

Exercises #6

CCS

[Ex. 4] Define a CCS process B_k^n that represents an in/out buffer with capacity n of which k positions are taken. Show that B_0^n is strongly bisimilar to n copies of B_0^1 that run in parallel.

Ex. 4, buffers

$$B_0^n \triangleq in.B_1^n$$

$$B_k^n \triangleq in.B_{k+1}^n + \overline{out}.B_{k-1}^n \text{ with } 0 < k < n$$

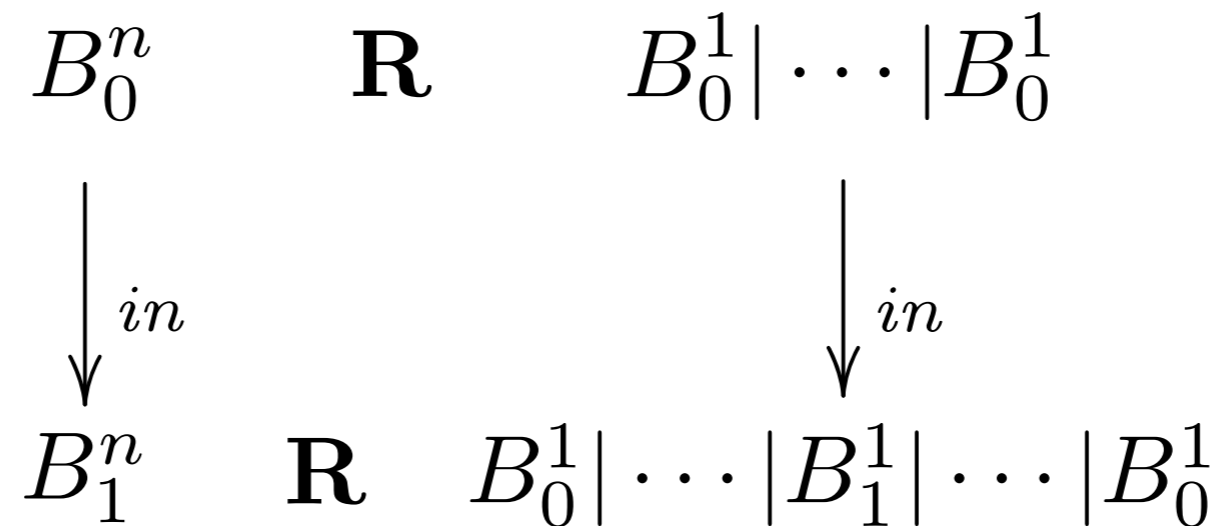
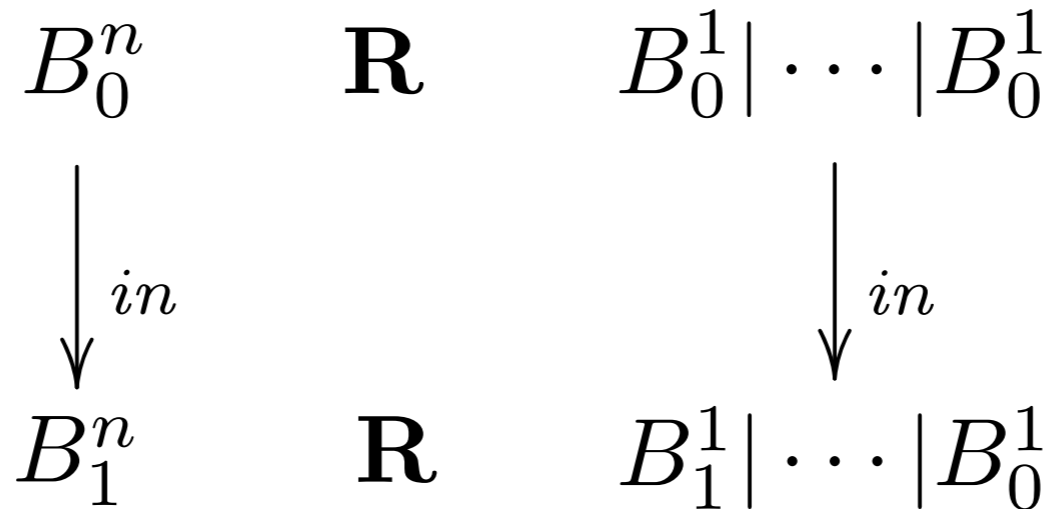
$$B_n^n \triangleq \overline{out}.B_{n-1}^n$$

we want to prove $B_0^n \simeq \underbrace{B_0^1 | \cdots | B_0^1}_n$

$$\mathbf{R} \triangleq \{(B_k^n, B_{k_1}^1 | \cdots | B_{k_n}^1) \mid \forall i. k_i \in \{0, 1\} \wedge \sum_{i=1}^n k_i = k\}$$

we prove that \mathbf{R} is a strong bisimulation

Ex. 4, buffers



Ex. 4, buffers

$$\begin{array}{l}
 0 < k < n \quad B_k^n \quad \mathbf{R} \quad B_{k_1}^1 | \cdots | B_{k_n}^1 \quad \sum_{i=1}^n k_i = k \\
 \downarrow \textit{in} \quad \downarrow \textit{in} \quad \exists k_i = 0 \\
 B_{k+1}^n \quad \mathbf{R} \quad B_{k_1}^1 | \cdots | B_{k_i+1}^n | \cdots | B_{k_n}^1
 \end{array}$$

