Ex.3 \( \exists M \in [M_0]. M(p_1) > 1 \)

2. no, because \( t_3 \) is dead (\( G \) has no arc labelled by \( t_3 \))
3. yes, all states have outgoing arcs
4. yes, because \( G \) is finite
5. yes, in each reachable marking there is at most one token in each place
6. no, the initial marking \( p_1 \) has no incoming arc

Ex.5 1. the Parikh vector is \( \vec{\sigma} = [3 2 3 2] \), thus \( M = M_0 + N \cdot \vec{\sigma} = 2p_2 + 2p_4 + 2p_5 \)
2. the Parikh vector is \( \vec{\sigma}' = [3 3 2 1] \), but \( M' = M_0 + N \cdot \vec{\sigma}' = \begin{bmatrix} -1 & 1 & 1 & 0 & 2 & 2 \end{bmatrix} \) is not a marking

Ex.6 1. yes, every place has exactly one incoming arc and one outgoing arc
2. \( I = [3 1 2 1 1 2] \)
3. \( I \cdot M_0 = 6 \) and \( I(p_1) = 3 \) thus for any \( M \in [M_0] \) we have \( M(p_1) \leq 6/3 = 2 \); note that by firing \( t_2 t_4 \) we get two tokens in \( p_1 \)

Ex.4 1. no because \( p_2 \) has two outgoing arcs
2. the firing sequence \( t_5 t_1 t_1 t_3 t_4 t_5 t_1 t_3 \) leads to \( 2p_3 \) which is a deadlock
3. no, because it is not deadlock free
4. places \( p_1, p_2, p_3, p_5 \) must be assigned the same weight \( x \) because of \( t_1, t_3, t_5 \); then because of \( t_2 \) we have \( x = x + y \), for \( y \) the weight of \( p_4 \), hence \( y = 0 \)
5. no

Ex.5 1. the Parikh vector is \( \vec{\sigma} = [3 1 2 2 2] \), thus \( M = M_0 + N \cdot \vec{\sigma} = p_3 + p_5 \)
2. the Parikh vector is $\vec{\sigma}' = [2 1 2 1 1]$, but $M' = M_0 + N \cdot \vec{\sigma} = [0 -1 2 1 1]$ is not a marking

Ex.6 1. $I = [2 1 1 1 1]$
2. $I \cdot M_0 = 2$ and $I(p_1) = 2$ thus for any $M \in [M_0]$ we have $M(p_1) \leq 2/2 = 1$
3. the only possible T-invariants are of the form $[x x x x 0 0]$
4. from (i) the system is bounded; from a theorem: “if a bounded system is live then it has a positive T-invariant”; since the system has no positive T-invariant then it is not live

November 3, 2016

Ex.4 1. no: $t_2$ and $t_5$ have different pre-sets with $p_2$ in common
2. a positive S-invariant is $I = [2 1 1 1 1 2 2]$
3. $I \cdot M = 4 \neq 3 = I \cdot M_0$
4. $t_1 t_2 t_3$ leads to the marking $2p_4 + p_5$ that is deadlock
5. no, because it is not deadlock free

Ex.5 1. the sequence $t_1 t_2$ leads to the marking $M_0 + p_1$
2. $I = [0 0 0 0 1 1 1]$ is semi-positive and $I \cdot M_0 = 0$
3. the Parikh vector is $\vec{\sigma} = [2 3 3 5 0 0 0]$, then $M = M_0 + N \cdot \vec{\sigma} = [1 3 0 -1 0 0 0]$ is not a marking

November 5, 2015

Ex.3 1. no, $p_1$ has two incoming arcs
2. yes, $I = [1 1 1 1 1]$ is a positive S-invariant
3. no, transitions $t_1, t_3, t_4$ must be assigned the same weight $x$; then we have $x = x + y$, for $y$ the weight of $t_2$, hence $y = 0$
4. no positive T-invariant + boundedness implies the system is not live

