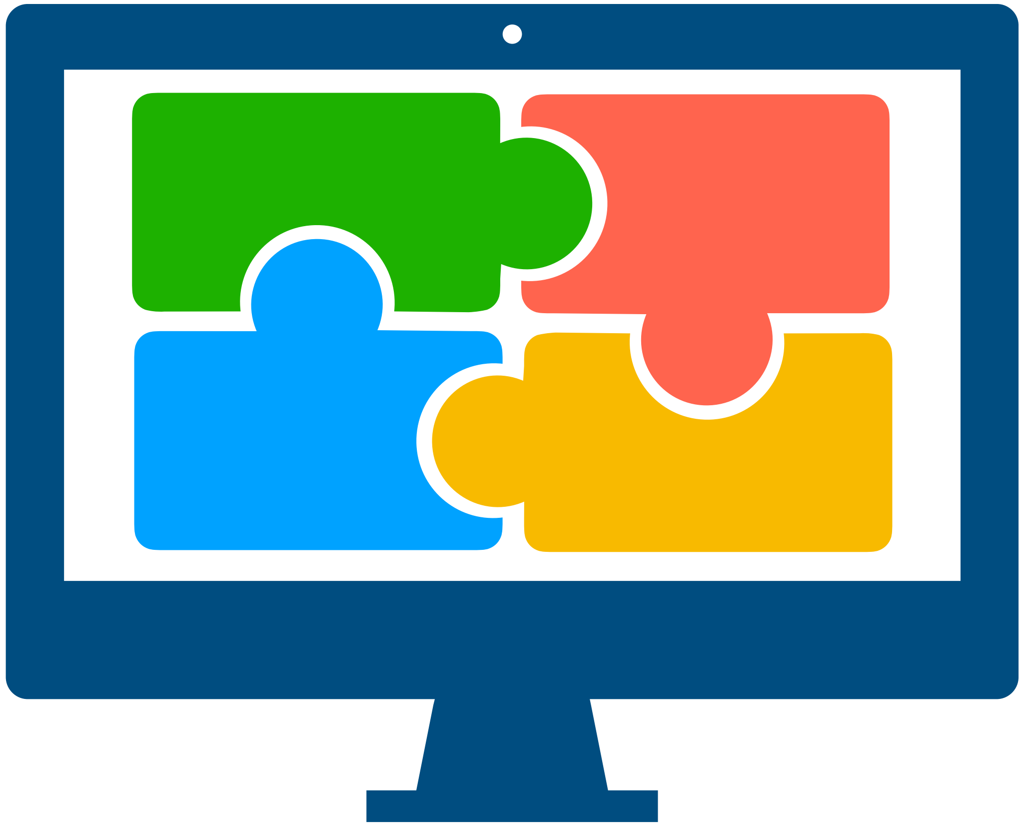


Exercises #6

CCS

[**Ex. 4**] Define a CCS process B^n_k that represents an in/out buffer with capacity n of which k positions are taken. Show that B^n_0 is strongly bisimilar to n copies of B^1_0 that run in parallel.

Ex. 4, buffers

$$B_0^n \triangleq in.B_1^n$$

$$B_k^n \triangleq in.B_{k+1}^n + \overline{out}.B_{k-1}^n \text{ with } 0 < k < n$$

$$B_n^n \triangleq \overline{out}.B_{n-1}^n$$

we want to prove $B_0^n \simeq \underbrace{B_0^1|\cdots|B_0^1}_{n}$

$$\mathbf{R} \triangleq \{(B_k^n, B_{k_1}^1|\cdots|B_{k_n}^1) \mid \forall i.\ k_i \in \{0,1\} \wedge \sum_{i=1}^{n} k_i = k\}$$

we prove that **R** is a strong bisimulation

Ex. 4, buffers

$$
\begin{array}{ccc}
B_0^n & \mathbf{R} & B_0^1 | \cdots | B_0^1 \\
\big\downarrow {\scriptstyle in} & & \big\downarrow {\scriptstyle in} \\
B_1^n & \mathbf{R} & B_1^1 | \cdots | B_0^1
\end{array}
$$

$$
\begin{array}{ccc}
B_0^n & \mathbf{R} & B_0^1 | \cdots | B_0^1 \\
\big\downarrow {\scriptstyle in} & & \big\downarrow {\scriptstyle in} \\
B_1^n & \mathbf{R} & B_0^1 | \cdots | B_1^1 | \cdots | B_0^1
\end{array}
$$