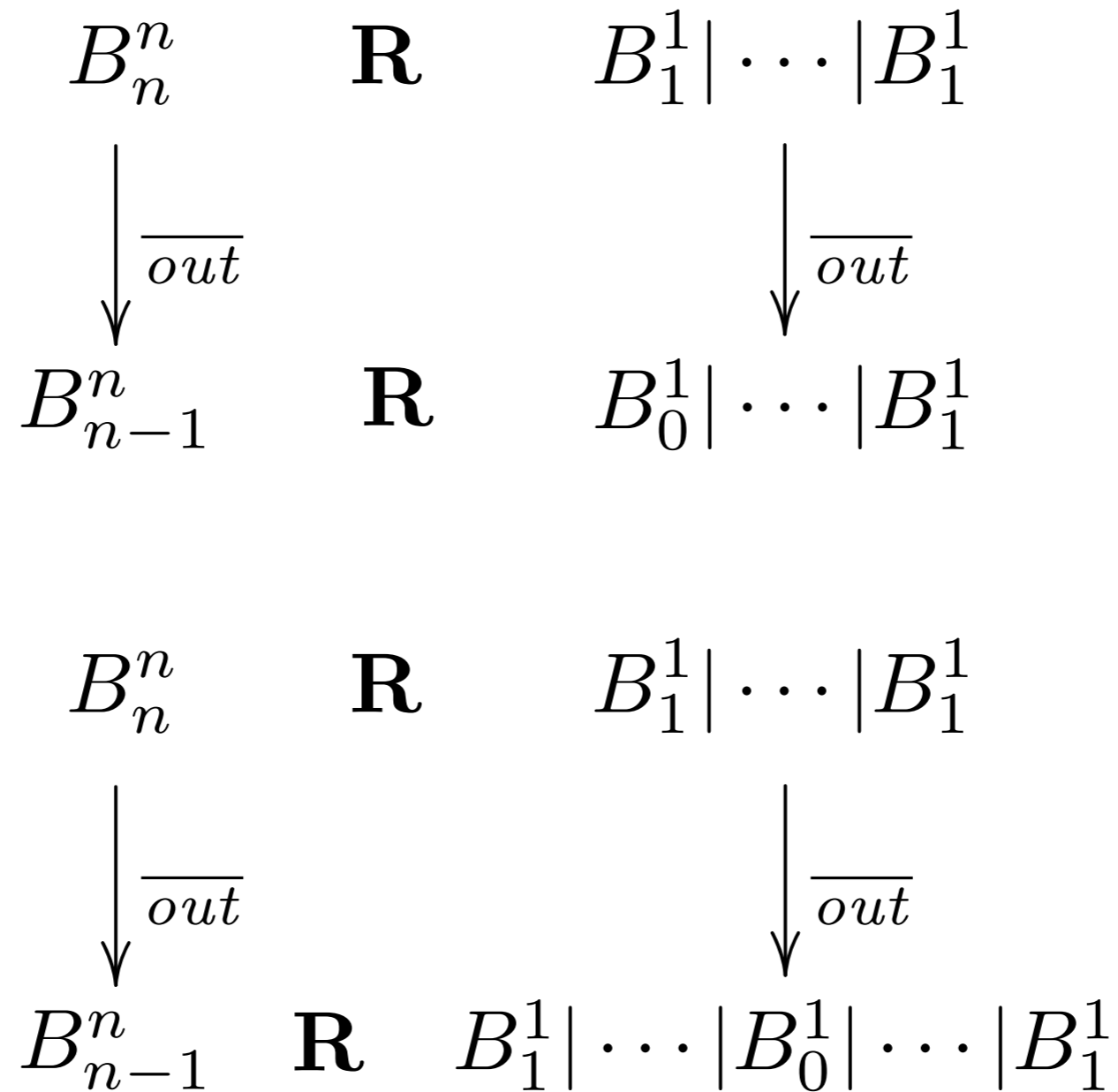
$$\begin{array}{l}
 B_k^n \quad \mathbf{R} \quad B_{k_1}^1 | \cdots | B_{k_n}^1 \quad \exists k_i = 0 \\
 \downarrow \textit{in} \quad \downarrow \textit{in} \\
 B_{k+1}^n \quad \mathbf{R} \quad B_{k_1}^1 | \cdots | B_{k_i+1}^n | \cdots | B_{k_n}^1
 \end{array}$$