Ex.4 1. the Parikh vector is $\vec{\sigma} = [4 2 3 4]$, then $M = M_0 + N \cdot \vec{\sigma} = [3 0 0 1] = 3p_1 + p_4$
2. $I = [1 1 1 1]$ is an S-invariant with $I \cdot M = 5 \neq 4 = I \cdot M_0$

Ex.5 1. yes
2. it has no initial place
3. not strongly connected: $p_0$ has no outgoing arc
4. $I = [2 1 1 2 1]$ is a positive S-invariant
5. from (iv) it is bounded; bounded + not strongly connected implies non live
November 7, 2014

Ex.3 1. the firing of $t_2$ leads to the marking $p_3$ that is deadlock
2. not live because not deadlock free
3. no, $t_3$ and $t_4$ have different pre-sets with $p_2$ in common
4. the firing sequence $t_1$ $t_4$ $t_6$ $t_5$ leads to $M_0 + p_3$

Ex.4 1. the Parikh vector is $\vec{\sigma} = [3 \ 1 \ 0 \ 3 \ 2 \ 3]$, then $M = M_0 + N \cdot \vec{\sigma} = [0 \ 0 \ 3 \ 1 \ 0] = 3p_3 + p_4$
2. the Parikh vector is $\vec{\sigma}' = [3 \ 0 \ 2 \ 3 \ 2 \ 2]$, then $M' = M_0 + N \cdot \vec{\sigma}' = [0 \ -2 \ 0 \ 3 \ 1]$ is not a marking

Ex.5 1. $I = [3 \ 1 \ 2 \ 1 \ 1 \ 2 \ 3]$ is a positive S-invariant
2. $I \cdot M = 6 \neq 5 = I \cdot M_0$

November 6, 2013

Ex.3 1. not live
2. not place-live
3. not deadlock free
4. bounded
5. safe
6. not cyclic

Ex.4 1. the Parikh vector is $\vec{\sigma} = [2 \ 0 \ 2 \ 1 \ 0 \ 1 \ 2]$, then $M = M_0 + N \cdot \vec{\sigma} = [0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0] = p_2 + p_5$

Ex.5 1. $I = [2 \ 2 \ 1 \ 1 \ 1]$ is a positive S-invariant from which we have $I \cdot M = 6 \neq 5 = I \cdot M_0$

November 7, 2012

Ex.3 The net is alike the producer-consumer example with bounded buffer (2 producers, 2 consumers, 2 slots):
1. live
2. deadlock free
3. bounded
4. not safe
5. cyclic

Ex.5 1. the Parikh vector is $\vec{\sigma} = [3 \ 2 \ 1 \ 1]$, then $M = M_0 + N \cdot \vec{\sigma} = [0 \ 2 \ 1 \ 0] = 2p_2 + p_3$
2. the Parikh vector is $\vec{\sigma}' = [4 \ 3 \ 2 \ 2]$, then $M = M_0 + N \cdot \vec{\sigma}' = [0 \ 2 \ 1 \ 0] = 2p_2 + p_3$
Ex.3 The analysis of the T-system relies on the identification of its circuits. Let:

\[ \gamma_0 = (p_5, t_4)(t_4, p_6)(p_6, t_3)(t_3, p_4)(p_4, t_2) \]
\[ \gamma_1 = (p_3, t_3)(t_3, p_4)(p_4, t_2) \]
\[ \gamma_2 = (p_1, t_1)(t_1, p_2)(p_2, t_2) \]

Note that \( M_0(\gamma_0) = 3 \), \( M_0(\gamma_1) = 2 \), and \( M_0(\gamma_2) = 1 \).

1. It is immediate to check that the T-system is strongly connected. By a known Lemma, since it is strongly connected, then it is bounded. Place \( p_5 \) is not 1-bounded, hence the T-system is not safe (e.g., take the firing \( M_0 \xrightarrow{t_2} M' \)).