Ex. 4, buffers

$$0 < k < n \qquad B^n_k \quad \mathbf{R} \quad B^1_{k_1}|\cdots|B^1_{k_n} \qquad \sum_{i=1}^{n} k_i = k$$

$$\downarrow in \qquad\qquad \downarrow in \qquad\qquad \exists k_i = 0$$

$$B^n_{k+1} \ \mathbf{R} \ B^1_{k_1}|\cdots|B^n_{k_i+1}|\cdots|B^1_{k_n}$$

$$B^n_k \quad \mathbf{R} \quad B^1_{k_1}|\cdots|B^1_{k_n} \qquad \exists k_i = 0$$

$$\downarrow in \qquad\qquad \downarrow in$$

$$B^n_{k+1} \ \mathbf{R} \ B^1_{k_1}|\cdots|B^n_{k_i+1}|\cdots|B^1_{k_n}$$

Ex. 4, buffers

$$0 < k < n \qquad B^n_k \quad \mathbf{R} \quad B^1_{k_1}|\cdots|B^1_{k_n} \qquad \sum_{i=1}^{n} k_i = k$$

$$\Big\downarrow \overline{out} \qquad\qquad \Big\downarrow \overline{out} \qquad\qquad \exists k_i = 1$$

$$B^n_{k-1} \;\; \mathbf{R} \;\; B^1_{k_1}|\cdots|B^n_{k_i-1}|\cdots|B^1_{k_n}$$

$$B^n_k \quad \mathbf{R} \quad B^1_{k_1}|\cdots|B^1_{k_n} \qquad \exists k_i = 1$$

$$\Big\downarrow \overline{out} \qquad\qquad \Big\downarrow \overline{out}$$

$$B^n_{k-1} \;\; \mathbf{R} \;\; B^1_{k_1}|\cdots|B^n_{k_i-1}|\cdots|B^1_{k_n}$$

Ex. 4, buffers

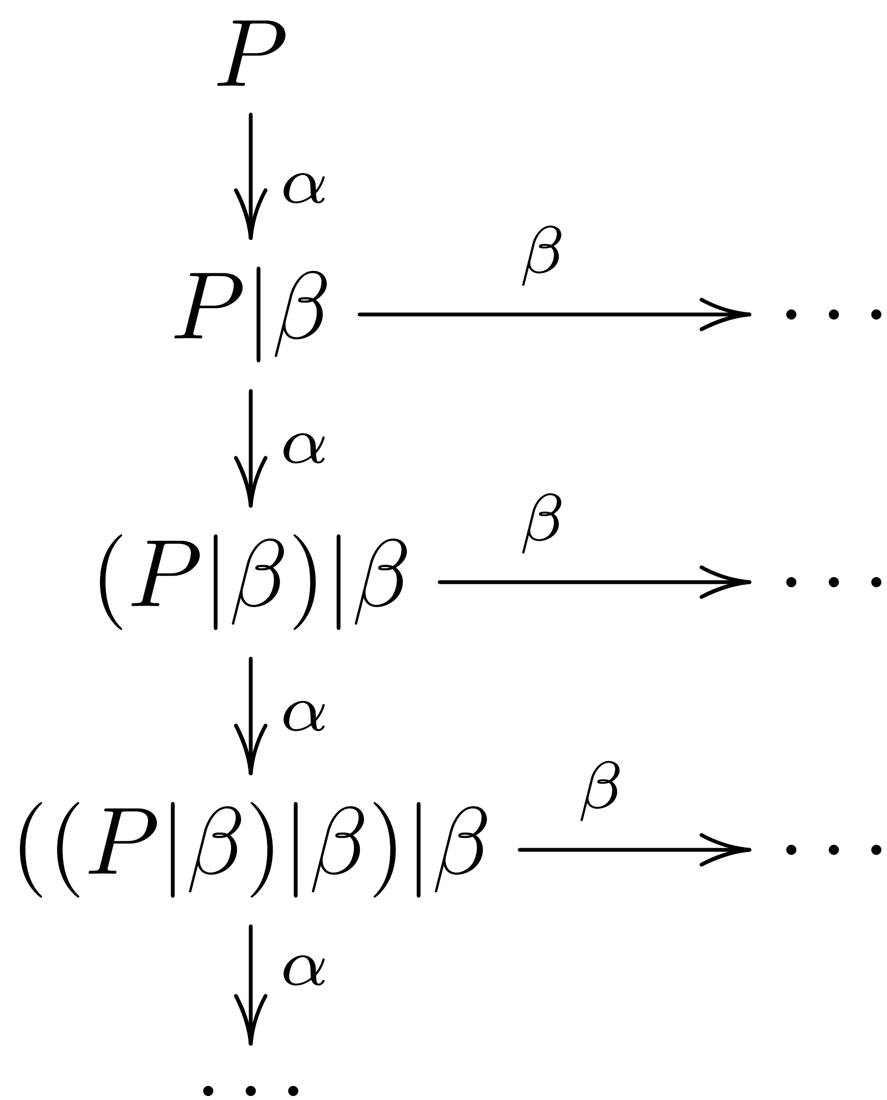
$$
\begin{array}{ccc}
B^n_n & \mathbf{R} & B^1_1 | \cdots | B^1_1 \\
\Big\downarrow \overline{out} & & \Big\downarrow \overline{out} \\
B^n_{n-1} & \mathbf{R} & B^1_0 | \cdots | B^1_1
\end{array}
$$

