

Algorithm Engineering -- EXERCISES

15 January 2024 – 1 hour

Name and Surname:

#matricola:

Question #1 [score 4+3+3] Given the set of strings

$$S = \{BAA, BAB, BACAA, BACAB, BACAD, BACB, CA, CB\},$$

index S via a two-level scheme with block size of 2 strings each and a Patricia trie in internal memory.

Then show how to perform:

- A **lexicographic** search for the string BB
- A **prefix** search for the string BAC.

Question #2 [score 5]. Given the sequence of integers (2, 5, 7), compress it via Interpolative Coding.

Question #3 [rank 5]. Given a sequence of strings (BACAB, ABB, BBC, DD, DF), sort them via multikey quicksort by assuming that the pivot is taken as the first string of each (sub-)sequence to be sorted.

Question #4 [score 6]. Given the text $T = ABRABRA$, apply the pipeline BWT+MTF+RLE0 (with Wheeler's code) and finally apply Arithmetic coding on the first 3 numbers of the output of this pipeline.

Question #5 [score 4] Assume you are given 5 strings (aa, ab, bb, bc, cc) and you wish to construct a minimal ordered perfect hash function (MOPHF).

Assume that $\text{rank}(c)$ is the ordered position of ' c ' in the alphabet, counting letter a from 1.

We let the two random functions required by the design of MOPHF as

$h_1(c' c'') = 2 * \text{rank}(c') + \text{rank}(c'') \bmod 11$ and $h_2(c' c'') = 3 * \text{rank}(c') * \text{rank}(c'') \bmod 11$. Construct the final $h(t)$.

Algorithm Engineering -- THEORY
15 January 2024 – 45 minutes

Name and Surname:

#matricola:

Question #1 [score 5+3]

- Prove that the expected length of an ordered sequence produced by the algorithm Snow Plow is $2M$.
- What is that expected length if the probability for an item to go in the “unsorted bucket” is $\frac{1}{4}$ instead of $\frac{1}{2}$?

Question #2 [score 5+4+3].

- Show how to COUNT in a text $T[1,n]$ all occurrences of a pattern $P[1,p]$, by assuming that T has been indexed via a Suffix Array data structure, built off-line and residing in memory.
- Show and prove the time complexity of the above COUNT operation.
- What is the I/O-cost of performing the RETRIEVAL of the positions of all pattern occurrences in the case that the Suffix Array is stored on disk?

Question #3 [rank 5+5]. Given two sorted lists of integers, say $L1$ and $L2$ of lengths n and m respectively:

- Describe the “doubling algorithm” to compute their intersection and state its time complexity.
- Prove the time complexity of the previous point.