Network flows

(Ahuja - Magnanti - Orlin : chapters 1, 2, 3 (essentially 3.5))

Review of basic concepts and used notation

Chapter 1 (until page 7, row + 3)

- Introduction to network flows and their application in nowaday life

  - how to move some entity (electricity, a consumer product, a person, a vehicle ...) from some points to other points in an underlying network in an "efficient" way;

  - typical of areas such as applied mathematics, computer science, engineering, management ... ;

Basic problems:

- Shortest path problem (assumed known!)
- Maximum flow problem: given arc capacities, how can we send as much flow (good) as possible from a source to a destination?
- Minimum cost flow problem: if we incur a cost per unit flow in a capacitated network, how can we send units of a good from some points to other points at a minimum cost?

NB: these are special LPs (and so, polynomially solvable); however, for efficiency reasons they are addressed directly via graph theory, and not via a LP perspective (although many concepts derive from LP theory!)

Minimum cost flow

- distribution of a product from plants to warehouses
- routing of vehicles along a street network .....,

→ See Chapter 2 (from 2.1 to 2.3 : TO READ) for basic notation and definitions of graph theory

Let $G = (N, A)$ directed network

- N set of n nodes
- A set of m directed arcs
- c_{ij} cost per unit flow on (i,j), $\forall (i,j) \in A$
- u_{ij} capacity of (i,j), $\forall (i,j) \in A$
  ("maximum" amount of flow)
- $b(i) \in \mathbb{Z}$ supply / demand of node i, $\forall i \in N$:

Assumption (the opposite w.r.t. RO course):

$b(i) > 0$ supply node

$b(i) < 0$ demand node (with demand $-b(i)$)

$b(i) = 0$ transshipment node

Decision variables (flow variables):

x_{ij} flow to push along (i,j), $\forall (i,j) \in A$

Mathematical model (LP):

(MCF) $\text{Min} \sum_{(i,j)\in A} c_{ij} x_{ij}$

Flow conservation constraints

$$\underbrace{\sum_{(i,j)\in FS(i)}}_{\text{Forward Star of } i} x_{ij} - \underbrace{\sum_{(j,i)\in BS(i)}}_{\text{Backward Star of } i} x_{ji} = b(i) \quad \forall i \in N$$

$$0 \le x_{ij} \le u_{ij} \quad \forall (i,j) \in A$$

l_{ij} in A.M.O.

Capacity constraints