$$
\begin{array}{ccc}
B^n_n & \mathbf{R} & B^1_1 | \cdots | B^1_1 \\
\Big\downarrow \overline{out} & & \Big\downarrow \overline{out} \\
B^n_{n-1} & \mathbf{R} & B^1_1 | \cdots | B^1_0 | \cdots | B^1_1
\end{array}
$$

[**Ex. 5**] Write a guarded CCS process whose LTS has infinitely many states without using parallel composition.

Ex. 5, infinite LTS

using par $\quad P \triangleq \mathbf{rec}\ x.\ \alpha.(x|\beta.\mathbf{nil}) \quad P \triangleq \alpha.(P|\beta)$



Ex. 5, infinite LTS

$$P \triangleq \mathbf{rec}\ x.\ (\alpha.x)\backslash\beta \qquad\qquad P \triangleq (\alpha.P)\backslash\beta$$

$$\begin{array}{c} P \\ \downarrow \alpha \\ P\backslash\beta \\ \downarrow \alpha \\ P\backslash\beta\backslash\beta \\ \downarrow \alpha \\ P\backslash\beta\backslash\beta\backslash\beta \\ \downarrow \alpha \\ \cdots \end{array}$$

all states are bisimilar

Ex. 5, infinite LTS

$$P \triangleq \mathbf{rec}\ x.\ \alpha_1.(x[\phi]) \qquad \phi(\alpha_i) = \alpha_{i+1}$$

$$P \xrightarrow{\alpha_1} P[\phi] \xrightarrow{\alpha_2} P[\phi][\phi] \xrightarrow{\alpha_3} P[\phi][\phi][\phi] \xrightarrow{\alpha_4} \cdots$$

Bisimulation

[**Ex. 6**] Prove that CCS strong bisimilarity is a congruence w.r.t. restriction, i.e., that for all p, q, α:

$$p \simeq q \;\Rightarrow\; p\backslash\alpha \simeq \; q\backslash\alpha$$

Ex. 6, congruence \a

$\mathbf{R} \triangleq \{(p\backslash\alpha, q\backslash\alpha) \mid p \simeq q\}$ we prove **R** is a strong bisimulation

take $(p\backslash\alpha, q\backslash\alpha) \in \mathbf{R}$ (with $p \simeq q$)

take $p\backslash\alpha \xrightarrow{\mu} p'$ we want to find $q\backslash\alpha \xrightarrow{\mu} q'$ with $p' \,\mathbf{R}\, q'$

by rule res) it must be $p \xrightarrow{\mu} p''$ with $\mu \notin \{\alpha, \overline{\alpha}\}$ and $p' = p''\backslash\alpha$

since $p \simeq q$ and $p \xrightarrow{\mu} p''$ we have $q \xrightarrow{\mu} q''$ with $p'' \simeq q''$

take $q' = q''\backslash\alpha$: by rule res) we have $q\backslash\alpha \xrightarrow{\mu} q'$ and $(p', q') \in \mathbf{R}$

take $q\backslash\alpha \xrightarrow{\mu} q'$ we want to find $p\backslash\alpha \xrightarrow{\mu} p'$ with $p' \,\mathbf{R}\, q'$

