Algorithm Engineering – exercises
16 January 2023 – time 60 minutes

Name and Surname: **#matricola:**

Question #1 [scores 4] Simulate the algorithm SnowPlow over the sequence 2,5,4,3,1,4,2, and show which sorted blocks it forms with a memory of size M=2.

Question #2 [scores 4+4] Given the ordered set of strings

$$S = \{ \text{AABA, AACAAAC, AACAACC, BABAA, BABBB, BACA} \}$$

- Build the Patricia trie PT for S.
- Show the steps executed to lexicographically search for the pattern P = AACBACD in the set S by means of the Patricia Trie.

Question #3 [scores 5] Given the ordered set of strings S = {AA, AC, BB, CC}. Compute the Minimal Ordered Perfect Hash for S by assuming the following two hash functions:

$$h_1(xy) = x+y \mod 7 \quad \text{and} \quad h_2(xy) = x+2*y \mod 7$$

in which x (resp. y) is the code of the first (resp. second) letter of a string of S, and the codes are: A=1, B=2, and C=3.
As an example, if the string is AC, then x=1 and y=3.

Question #4 [scores 5]. Given the string "ABABAC" compress it by using the pipeline BWT + MTF + RLE0 + Huffman, where MTF counts letter's positions from 0, and RLE0 uses the Wheeler's code.

Question #5 [scores 3+3+2]. You are given the binary tree T described by the following list of edges, where (x,y) indicates that the edge is from node x to node y, and edges are ordered from left to right (hence node c has only the left child):

$$(a,b)(a,c)(b,d)(b,e)(c,f)(e,g)(e,h)$$

a) Provide a succinct encoding of T's structure in a binary array B, and the storage of T's labels in an array L;
b) Show which operations are executed to percolate the path:
      a (root) -> c (right children) -> f (left children)
   by using just the array B.
c) How can you discover that the node labeled f is a leaf of the original tree T ?

Algorithm Engineering – theory
16 January 2023 – time 60 minutes

Name and Surname: #matricola:

Question #1 [scores 6] Sketch the LSD radix-sort algorithm, state its time complexity, and prove that the algorithm is correct.

Question #2 [scores 5+6]
- Define what is a class of universal hash functions
- Provide an example and prove its universality

Question #3 [scores 5] Describe the data structure used to support the rank operation over a binary array B[1,n] in constant time, and evaluate its space occupancy.

Question #4 [scores 4+4]
- Define formally what is the suffix array SA of a text string T[1,n]
- Define formally what is its corresponding LCP array.