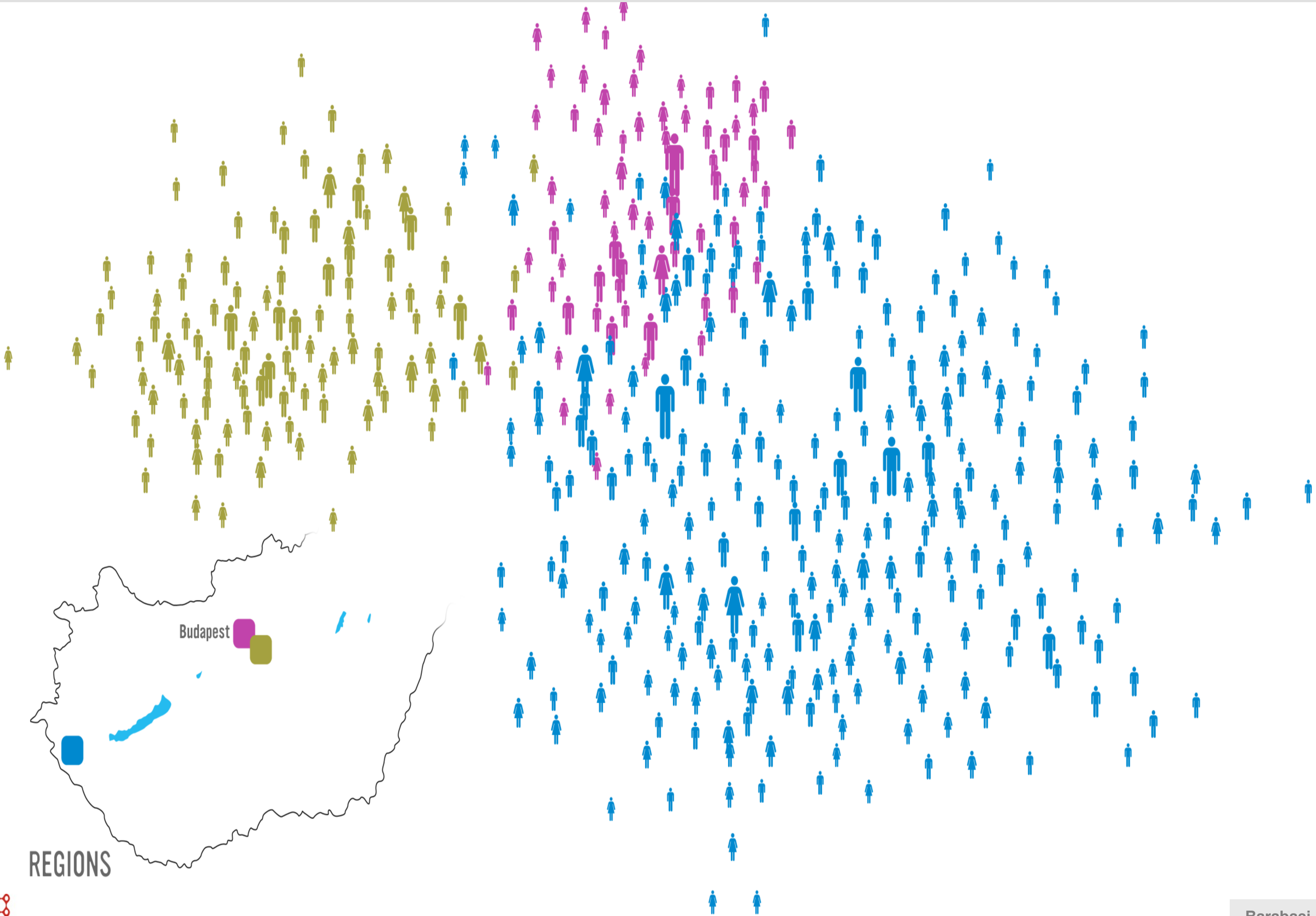


The “Social Graph” behind Facebook

Keith Shepherd's "Sunday Best". http://baseballart.com/2010/07/shades-of-greatness-a-story-that-needed-to-be-told/

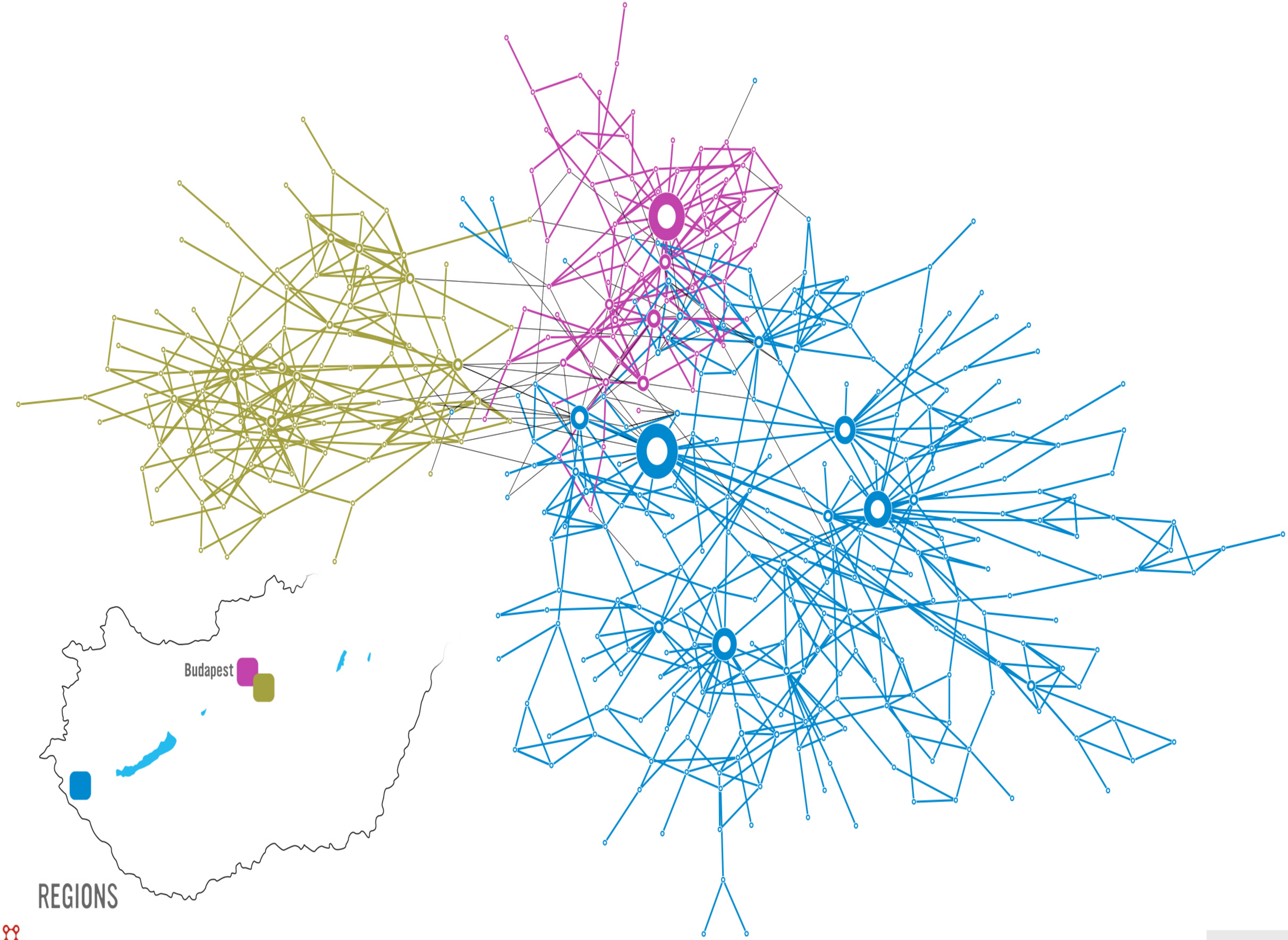



Mapping Organizations



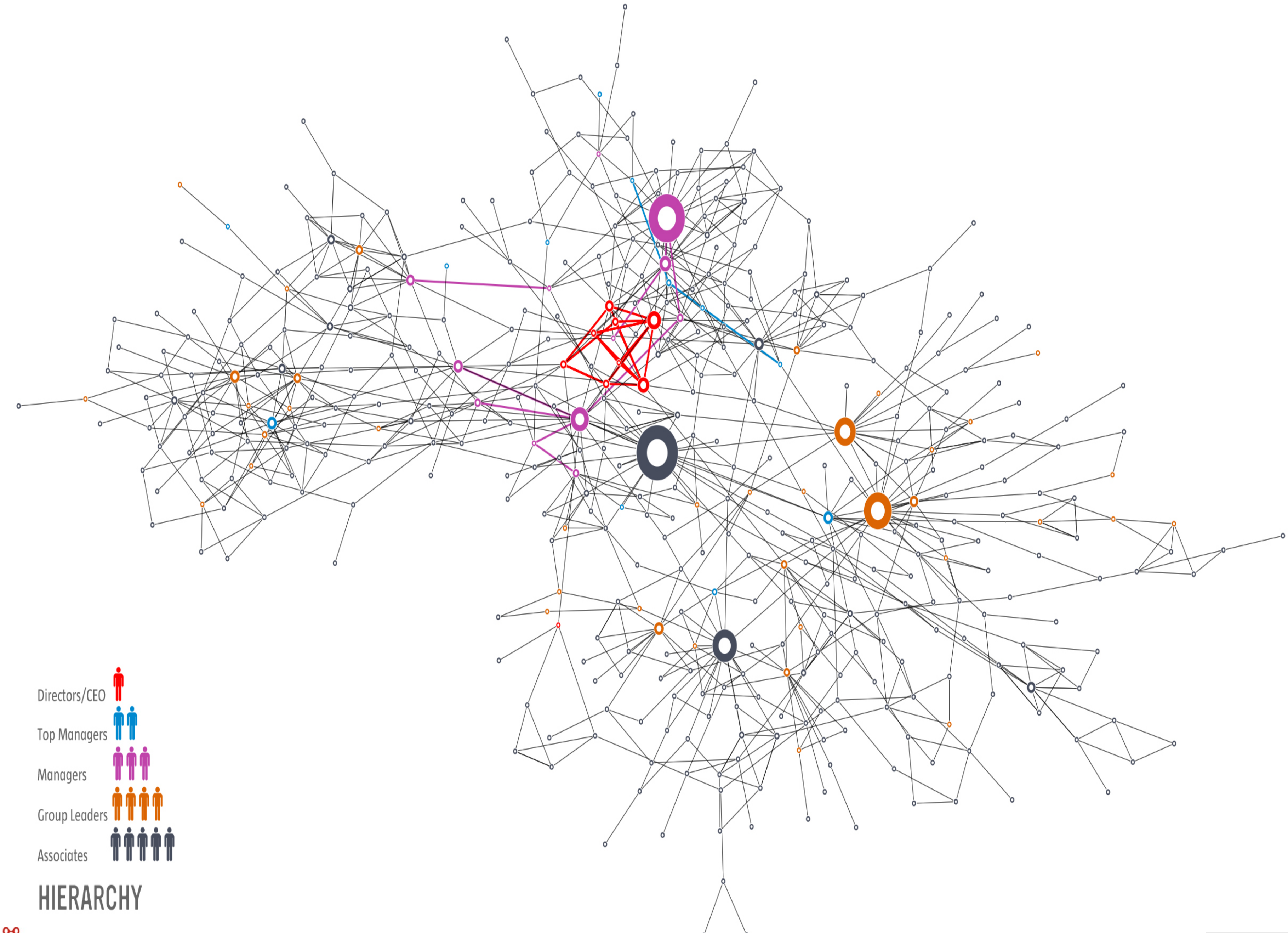
maven7
connecting knowledge

Barabasi Lab



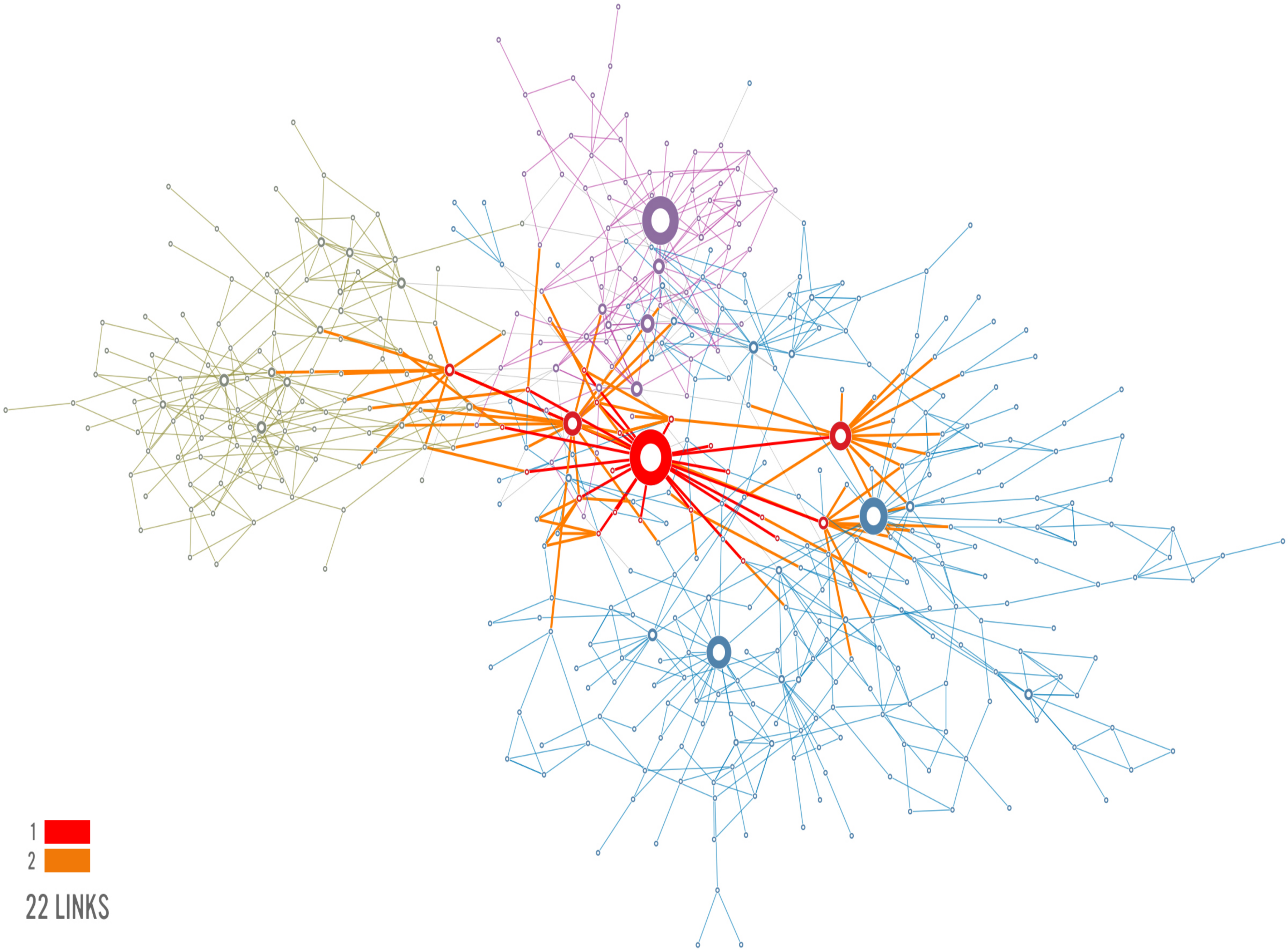
maven7
connecting knowledge

Barabasi Lab



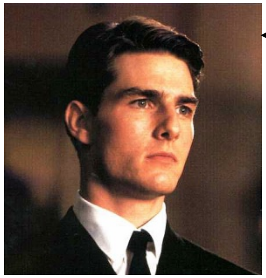
maven7 connecting knowledge

Barabasi Lab


1
2
22 LINKS

COLLABORATION NETWORKS: ACTOR NETWORK

Nodes: actors
Links: cast jointly

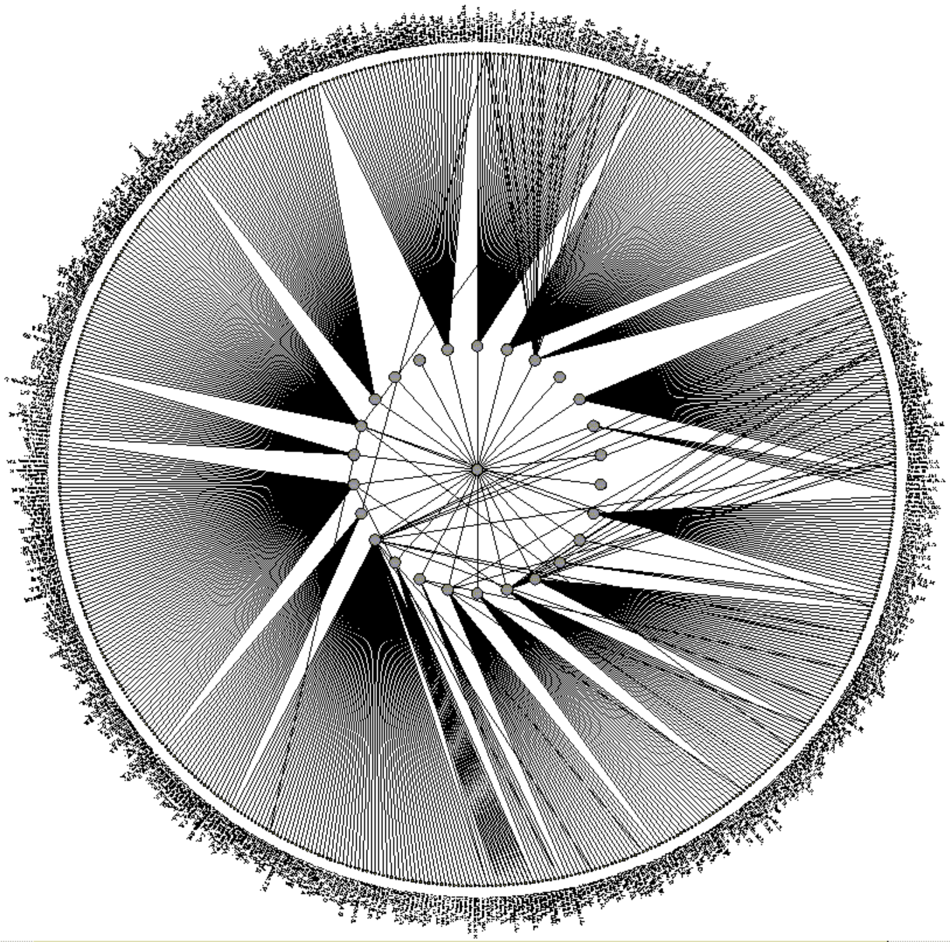


Days of Thunder (1990)
Far and Away (1992)
Eyes Wide Shut (1999)

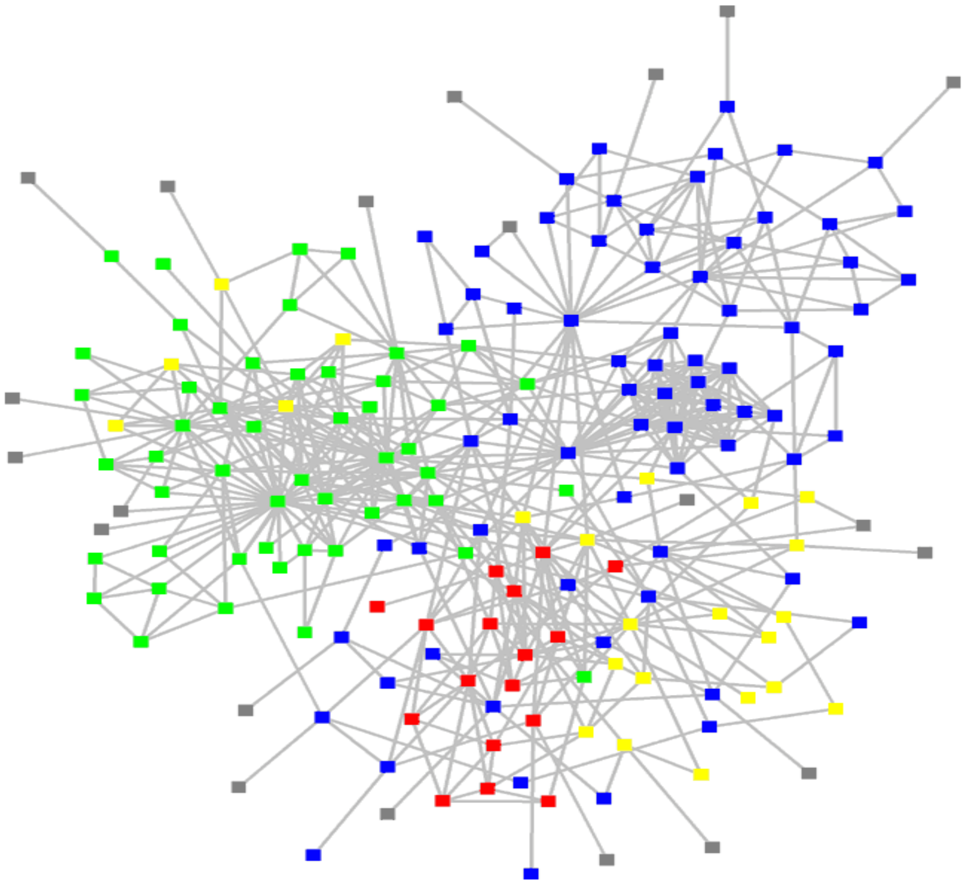
N = 212,250 actors $\langle k \rangle$ =28.78

COLLABORATION NETWORKS: SCIENCE CO-AUTHORSHIP

Nodes: scientist (authors)
Links: write paper together



STRUCTURE OF AN ORGANIZATION



www.orgnet.com

: departments

: consultants

: external experts

BUSINESS TIES IN US BIOTECH-INDUSTRY

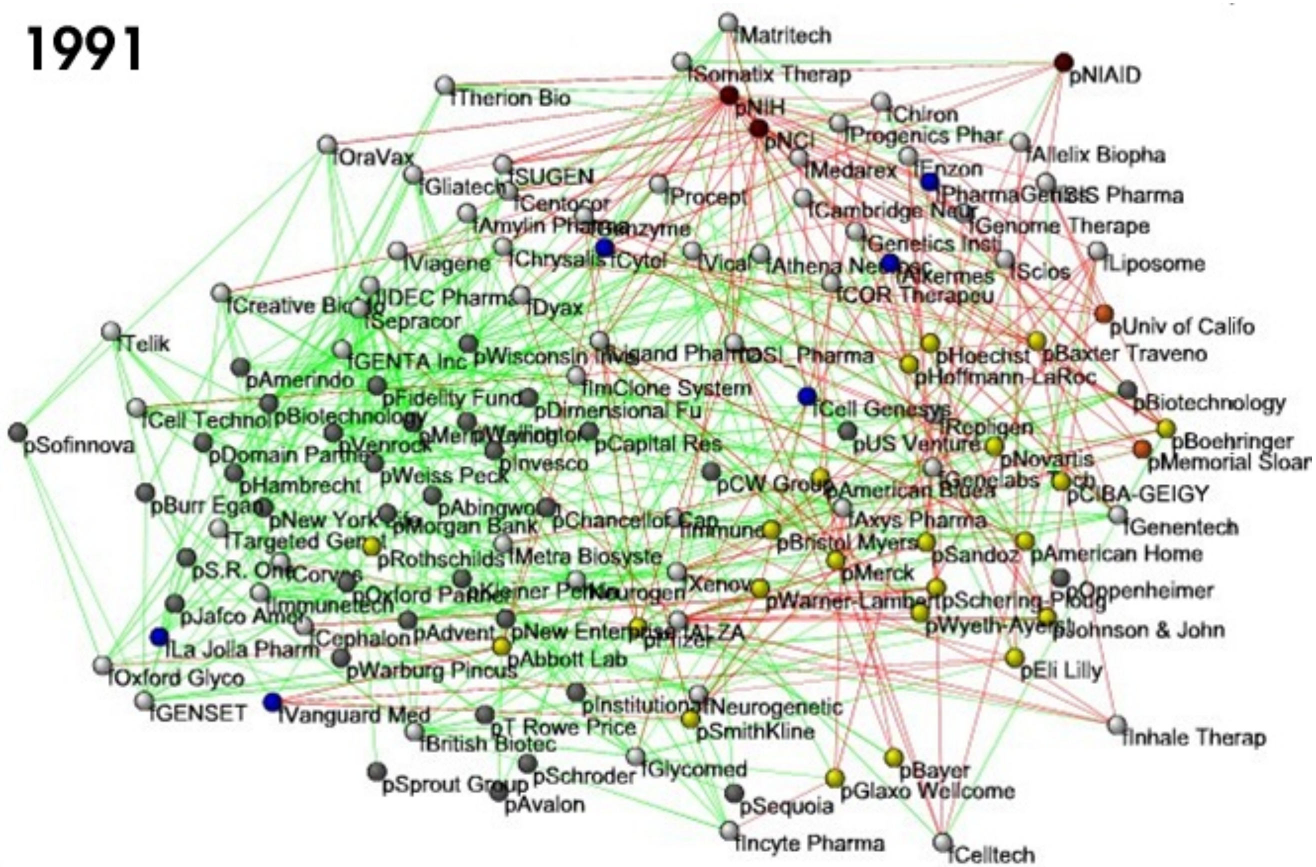
Nodes:

- Companies
- Investment
- Pharma
- Research Labs
- Public
- Biotechnology

Links:

- Collaborations
- Financial
- R&D

1991

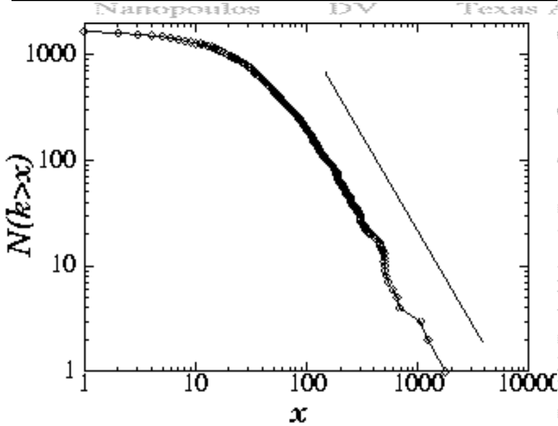


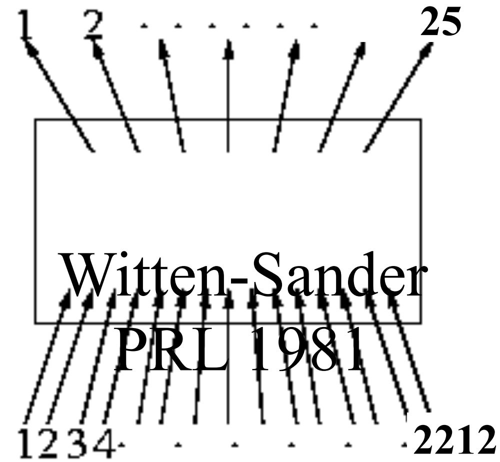
http://ecclectic.ss.uci.edu/~drwhite/Movie

Information networks: the Web and Science Citation Indexes

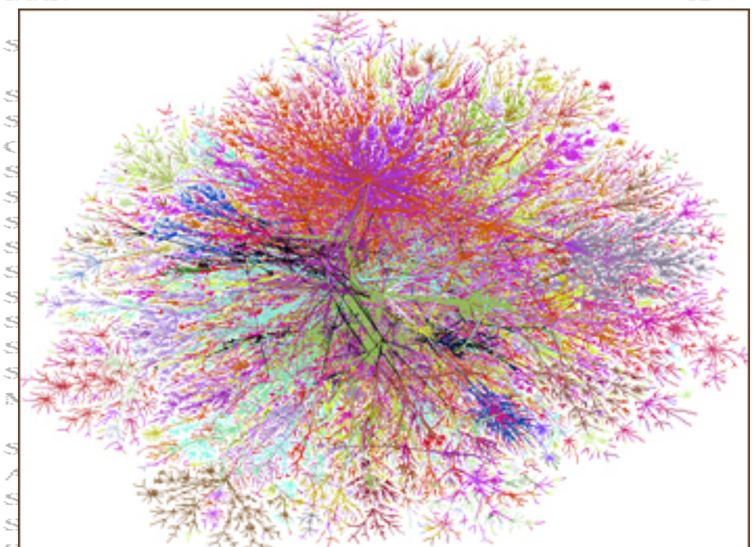
Nodes: papers
Links: citations

1736 PRL papers (1988)

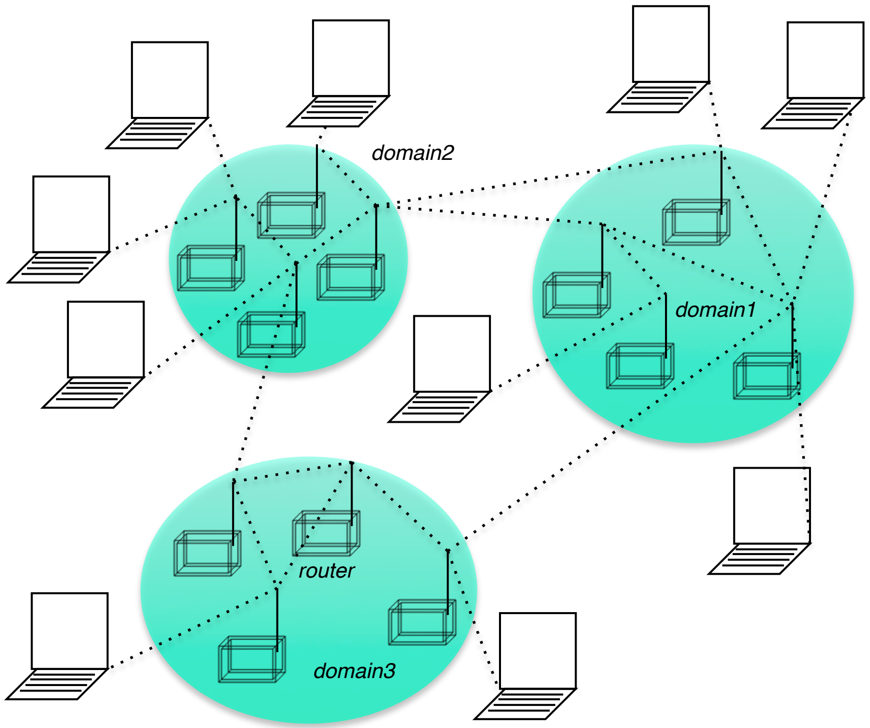


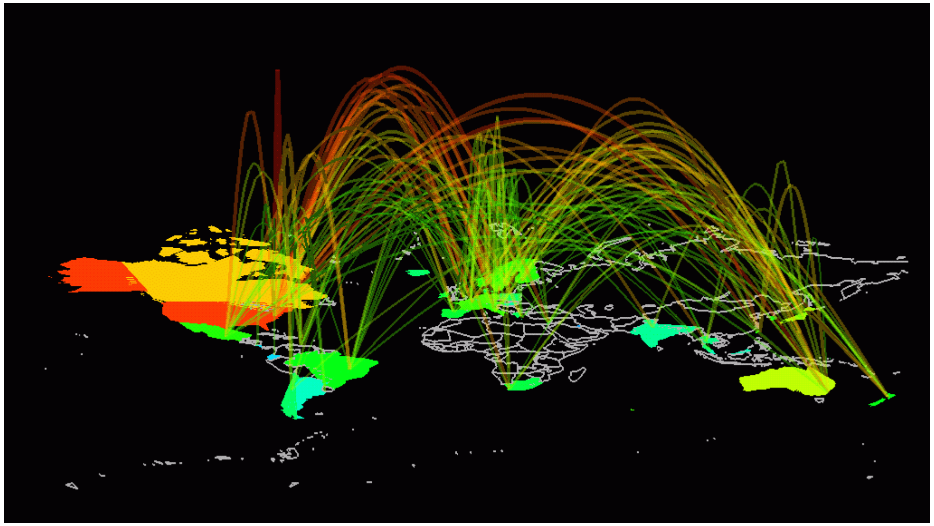


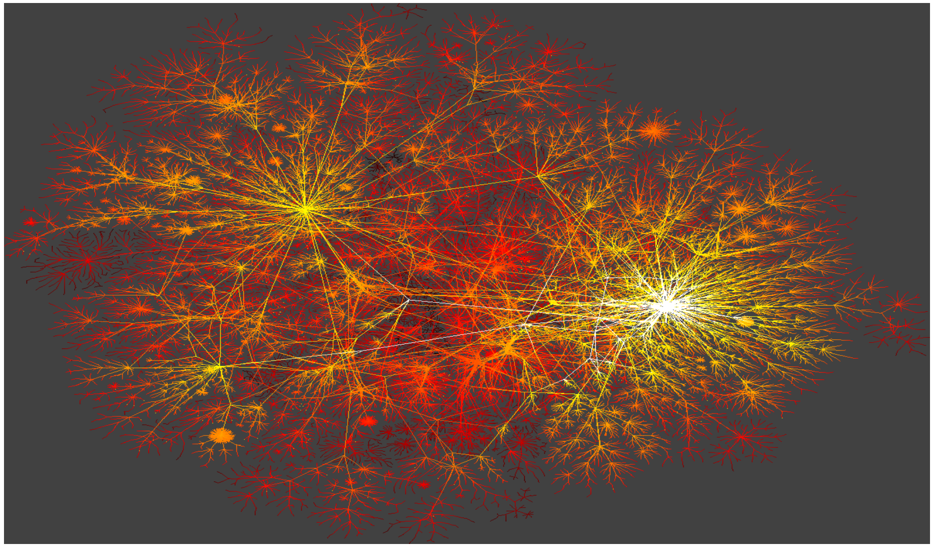
Nodes: web pages
Links: ditto ;-)

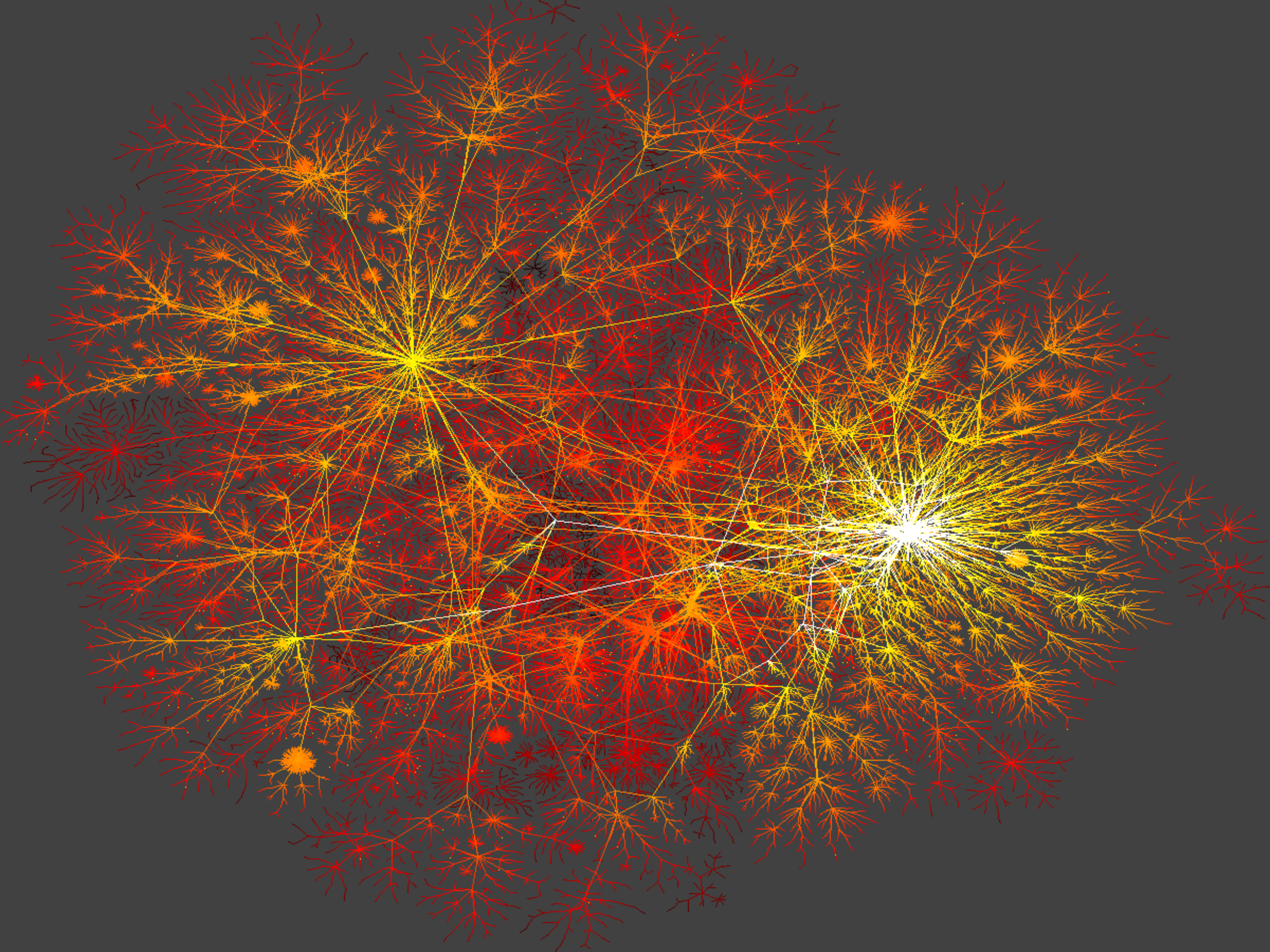


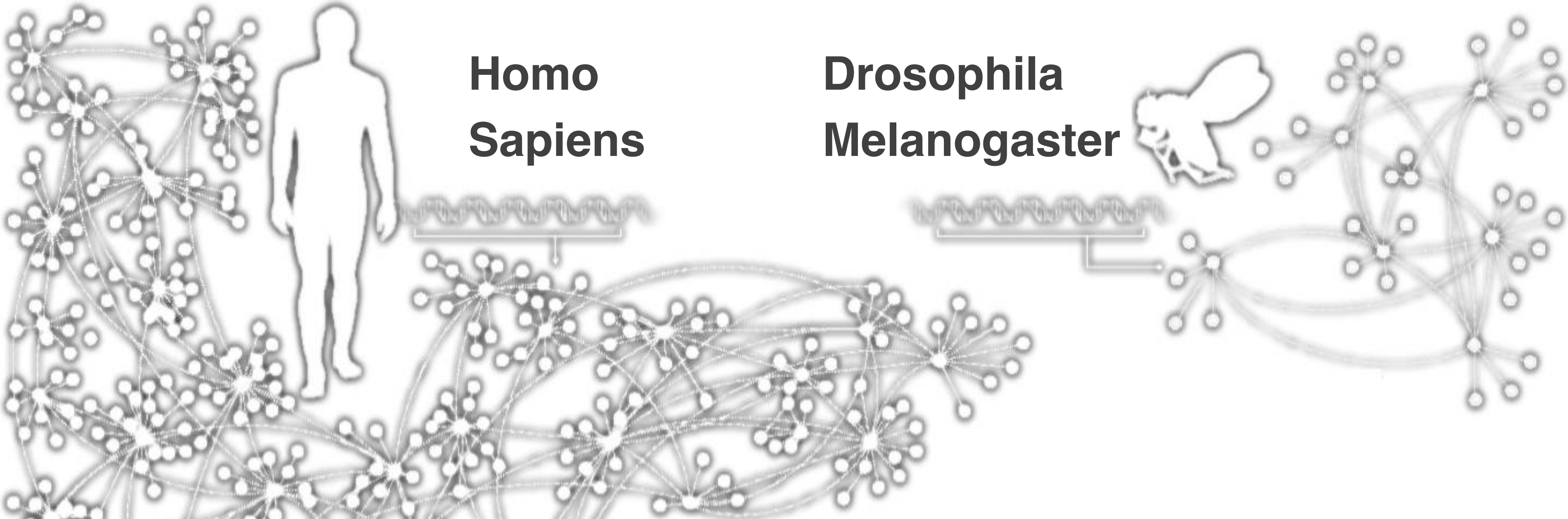
INTERNET

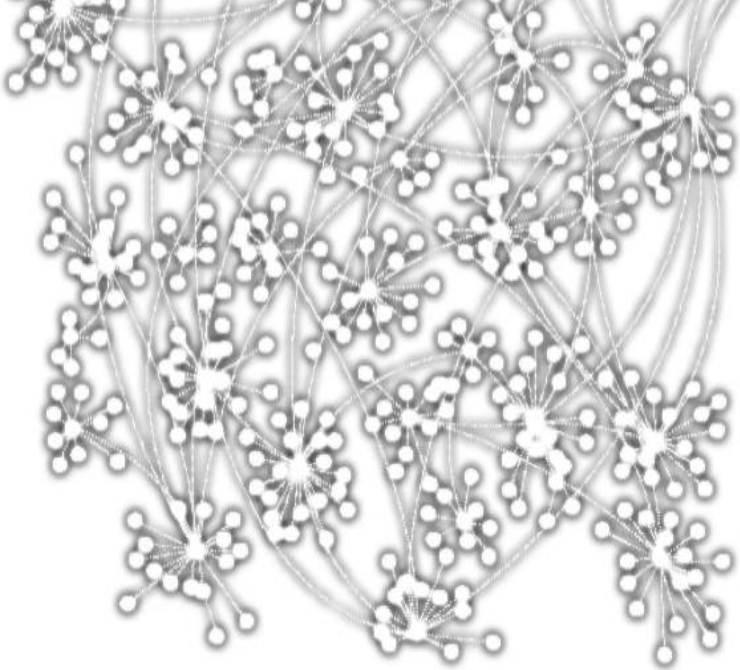










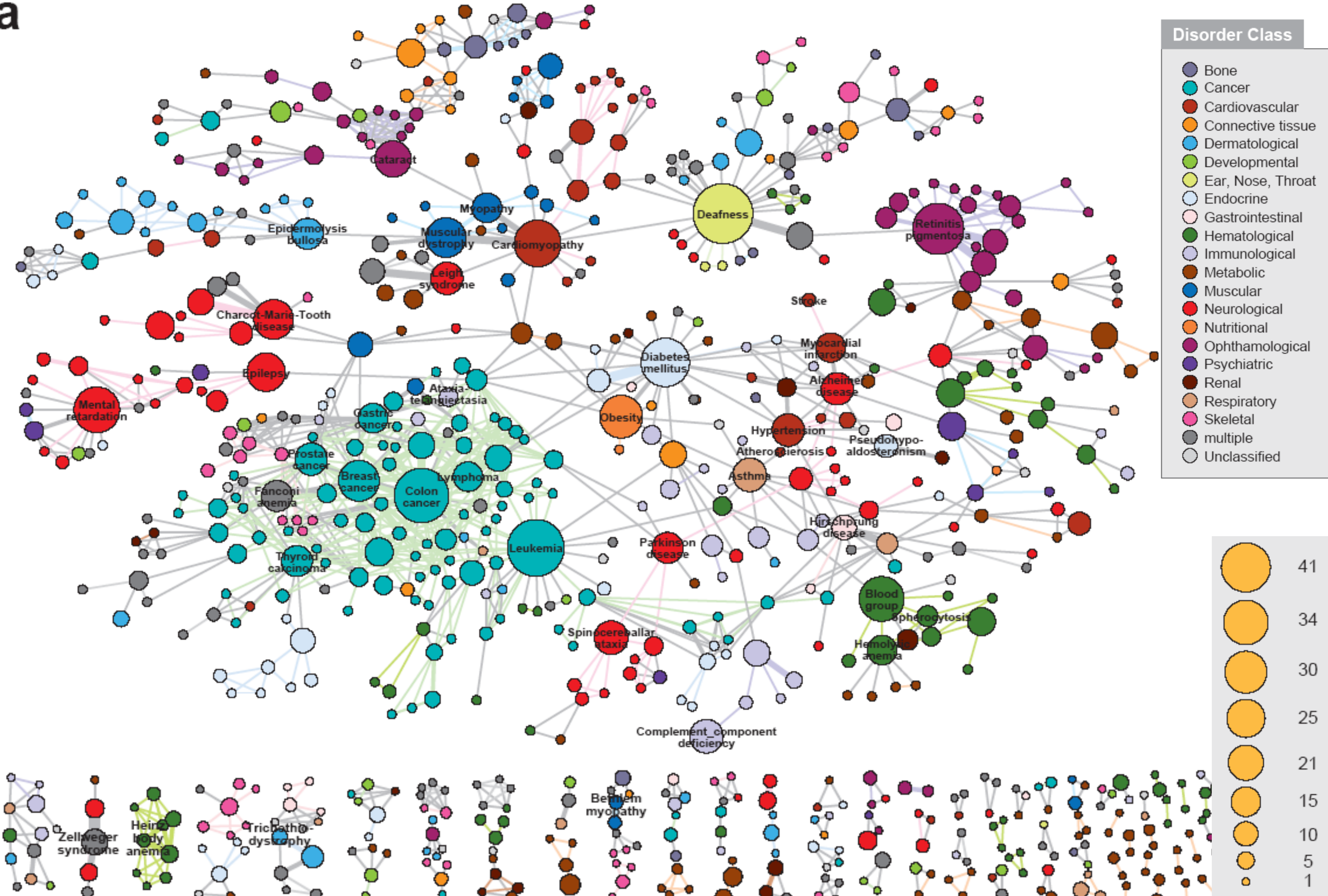


Complex systems

Made of many non-identical **elements** connected by diverse **interactions**.

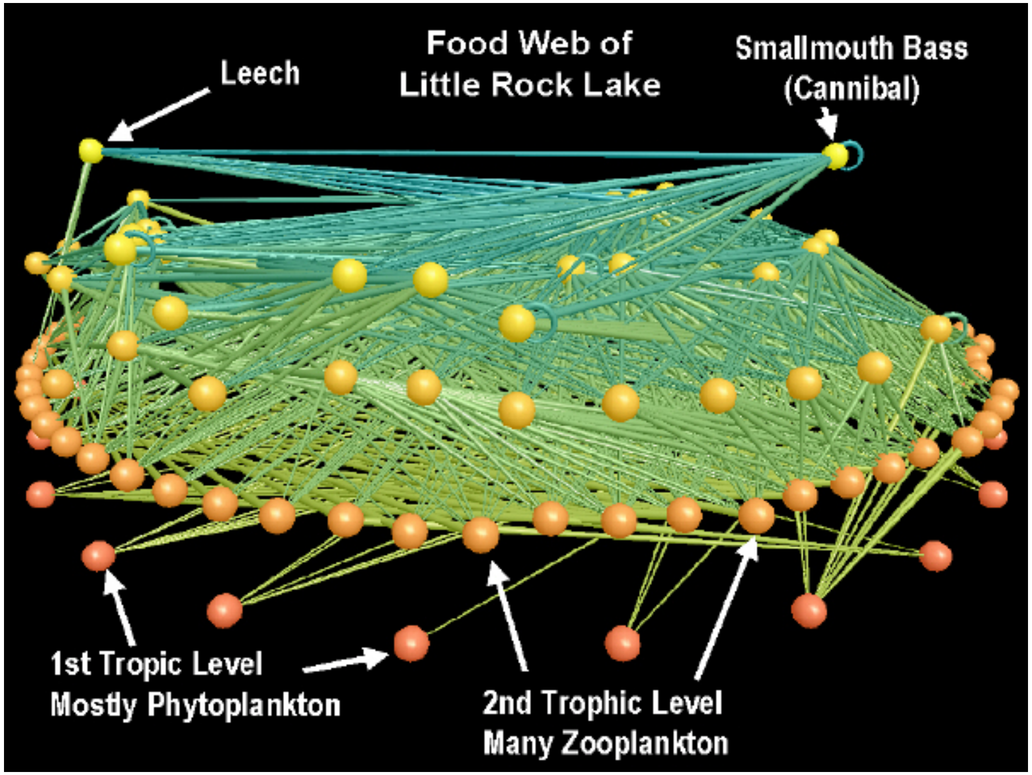
NETWORK

HUMAN DISEASE NETWORK



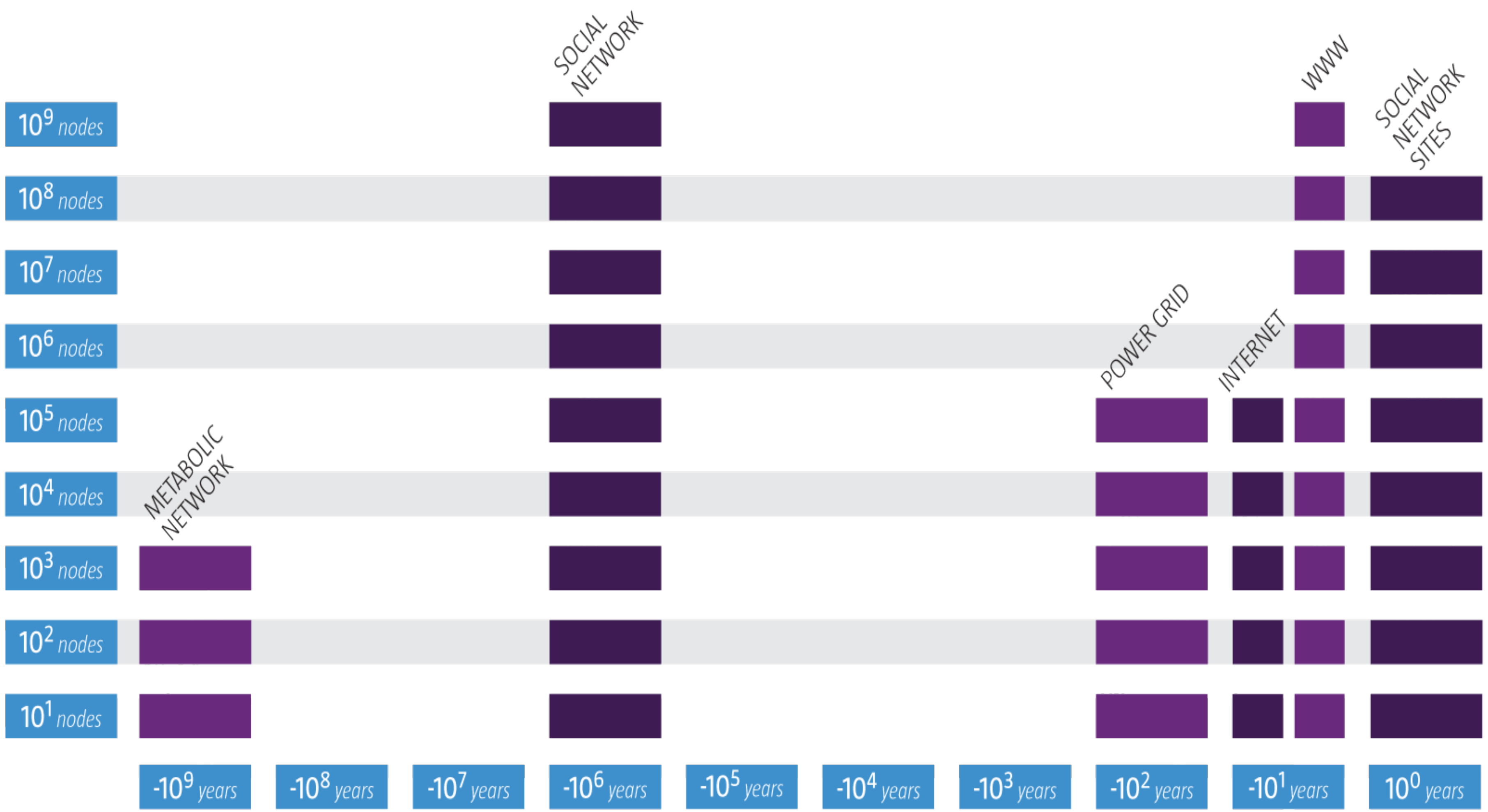
Biological networks: Food Web

Nodes: species
Links: trophic interactions



R. Sole (cond-mat/0011195) R.J. Williams, N.D. Martinez *Nature* (2000)

THE LIFE OF NETWORKS



THE EMERGENCE OF NETWORK SCIENCE

Data Availability:

Movie Actor Network, 1998;
World Wide Web, 1999.
C elegans neural wiring diagram 1990
Citation Network, 1998
Metabolic Network, 2000;
PPI network, 2001

Universality:

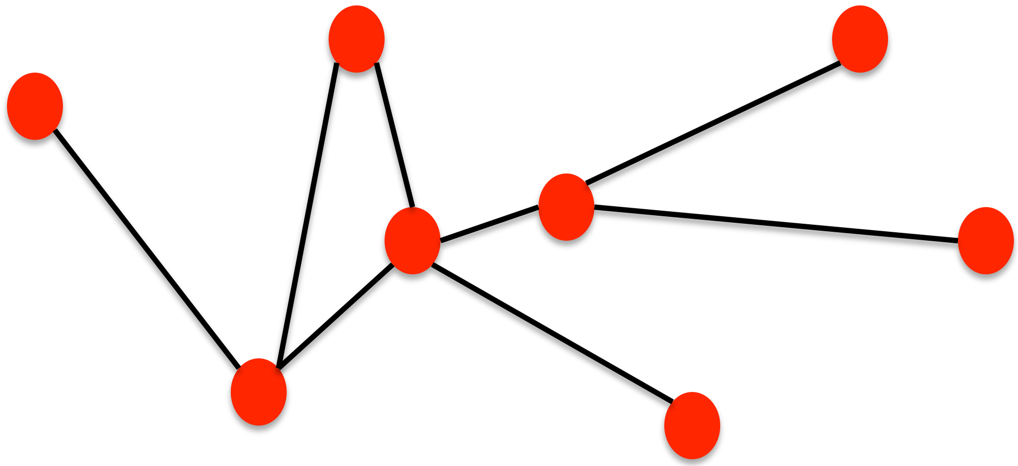
The architecture of networks emerging in various domains of science, nature, and technology are more similar to each other than one would have expected.

