

(L) Latency (T_c completion times)

Service time (T_s)

$S_p(m)$

Speedup

$S_c(m)$

$m \rightarrow$ par. degree
 $m \rightarrow$ # tasks

Efficiency

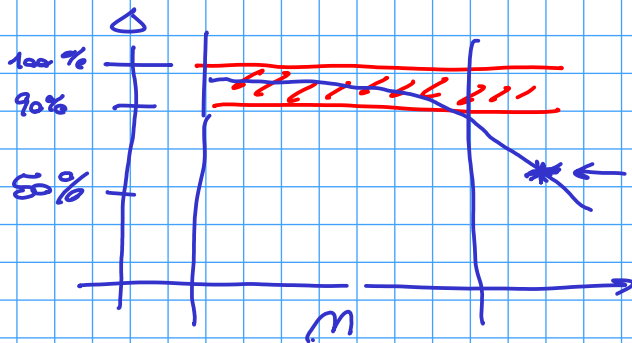
$$E(m) = \frac{T_{id}(m)}{T_{par}(m)}$$

$$T_{id}(m) = \frac{T_{seq}}{n}$$

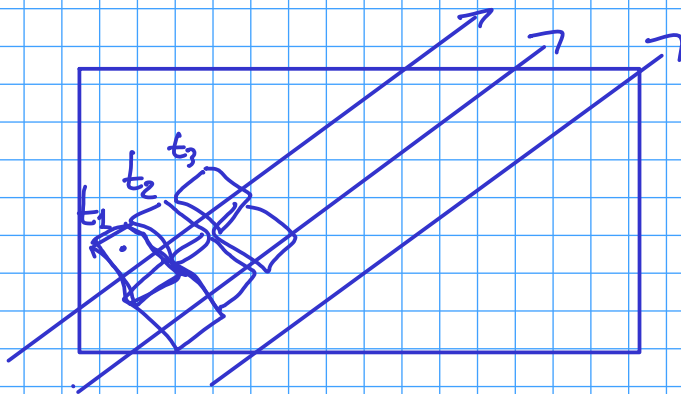
$$E(m) = \frac{T_{seq}/m}{T(m)}$$

$$= \frac{T_{seq}}{m \cdot T(m)}$$

$$E(m) = \frac{S_p(m)}{m}$$



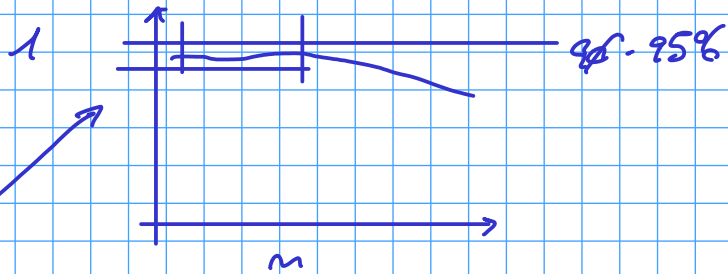
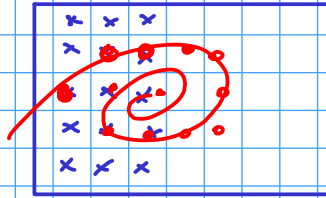
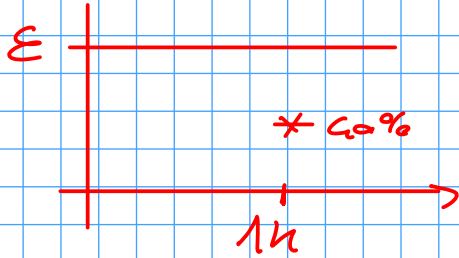
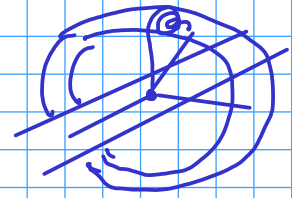
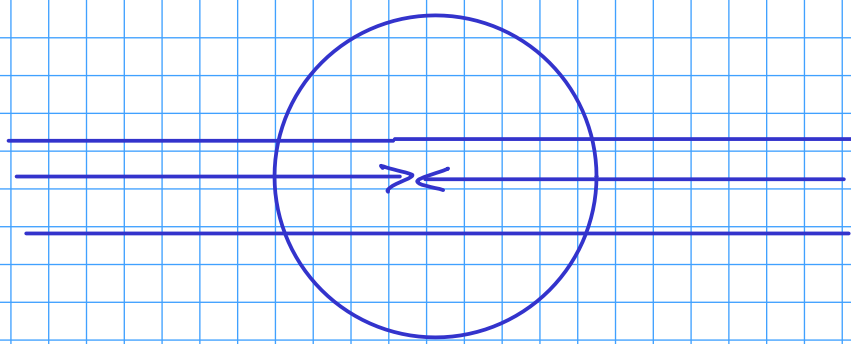
good range for E