Ex. 4, buffers

$$\begin{array}{l}
 0 < k < n \quad B_k^n \quad \mathbf{R} \quad B_{k_1}^1 | \cdots | B_{k_n}^1 \quad \sum_{i=1}^n k_i = k \\
 \downarrow \overline{\text{out}} \quad \downarrow \overline{\text{out}} \\
 B_{k-1}^n \quad \mathbf{R} \quad B_{k_1}^1 | \cdots | B_{k_i-1}^n | \cdots | B_{k_n}^1 \quad \exists k_i = 1
 \end{array}$$

$$\begin{array}{l}
 B_k^n \quad \mathbf{R} \quad B_{k_1}^1 | \cdots | B_{k_n}^1 \quad \exists k_i = 1 \\
 \downarrow \overline{\text{out}} \quad \downarrow \overline{\text{out}} \\
 B_{k-1}^n \quad \mathbf{R} \quad B_{k_1}^1 | \cdots | B_{k_i-1}^n | \cdots | B_{k_n}^1
 \end{array}$$

Ex. 4, buffers



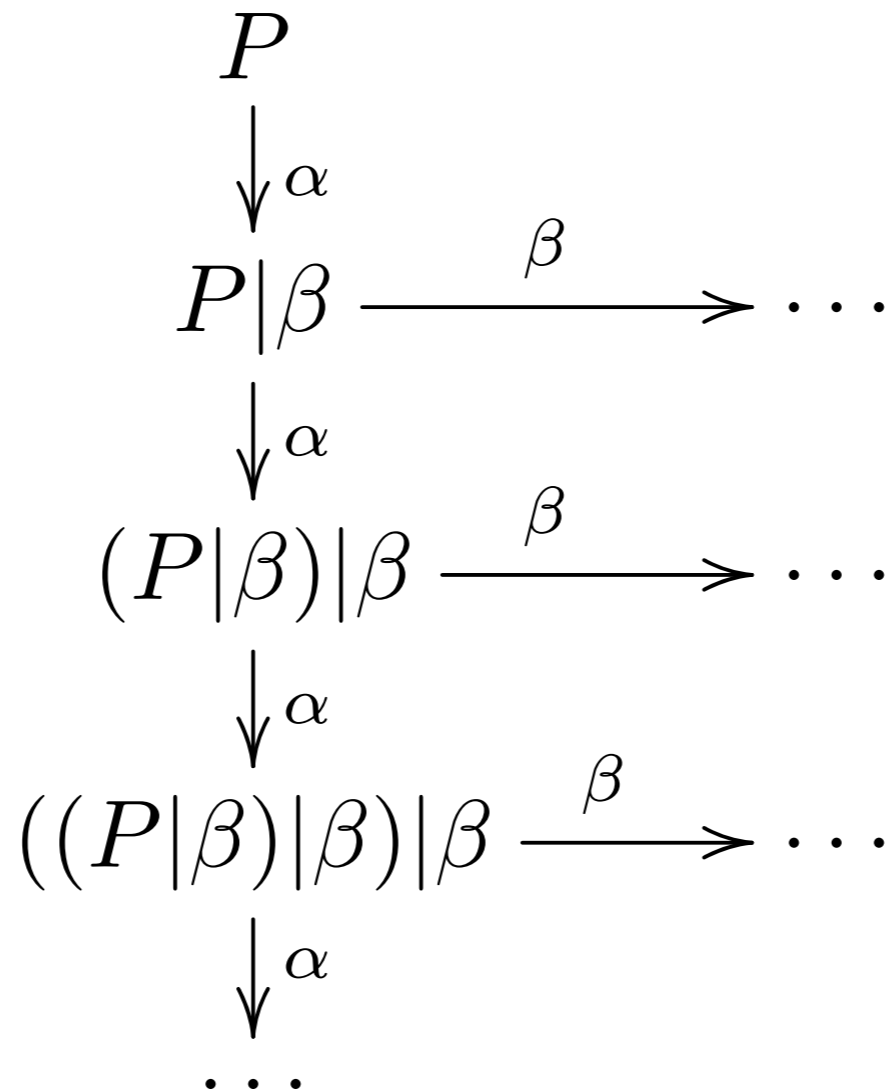
[Ex. 5] Write a guarded CCS process whose LTS has infinitely many states without using parallel composition.