The (urgent) need to understand complexity:

Despite the challenges complex systems offer us, we cannot afford to not address their behavior, a view increasingly shared both by scientists and policy makers. Networks are not only essential for this journey, but during the past decade some of the most important advances towards understanding complexity were provided in context of network theory.

Networks and graphs

COMPONENTS OF A COMPLEX SYSTEM



- **components**: nodes, vertices — N
- **interactions**: links, edges — L
- **system**: network, graph — (N,L)

NETWORKS OR GRAPHS?

network often refers to real systems

- www,
- social network
- metabolic network.

Language: (Network, node, link)

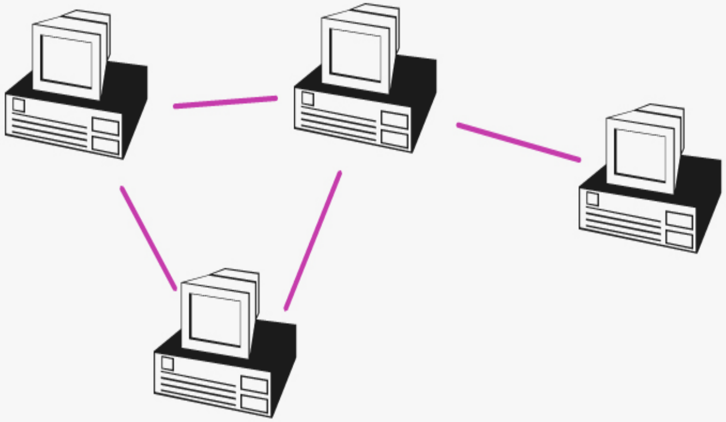
graph: mathematical representation of a network

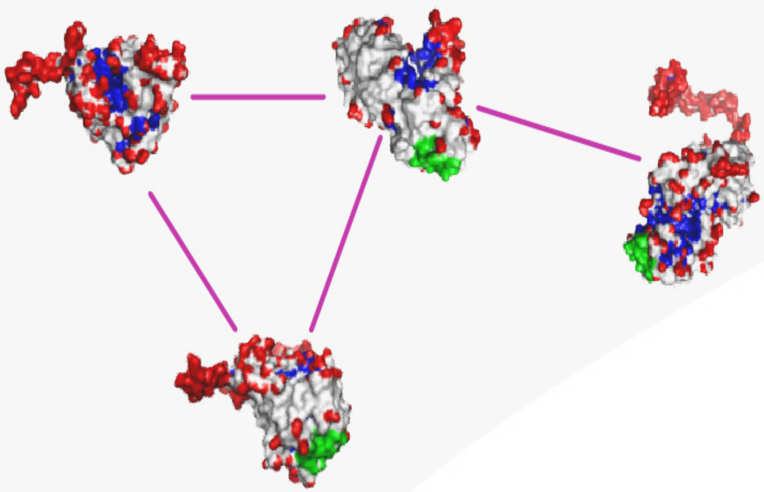
- web graph,
- social graph (a Facebook term)

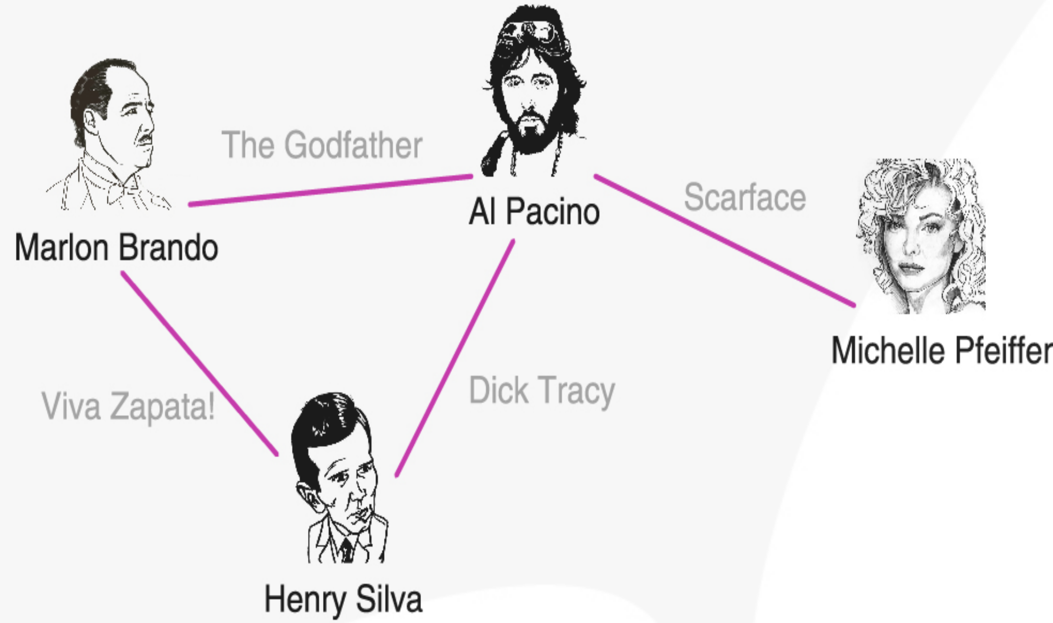
Language: (Graph, vertex, edge)

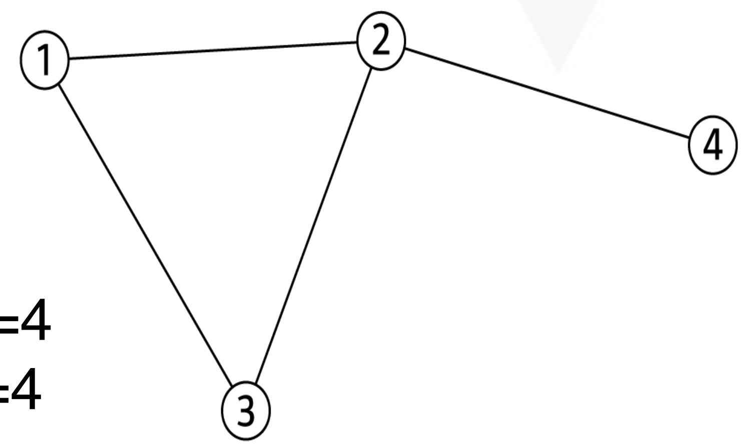
We will try to make this distinction whenever it is appropriate, but in most cases we will use the two terms interchangeably.

A COMMON LANGUAGE









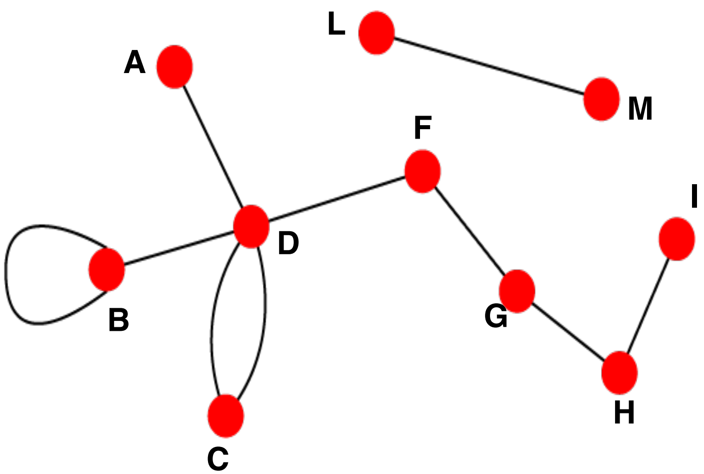
N=4
L=4

UNDIRECTED VS. DIRECTED NETWORKS

Undirected

Links: undirected (*symmetrical*)

Graph:

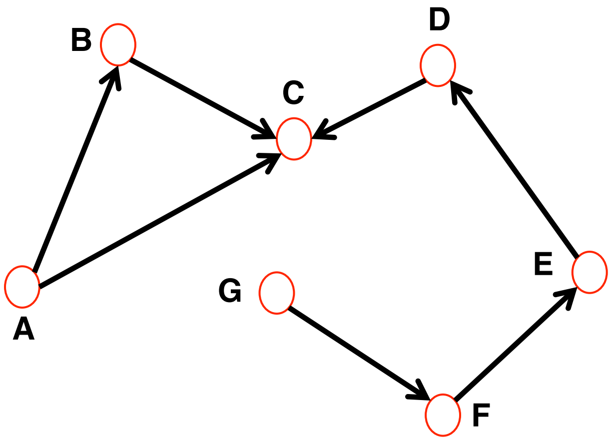


Undirected links :
coauthorship links
Actor network
protein interactions

Directed

Links: directed (*arcs*).

Digraph = directed graph:



An undirected link is the superposition of two opposite directed links.

Directed links :
URLs on the www
phone calls
metabolic reactions

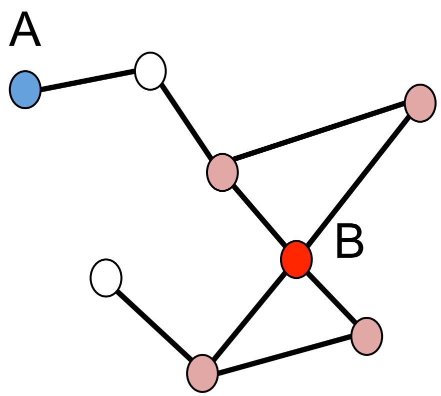
Reference Networks

NETWORK	NODES	LINKS	DIRECTED UNDIRECTED	N	L
Internet	Routers	Internet connections	Undirected	192,244	609,066
WWW	Webpages	Links	Directed	325,729	1,497,134
Power Grid	Power plants, transformers	Cables	Undirected	4,941	6,594
Mobile Phone Calls	Subscribers	Calls	Directed	36,595	91,826
Email	Email addresses	Emails	Directed	57,194	103,731
Science Collaboration	Scientists	Co-authorship	Undirected	23,133	93,439
Actor Network	Actors	Co-acting	Undirected	702,388	29,397,908
Citation Network	Paper	Citations	Directed	449,673	4,689,479
E. Coli Metabolism	Metabolites	Chemical reactions	Directed	1,039	5,802
Protein Interactions	Proteins	Binding interactions	Undirected	2,018	2,930

Degree, Average Degree and Degree Distribution

NODE DEGREES

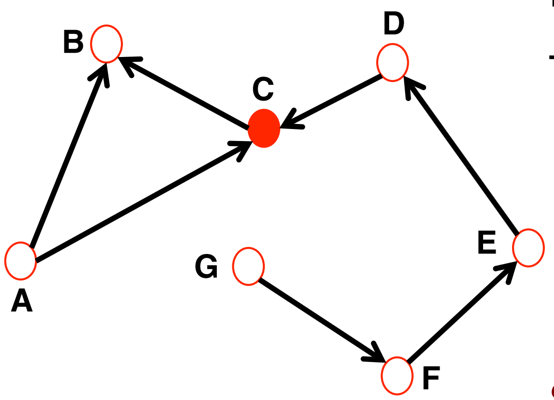
Undirected



Node degree: the number of links connected to the node.

$$k_A = 1 \qquad k_B = 4$$

Directed



In *directed networks* we can define an in-degree and out-degree.

The (total) degree is the sum of in- and out-degree.

$$k_C^{in} = 2 \quad k_C^{out} = 1 \qquad k_C = 3$$

Source: a node with $k^{in} = 0$; Sink: a node with $k^{out} = 0$.

A BIT OF STATISTICS

BRIEF STATISTICS REVIEW

Four key quantities characterize a sample of N values $x_1, \ldots, x_N$:

Average (mean):

$$\langle x \rangle = \frac{x_1 + x_2 + \ldots + x_N}{N} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

The nth *moment*:

$$\langle x^n \rangle = \frac{x_1^n + x_2^n + \ldots + x_N^n}{N} = \frac{1}{N}\sum_{i=1}^{N} x_i^n$$

Standard deviation:

$$\sigma_x = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(x_i - \langle x \rangle\right)^2}$$

Distribution of x:

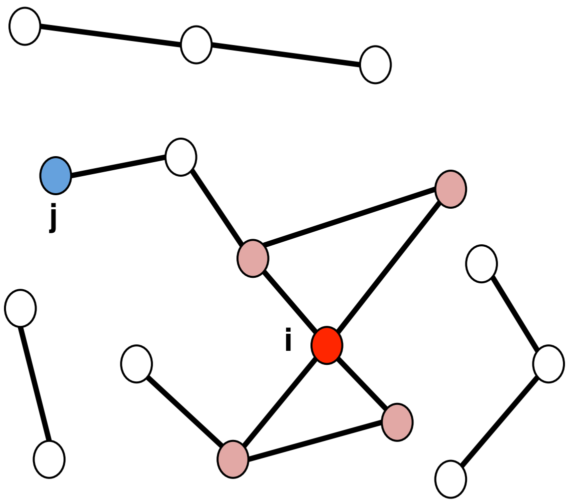
$$p_x = \frac{1}{N}\sum_{i} \delta_{x,x_i}$$

where p_x follows

$$\sum_i p_x = 1 \left(\int p_x \, dx = 1 \right)$$

AVERAGE DEGREE

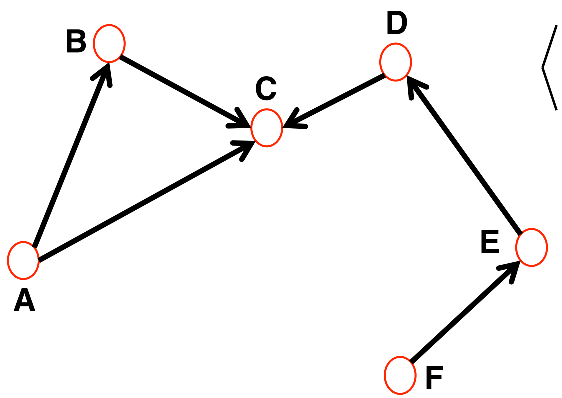
Undirected



$$\langle k \rangle \equiv \frac{1}{N}\sum_{i=1}^{N} k_i \qquad \langle k \rangle \equiv \frac{2L}{N}$$

N – the number of nodes in the graph

Directed



$$\langle k^{in} \rangle \equiv \frac{1}{N}\sum_{i=1}^{N} k_i^{in}, \quad \langle k^{out} \rangle \equiv \frac{1}{N}\sum_{i=1}^{N} k_i^{out}, \quad \langle k^{in} \rangle = \langle k^{out} \rangle$$

$$\langle k \rangle \equiv \frac{L}{N}$$

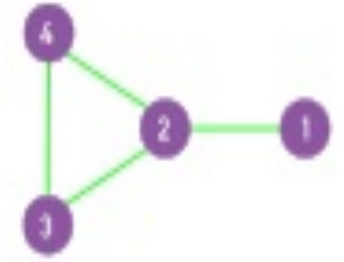
Average Degree

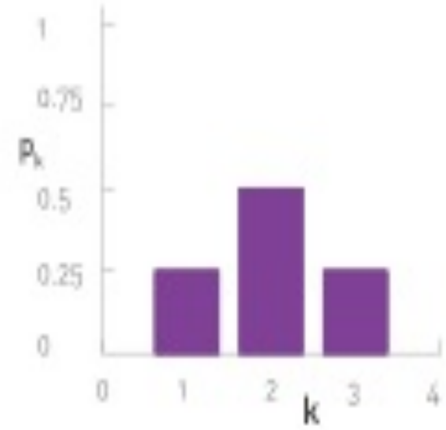
NETWORK	NODES	LINKS	DIRECTED UNDIRECTED	N	L	$\langle k \rangle$
Internet	Routers	Internet connections	Undirected	192,244	609,066	6.33
WWW	Webpages	Links	Directed	325,729	1,497,134	4.60
Power Grid	Power plants, transformers	Cables	Undirected	4,941	6,594	2.67
Mobile Phone Calls	Subscribers	Calls	Directed	36,595	91,826	2.51
Email	Email addresses	Emails	Directed	57,194	103,731	1.81
Science Collaboration	Scientists	Co-authorship	Undirected	23,133	93,439	8.08
Actor Network	Actors	Co-acting	Undirected	702,388	29,397,908	83.71
Citation Network	Paper	Citations	Directed	449,673	4,689,479	10.43
E. Coli Metabolism	Metabolites	Chemical reactions	Directed	1,039	5,802	5.58
Protein Interactions	Proteins	Binding interactions	Undirected	2,018	2,930	2.90

DEGREE DISTRIBUTION

Degree distribution

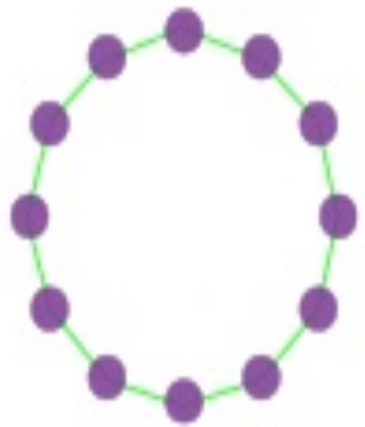
P(k): probability that a
randomly chosen node
has degree *k*

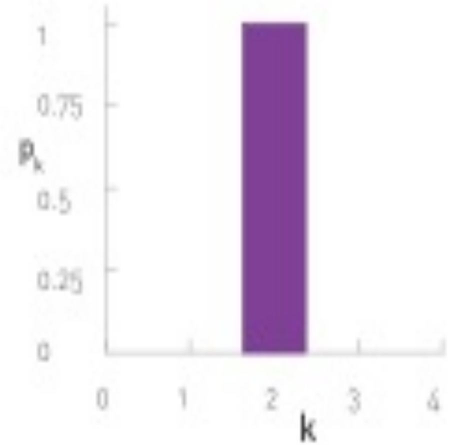




N_k = # nodes with degree k

$P(k) = N_k / N$ ➔ plot





DEGREE DISTRIBUTION

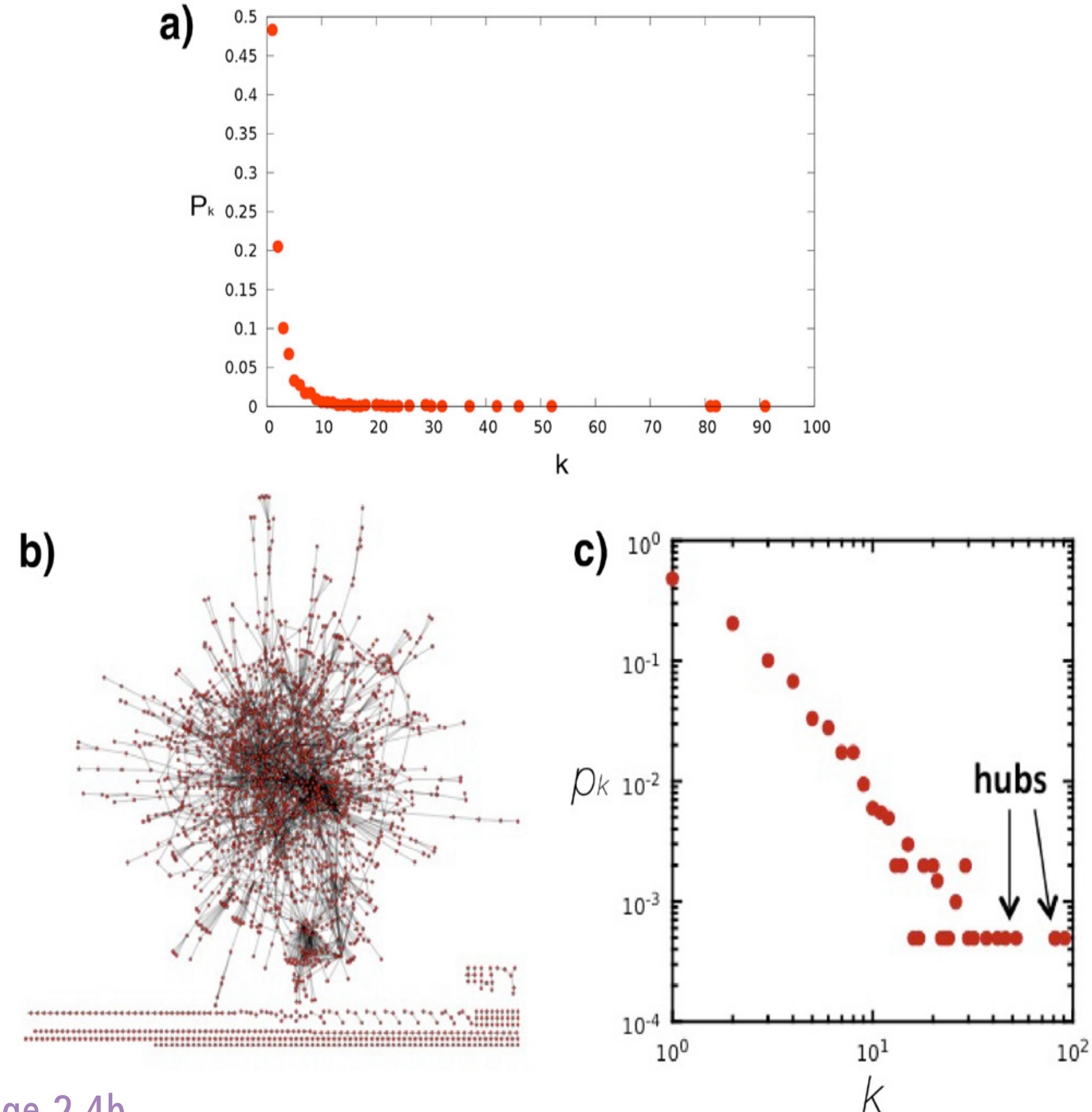


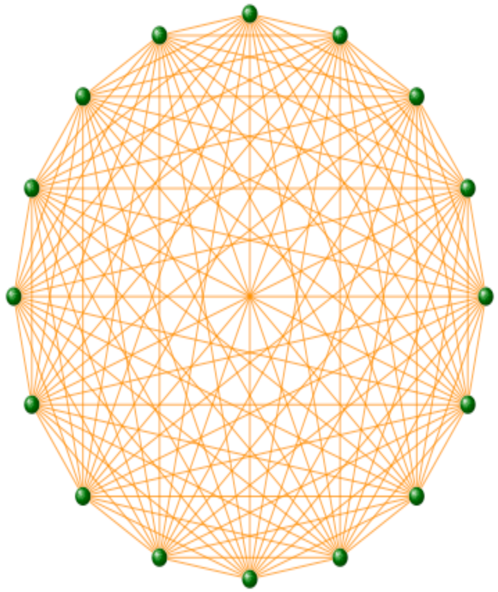
Image 2.4b

Real networks are sparse

COMPLETE GRAPH

The maximum number of links a network of N nodes can have is:

$$L_{\max} = \binom{N}{2} = \frac{N(N-1)}{2}$$



A graph with degree $L=L_{max}$ is called a complete graph, and its average degree is **<k>=N-1**

