

# Basic Network Design

①

## Problems

(Pozzo - Medhi :  $ui$  [4.1, 4.2, 4.3, 4.4])

- Basic problems associated with normal (or nominal) state of communication networks
- All resources fully available

## Notation

$d$  : denotes demand between an origin and a destination  $\equiv$  also commodity

$D$  : set of demands (commodities)

$P_d$  : candidate paths to satisfy  $d$

$h_d$  : volume of demand of  $d$ ,  $\forall d \in D$

$c_{ij}$  : unit cost of link  $(i, j)$ ,  $\forall (i, j) \in A$

[  $c_{ij} \sim \xi_e$  in  $P-N$ ,  
with  $e = (i, j)$  ]

# Uncapacitated network design (4.1.1) <sup>②</sup>

## Variables

- $x_{dp}$ : flow on path  $p \in P_d$ ,  $\forall d \in D$ ,  $p \in P_d$

< link-path formulation >

- $y_{ij}$ : capacity to be associated with link  $(i,j)$ ,  $\forall (i,j) \in A$

$$\text{Min } \sum_{(i,j) \in A} c_{ij} y_{ij} \quad (\text{bandwidth cost})$$

$$\sum_{p \in P_d} x_{dp} = h_d \quad \forall d \in D \quad (\text{demand satisfaction})$$

$$\sum_{d \in D} \sum_{p \in P_d} \delta_{ij}^{(dp)} x_{dp} \leq y_{ij} \quad \forall (i,j) \in A$$

$(\delta_{edp} \text{ is P-M})$

(capacity constraints)

$$x_{dp} \geq 0 \quad \forall d \in D, \forall p \in P_d$$

where

$$\delta_{ij}^{(dp)} = \begin{cases} 1 & \text{if } (i,j) \in p, p \in P_d \\ 0 & \text{otherwise} \end{cases}$$

Obs:

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- 1) LP model: "easy", but not tractable in this form
- 2) in the optimal solution,  $y_{ij}$  are equal to the "load" of  $(i,j)$ ,  $\forall (i,j) \in A$
- 3) indeed, an optimal solution can be obtained by sending  $h_d$ ,  $\forall d$ , on a shortest path in  $P_d$  (w.r.t.  $\{c_{ij}\}$ ):

shortest path allocation

rule

easy!

- 4) the corresponding node-link formulation is (MCF1), by replacing  $u_{ij}$  by variables  $y_{ij}$ ,  $\forall (i,j) \in A$ .

< only model at page 106 >