Ex. 5, infinite LTS

using par

$$P \triangleq \mathbf{rec} \ x. \ \alpha.(x|\beta.\mathbf{nil})$$

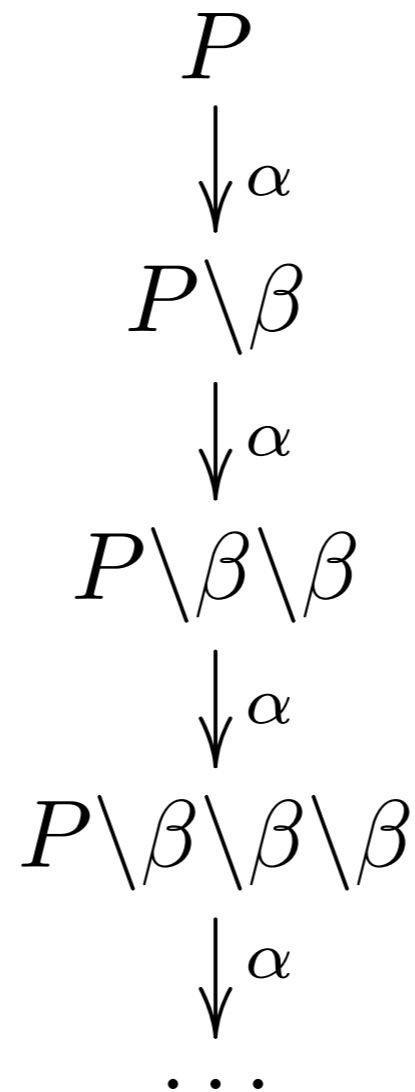
$$P \triangleq \alpha.(P|\beta)$$



Ex. 5, infinite LTS

$$P \triangleq \mathbf{rec} \ x. (\alpha.x) \setminus \beta$$

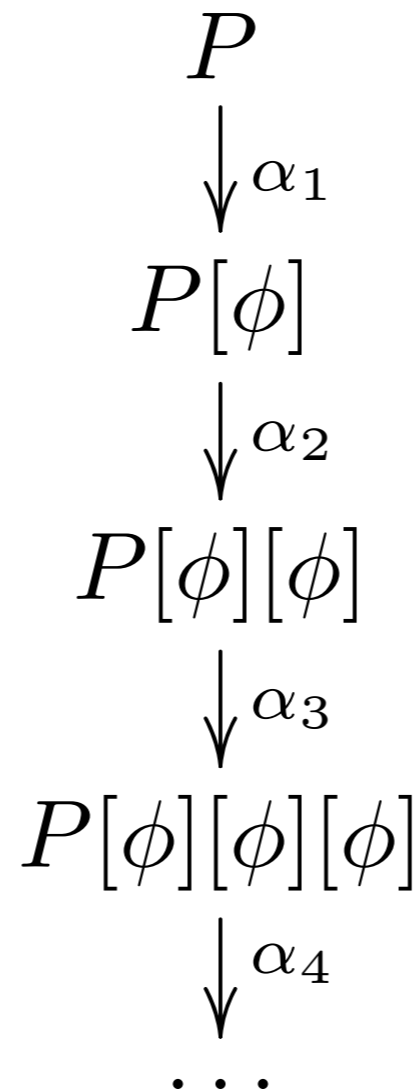
$$P \triangleq (\alpha.P) \setminus \beta$$



all states are bisimilar

Ex. 5, infinite LTS

$$P \triangleq \mathbf{rec} \ x. \ \alpha_1.(x[\phi]) \quad \phi(\alpha_i) = \alpha_{i+1}$$



Bisimulation

[Ex. 6] Prove that CCS strong bisimilarity is a congruence w.r.t. restriction, i.e., that for all p, q, α :

$$p \simeq q \Rightarrow p \setminus \alpha \simeq q \setminus \alpha$$

Ex. 6, congruence $\backslash \alpha$

$\mathbf{R} \triangleq \{(p \backslash \alpha, q \backslash \alpha) \mid p \simeq q\}$ we prove \mathbf{R} is a strong bisimulation

take $(p \backslash \alpha, q \backslash \alpha) \in \mathbf{R}$ (with $p \simeq q$)

take $p \backslash \alpha \xrightarrow{\mu} p'$ we want to find $q \backslash \alpha \xrightarrow{\mu} q'$ with $p' \mathbf{R} q'$

by rule res) it must be $p \xrightarrow{\mu} p''$ with $\mu \notin \{\alpha, \bar{\alpha}\}$ and $p' = p'' \backslash \alpha$

since $p \simeq q$ and $p \xrightarrow{\mu} p''$ we have $q \xrightarrow{\mu} q''$ with $p'' \simeq q''$

take $q' = q'' \backslash \alpha$: by rule res) we have $q \backslash \alpha \xrightarrow{\mu} q'$ and $(p', q') \in \mathbf{R}$

take $q \backslash \alpha \xrightarrow{\mu} q'$ we want to find $p \backslash \alpha \xrightarrow{\mu} p'$ with $p' \mathbf{R} q'$