analogous to the previous case

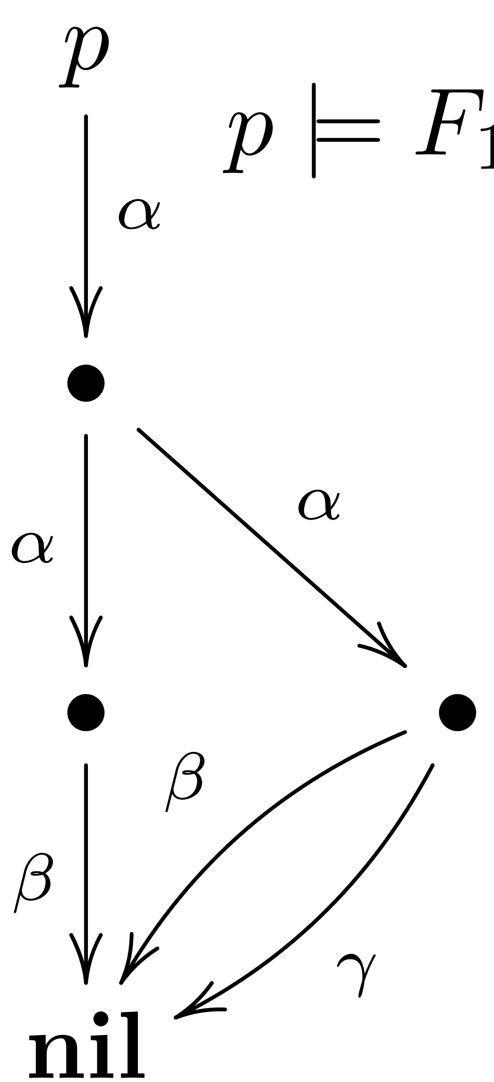
[**Ex. 7**] Prove that the CCS agents

$$p \stackrel{\text{def}}{=} \alpha.(\alpha.\beta.\mathbf{nil} + \alpha.(\beta.\mathbf{nil} + \gamma.\mathbf{nil})) \qquad \text{and} \qquad q \stackrel{\text{def}}{=} \alpha.(\alpha.\beta.\mathbf{nil} + \alpha.\gamma.\mathbf{nil})$$

are not strong bisimilar.

Ex. 7, non bisimilar

$p \triangleq \alpha.(\alpha.\beta + \alpha.(\beta + \gamma))$ $\qquad$ $q \triangleq \alpha.(\alpha.\beta + \alpha.\gamma)$



$$F_1 \triangleq \Diamond_\alpha \Diamond_\alpha (\Diamond_\beta \mathbf{tt} \wedge \Diamond_\gamma \mathbf{tt})$$

$p \models F_1$ $\qquad$ $q \not\models F_1$

$$F_2 \triangleq \Box_\alpha \Box_\alpha \Diamond_\beta \mathbf{tt}$$

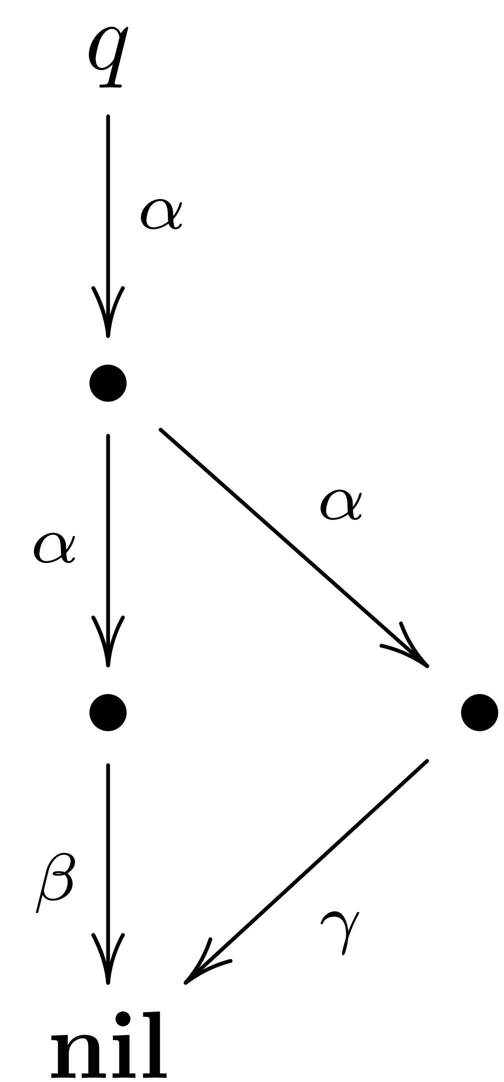
$p \models F_2$ $\qquad$ $q \not\models F_2$

$$F_2^c \triangleq \Diamond_\alpha \Diamond_\alpha \Box_\beta \mathbf{ff}$$

$p \not\models F_2^c$ $\qquad$ $q \models F_2^c$

$$F_1^c \triangleq \Box_\alpha \Box_\alpha (\Box_\beta \mathbf{ff} \vee \Box_\gamma \mathbf{ff})$$