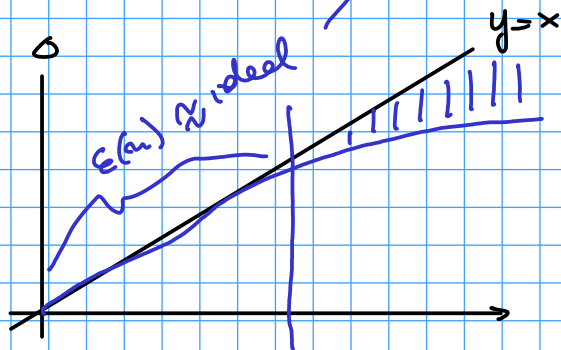
forall "photo" x

$F(x)$

$t < (t_i - t_{i-1})$



$$E(m) = \frac{SP(m)}{m}$$



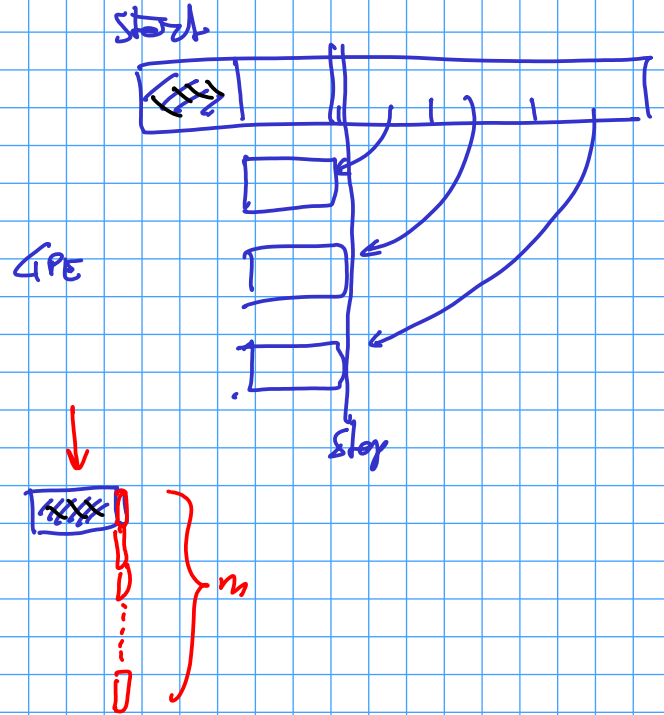
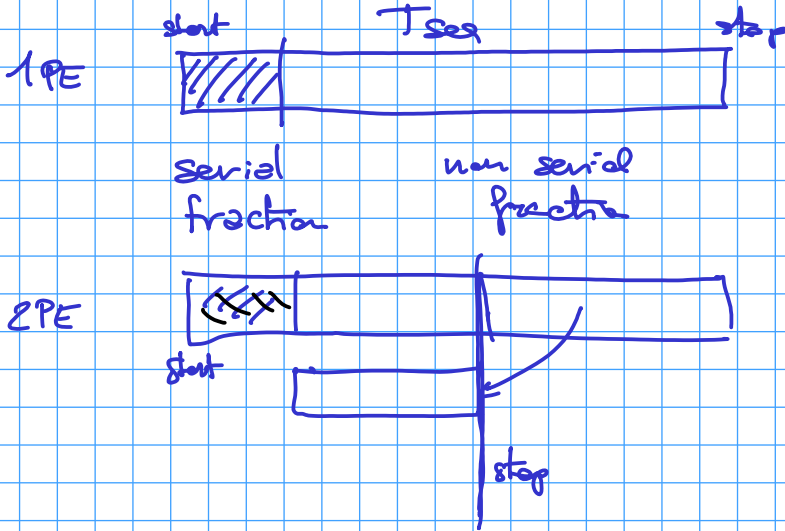
$$SP(m) = \frac{T_{seq}}{T(m)}$$