analogous to the previous case

[**Ex. 7**] Prove that the CCS agents

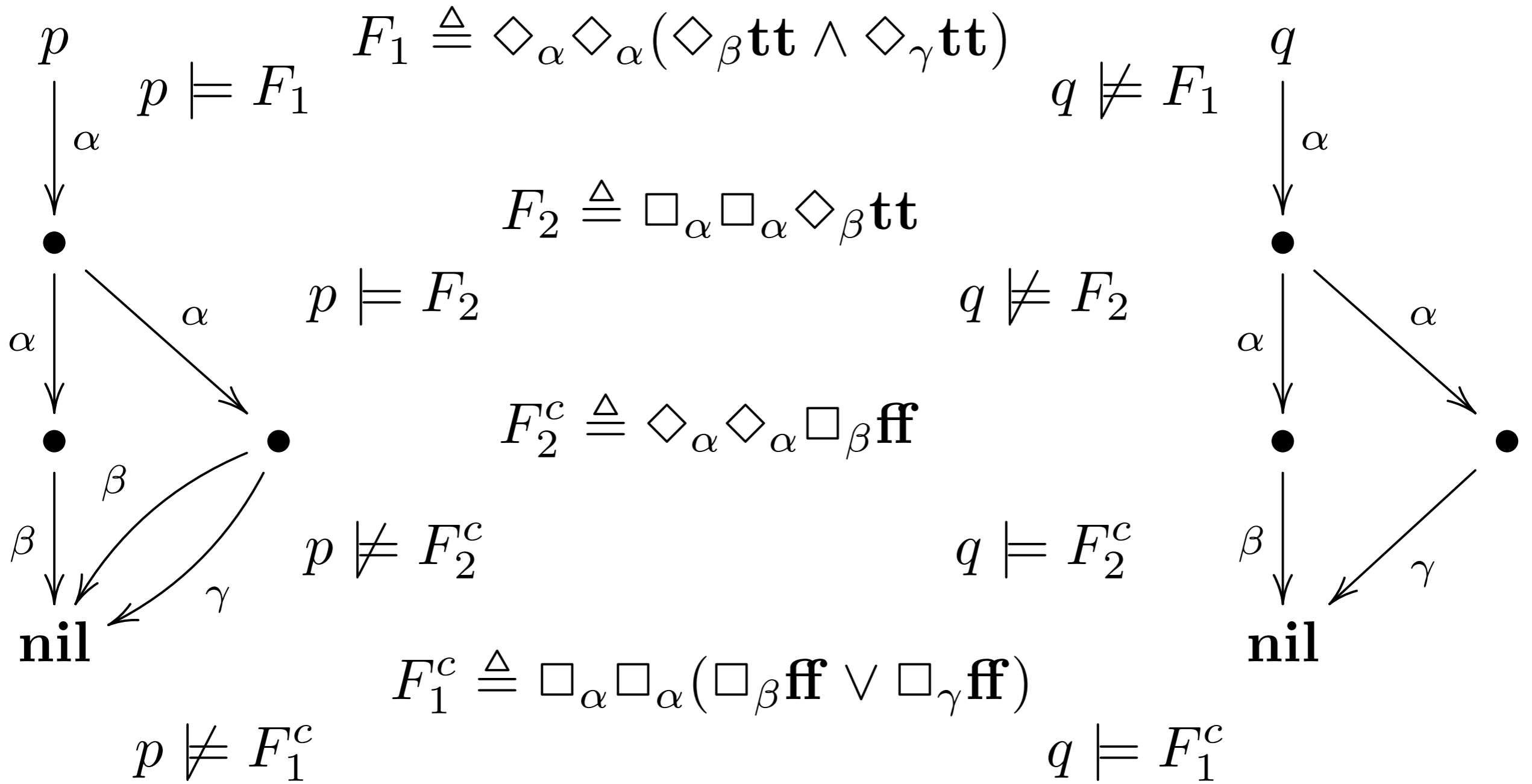
$$p \stackrel{\text{def}}{=} \alpha.(\alpha.\beta.\mathbf{nil} + \alpha.(\beta.\mathbf{nil} + \gamma.\mathbf{nil})) \quad \text{and} \quad q \stackrel{\text{def}}{=} \alpha.(\alpha.\beta.\mathbf{nil} + \alpha.\gamma.\mathbf{nil})$$

are not strong bisimilar.

Ex. 7, non bisimilar

$$p \triangleq \alpha.(\alpha.\beta + \alpha.(\beta + \gamma))$$

$$q \triangleq \alpha.(\alpha.\beta + \alpha.\gamma)$$



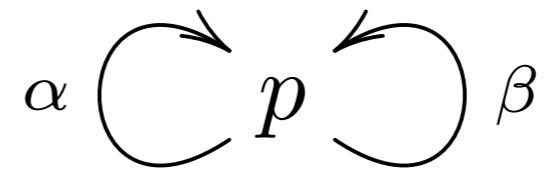
[Ex. 8] Let us consider the guarded CCS processes

$$p \stackrel{\text{def}}{=} \mathbf{rec} \ x.(\alpha.x + \beta.x) \quad q \stackrel{\text{def}}{=} \mathbf{rec} \ y.(\bar{\alpha}.\mathbf{nil} + \gamma.y) \quad r \stackrel{\text{def}}{=} \mathbf{rec} \ z.(\bar{\beta}.\mathbf{nil} + \bar{\gamma}.z)$$

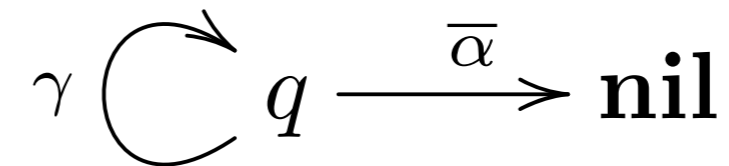
1. Draw the LTSs of the processes p , q , r and $s \stackrel{\text{def}}{=} (p|q|r) \setminus \alpha \setminus \beta \setminus \gamma$.
2. Show that s is strong bisimilar to the process $t \stackrel{\text{def}}{=} \mathbf{rec} \ w.(\tau.w + \tau.\tau.\mathbf{nil})$.