$p \not\models F_1^c$ $\qquad$ $q \models F_1^c$



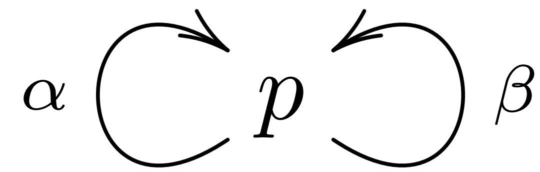
[Ex. 8] Let us consider the guarded CCS processes

$$p \stackrel{\text{def}}{=} \textbf{rec}\ x.(\alpha.x + \beta.x) \quad q \stackrel{\text{def}}{=} \textbf{rec}\ y.(\overline{\alpha}.\textbf{nil} + \gamma.y) \quad r \stackrel{\text{def}}{=} \textbf{rec}\ z.(\overline{\beta}.\textbf{nil} + \overline{\gamma}.z)$$

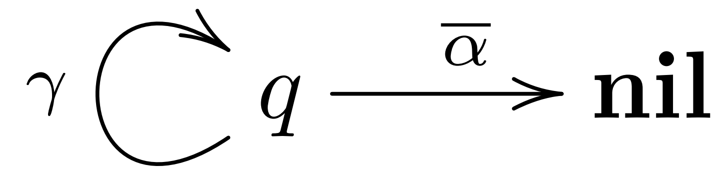
1. Draw the LTSs of the processes p, q, r and $s \stackrel{\text{def}}{=} (p|q|r)\backslash\alpha\backslash\beta\backslash\gamma$.
2. Show that s is strong bisimilar to the process $t \stackrel{\text{def}}{=} \textbf{rec}\ w.(\tau.w + \tau.\tau.\textbf{nil})$.

Ex. 8, bisimilar

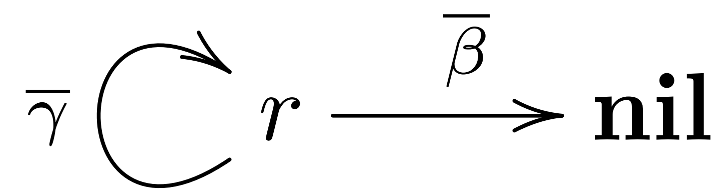
$$p \triangleq \mathbf{rec}\ x.(\alpha.x + \beta.x)$$



