

Basic Network Design

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Problems

(Pozzo - Medhi : ui [4.1, 4.2, 4.3, 4.4])

- Basic problems associated with normal (or nominal) state of communication networks
- All resources fully available

Notation

d : denotes demand between an origin and a destination \equiv also commodity

D : set of demands (commodities)

P_d : candidate paths to satisfy d

R_d : volume of demand of d , $\forall d \in D$

c_{ij} : unit cost of link (i, j) , $\forall (i, j) \in A$

[$c_{ij} \sim \xi_e$ in $P-N$,
with $e = (i, j)$]

Uncapacitated network design (4.1.1) ⁽²⁾

Variables

- x_{dp} : flow on path $p \in P_d$, $\forall d \in D$, $p \in P_d$

< link-path formulation >

- y_{ij} : capacity to be associated with link (i,j) , $\forall (i,j) \in A$

$$\text{Min } \sum_{(i,j) \in A} c_{ij} y_{ij} \quad (\text{bandwidth cost})$$

$$\sum_{p \in P_d} x_{dp} = h_d \quad \forall d \in D \quad (\text{demand satisfaction})$$

$$\sum_{d \in D} \sum_{p \in P_d} \delta_{ij}^{(dp)} x_{dp} \leq y_{ij} \quad \forall (i,j) \in A$$

(δ_{edp} is P-M)

(capacity constraints)

$$x_{dp} \geq 0 \quad \forall d \in D, \forall p \in P_d$$

where

$$\delta_{ij}^{(dp)} = \begin{cases} 1 & \text{if } (i,j) \in p, p \in P_d \\ 0 & \text{otherwise} \end{cases}$$

Obs:

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- 1) LP model: "easy", but not tractable in this form
- 2) in the optimal solution, y_{ij} are equal to the "load" of (i,j) , $\forall (i,j) \in A$
- 3) indeed, an optimal solution can be obtained by sending f_d , $\forall d$, on a shortest path in P_d (w.r.t. $\{c_{ij}\}$):

shortest path allocation

rule

easy!

- 4) the corresponding node-link formulation is (MCF1), by replacing u_{ij} by variables y_{ij} , $\forall (i,j) \in A$.

< only model at page 106 >