$$\binom{k}{p} \text{ FLOPS}$$

FLOPS/WATT

AMIAHL

'60



$m \rightarrow \infty$

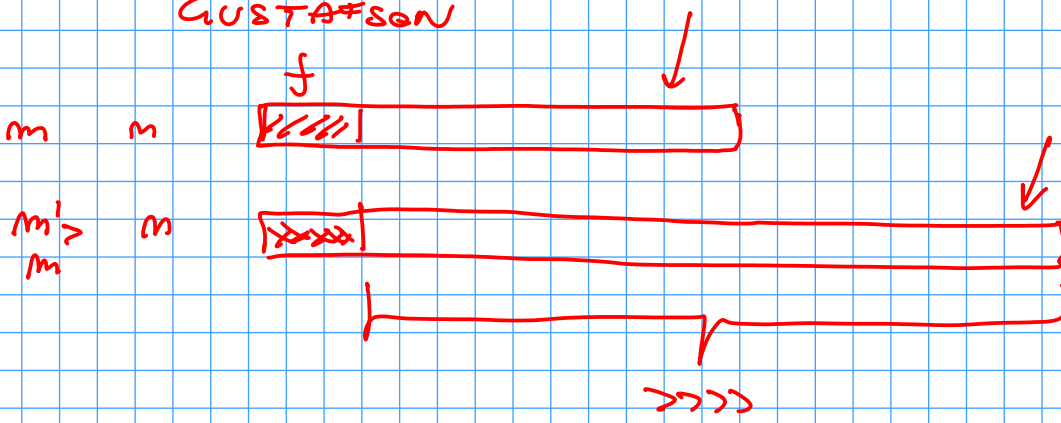
f serial fraction % of $T_{seq} \in [0, 1]$

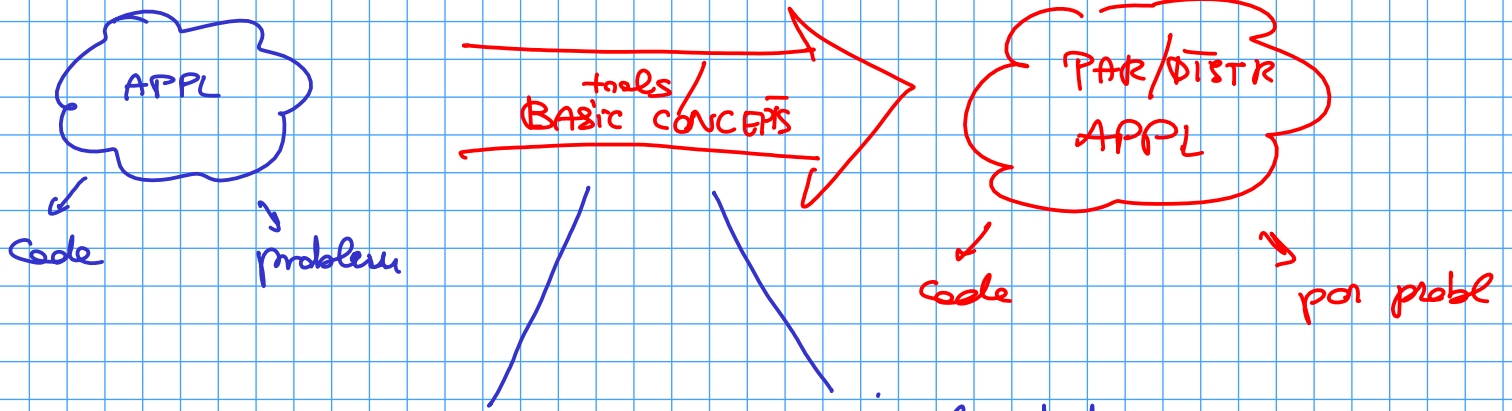
$$T_{seq} = (f) T_{seq} + (1-f) T_{seq}$$