$$q \triangleq \mathbf{rec}\ y.(\overline{\alpha} + \gamma.y)$$



$$r \triangleq \mathbf{rec}\ z.(\overline{\beta} + \overline{\gamma}.z)$$

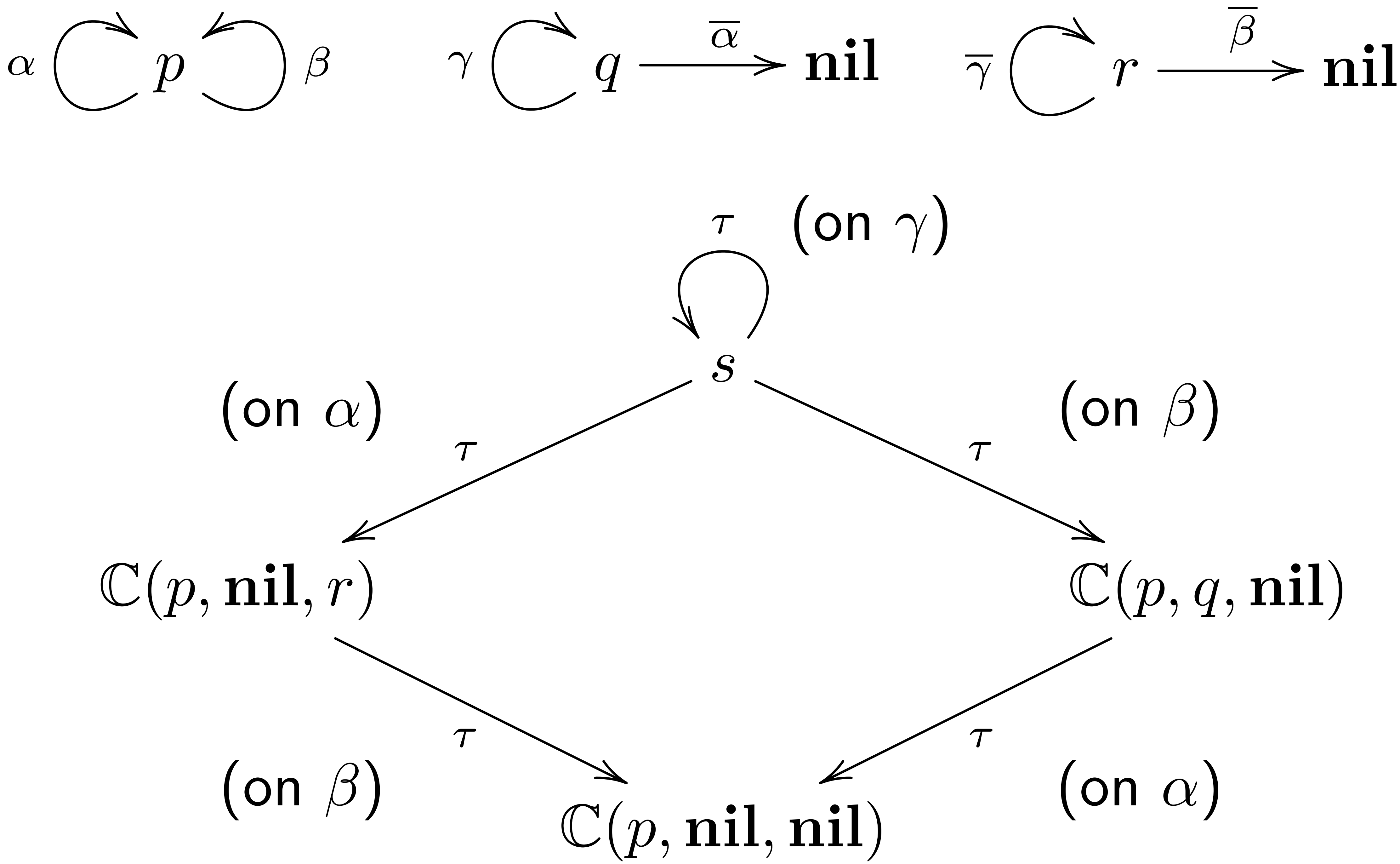


$$s \triangleq (p|q|r)\backslash\alpha\backslash\beta\backslash\gamma$$

$$\mathbb{C}(p,q,r) \triangleq (p|q|r)\backslash\alpha\backslash\beta\backslash\gamma$$