2. It is immediate to check that all places belong to \( \gamma_0 \) or \( \gamma_1 \) or \( \gamma_2 \). By a known Theorem, a T-system is live iff all of its circuits are marked at \( M_0 \). Since \( \gamma_0 \), \( \gamma_1 \) and \( \gamma_2 \) are all marked, we can conclude that the T-system is live. Note that, by a known Theorem, a live T-system is \( k \)-bounded iff every place \( p \) belongs to some circuit \( \gamma_p \) such that \( M_0(\gamma_p) \leq k \). We can exploit this property to confirm that the T-system is not safe by noting that no such circuit \( \gamma \) with \( M_0(\gamma) \leq 1 \) can be found for \( p_5 \).

3. The fundamental property of T-systems guarantees that the token count of a circuit is invariant under any firing. By noting that \( M(\gamma_0) = M(p_5) + M(p_6) + M(p_4) = 2 \neq 3 = M_0(\gamma_0) \) we can conclude that \( M \) is not reachable from \( M_0 \).

Ex.4 The set \( M \) is stable. In fact, the fundamental property of S-systems guarantees that the token count is invariant under any firing. Therefore, taken any \( M \in M \), and any firing \( M \xrightarrow{t} M' \), we know that \( M'(P) = M(P) = M_0(P) \) and thus \( M' \in M \).

By the Reachability Lemma for S-system, \( M = \{ M_0 \} \) iff the S-system is strongly connected.

Ex.5 For each vector \( I_i \) we need to check that

\[ \forall t \in T. \sum_{p \in \bullet t} I_i(p) = \sum_{p \in \bullet t} I_i(p) \]

- \( I_1 = [1 1 0 0 0] \) is an S-invariant.
- \( I_2 = [0 0 1 1 1] \) is not an S-invariant, because the above equality does not hold, e.g., for \( t_1 \).
- \( I_3 = [2 2 1 2 1] \) is an S-invariant.
For each vector $\mathbf{J}_i$, we need to check that

$$\forall p \in P. \sum_{t \in \bullet p} \mathbf{J}_i(t) = \sum_{t \in p \bullet} \mathbf{J}_i(t)$$

- $\mathbf{J}_1 = [1 \ 2 \ 2 \ 1]$ is not a T-invariant, because the above equality does not hold, e.g., for $p_4$.
- $\mathbf{J}_2 = [1 \ 1 \ 1 \ 0]$ is a T-invariant.
- $\mathbf{J}_3 = [0 \ 1 \ 0 \ 1]$ is not a T-invariant, because the above equality does not hold, e.g., for $p_3$.

May 2, 2011

Ex.3  1. S-net
     2. not a T-net
     3. free-choice (because S-net)

Ex.4  1. there can exist nets with a semi-positive S-invariant but with some unbounded place (e.g., producer-consumer example with unbounded buffer)
     2. the existence of a positive S-invariant implies the boundedness but not the safeness (e.g., put two tokens in the same place in the initial marking)
     3. the existence of a positive S-invariant implies the boundedness (known theorem)

Ex.5 For each vector $\mathbf{I}_i$, we need to check that

$$\forall t \in T. \sum_{p \in \bullet t} \mathbf{I}_i(p) = \sum_{p \in t \bullet} \mathbf{I}_i(p)$$

1. $\mathbf{I}_1 = [1 \ 1 \ 0 \ 0 \ 0]$ is not an S-invariant, because the above equality does not hold, e.g., for $t_3$.
2. $\mathbf{I}_2 = [0 \ 0 \ 1 \ 1 \ 1]$ is an S-invariant.
3. $\mathbf{I}_3 = [1 \ 1 \ 2 \ 2 \ 1]$ is an S-invariant.
4. $\mathbf{I}_4 = [2 \ 2 \ 1 \ 1 \ 1]$ is not an S-invariant, because the above equality does not hold, e.g., for $t_3$.
5. $\mathbf{I}_5 = [1 \ 1 \ 1 \ 1 \ 0]$ is an S-invariant ($\mathbf{I}_5 = \mathbf{I}_3 - \mathbf{I}_2$).
6. $\mathbf{I}_6 = [0 \ 1 \ 0 \ 1 \ 1]$ is not an S-invariant, because the above equality does not hold, e.g., for $t_1$. 