REAL NETWORKS ARE SPARSE

Most networks observed in real systems are sparse:

$$L \ll L_{max}$$

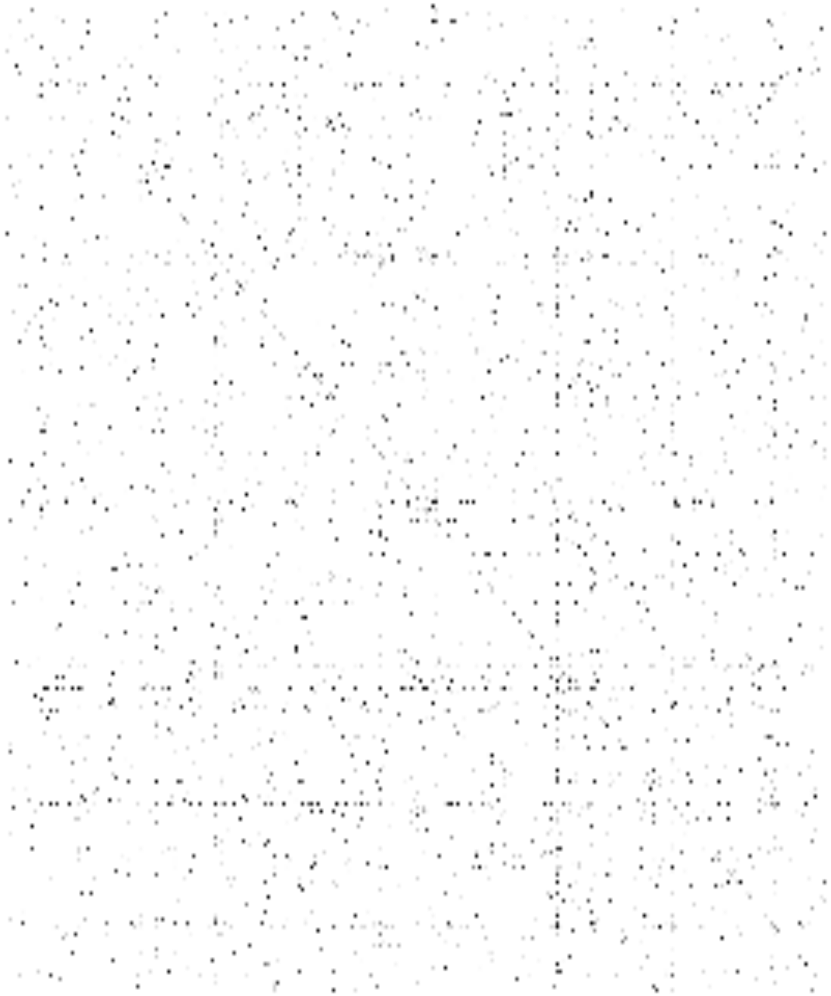
or

$$\langle k \rangle \ll N-1.$$

WWW (ND Sample):	N=325,729;	$L=1.4\ 10^6$	$L_{max}=10^{12}$	$\langle k \rangle=4.51$
Protein (*S. Cerevisiae*):	N= 1,870;	L=4,470	$L_{max}=10^7$	$\langle k \rangle=2.39$
Coauthorship (Math):	N= 70,975;	$L=2\ 10^5$	$L_{max}=3\ 10^{10}$	$\langle k \rangle=3.9$
Movie Actors:	N=212,250;	$L=6\ 10^6$	$L_{max}=1.8\ 10^{13}$	$\langle k \rangle=28.78$

(Source: Albert, Barabasi, RMP2002)

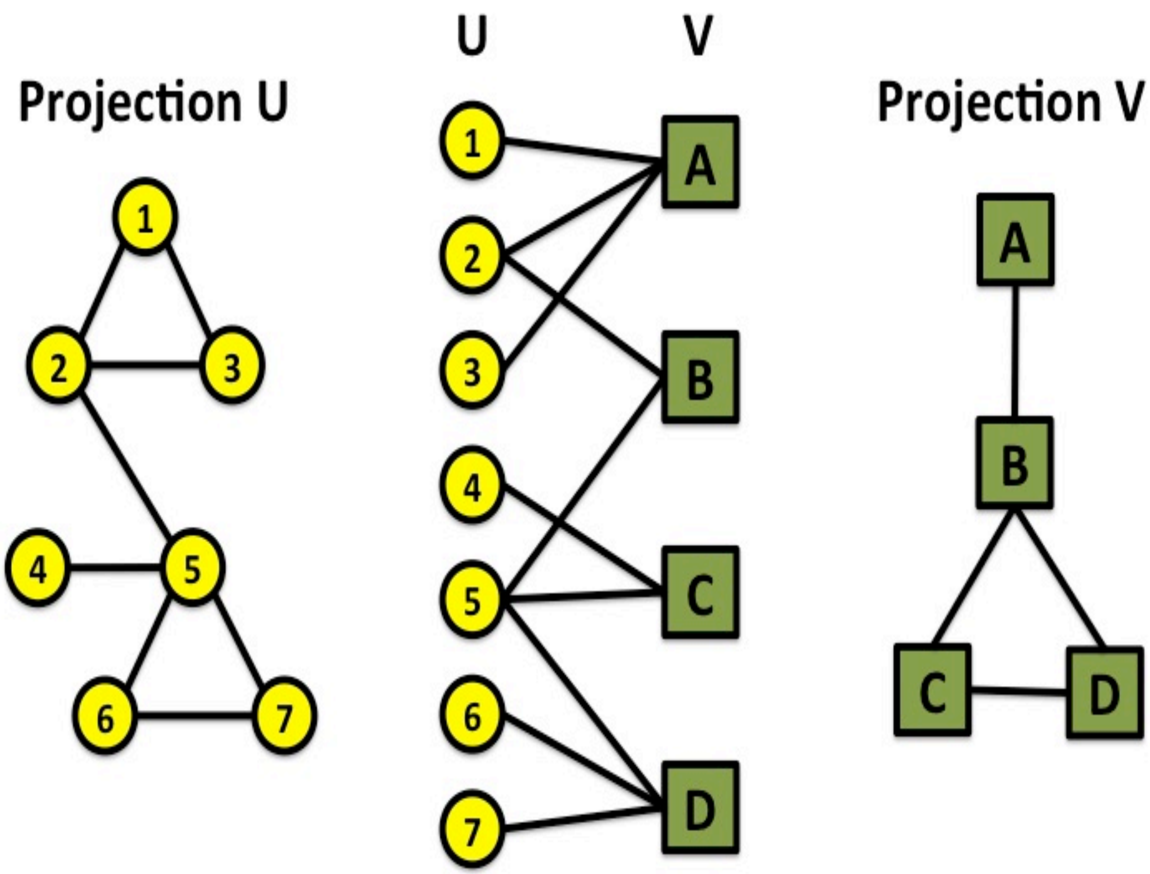
ADJACENCY MATRICES ARE SPARSE



BIPARTITE NETWORKS

BIPARTITE GRAPHS

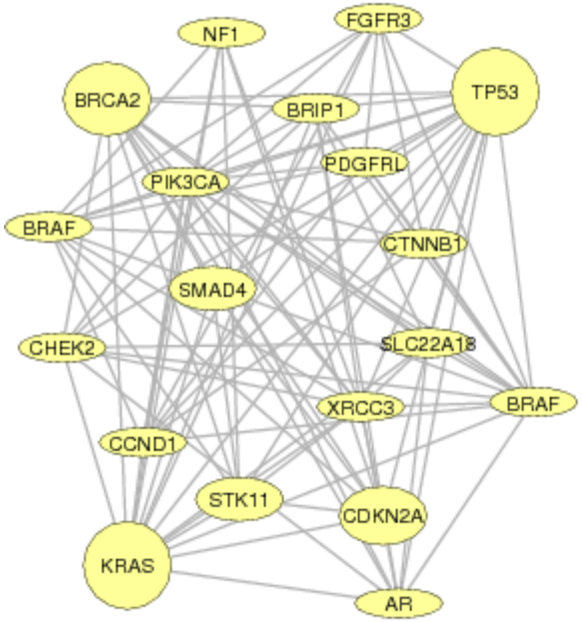
bipartite graph (or **bigraph**) is a graph whose nodes can be divided into two disjoint sets *U* and *V* such that every link connects a node in *U* to one in *V*; that is, *U* and *V* are independent sets.



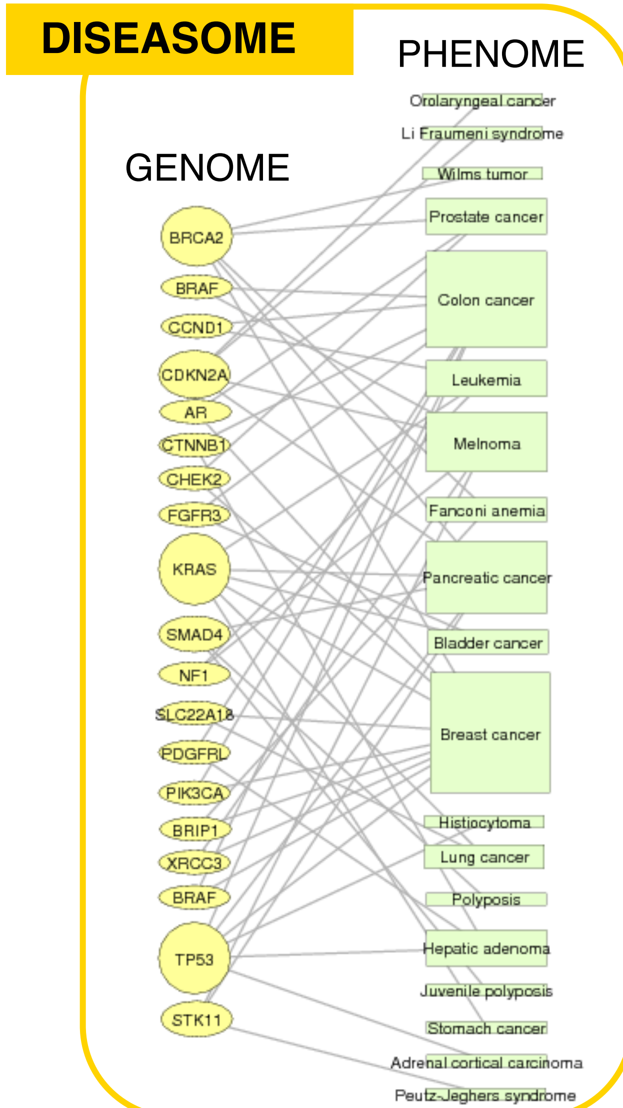
Examples:

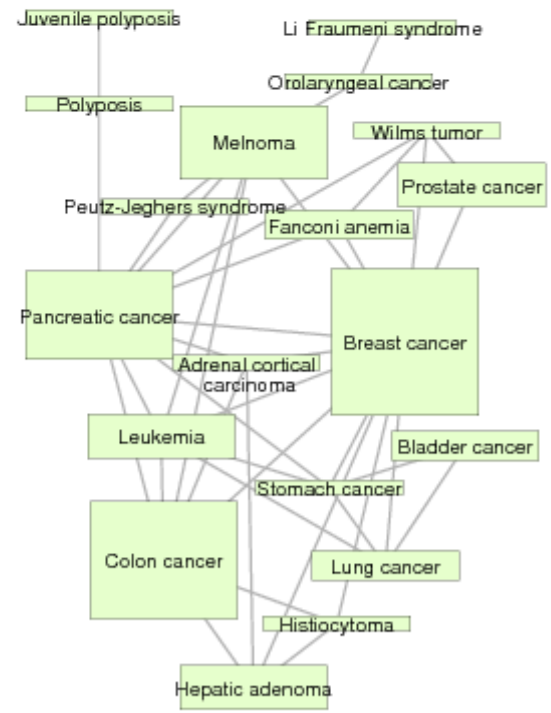
Hollywood actor network
Collaboration networks
Disease network (diseasome)

GENE NETWORK – DISEASE NETWORK



Gene network

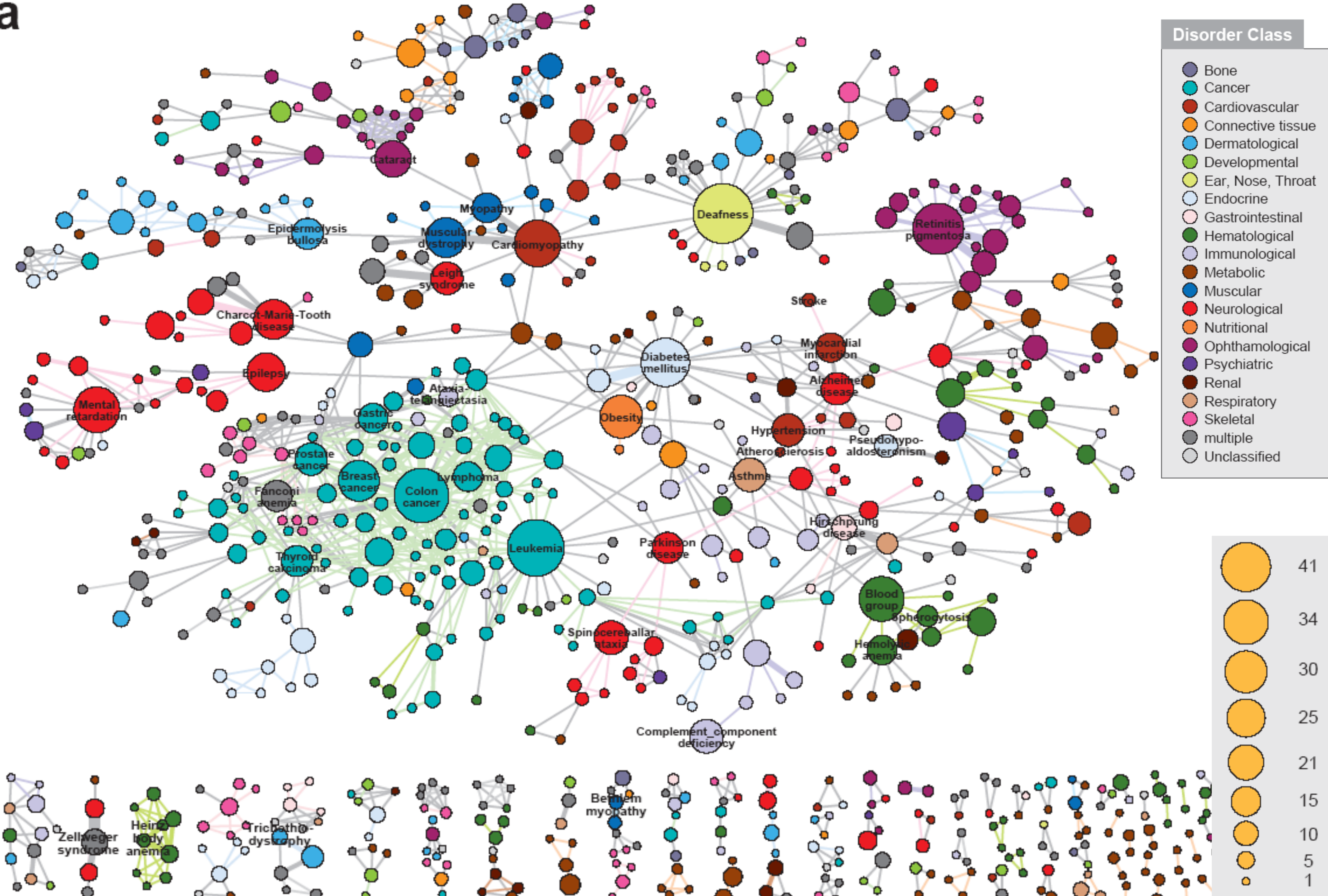




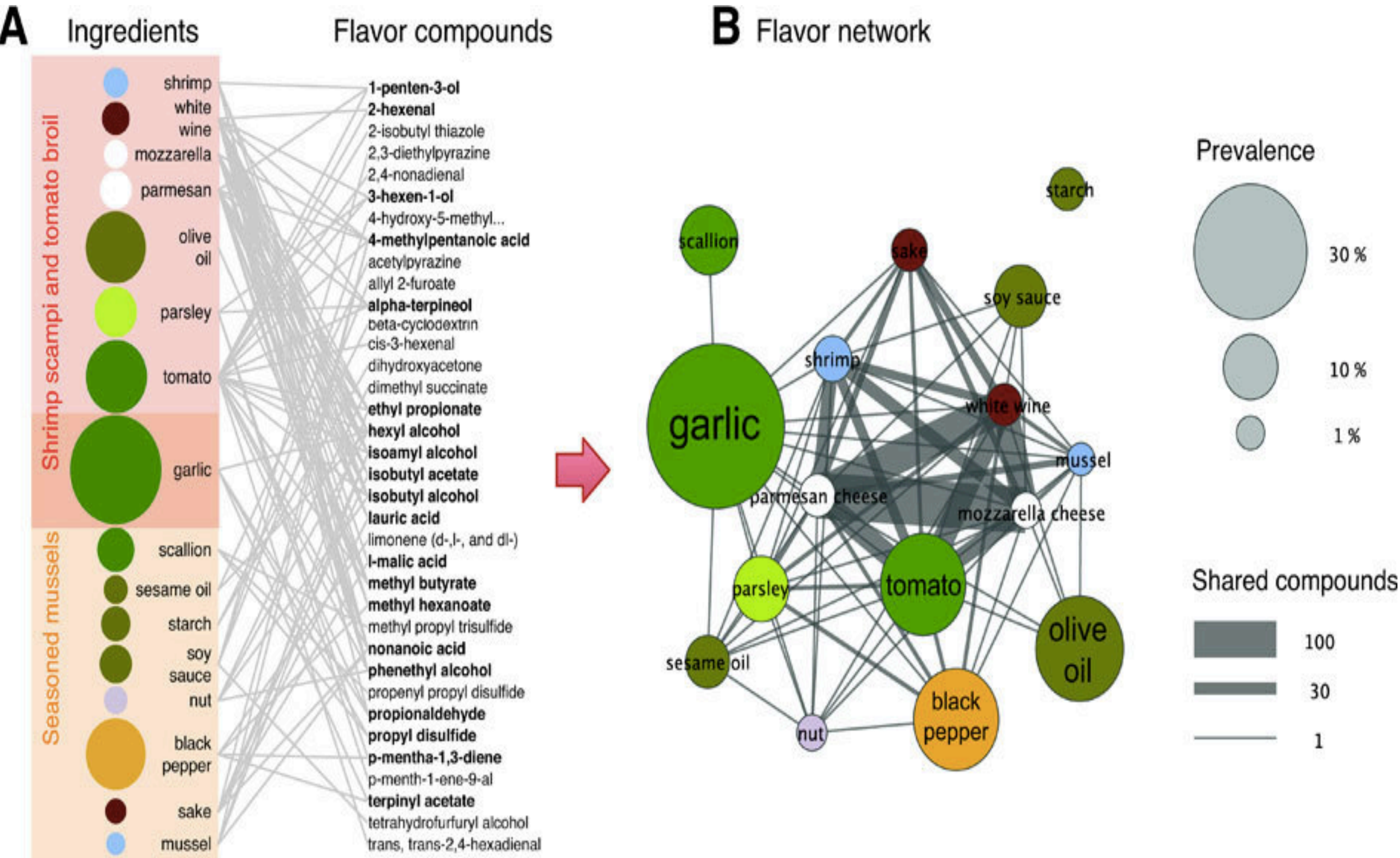
Disease network

Goh, Cusick, Valle, Childs, Vidal & Barabási, PNAS (2007)

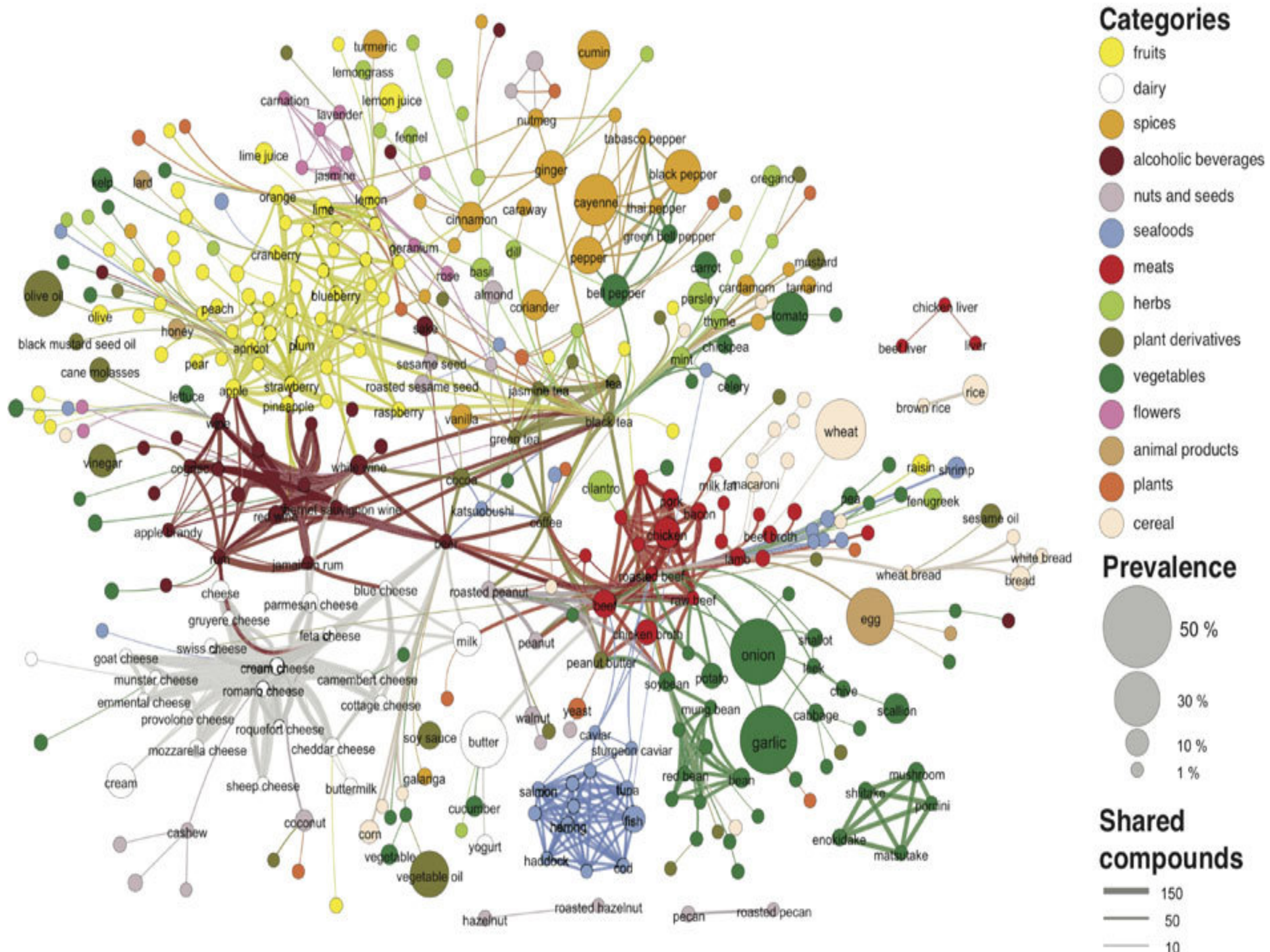
HUMAN DISEASE NETWORK



Ingredient-Flavor Bipartite Network



Y.-Y. Ahn, S. E. Ahnert, J. P. Bagrow, A.-L. Barabási
Flavor network and the principles of food pairing , Scientific Reports 196, (2011).



