Business Processes Modelling

MPB (6 cfu, 295AA)

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15 - Sound by construction
Object

We show a technique to build sound Workflow nets
Soundness proof by construction

Idea
1. Find a suitable set of "building blocks"
   they are (small) workflow nets
   that can be (easily) proved
   to be **sound** and
   to be **safe** (1-bounded)

2. Define composition patterns so that
   by composing **safe and sound** WF nets
   we get **safe and sound** WF nets
Sound and safe by composition

Let \( N, N' \) be two safe and sound workflow nets
Sound and safe by composition

Let $t$ be a task of $N$ with exactly one input and one output place.
Sound and safe by composition

Let $N[N'/t]$ denote the net obtained by replacing the task $t$ in $N$ by $N'$.
Sound and safe by composition

The net $N[N'/t]$ is a sound and safe workflow net (proof omitted)
Proof sketch

Intuitively
a sound workflow net behaves as a transition:
it takes one token from its input place and
it produces one token to its output place
(but not atomically)

Formally
the crux of the proof is showing a bijective correspondence
between
markings of the composed net N[N'/t]
and the pairs of markings in N and N'
Some Building Blocks 1

- Basic sequence
- Implicit XOR
- Iteration
Some Building Blocks 2

- explicit XOR-split
- explicit XOR-join
Some Building Blocks 3

But you can define more blocks on your own
Example: refinement
Example: refinement
Example: refinement
Example: refinement
Example: refinement
Example: refinement
Example: abstraction

Prove that the net below is a safe and sound workflow net
Example: abstraction

Prove that the net below is a safe and sound workflow net

explicit XOR block
Example: abstraction

Prove that the net below is a safe and sound workflow net

sequence
Example: abstraction

Prove that the net below is a safe and sound workflow net.
Example: abstraction

Prove that the net below is a safe and sound workflow net
Example: abstraction

Prove that the net below is a safe and sound workflow net

parallel (AND) block
Example: abstraction

Prove that the net below is a safe and sound workflow net
Exercise

Prove that the net below is a safe and sound workflow net
Exercise

Prove that the net below is a safe and sound workflow net (hint: "desugar" it)
We would like to progressively refine transitions with multiple incoming and outgoing arcs.
Lemma: Let $N$ be a sound WF net. If $(i,t) \in F$ then the pre-set of $t$ is $\{i\}$

(otherwise $t$ would be a dead transition)

Lemma: Let $N$ be a sound WF net. If $(t,o) \in F$ then the post-set of $t$ is $\{o\}$

(otherwise $t$ would be dead or proper completion would not hold)
**General replacement**

Let \( T_{i'} = \{ u \mid \bullet u = \{i'\} \} \). (initial transitions of \( N' \))

Let \( T_{o'} = \{ v \mid v\bullet = \{o'\} \} \). (final transitions of \( N' \))

- If \((p, t) \in F_N, u \in T_{i'}\) then \((p, u) \in F_{N'[N'/t]}\)
- If \((t, q) \in F_N, v \in T_{o'}\) then \((v, q) \in F_{N'[N'/t]}\)

The net \( N[N'/t] \) is a **sound** and **safe** workflow net.
General replacement

Let $T_{i'} = \{ u \mid \bullet u = \{ i' \} \}$.  (initial transitions of $N'$)
Let $T_{o'} = \{ v \mid v\bullet = \{ o' \} \}$.  (final transitions of $N'$)

If $(p, t) \in F_N, u \in T_{i'}$ then $(p, u) \in F_{N[N'/t]}$
If $(t, q) \in F_N, v \in T_{o'}$ then $(v, q) \in F_{N[N'/t]}$

The net $N[N'/t]$ is a sound and safe workflow net
Some Building Blocks 4

But you can define more blocks on your own

AND (parallel)
Example
Example
Exercise

Prove that the net below is a safe and sound workflow net.