$$sp(n) = \frac{T_{seq}}{f T_{seq} + \frac{(1-f) T_{seq}}{n}}$$

$$\lim_{n \rightarrow \infty} sp(n) = \frac{T_{seq}}{f T_{seq}} = \frac{1}{f}$$

GUSTAFSON

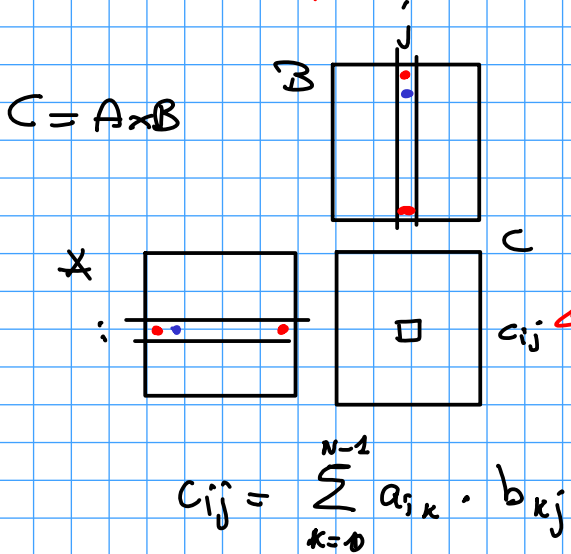




find what we can (usefully) do in parallel/distributed to "improve" implementation

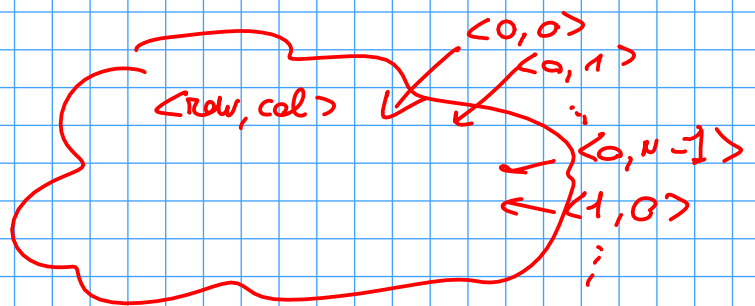
- concurrent activities } threads / processes
- communication
- synchronization

Matrix Multiplication



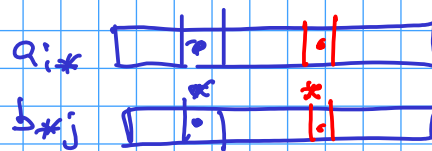
Can I do $c_{ij} = \sum \dots$ relatively to $\langle i, j \rangle$ and $\langle i', j' \rangle$ $i' \neq i$ $j' \neq j$ concurrently?

$\dots \times \dots + \dots \times \dots + \dots \times \dots$



computation of each c_{ij} may be done concurrently (write c_{ij})

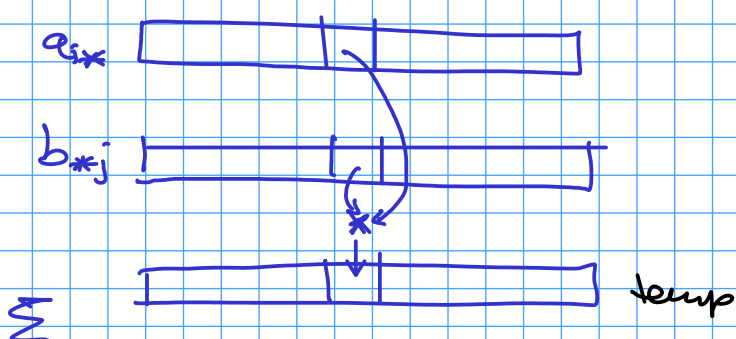
$$c_{ij} = \sum_k a_{ik} \cdot b_{kj}$$



act_i R(act_i) = { data read by } may be concurrent?
act_j W(act_i) = { data written by } YES!