Ex. 8, bisimilar

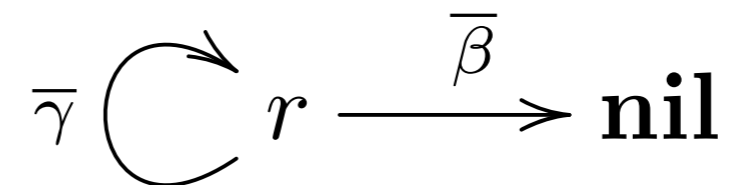
$$p \triangleq \mathbf{rec} \ x.(\alpha.x + \beta.x)$$



$$q \triangleq \mathbf{rec} \ y.(\bar{\alpha} + \gamma.y)$$



$$r \triangleq \mathbf{rec} \ z.(\bar{\beta} + \bar{\gamma}.z)$$

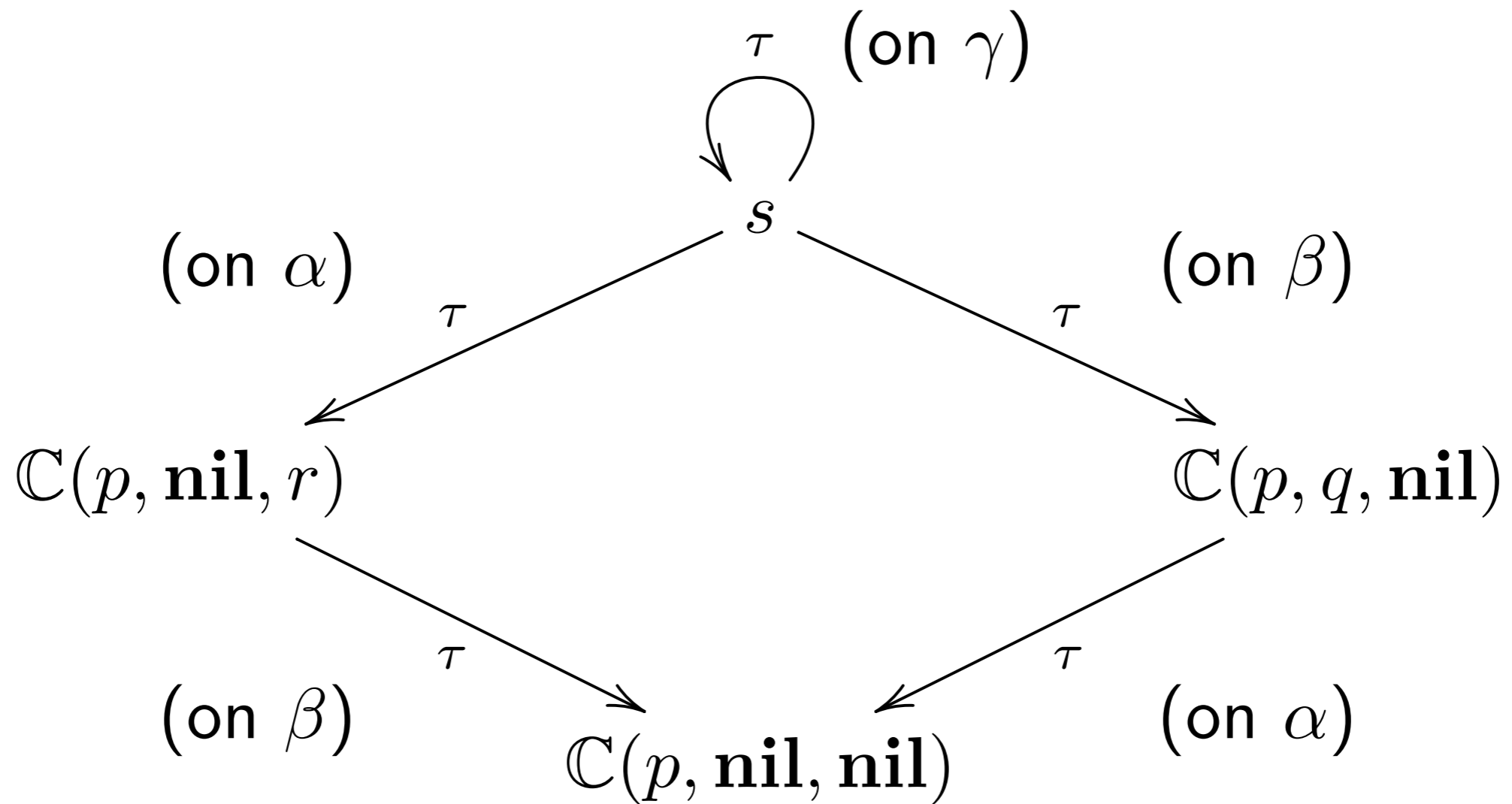
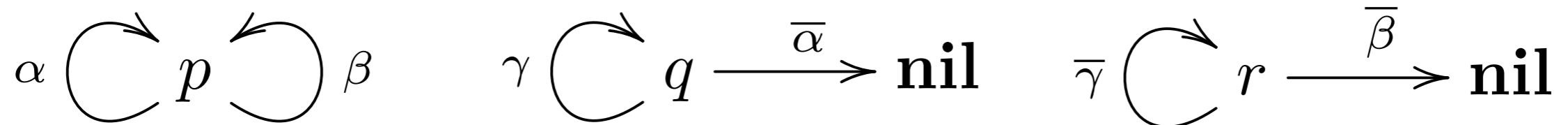


