Branch and Bound

(Wolsey : Chapter 7 (7.1, 7.2))

Consider

$$(P) \qquad z = \max_{x \in S} c(x)$$

How can we break (P) into a series of smaller (and easier) problems, solve the smaller problems, and then put the information together to solve (P)?

Divide and conquer approach

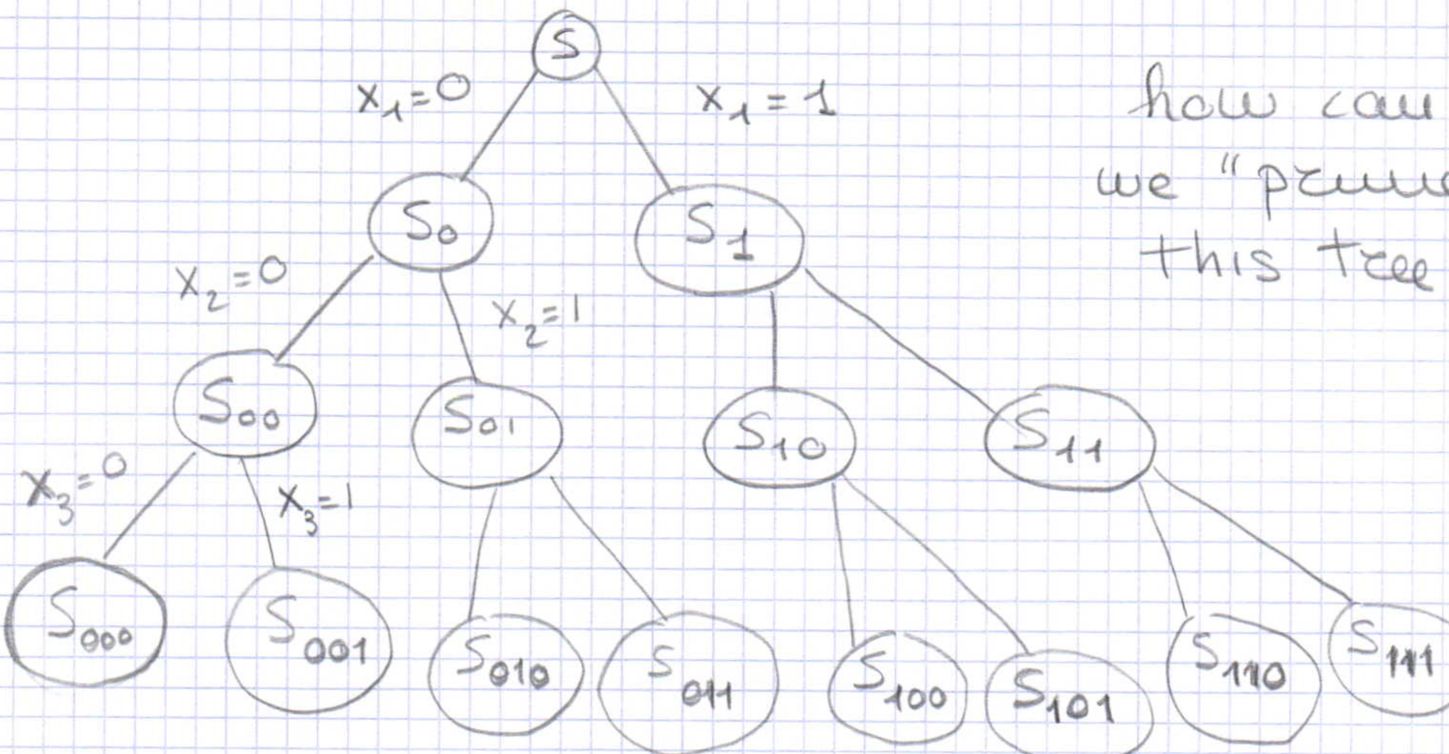
Proposition : Let $S = S_1 \cup S_2 \cup \ldots \cup S_K$ be a decomposition of S, and let $z^k = \max\{c(x) : x \in S_k\}$, $k = 1, \ldots K$.
Then $z = \max_k z^k$.

- A typical way is to decompose S via an enumeration tree
- However: a complete enumeration is usually impossible (we can not divide indefinitely)

<u>So</u>:

- how can we use some bounds on $\{z^k\}$ intelligently?
- how can we put together bound information?

<u>example</u>: binary enumeration tree for $S \subseteq \{0,1\}^3$



how can we "prune" this tree?

Implicit enumeration

<u>Proposition</u>: Let $S = S_1 \cup .. \cup S_K$ be a decomposition of S, and let $z^k = \max\{c(x) : x \in S_k\}$, $\bar{z}^k$ be an upper bound on z^k, and $\underline{z}^k$ be a lower bound on z^k, $k = 1, \ldots K$.

Then:

$$\boxed{\bar{z} = \max_k \bar{z}^k}$$ is an <u>upper bound on z</u>

and

$$\boxed{\underline{z} = \max_k \underline{z}^k}$$ is a <u>lower bound on z</u>

- So, bound information (partial information) about subproblems can be put together to derive bounds on z!

- What can be deduced from these bounds, and which sets need further examination to compute z?