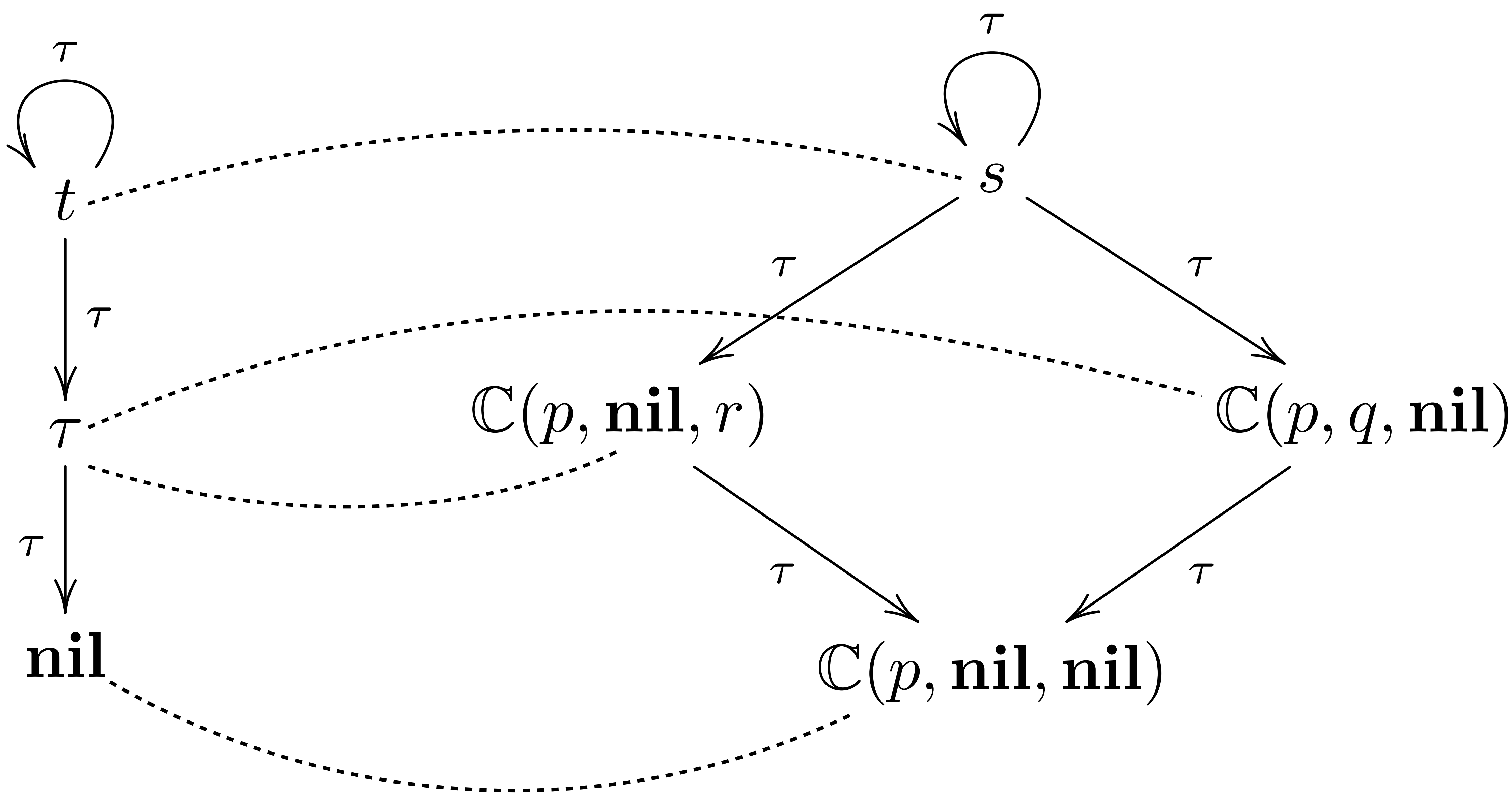
$$s \triangleq \mathbb{C}(p,q,r)$$

Ex. 8, bisimilar



Ex. 8, bisimilar

$t \triangleq \mathbf{rec}\ w.(\tau.w + \tau.\tau)$



$$\mathbf{R} \triangleq \{\ \{s, t\}\ ,\ \{\tau, \mathbb{C}(p, \mathbf{nil}, r), \mathbb{C}(p, q, \mathbf{nil})\}\ ,\ \{\mathbf{nil}, \mathbb{C}(p, \mathbf{nil}, \mathbf{nil})\}\ \}$$

R is a strong bisimulation

[**Ex. 9**] Prove that the following property is valid for any agent p, where $\approx$ is the weak bisimilarity:

$$p + \tau.p \approx \tau.p$$

Ex. 9, weak bisimilar

$$\mathbf{R} \triangleq \{(p + \tau.p, \tau.p) \mid p \in \mathcal{P}\} \cup Id$$

we check that **R** is a weak bisimulation

no need to check pairs in Id

$$\begin{array}{ccccc} p & & p+\tau.p & \mathbf{R} & \tau.p \\ \downarrow \mu & & \downarrow \mu & & \Downarrow \mu \\ p' & & p' & \mathbf{R} & p' \end{array}$$

$$\begin{array}{ccc} p+\tau.p & \mathbf{R} & \tau.p \\ \downarrow \tau & & \Downarrow \tau \\ p & \mathbf{R} & p \end{array}$$

Ex. 9, weak bisimilar

$$\mathbf{R} \triangleq \{(p + \tau.p, \tau.p) \mid p \in \mathcal{P}\} \cup Id$$

we check that **R** is a weak bisimulation

no need to check pairs in Id

$$\begin{array}{ccc} p + \tau.p & \mathbf{R} & \tau.p \\ \Downarrow \tau & & \downarrow \tau \\ p & \mathbf{R} & p \end{array}$$