$$s \triangleq (p|q|r) \setminus \alpha \setminus \beta \setminus \gamma$$

$$\mathbb{C}(p, q, r) \triangleq (p|q|r) \setminus \alpha \setminus \beta \setminus \gamma$$

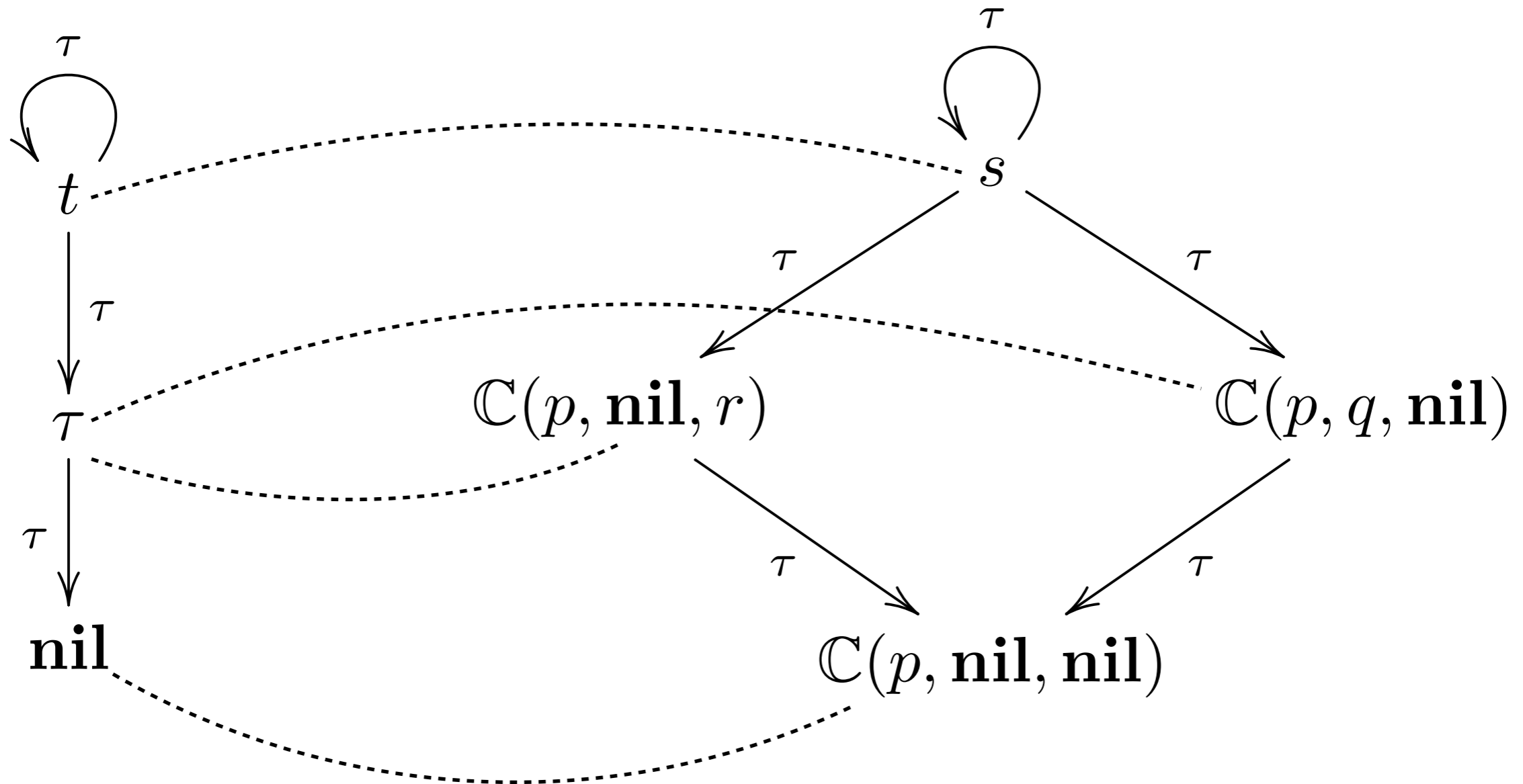
$$s \triangleq \mathbb{C}(p, q, r)$$

Ex. 8, bisimilar



Ex. 8, bisimilar

$$t \triangleq \mathbf{rec} w.(\tau.w + \tau.\tau)$$



$$\mathbf{R} \triangleq \{ \{s, t\}, \{\tau, \mathbb{C}(p, \mathbf{nil}, r), \mathbb{C}(p, q, \mathbf{nil})\}, \{\mathbf{nil}, \mathbb{C}(p, \mathbf{nil}, \mathbf{nil})\} \}$$