Categories
fruits
dairy
spices
alcoholic beverages
nuts and seeds
seafoods
meats
herbs
plant derivatives
vegetables
flowers
animal products
plants
cereal
Prevalence
50 %
30 %
10 %
1 %
Shared compounds
150
50
10
turmeric
lemongrass
carnation
lavender
lemon juice
cumin
nutmeg
fennel
tabasco pepper
lime juice
jasmine
ginger
black pepper
kelp
lard
orange
lime
lemon
cinnamon
caraway
cayenne
thai pepper
oregano
cranberry
geranium
green bell pepper
dill
pepper
carrot
mustard
rose
basil
olive oil
blueberry
almond
coriander
bell pepper
parsley
cardamom
tamarind
olive
peach
sake
thyme
tomato
chicken liver
honey
black mustard seed oil
apricot
plum
chickpea
beef liver
liver
sesame seed
pear
mint
cane molasses
roasted sesame seed
jasmine tea
tea
celery
lettuce
apple
strawberry
rice
pineapple
brown rice
wine
raspberry
vanilla
black tea
wheat
green tea
vinegar
cognac
white wine
raisin
shrimp
cocoa
cilantro
milk fat
macaroni
pork
pea
fenugreek
katsuobushi
cabernet sauvignon wine
red wine
bacon
sesame oil
coffee
chicken
apple brandy
beef broth
beer
lamb
white bread
rum
jamaican rum
roasted beef
wheat bread
bread
cheese
blue cheese
roasted peanut
parmesan cheese
beef
raw beef
egg
gruyere cheese
feta cheese
milk
chicken broth
swiss cheese
peanut
shallot
onion
goat cheese
cream cheese
peanut butter
leek
munster cheese
camembert cheese
soybean
potato
chive
emmental cheese
romano cheese
cottage cheese
mung bean
scallion
walnut
yeast
cabbage
provolone cheese
roquefort cheese
soy sauce
butter
caviar
garlic
mozzarella cheese
cheddar cheese
sturgeon caviar
mushroom
red bean
bean
galanga
cream
sheep cheese
buttermilk
salmon
tuna
shiitake
porcini
cucumber
fish
herring
cashew
coconut
enokidake
corn
yogurt
matsutake
vegetable
haddock
cod
vegetable oil
roasted hazelnut
roasted pecan
hazelnut
pecan

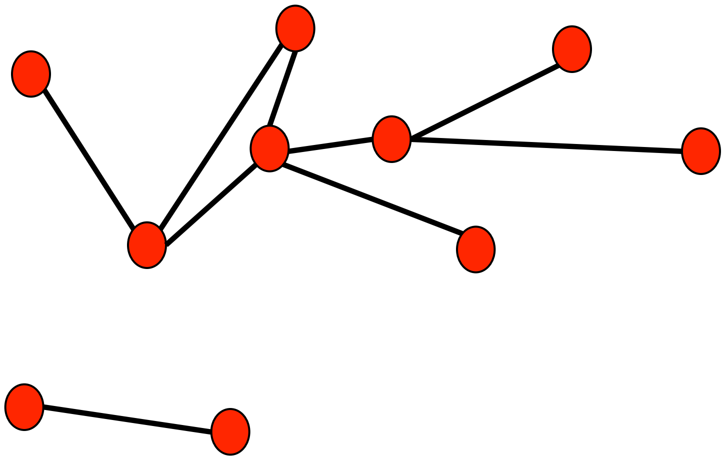
Basic network measures

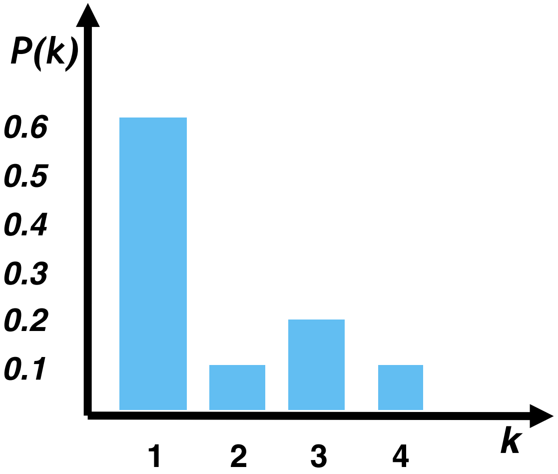
Degree of a node
Distance between two nodes
Clustering among three nodes

Degree distribution P(k): probability that a randomly chosen vertex has degree k

N_k = # nodes with degree k

$P(k) = N_k / N$ ➔ plot



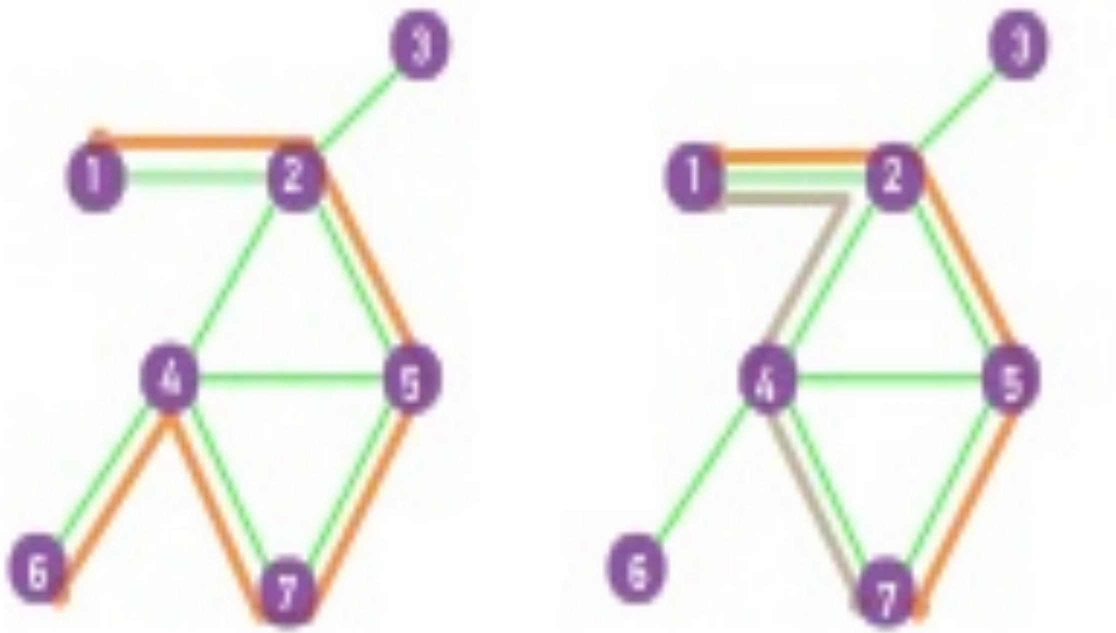


PATHS

A *path is* a sequence of nodes in which each node is adjacent to the next one

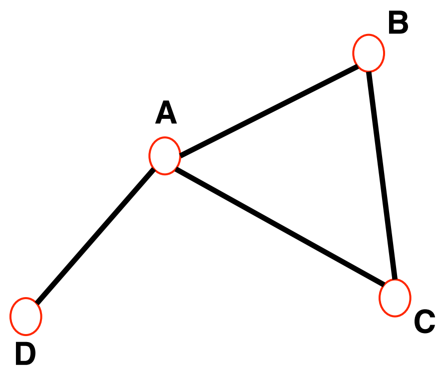
$P_{i0,in}$ of length *n* between nodes i_0 and i_n is an ordered collection of *n+1* nodes and *n* links

$$P_n = \{i_0, i_1, i_2, \ldots, i_n\} \qquad P_n = \{(i_0, i_1), (i_1, i_2), (i_2, i_3), \ldots, (i_{n-1}, i_n)\}$$



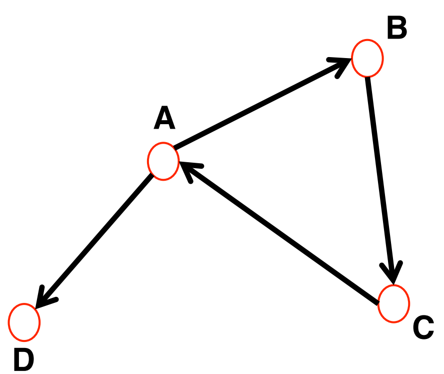
- In a directed network, the path can follow only the direction of an arrow.

Shortest Path, Geodesic Path



The *distance (shortest path, geodesic path)* between two nodes is defined as the number of edges along the shortest path connecting them.

*If the two nodes are disconnected, the distance is infinity.



In directed graphs each path needs to follow the direction of the arrows.

Thus in a digraph the distance from node A to B (on an AB path) is generally different from the distance from node B to A (on a BCA path).

NETWORK DIAMETER AND AVERAGE DISTANCE

Diameter: d_{max} the maximum distance between any pair of nodes in the graph.

Average path length/distance, <d>, for a connected graph:

where d_{ij} is the distance from node i to node j

$$\langle d \rangle \equiv \frac{1}{2L_{\max}} \sum_{i,j \neq i} d_{ij}$$

In an *undirected graph* $d_{ij} = d_{ji}$, *so* we only need to count them once:

$$\langle d \rangle \equiv \frac{1}{L_{\max}} \sum_{i,j>i} d_{ij}$$

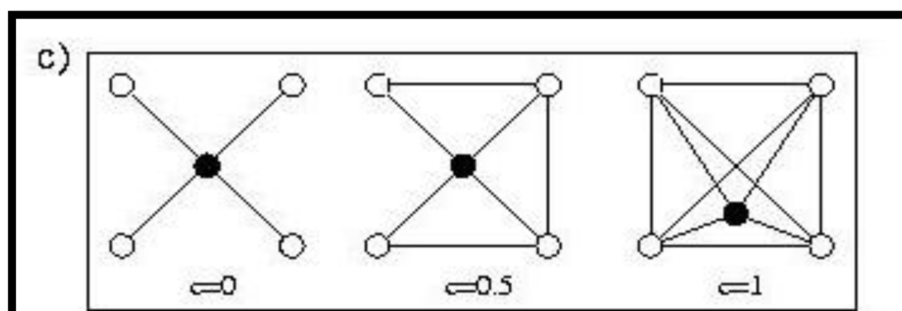
CLUSTERING COEFFICIENT

* Clustering coefficient:

what portion of your neighbors are connected?

* Node i with degree k_i
* C_i in [0,1]

$$C_i = \frac{2e_i}{k_i(k_i - 1)}$$



KEY MEASURES

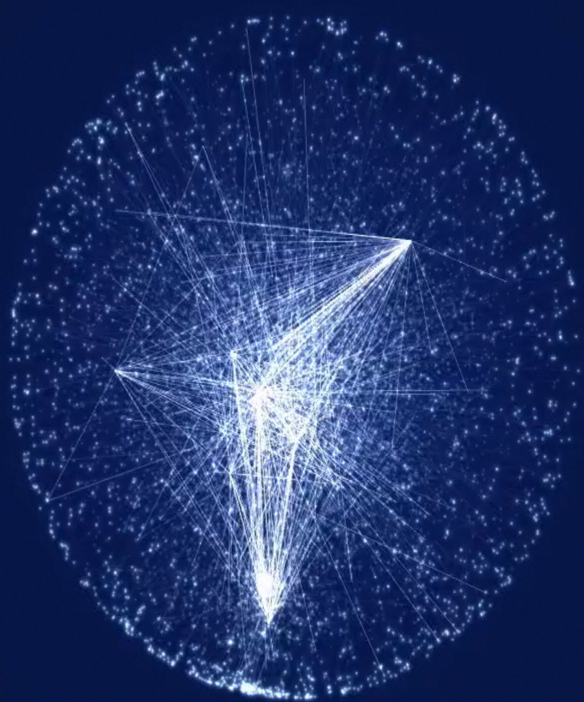
Degree distribution: **P(k)**

Path length: ***l***

Clustering coefficient:

$$\boxed{C_i = \frac{2e_i}{k_i(k_i - 1)}}$$

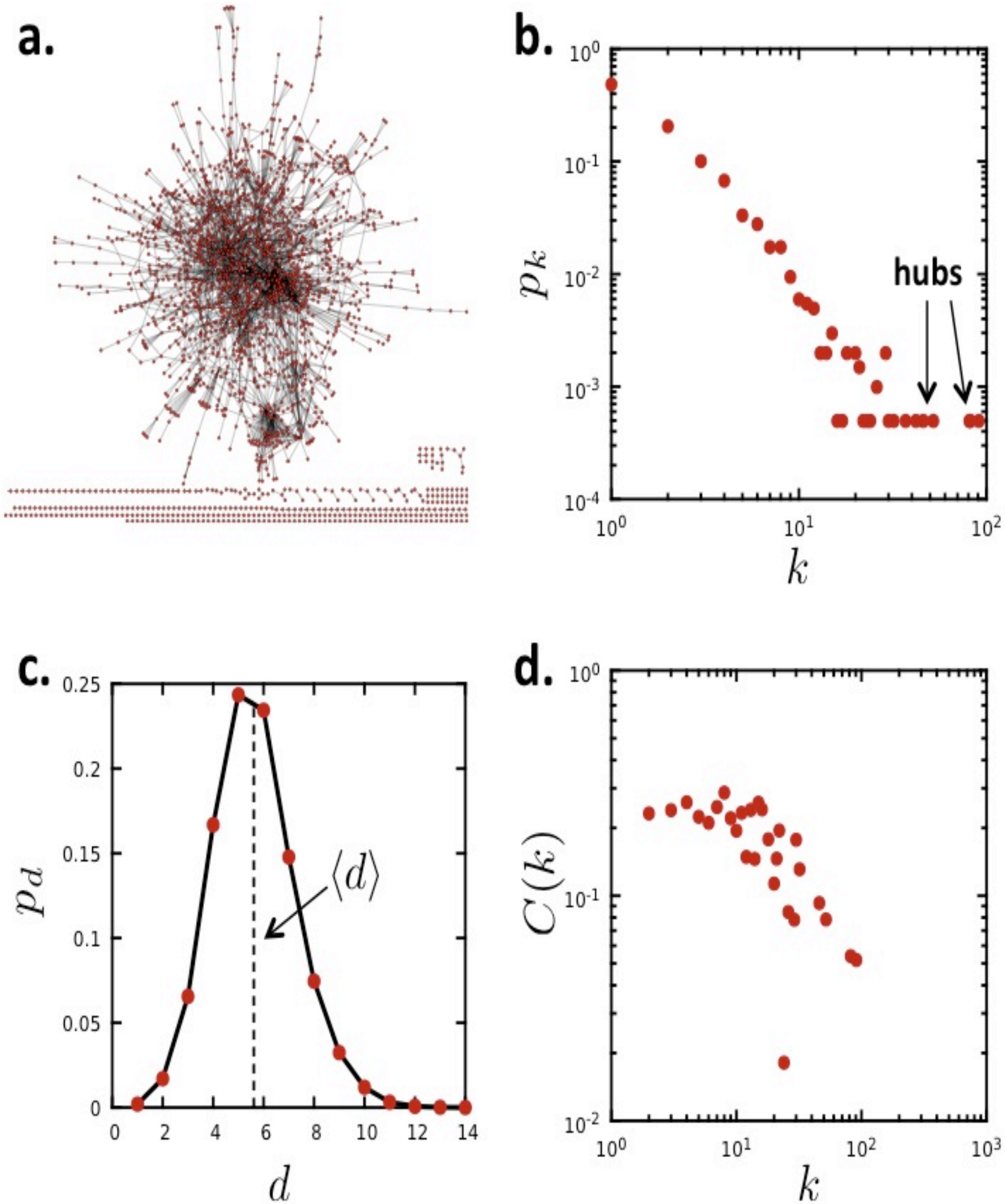
A CASE STUDY: PROTEIN-PROTEIN INTERACTION NETWORK



Undirected network
N=2,018 proteins as nodes
L=2,930 binding interactions as links.
Average degree <k>=2.90.

Not connected: 185 components
the largest (giant component)
1,647 nodes

A CASE STUDY: PROTEIN-PROTEIN INTERACTION NETWORK



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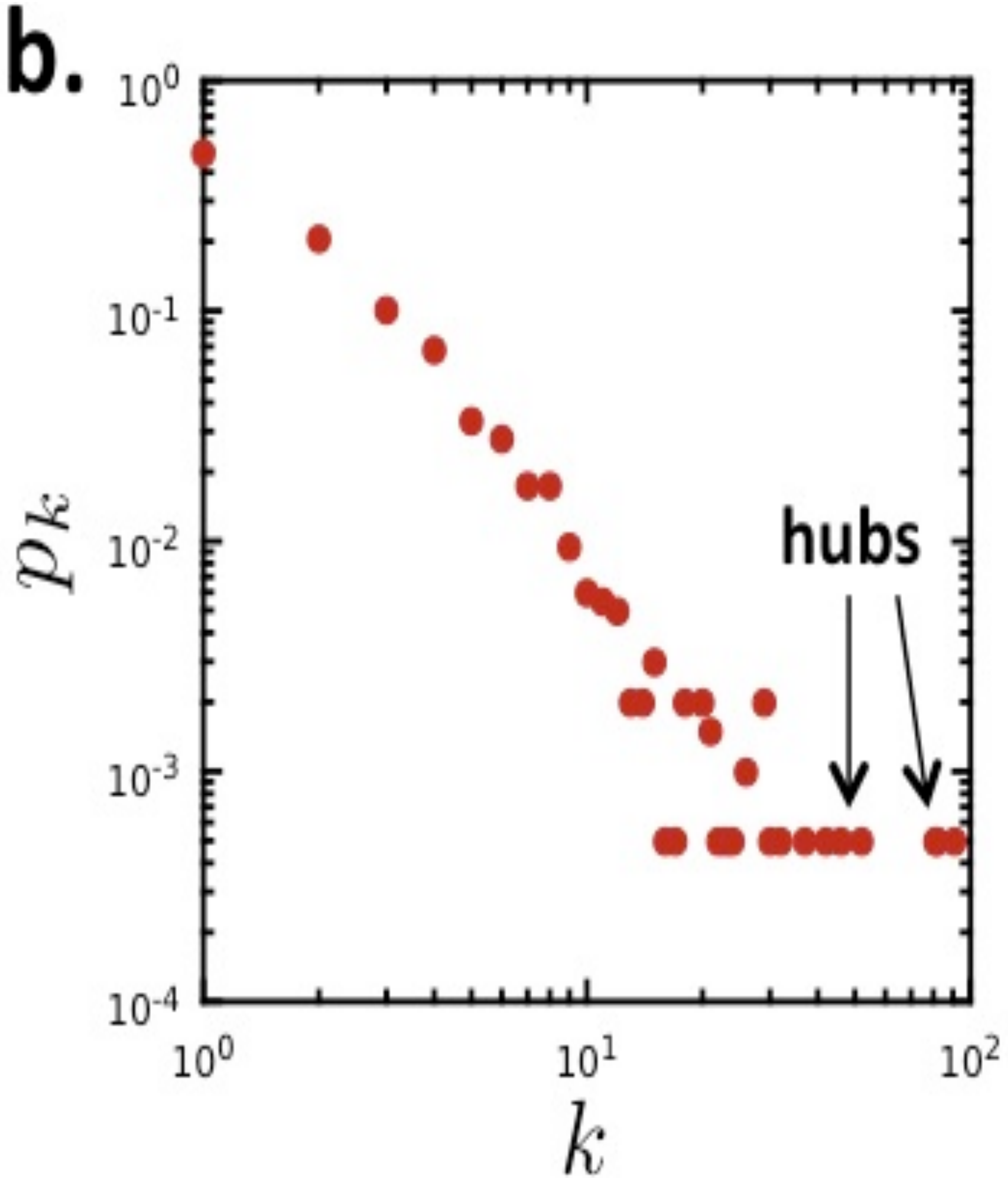
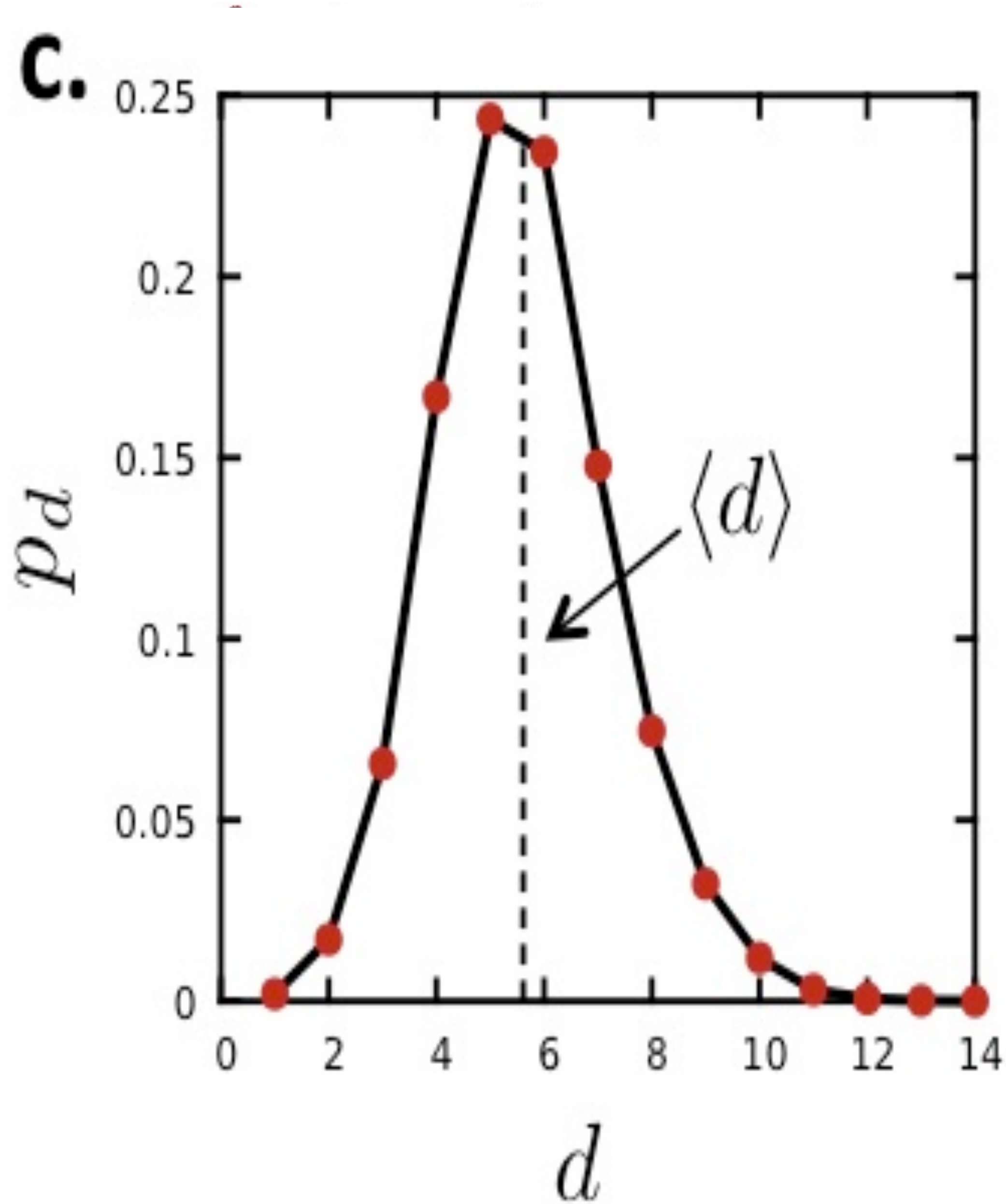


p_k is the probability that a node has degree **k**.

N_k = # nodes with degree k

$p_k = N_k / N$

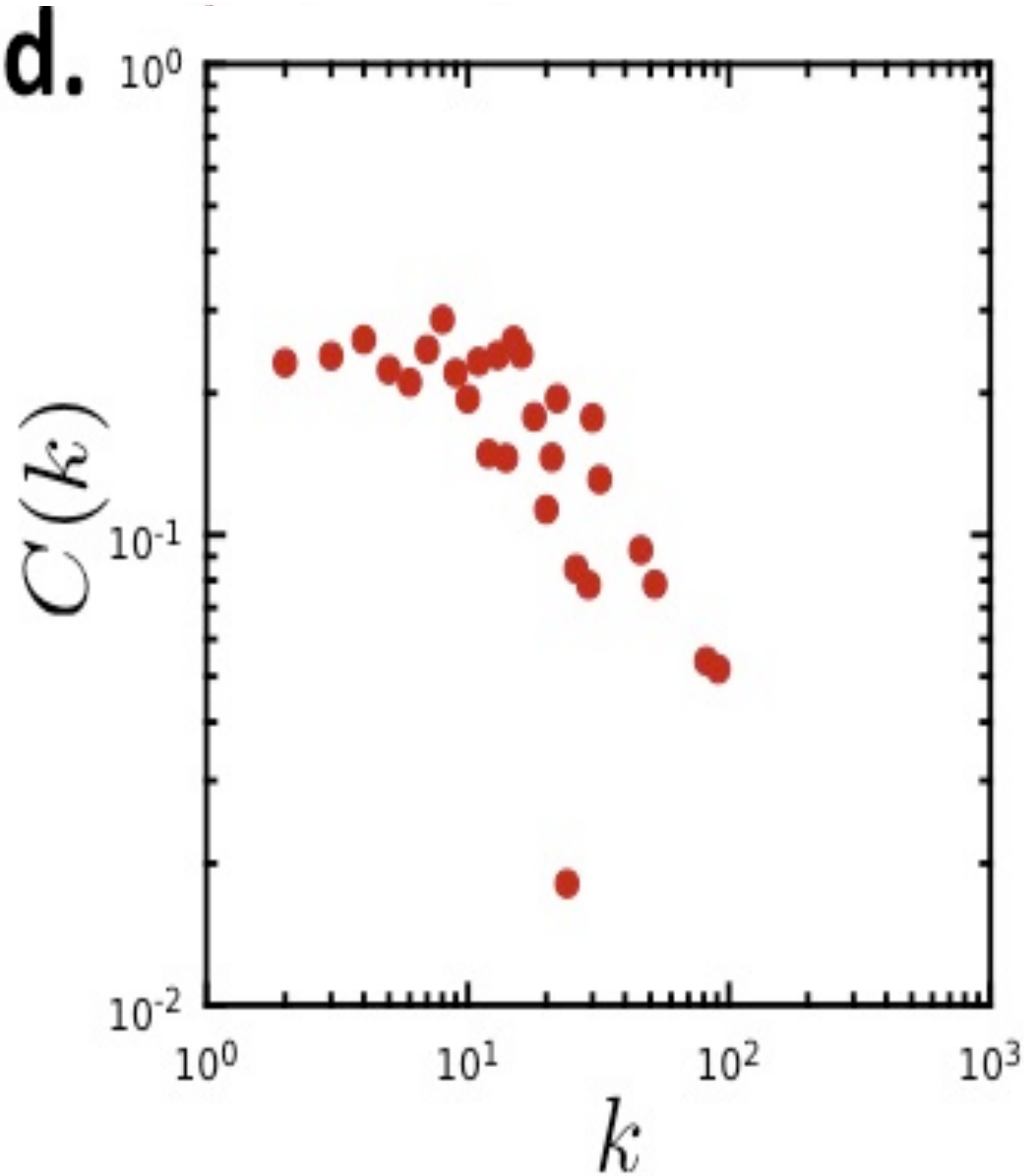
A CASE STUDY: PROTEIN-PROTEIN INTERACTION NETWORK

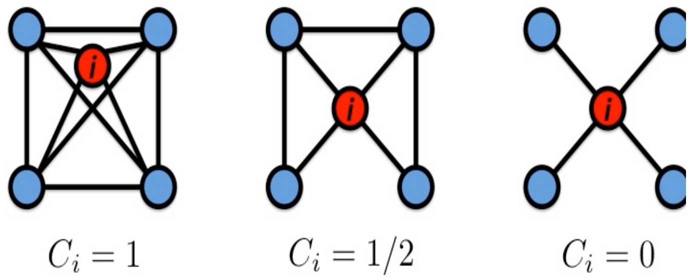


d_{max}=14

<d>=5.61

A CASE STUDY: PROTEIN-PROTEIN INTERACTION NETWORK





$$C_i = \frac{2e_i}{k_i(k_i - 1)}$$

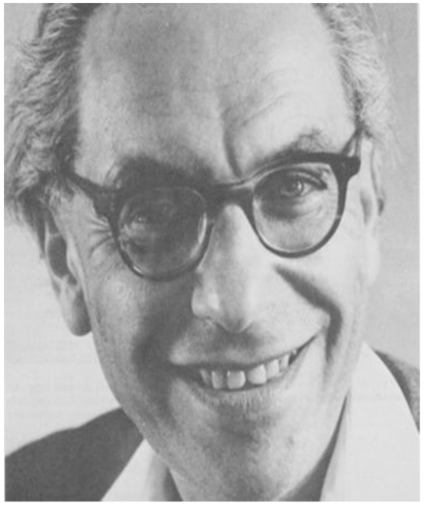
<C>=0.12

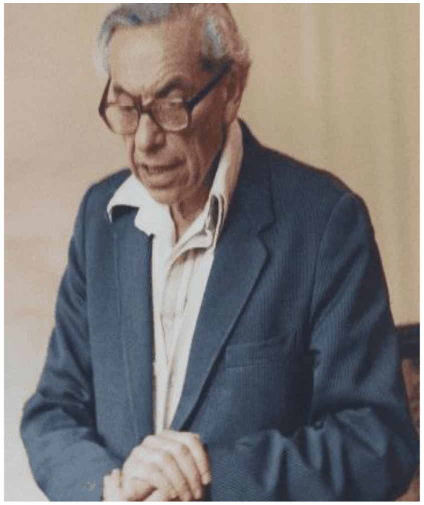
Random graphs

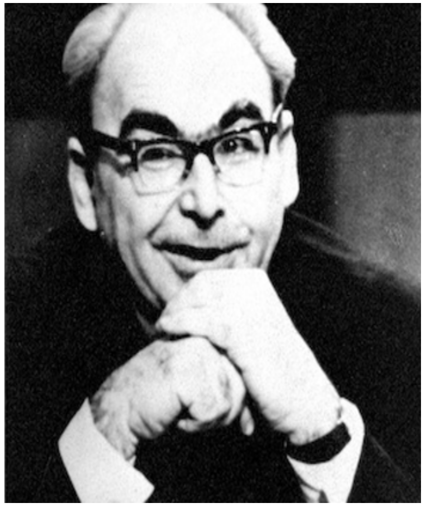
What are the expected basic measures emerging from random?

