

**Algorithm Engineering – exercises**  
**05 June 2023 – time 60 minutes**

**Name and Surname:**

**#matricola:**

**Question #1 [scores 8]** Given the string  $S = abababc$  show the result of the algorithmic pipeline BWT + MTF + RLE0 + Huffman, where RLE0 is the application of the special RunLengthEncoding algorithm over 0-runs and the Wheeler code.

**Question #2 [scores 4+5]** Given the sequence of integers (11, 14, 16, 19, 20, 21, 22), show how to encode them based on

- Elias-Fano Code
- Interpolative Code (just one level of recursion, hence just 3 numbers)

**Question #3 [scores 5+3]**

- Construct two (maximum-priority) treaps over the set of pairs  $\langle \text{key}, \text{priority} \rangle$ :
  - T1 contains  $\{ (A,8), (B,2), (C,9), (D,4) \}$
  - T2 contains  $\{ (H,3), (M,7), (G,0), (L,1) \}$

where we assume that letters (keys) are alphabetically ordered.

- Then show the result of merging T1 with T2.

**Question #4 [scores 5].** Decode the compressed sequence  $\langle 4, 011110 \rangle$  produced by arithmetic code, by assuming probabilities  $P[a]=P[c]=1/4$  and  $P[b]=1/2$ .

**Algorithm Engineering – theory**  
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**Question #1 [scores 5+5+5+5]**

- State and prove the lower-bound to the time complexity of sorting  $n$  strings of total length  $N$ , built over an alphabet of size  $\sigma$ .
- Write the pseudocode of the Multikey Quicksort to sort  $n$  strings of total length  $N$ , built over an alphabet of size  $\sigma$ .
- Prove that the Multikey Quicksort is time optimal.
- Prove the time complexity of the optimal Radix sort when applied over a set of  $n$  binary strings of total length  $N$ .

**Question #2 [scores 3+4]**

- Given a binary array  $B$  show the Rank data structure.
- Compute its space complexity in bits.

**Question #3 [scores 3]** Given two sets  $A$  and  $B$ , stored in two different servers, show how to compute their intersection by deploying a Bloom Filter and just one-communication round, and possibly making errors.