$R(act_i) \cap W(act_j) = \emptyset$
 $W(act_i) \cap R(act_j) = \emptyset$
 $W(act_i) \cap N(act_j) = \emptyset$

$\left. \begin{array}{l} \\ \\ \end{array} \right\} \Rightarrow$ *dec. activities*



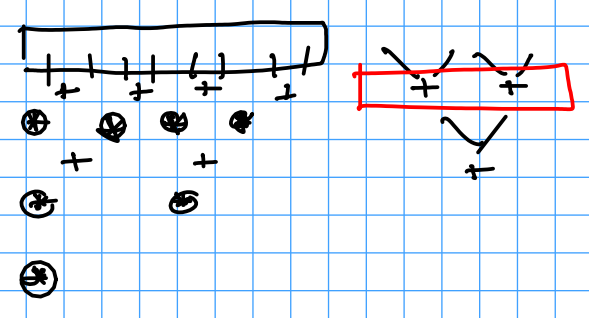
$+$: $(a_i + b) + c = a + (b + c)$
 $(a + b) = (b + c)$

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sum = 0;
for (i = 0; i < N; i++)
    sum = sum + temp[i];

```

(A tree diagram with nodes containing '+' and '' symbols is drawn to the left of the code.)*



Solve linear system

$$\begin{cases} a_{00}x_0 + a_{01}x_1 + \dots + a_{0,n-1}x_{n-1} = b_0 \\ a_{10}x_0 + \dots = b_1 \\ \vdots \\ a_{n-2,0}x_0 + \dots = b_{n-2} \end{cases}$$

$$\begin{bmatrix} a_{00} & \dots & a_{0,n-1} & b_0 \\ \vdots & & & \vdots \\ a_{n-2,0} & \dots & a_{n-2,n-1} & b_{n-2} \end{bmatrix}$$

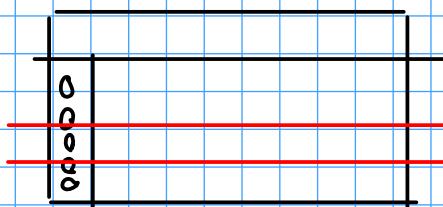
$$a_{00} \dots b_0$$

$$R_0$$

$$a_{01}$$

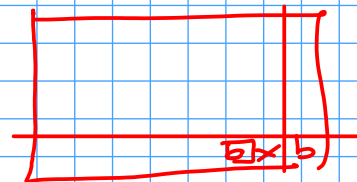
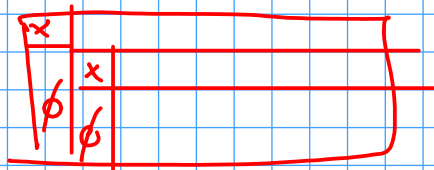
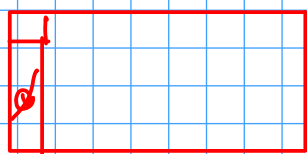
$$R_1 = R_1 - \left(\frac{a_{01}}{a_{00}}\right)R_0$$

$$\begin{array}{c} \hline a_{00} \dots b_0 \\ \emptyset \quad a_{11} \dots b_1 \end{array}$$



$$R_i = \{R_0, R_i\}$$

$$W = \{R_i\}$$



$$i \neq j$$

$$R_i = \{R_0, R_i\} \quad \{R_0, R_j\}$$

$$W_j = \{R_j\} \quad \{R_j\}$$

$$R_i \cap W_j$$

$$W_i \cap R_j$$

$$W_i \cap W_j$$