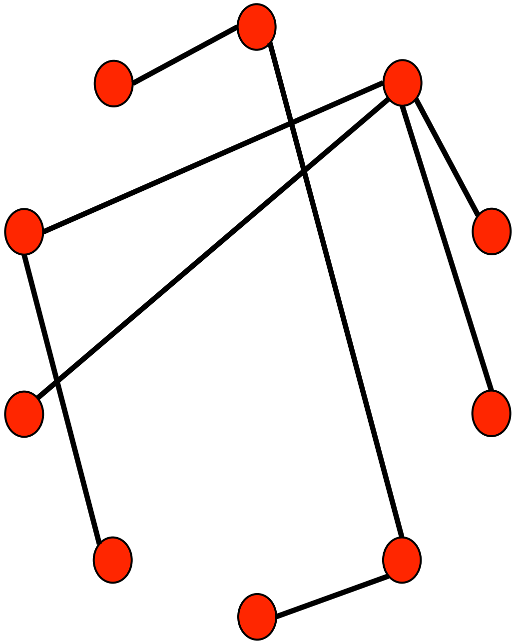
RANDOM NETWORK MODEL

Pául Erdös
(1913-1996)









Erdös-Rényi model (1960)

Connect with probability p

p=1/6 N=10

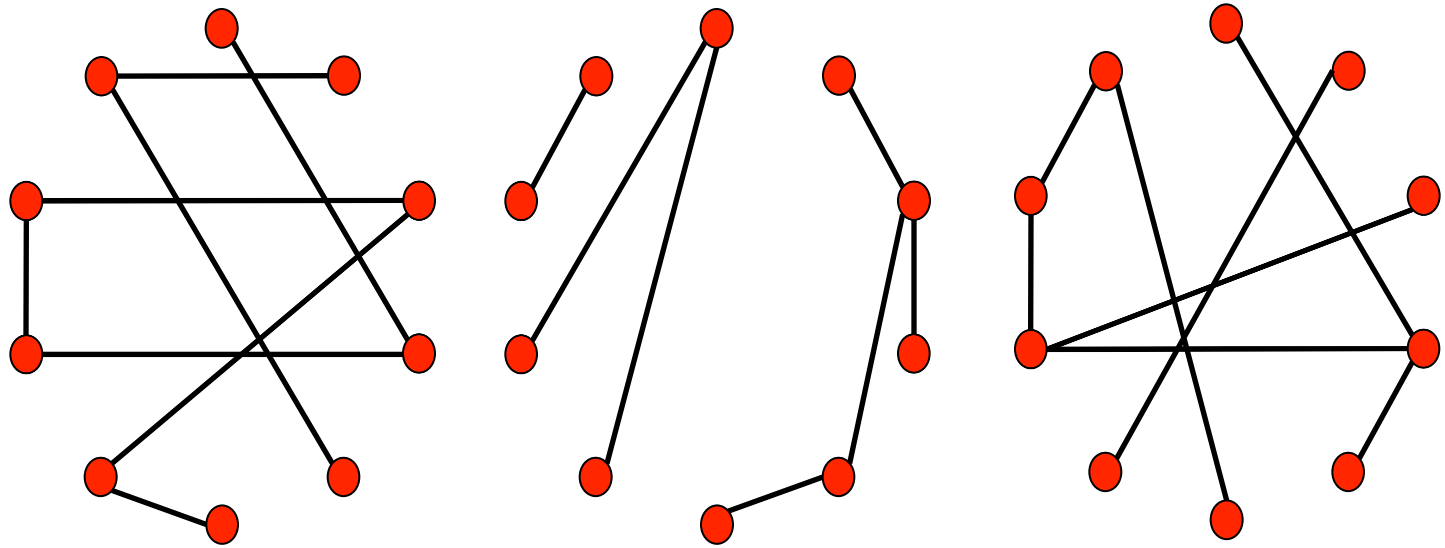
$\langle k \rangle \sim 1.5$

RANDOM NETWORK MODEL

Definition: A **random graph** is a graph of N labeled nodes where each pair of nodes is connected by a preset probability **p**.

RANDOM NETWORK MODEL

N and ***p*** do not uniquely define the network– we can have many different realizations of it. **How many?**



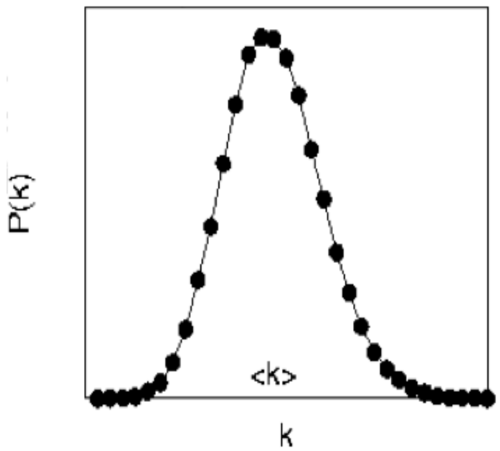
N=10
p=1/6

The probability to form a *particular* graph **G(N,L)** is

$$P(G(N,L)) = p^L (1-p)^{\frac{N(N-1)}{2}-L}$$

That is, each graph **G(N,L)** appears with probability **P(G(N,L))**.

DEGREE DISTRIBUTION OF A RANDOM GRAPH



$$P(k) = \binom{N-1}{k} p^k (1-p)^{(N-1)-k}$$

Select k nodes from N-1

probability of having *k* edges

probability of missing N-1-k edges

$$< k >= p(N-1) \qquad \sigma_k^2 = p(1-p)(N-1)$$

$$\frac{\sigma_k}{< k >} = \left[\frac{1-p}{p}\frac{1}{(N-1)}\right]^{1/2} \approx \frac{1}{(N-1)^{1/2}}$$

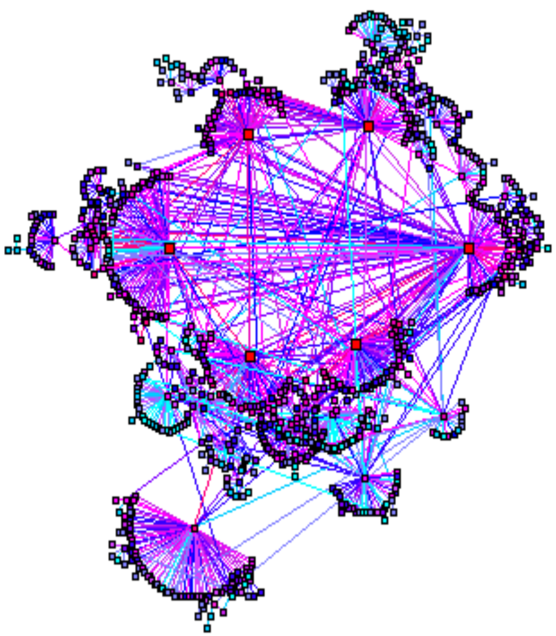
As the network size increases, the distribution becomes increasingly narrow—we are increasingly confident that the degree of a node is in the vicinity of <k>.

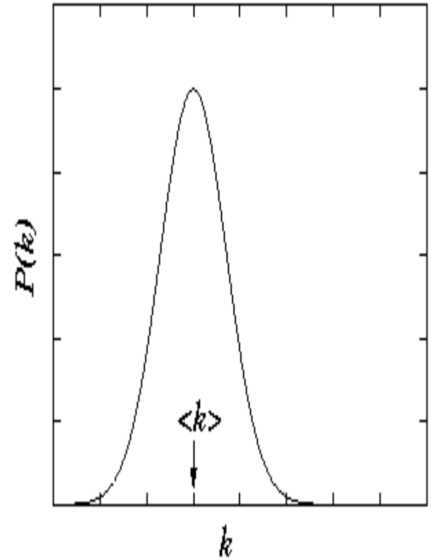
WORLD WIDE WEB

Nodes: **WWW documents**
Links: **URL links**

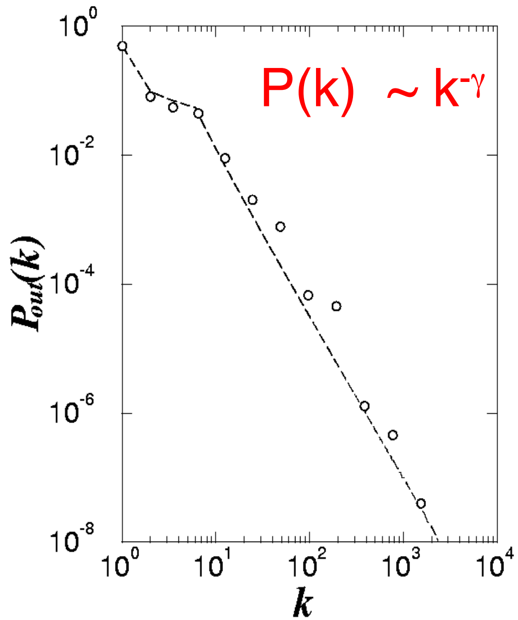
Over 3 billion documents

ROBOT: collects all URL's found in a document and follows them recursively



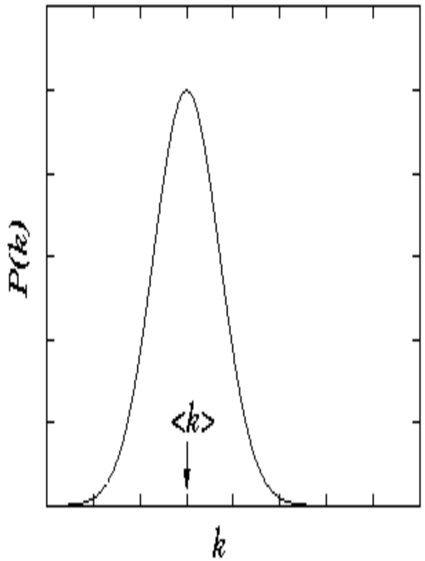


Expected

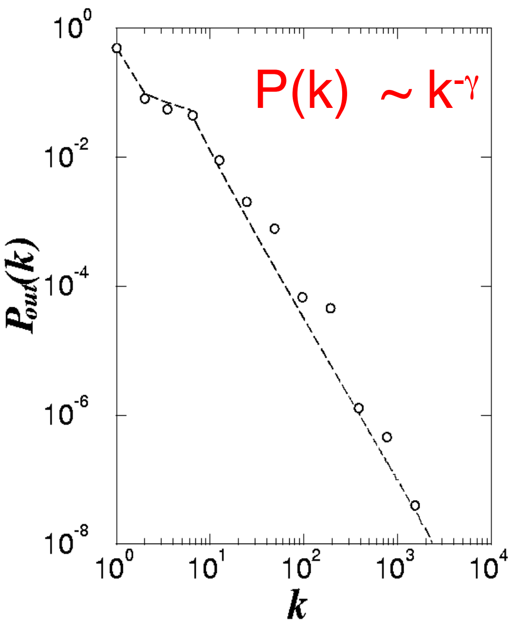


Found

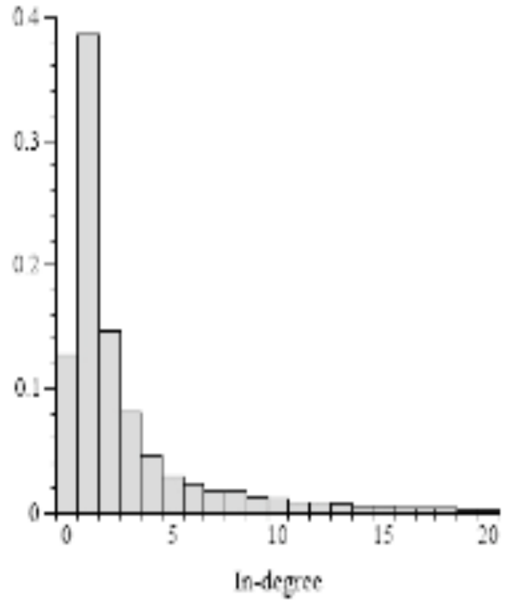
Degree distribution of the WWW

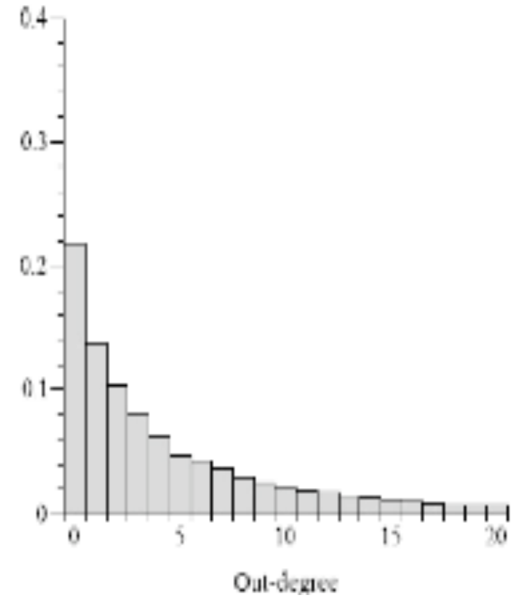


Expected



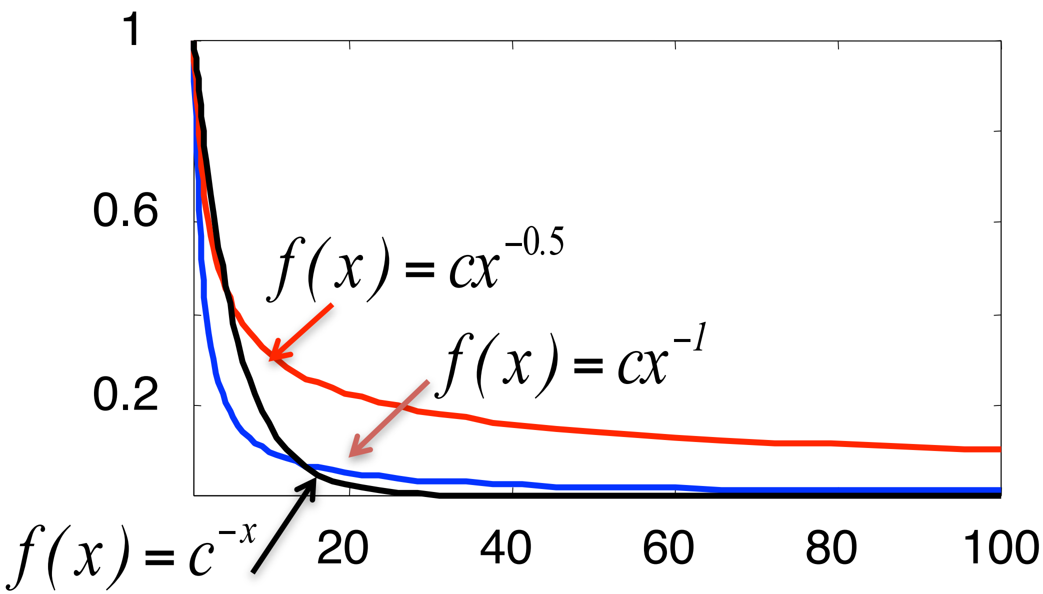
Found





R. Albert, H. Jeong, A-L Barabasi, *Nature*, 401 130 (1999).

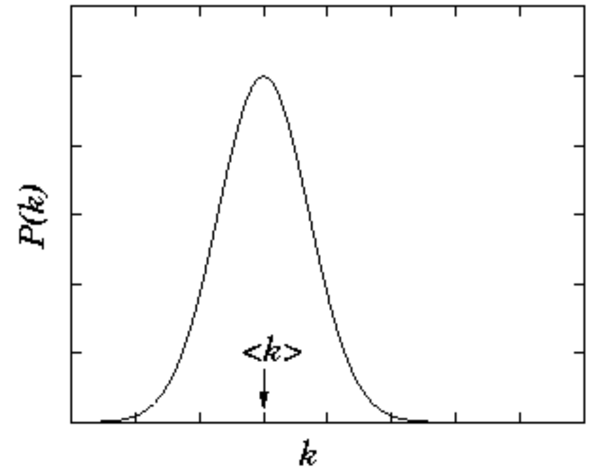
The difference between a power law and an exponential distribution



Above a certain x value, the power law is always higher than the exponential.

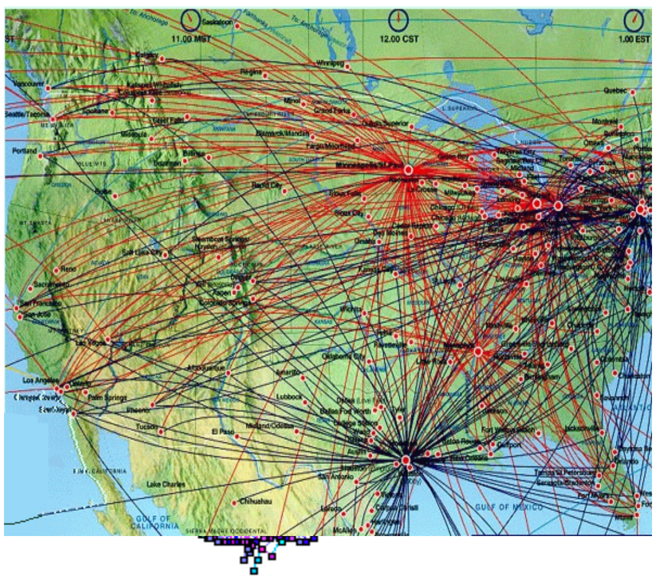
What does the difference mean? Visual representation.

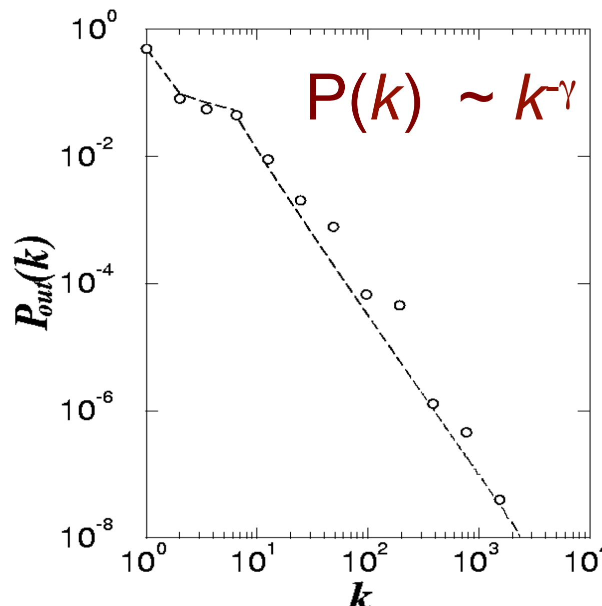
Exponential Network



Expected

Scale-free Network

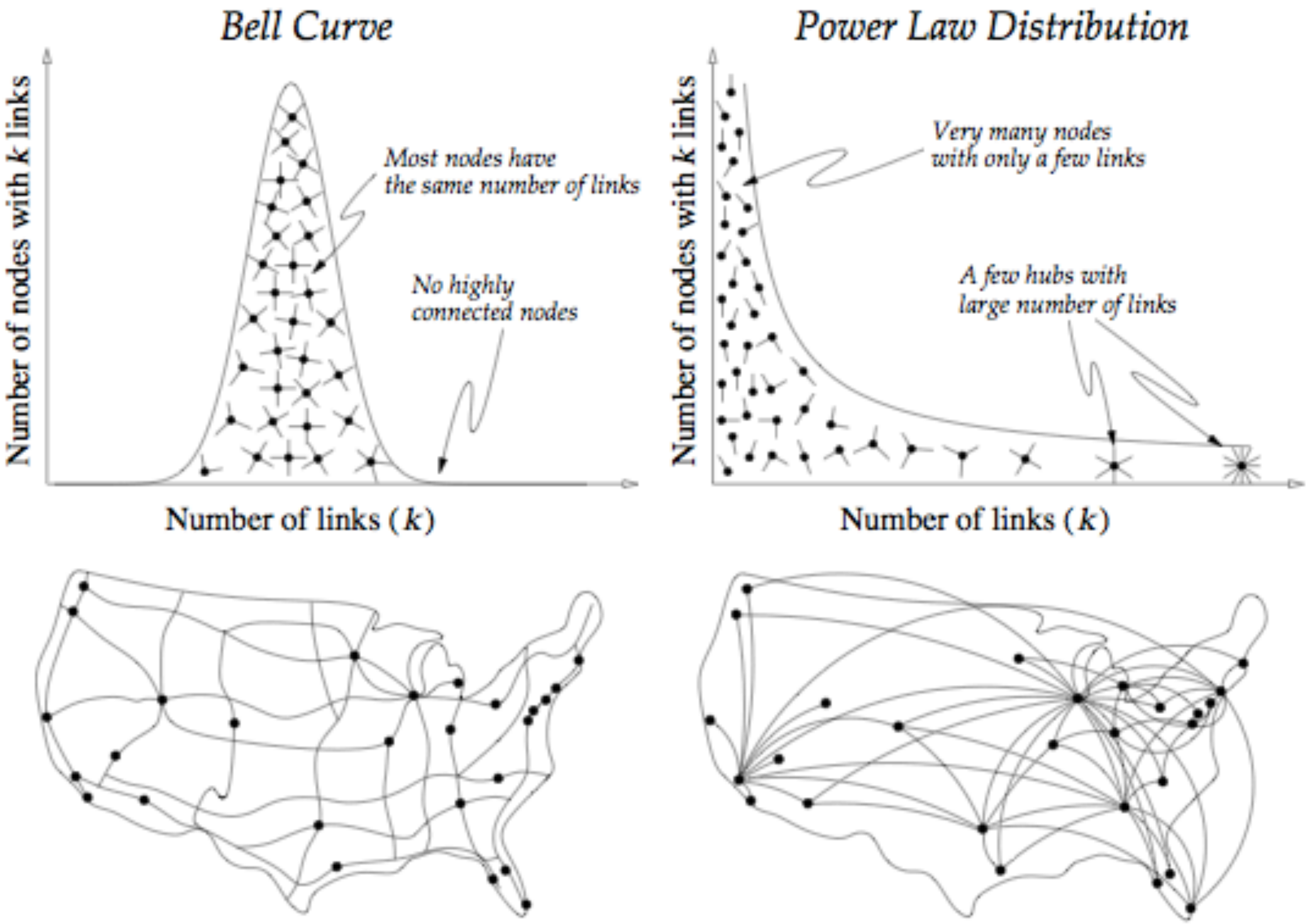




Found

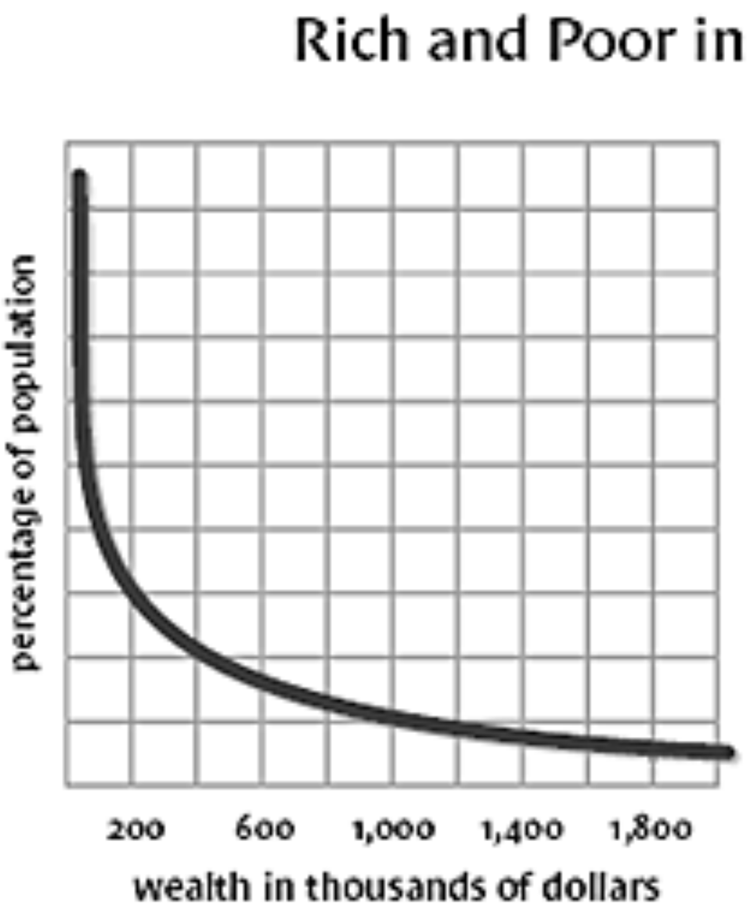
R. Albert, H. Jeong, A-L Barabasi, *Nature*, 401 130 (1999).

