Programming for Data Science (11/1/2024)

30% of the points are assigned to quality of documentation and/or comments to solutions.
Solutions must include tests of executions of the developed *functions.*

Name files as "<your matricola>_<firstname>_<lastname>_ex1.py" for Exercise 1, and "<your matricola>_<firstname>_<lastname>_ex2.c" for the second exercise.

Upload the TWO files in a folder
(named with your student number and your last name) at the following URL: *Upload here*
(access GDrive using your university credentials)

Exercise 1. (Math, on paper)

Complete the following descriptions for sets of Natural numbers including, respectively, only even numbers and prime numbers:
1a. Even = {x…….. | ……. } // use the congruence relation modulo
1b. Primes= {x …….. | ……..} // use the a divides b (denoted a | b) relation

Formalize in first order logic:
1c. "There is a number that is both even and prime"
1d. "All odd natural numbers are greater than zero"

Let MEn be the set of matrices nxn such that all elements in the matrices are even. Let M1, M2 ∈ MEn
1e. Does M1+M2 ∈ MEn ? Justify your answer
1f. Does M1 * M2 ∈ MEn ? Justify your answer
1g. Is the determinant of M1 even? Justify your answer
1h. Is the rank of M1 at most n-1? Justify your answer

SOLUTIONS

1a. Even = {x ∈ N | x ≡ 0 (mod 2) }
1b. Primes= {x ∈ N | ∀ y (y | x → y=x or y=1}
1c. Let p(x) denote x is prime, and e(x) denote x is even: ∃ x. (p(X) ∧ e(x)) "
1d. Let o(x) denote x is odd eand let the universe be the natural numbers: ∀ x. (o(x) → x>0)
1e. Yes, since each element c_{ij} of M1+M2 is the sum of $a_{ij} \in$ M1 and $b_{ij} \in$ M2 and the sum of even numbers is even
1f. Yes, since each element c_{ij} of M1 * M2 is obtained with sum and product of even numbers
1g. Yes, since it is is obtained with sum and product of even numbers
1h. No, consider the counter-example $\begin{bmatrix} 2 & 6 \\ 4 & 12 \end{bmatrix}$ in general, if you take a non-singular matrix (i.e. a full rank matrix) and multiply all elements by 2, the rank does not change and all elements are now even.