

Optimization methods to

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multicommodity flows

(Ahuja - Magnanti - Orlin : 17.4 - 17.5 - 17.6 - 17.7)

① Lagrangian relaxation

(Ahuja - Magnanti - Orlin : 17.4)

Recall the general LP model:

$$z = \text{Min} \sum_{k=1}^K \sum_{(i,j) \in A} c_{ij}^k x_{ij}^k$$

(MCF 1)

$$\sum_{(i,j) \in FS(i)} x_{ij}^k - \sum_{(j,i) \in BS(i)} x_{ji}^k = b_i^k \quad \forall i \in N$$

$k=1, \dots, K$

$$\sum_{k=1}^K x_{ij}^k \leq u_{ij} \quad \forall (i,j) \in A$$

π_{ij}

$$x_{ij}^k \geq 0 \quad \forall (i,j) \in A, k=1, \dots, K$$

where:

K : number of commodities

x_{ij}^k : amount of flow pushed along (i,j) for commodity k

To apply Lagrangian relaxation, associate a nonnegative Lagrangian multiplier π_{ij} with each bundle constraint:

$$\bar{z}(\pi) = \min \sum_{k=1}^K \sum_{(i,j) \in A} (c_{ij}^k + \pi_{ij}) x_{ij}^k - \sum_{(i,j) \in A} \pi_{ij} u_{ij}$$

$$\sum_{(i,j) \in FS(i)} x_{ij}^k - \sum_{(j,i) \in BS(i)} x_{ji}^k = b_i^k \quad \forall i \in N, k=1, \dots, K$$

$$x_{ij}^k \geq 0 \quad \forall (i,j) \in A, k=1, \dots, K$$

For given $\{\pi_{ij}\}$ the term $\sum_{(i,j) \in A} \pi_{ij} u_{ij}$ is constant.

Therefore, the resulting problem decomposes into separate minimum cost flow problems, one for each commodity, with costs $\{c_{ij}^k + \pi_{ij}\}$.

Note that, since (MCF1) is a linear program, if we solve the corresponding Lagrangian Dual we get

$$\bar{z} = \max_{\pi: \pi_{ij} \geq 0} z(\pi)$$

For example, if we use the Subgradient ⁽³⁾ algorithm to solve the Lograngian Dual, then at each step we can update the Lograngian multipliers according to:

$$\pi_{ij}^{\epsilon+1} = \max \left\{ \pi_{ij}^{\epsilon} + \lambda_{\epsilon} \left(\sum_{k=1}^K x_{ij}^{*k} - u_{ij} \right), 0 \right\},$$

↑
step size

where :

- ϵ current iteration
- $\{x_{ij}^{*k}\}$ optimal solution for commodity k

Recall, in fact, that $\left(\sum_{k=1}^K x_{ij}^{*k} - u_{ij} \right)$ is a subgradient of the Lograngian function at $\{ \pi_{ij}^{\epsilon} \}$.

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example (pages 661-662-663)