WORLD WIDE WEB

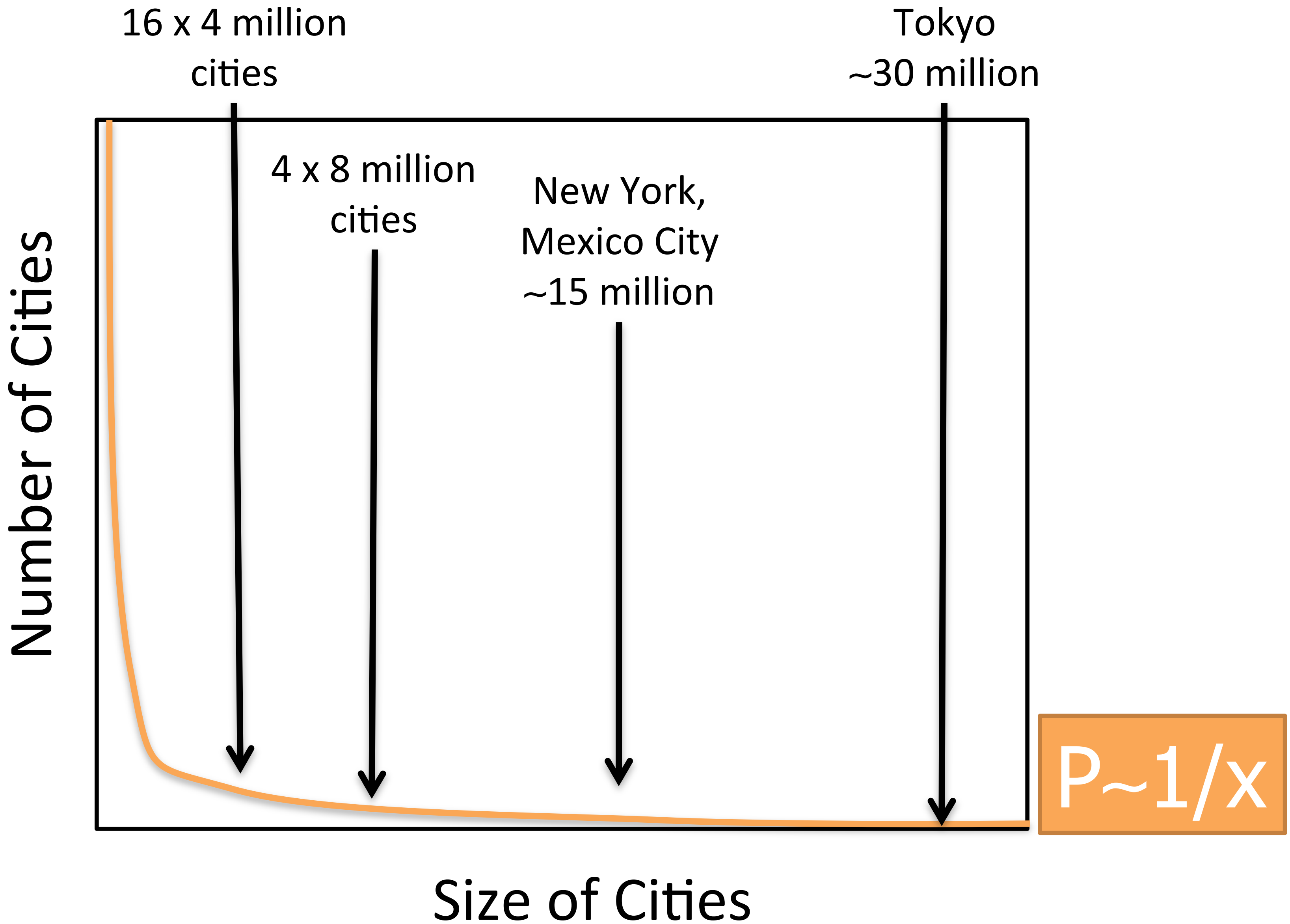


PARETO DISTRIBUTION OF WEALTH

Vilfredo Pareto (1848-1923)



This plot of household wealth in the United States, taken from 1998 census figures, clearly shows a distribution of rich and poor forming a Pareto curve. The highest percentage of households fall at the lower levels of wealth, but at the higher end, the curve drops off relatively slowly, displaying Pareto's "fat-tailed" pattern.


16 x 4 million cities
Tokyo ~30 million
4 x 8 million cities
New York, Mexico City ~15 million
Number of Cities
P~1/x
Size of Cities

NO OUTLIERS IN A RANDOM SOCIETY

$$P(k) = e^{-<k>} \frac{<k>^k}{k!}$$

→The most connected individual has degree k_{max}~1,185
→The least connected individual has degree k_{min} ~ 816

The probability to find an individual with degree k>2,000 is 10^{-27}. Hence the chance of finding an individual with 2,000 acquaintances is so tiny that such nodes are virtually inexistent in a random society.

→a random society would consist of mainly average individuals, with everyone with roughly the same number of friends.
→It would lack outliers, individuals that are either highly popular or recluse.

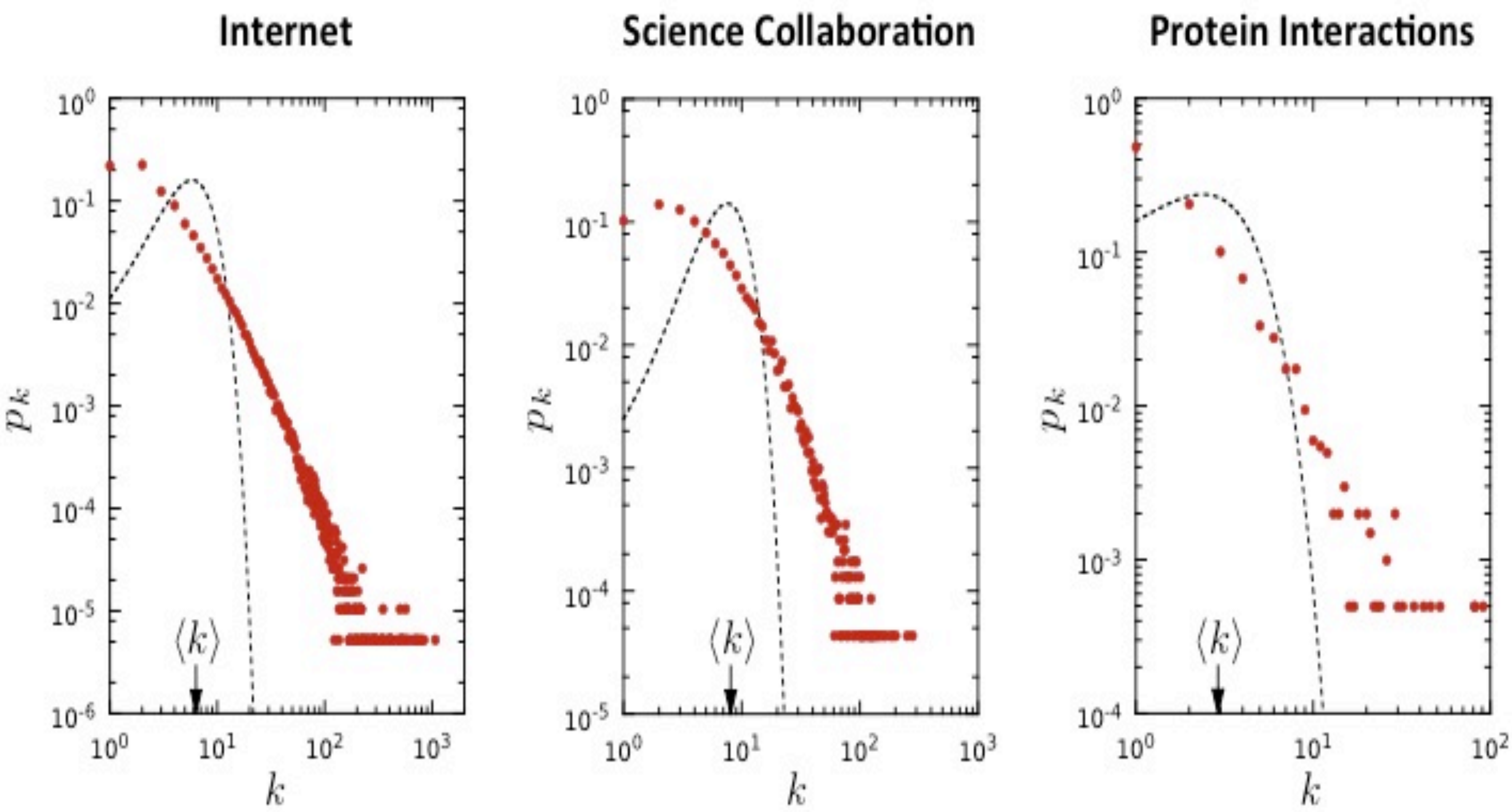
After Bill enters the arena the average wealth of the public ~ $1,000,000

~ $100 billion

10^5 people, 10^5 $ average wealth per capita

FACING REALITY: Degree distribution of real networks

$$P(k) = e^{-<k>} \frac{<k>^k}{k!}$$



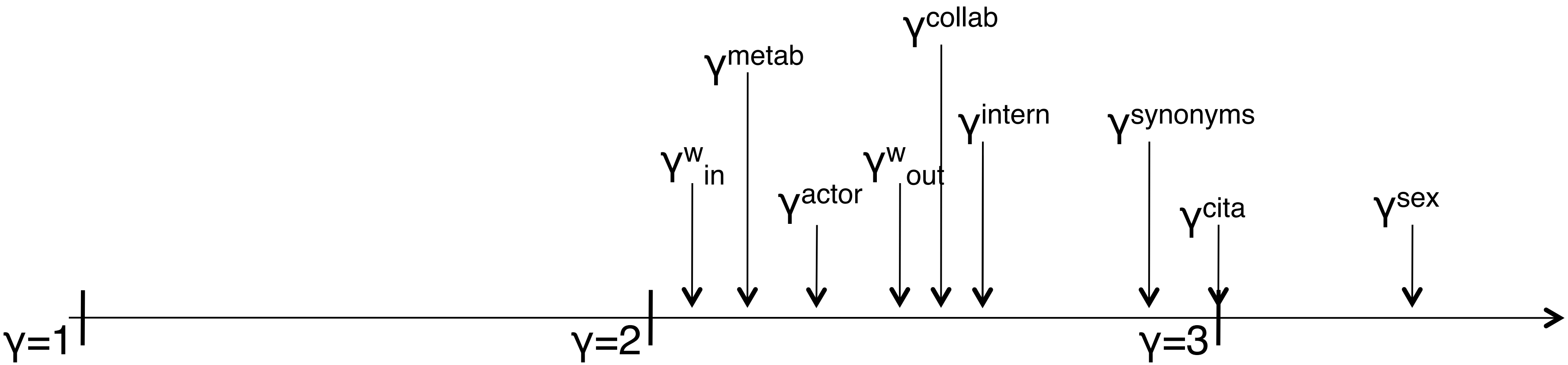
UNIVERSALITY

Network	Size	$\langle k \rangle$	κ	γ_{out}	γ_{in}
WWW	325 729	4.51	900	2.45	2.1
WWW	4×10^7	7		2.38	2.1
WWW	2×10^8	7.5	4000	2.72	2.1
WWW, site	260 000				1.94
Internet, domain*	3015–4389	3.42–3.76	30–40	2.1–2.2	2.1–2.2
Internet, router*	3888	2.57	30	2.48	2.48
Internet, router*	150 000	2.66	60	2.4	2.4
Movie actors*	212 250	28.78	900	2.3	2.3
Co-authors, SPIRES*	56 627	173	1100	1.2	1.2
Co-authors, neuro.*	209 293	11.54	400	2.1	2.1
Co-authors, math.*	70 975	3.9	120	2.5	2.5
Sexual contacts*	2810			3.4	3.4
Metabolic, *E. coli*	778	7.4	110	2.2	2.2
Protein, *S. cerev.**	1870	2.39		2.4	2.4
Ythan estuary*	134	8.7	35	1.05	1.05
Silwood Park*	154	4.75	27	1.13	1.13
Citation	783 339	8.57			3
Phone call	53×10^6	3.16		2.1	2.1
Words, co-occurrence*	460 902	70.13		2.7	2.7
Words, synonyms*	22 311	13.48		2.8	2.8

Networks:
The exponents vary from system to system.
Most are between 2 and 3

Universality:
the emergence of common features across different networks. Like the scale-free property.

VARIANCE DIVERGES!



$\langle k^2 \rangle$ diverges	$\langle k^2 \rangle$ finite

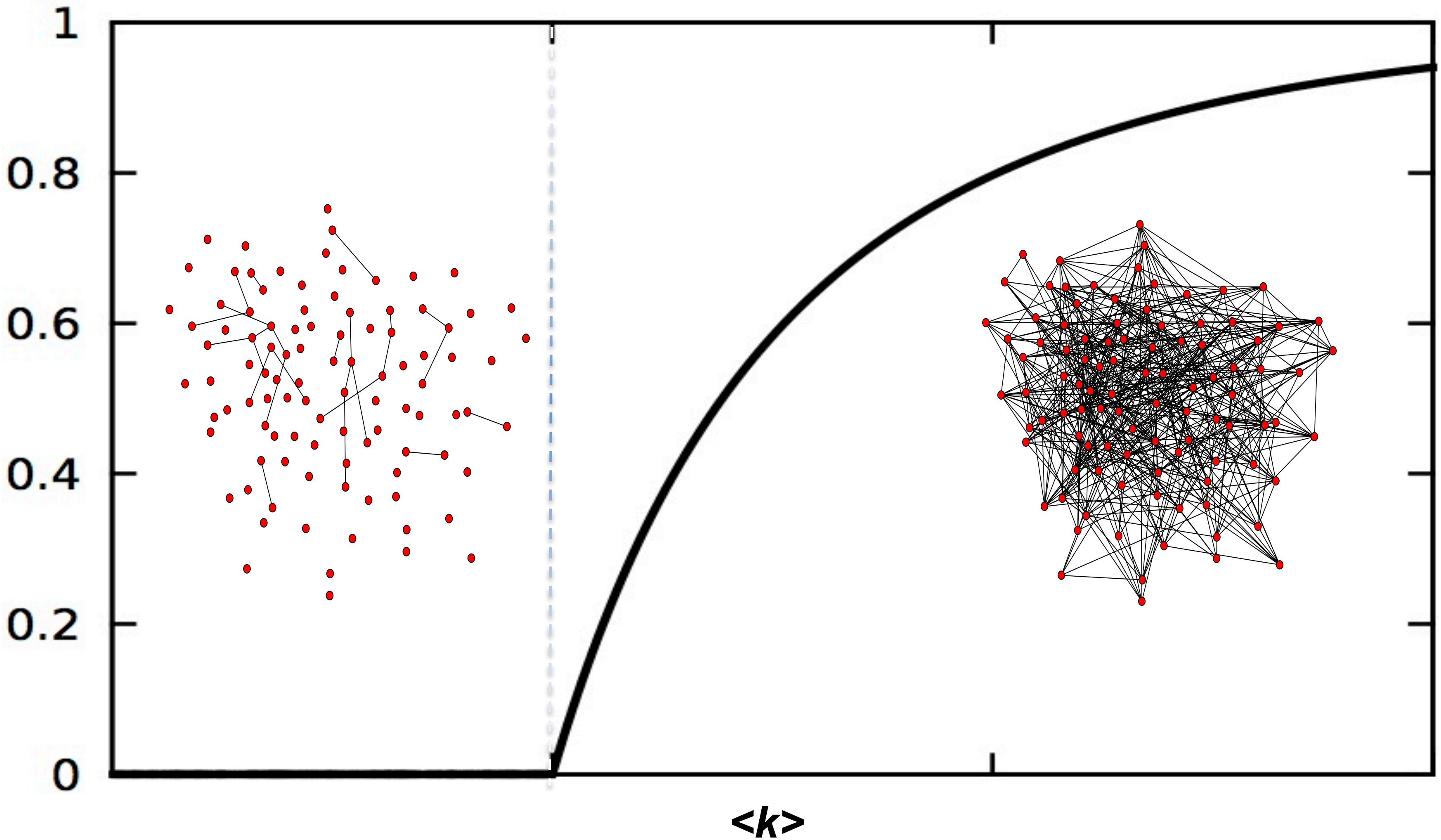
Regime full of anomalies…	The scale-free behavior is relevant	Behaves like a random network

Why are most exponents in this regime?

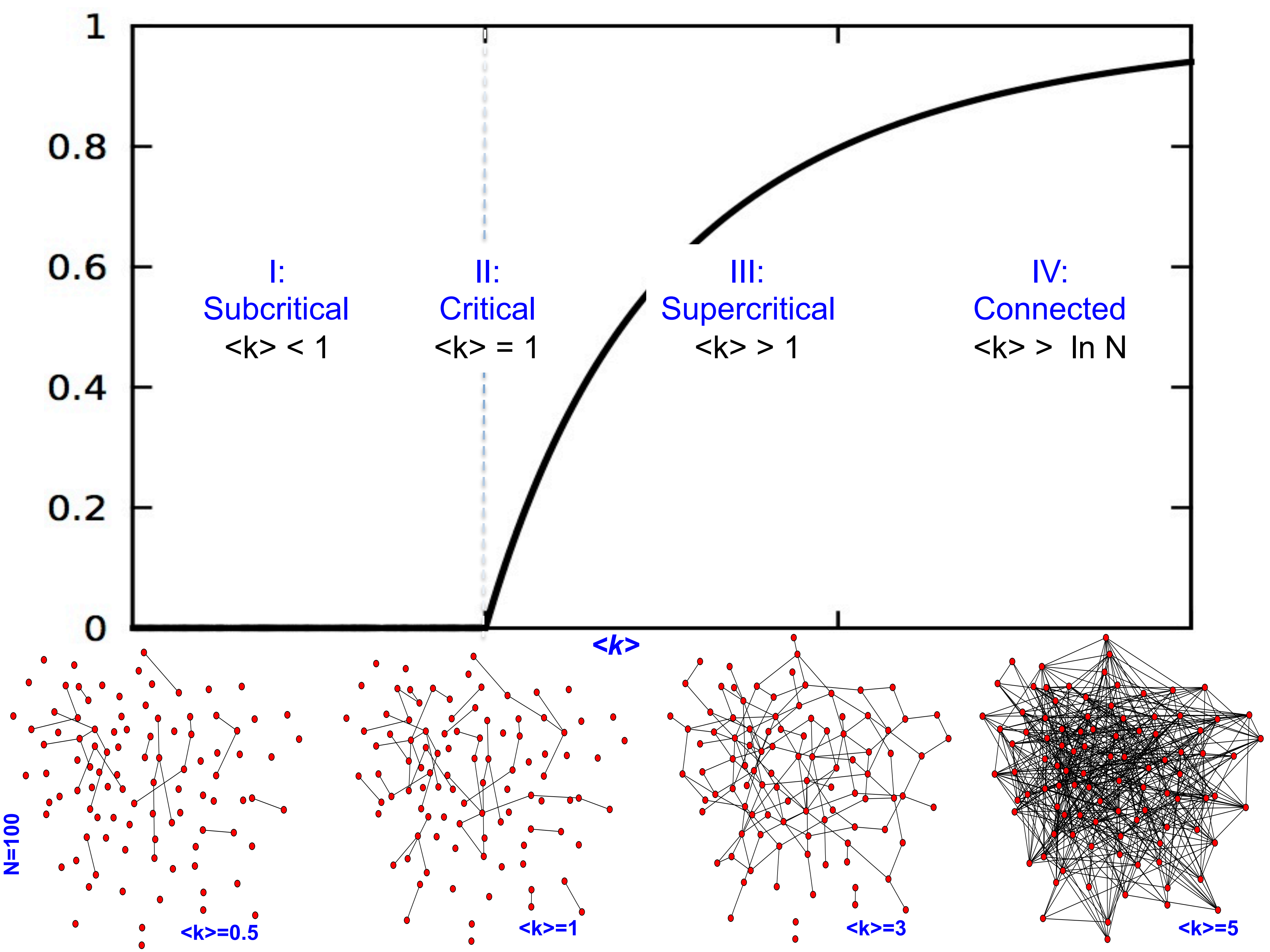
The evolution of a random network

EVOLUTION OF A RANDOM NETWORK

disconnected nodes ➔ **NETWORK.**

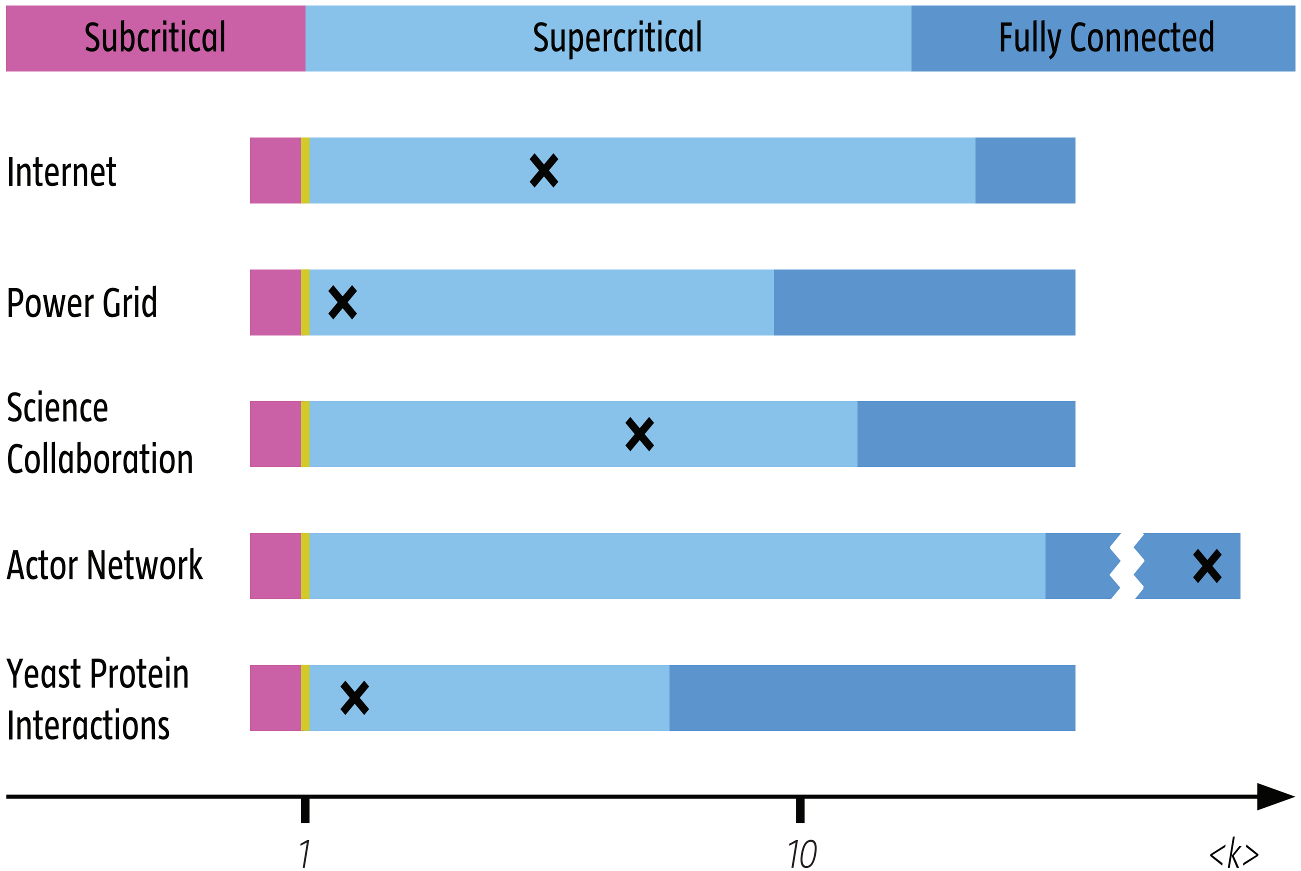


How does this transition happen?



