

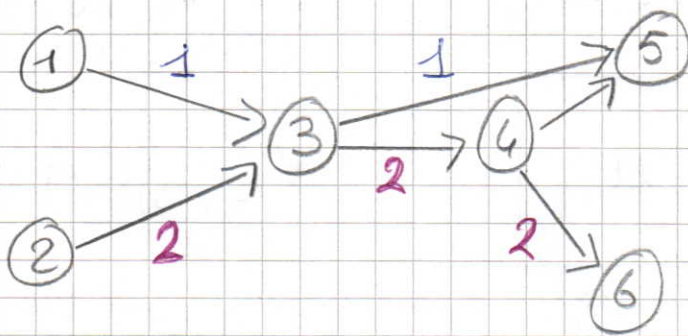
Multicommodity flows

①

(Ahuja - Magnanti - Orlin : 17.1)

In many application contexts, several commodities share network resources (common arc capacities)

example



$$u_{ij} = 2$$

$$\forall (i,j) \in A$$

commodity 1 : send one message from 1 to 5

commodity 2 : send two messages from 2 to 6

Obs: commodity 1 can not use the path (1 3 4 5), due to commodity 2 (which must use arc (3,4), of capacity 2)

General LP model:

(2)

x_{ij}^k : amount of flow pushed along (i, j) for commodity k

$$(MCF_1) \quad \text{Min} \quad \sum_{k=1}^K \sum_{(i,j) \in A} c_{ij}^k x_{ij}^k$$

$$\sum_{(i,j) \in FS(i)} x_{ij}^k - \sum_{(j,i) \in BS(i)} x_{ji}^k = b_i^k \quad \forall i \in N$$

$k=1, \dots, K$

$$0 \leq x_{ij}^k \leq u_{ij}^k$$

$$\forall (i,j) \in A$$

$k=1, \dots, K$

$$\sum_{k=1}^K x_{ij}^k \leq u_{ij}$$

$$\forall (i,j) \in A$$

Bundle
constraints

common arc
capacities

where:

K : number of commodities

u_{ij}^k : individual flow bounds
(maybe $+\infty$)

Assumptions:

1) Linear costs

This is not a proper assumption e.g. in traffic networks, where often the cost function is nonlinear, e.g. to model "congestion":

e.g.
$$\text{Min } \sum_{(i,j) \in A} \frac{x_{ij}^2}{(u_{ij} - x_{ij})}$$

where
$$x_{ij} = \sum_{k=1}^K x_{ij}^k$$

* when the flow approaches the arc capacity, then the "delay" goes to $+\infty \dots$

2) the model assumes that $\{x_{ij}^k\}$ can be fractional

This is not appropriate in several application contexts; however:

* Multicommodity flows do not satisfy the integrality property *