

Cutting Plane Algorithms

①

(Wolsey : Chapter 8)

Consider the Integer Linear Program (ILP):

$$(IP) \quad \begin{array}{l} \max c x \\ Ax \leq b \\ x \in \mathbb{Z}^n \end{array} \quad \left. \vphantom{\begin{array}{l} \max c x \\ Ax \leq b \\ x \in \mathbb{Z}^n \end{array}} \right\} x \in X$$

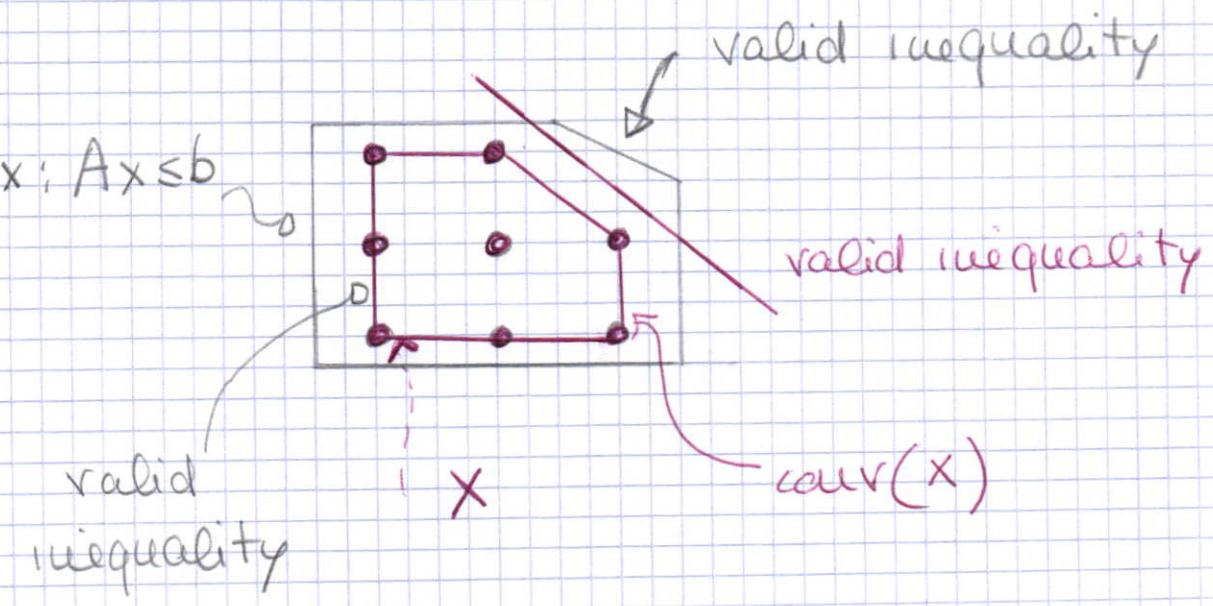
We know that the convex envelope of X , $\text{conv}(X)$ is a polyhedron. So, the optimal solution of (P) is an optimal solution of the LP:

$$\begin{array}{l} \max c x \\ x \in \text{conv}(X) \end{array}$$

Unfortunately, for NP-Hard problems we do not have an explicit, or a "good", description of $\text{conv}(X)$:

< how to find effective ways to approximate $\text{conv}(X)$? >

Definition : an inequality $\pi x \leq \pi_0$ is a valid inequality for X if $\pi x \leq \pi_0$ $\forall x \in X$



Simple valid inequalities

These are logical inequalities

1) 0-1 Knapsack set

$$X = \{ x \in \{0, 1\}^5 : 3x_1 - 4x_2 + 2x_3 - 3x_4 + x_5 \leq -2 \}$$

• if $x_2 = x_4 = 0$, then the l.h.s is $3x_1 + 2x_3 + x_5$, which is ≥ 0 but the r.h.s. is -2 impossible!

So $x_2 + x_4 \geq 1$ is valid for X

• similarly, if $x_1 = 1$ and $x_2 = 0$;

(3)

l.h.s, $3 + 2x_3 - 3x_4 + x_5 \geq 3 - 3 = 0$

but the r.h.s is -2

impossible!

So $x_1 \leq x_2$ is valid for X

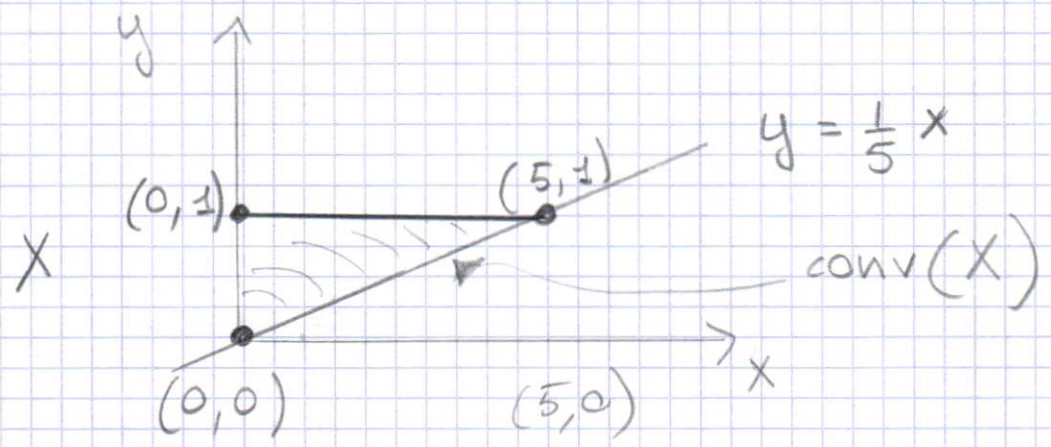
2) Mixed 0-1 set

$$X = \{(x, y) : x \leq 9999y, 0 \leq x \leq 5, y \in \{0, 1\}\}$$

Indeed:

$$X = \{(0, 0), (x, 1) \text{ with } 0 \leq x \leq 5\}$$

So $x \leq 5y$ is valid for X



Note that $\text{conv}(X) = \{(x, y) : 0 \leq x \leq 5, 0 \leq y \leq 1, x \leq 5y\}$