1
0.8
0.6
0.4
0.2
0
I:
Subcritical
<k> < 1
II:
Critical
<k> = 1
III:
Supercritical
<k> > 1
IV:
Connected
<k> > ln N
<k>
N=100
<k>=0.5
<k>=1
<k>=3
<k>=5

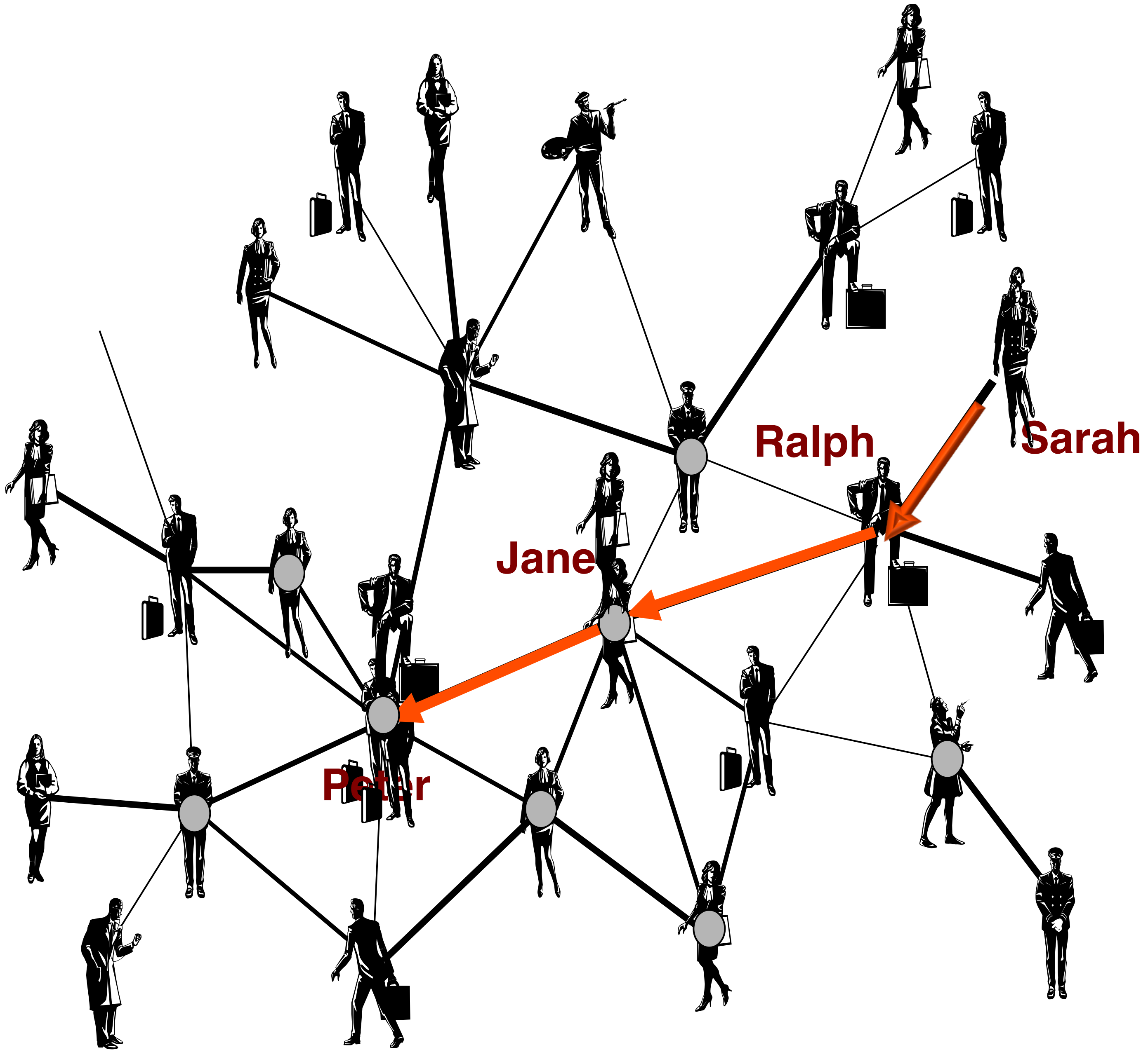
Real networks are supercritical



Network	N	L	$\langle k\rangle$	$\ln N$
Internet	192,244	609,066	6.34	12.17
Power Grid	4,941	6,594	2.67	8.51
Science Collaboration	23,133	186,936	8.08	10.04
Actor Network	212,250	3,054,278	28.78	12.27
Yeast Protein Interactions	2,018	2,930	2.90	7.61

Small world property

SIX DEGREES | small worlds



Frigyes Karinthy, 1929
Stanley Milgram, 1967

SIX DEGREES

1929: Frigyes Kartinthy

Frigyes Karinthy (1887-1938)
Hungarian Writer

1929: *Minden másképpen van* (Everything is Different)
Láncszemek (Chains)

“Look, Selma Lagerlöf just won the Nobel Prize for Literature, thus she is bound to know King Gustav of Sweden, after all he is the one who handed her the Prize, as required by tradition. King Gustav, to be sure, is a passionate tennis player, who always participates in international tournaments. He is known to have played Mr. Kehrling, whom he must therefore know for sure, and as it happens I myself know Mr. Kehrling quite well.”

"The worker knows the manager in the shop, who knows Ford; Ford is on friendly terms with the general director of Hearst Publications, who last year became good friends with Arpad Pasztor, someone I not only know, but to the best of my knowledge a good friend of mine. So I could easily ask him to send a telegram via the general director telling Ford that he should talk to the manager and have the worker in the shop quickly hammer together a car for me, as I happen to need one."

1967: Stanley Milgram

HOW TO TAKE PART IN THIS STUDY

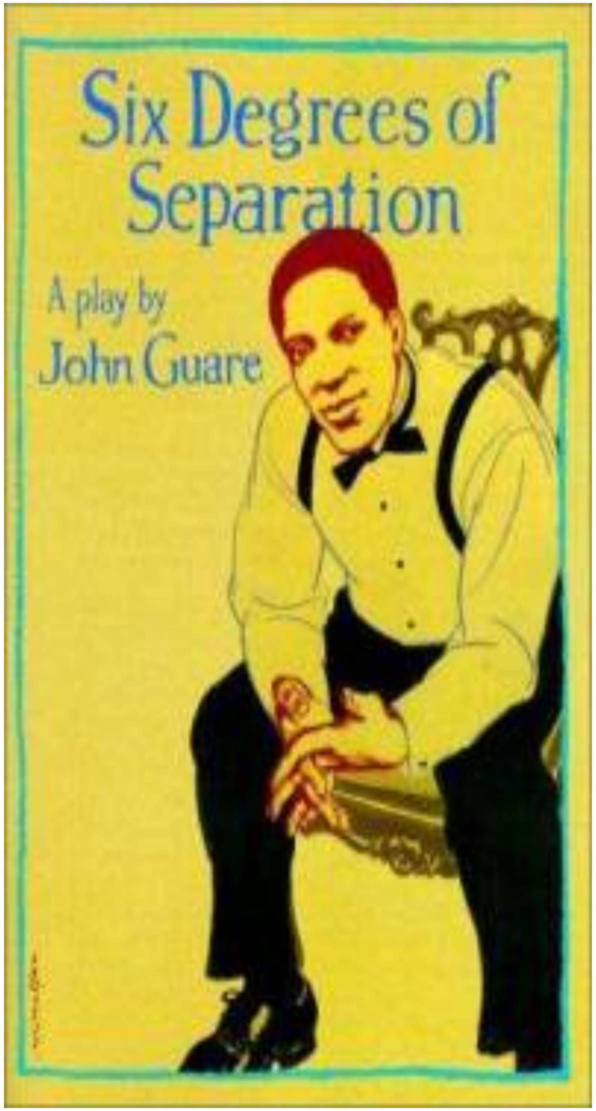
1. ADD YOUR NAME TO THE ROSTER AT THE BOTTOM OF THIS SHEET, so that the next person who receives this letter will know who it came from.

2. DETACH ONE POSTCARD. FILL IT AND RETURN IT TO HARVARD UNIVERSITY. No stamp is needed. The postcard is very important. It allows us to keep track of the progress of the folder as it moves toward the target person.

3. IF YOU KNOW THE TARGET PERSON ON A PERSONAL BASIS, MAIL THIS FOLDER DIRECTLY TO HIM (HER). Do this only if you have previously met the target person and know each other on a first name basis.

4. IF YOU DO NOT KNOW THE TARGET PERSON ON A PERSONAL BASIS, DO NOT TRY TO CONTACT HIM DIRECTLY. INSTEAD, MAIL THIS FOLDER (POST CARDS AND ALL) TO A PERSONAL ACQUAINTANCE WHO IS MORE LIKELY THAN YOU TO KNOW THE TARGET PERSON. You may send the folder to a friend, relative or acquaintance, but it must be someone you know on a first name basis.

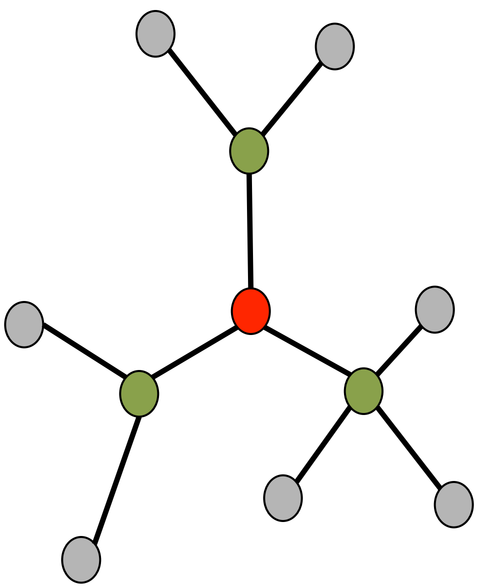
1991: John Guare



"Everybody on this planet is separated by only six other people. Six degrees of separation. Between us and everybody else on this planet. The president of the United States. A gondolier in Venice…. It's not just the big names. It's anyone. A native in a rain forest. A Tierra del Fuegan. An Eskimo. I am bound to everyone on this planet by a trail of six people. It's a profound thought. How every person is a new door, opening up into other worlds."

DISTANCES IN RANDOM GRAPHS

Random graphs tend to have a tree-like topology with almost constant node degrees.



- nr. of first neighbors: $N_1 \cong \langle k \rangle$
- nr. of second neighbors: $N_2 \cong \langle k \rangle^2$
- nr. of neighbours at distance d: $N_d \cong \langle k \rangle^d$
- estimate maximum distance:

$$1 + \sum_{l=1}^{l_{max}} \langle k \rangle^i = N \quad \Rightarrow \quad l_{max} = \frac{\log N}{\log \langle k \rangle}$$

DISTANCES IN RANDOM GRAPHS

compare with real data

$$l_{max} = \frac{\log N}{\log \langle k \rangle}$$

Network	Size	⟨k⟩	l	l_{rand}	C	C_{rand}	Reference	Nr
www, site level, undir	153127	35.21	3.1	3.35	0.1078	0.00023	Adamic, 1999	1
Internet, domain level	3015-6209	3.52-4.11	3.7-3.76	6.36-6.18	0.18-0.3	0.001	Yook e al., 2001a, Pastor-Satorras et al., 2001	2
Movie actors	225226	61	3.65	2.99	0.79	0.00027	Watts and Strogatz,1998	3
LANL co-authorship	52909	9.7	5.9	4.79	0.43	1.8×10^{-4}	Newman, 2001a, 2001b, 2001c	4
MEDLINE eo-authorship	1520251	18.1	4.6	4.91	0.066	1.1×10^{-5}	Newman, 2001a, 2001b, 2001c	5
SPIRES co-authorship	56627	173	4.0	2.12	0.726	0.003	Newman, 2001a, 2001b, 2001c	6
NCSTRL co-authorship	11994	3.59	9.7	7.34	0.496	3×10^{-4}	Newman, 2001a, 2001b, 2001c	7
Math. co-authorship	70975	3.9	9.5	8.2	0.59	5.4×10^{-5}	Barabasi et al, 2001	8
Neurosci. co-authorship	209293	11.5	6	5.01	0.76	5.5×10^{-5}	Barabasi et al, 2001	9
E. coli, sustrate graph	282	7.35	2.9	3.04	0.32	0.026	Wagner and Fell, 2000	10
E. coli, reaction graph	315	28.3	2.62	1.98	0.59	0.09	Wagner and Fell, 2000	11
Ythan estuary food web	134	8.7	2.43	2.26	0.22	0.06	Montoya and Sole, 2000	12
Silwood Park food web	154	4.75	3.40	3.23	0.15	0.03	Montoya and Sole, 2000	13
Words, co-occurrence	460902	70.13	2.67	3.03	0.437	0.0001	Ferrer i Cancho and Sole, 2001	14
Words, synonyms	22311	13.48	4.5	3.84	0.7	0.0006	Yook et al. 2001b	15
Power grid	4941	2.67	18.7	12.4	0.08	0.005	Watts and Strogatz, 1998	16
C.Elegans	282	14	2.65	2.25	0.28	0.05	Watts and Strogatz, 1998	17

Given the huge differences in scope, size, and average degree, the agreement is excellent.

CLUSTERING COEFFICIENT

$$C_i \equiv \frac{2n_i}{k_i(k_i - 1)}$$

Since edges are independent and have the same probability *p*,

$$n_i \cong p \frac{k_i(k_i - 1)}{2} \qquad \Rightarrow \qquad C \cong p = \frac{<k>}{N}$$

The clustering coefficient of random graphs is small.

For fixed degree C decreases with the system size N.

13.47 from Newman 2010

CLUSTERING IN RANDOM GRAPHS

compare with real data

Network	Size	⟨k⟩	l	l_{rand}	C	C_{rand}	Reference	Nr
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Erdös-Rényi MODEL (1960)

•**Degree distribution**

Binomial, Poisson (exponential tails)

•**Clustering coefficient**

Vanishing for large network sizes

•**Average distance among nodes**

Logarithmically small

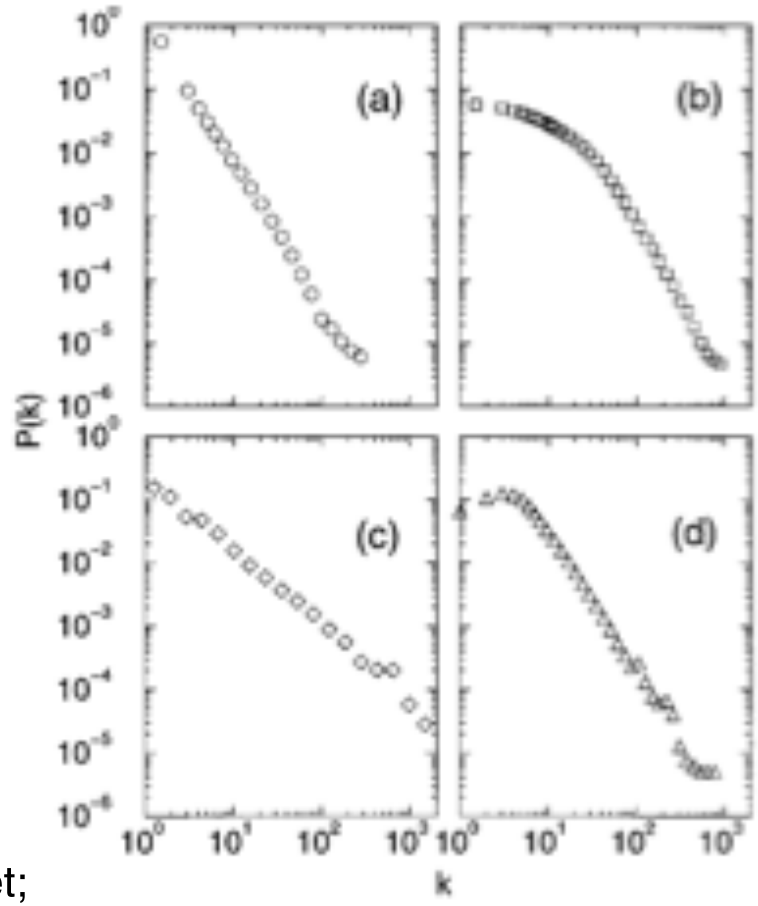
Are real networks like random graphs?

NO!

THE DEGREE DISTRIBUTION

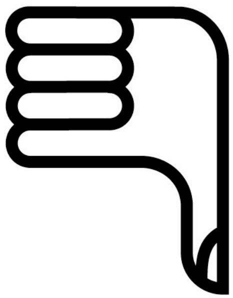
Prediction:

$$P_{rand}(k) \cong C_{N-1}^{k} p^{k} (1-p)^{N-1-k}$$



Data:

$$P(k) \approx k^{-\gamma}$$



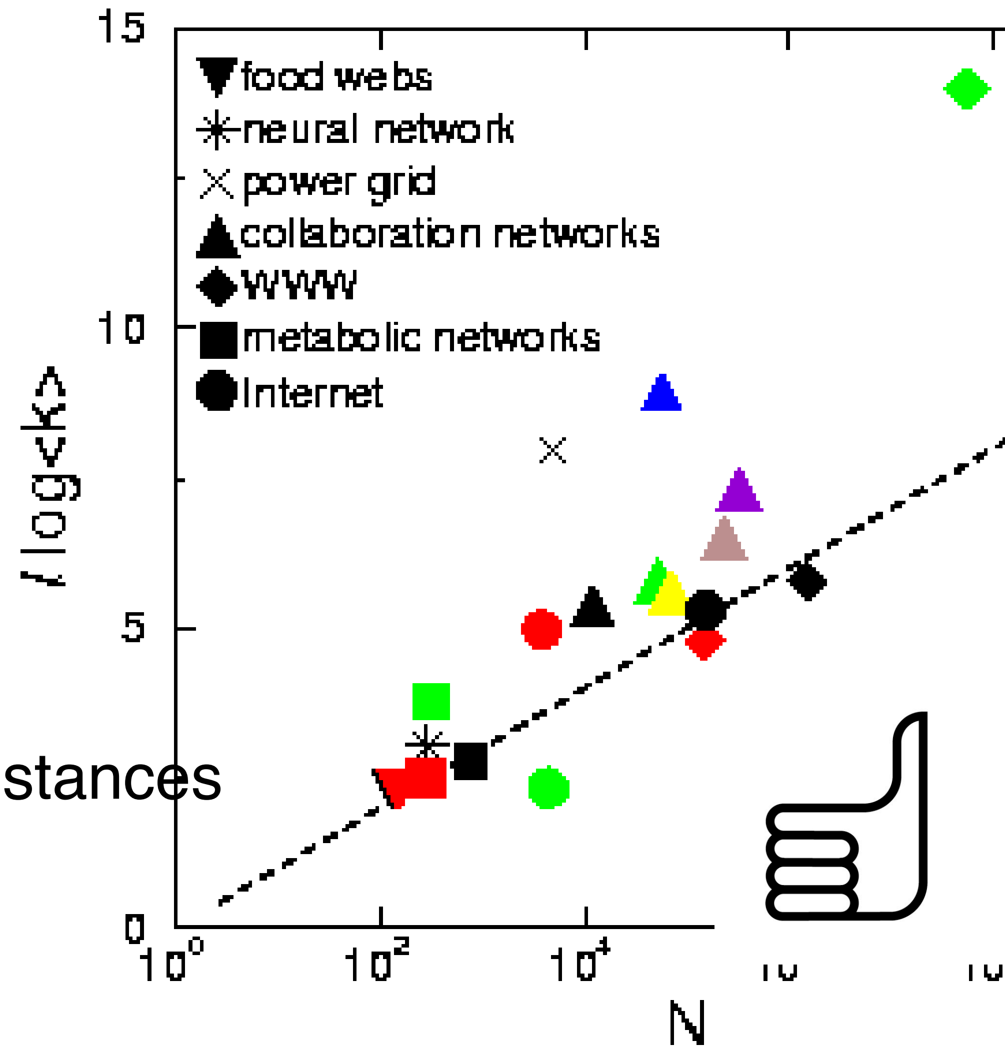
(a) Internet;
(b) Movie Actors;
(c) Coauthorship, high energy physics;
(d) Coauthorship, neuroscience

PATH LENGTHS IN REAL NETWORKS

Prediction:

$$l_{rand} = \frac{\log N}{\log \langle k \rangle}$$

Data:



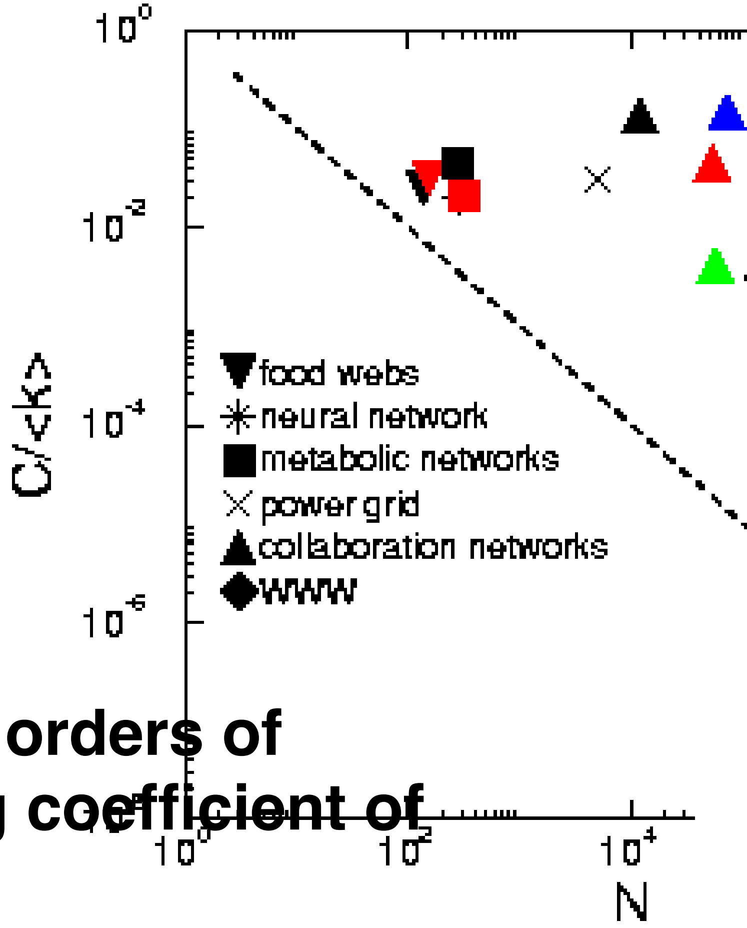
Real networks have short distances like random graphs.

CLUSTERING COEFFICIENT

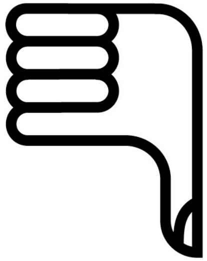
Prediction:

$$C_{rand} = \frac{\langle k \rangle}{N}$$

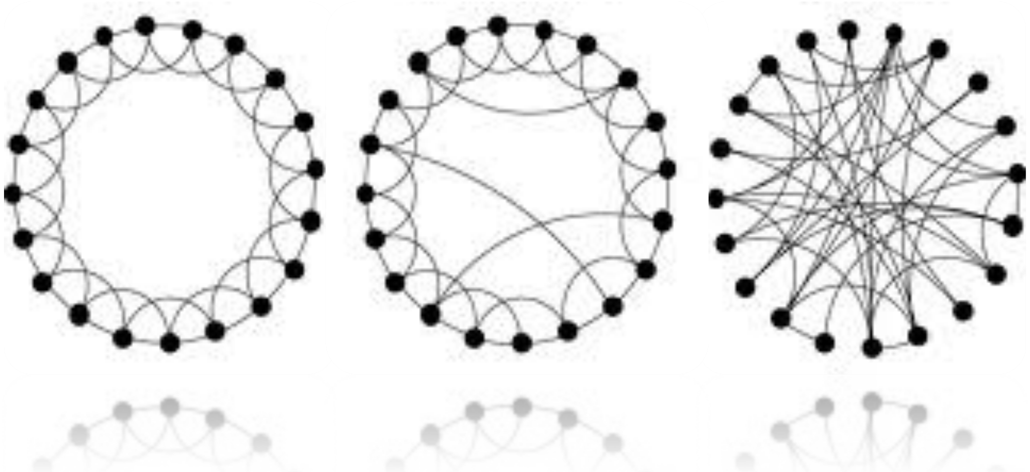
Data:

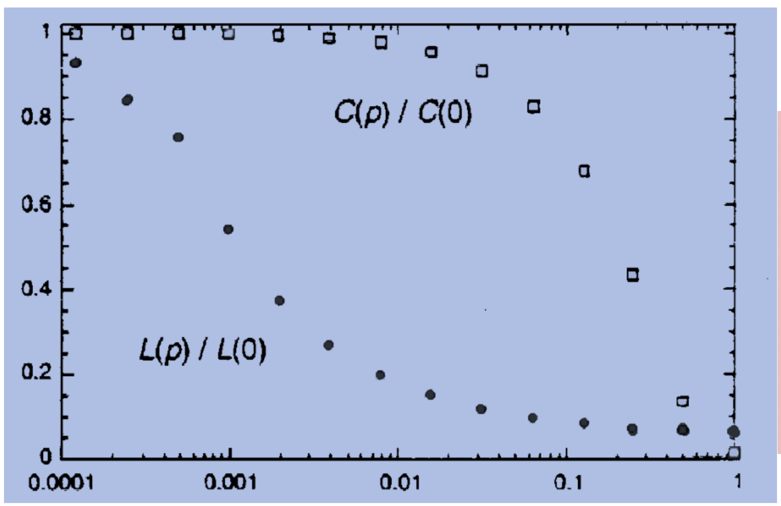


C_{rand} underestimates with orders of magnitudes the clustering coefficient of real networks.



Models for «real» networks: small world



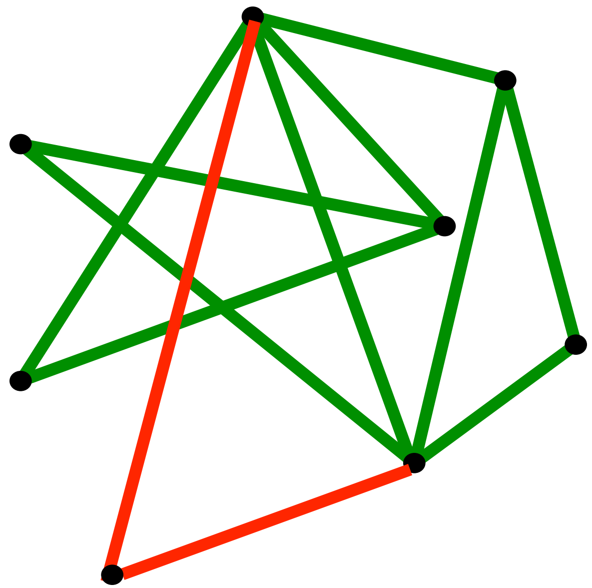


The Watts Strogatz Model:
It takes a lot of randomness to ruin the clustering, but a very small amount to overcome locality

Models for real networks: Preferential Attachment

Where will the new node link to?
ER, WS models: choose randomly.

New nodes prefer to link to highly connected nodes (www, citations, IMDB).



PREFERENTIAL ATTACHMENT:

the probability that a node connects to a node with *k* links is proportional to *k*.

$$\Pi(k_i) = \frac{k_i}{\Sigma_j k_j}$$

Barabási & Albert, *Science* **286,** 509 (1999)