R is a strong bisimulation

[Ex. 9] Prove that the following property is valid for any agent p , where \approx is the weak bisimilarity:

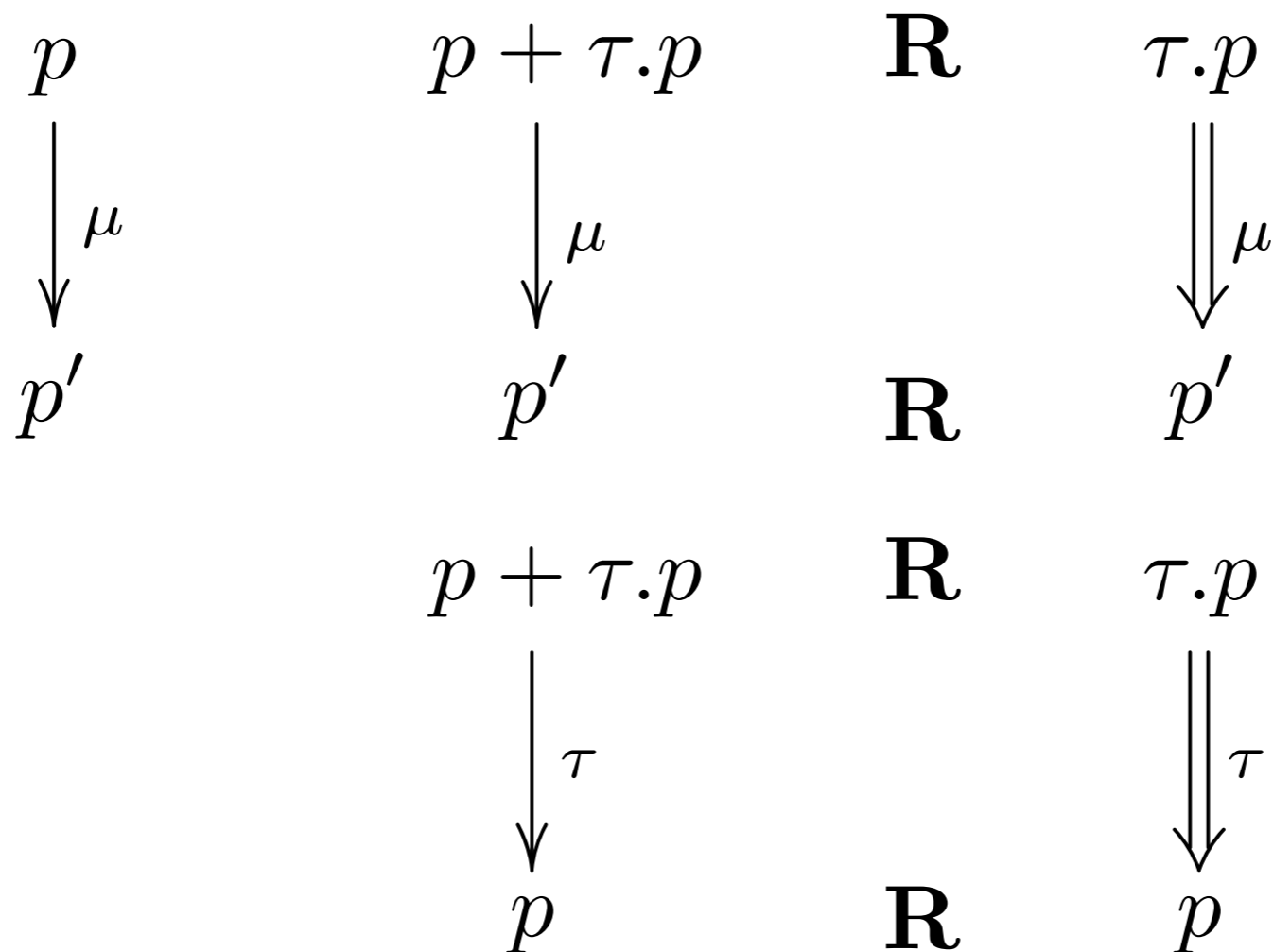
$$p + \tau.p \approx \tau.p$$

Ex. 9, weak bisimilar

$$\mathbf{R} \triangleq \{(p + \tau.p, \tau.p) \mid p \in \mathcal{P}\} \cup Id$$

we check that \mathbf{R} is a weak bisimulation

no need to check pairs in Id



Ex. 9, weak bisimilar

$$\mathbf{R} \triangleq \{(p + \tau.p, \tau.p) \mid p \in \mathcal{P}\} \cup Id$$

we check that \mathbf{R} is a weak bisimulation

no need to check pairs in Id

$$\begin{array}{ccc} p + \tau.p & \mathbf{R} & \tau.p \\ \downarrow \tau & & \downarrow \tau \\ p & \mathbf{R} & p \end{array}$$