Liveness, formally

$$(P, T, F, M_0)$$

$$\forall t \in T, \quad \forall M \in [\, M_0 \,\rangle, \quad \exists M' \in [\, M \,\rangle, \quad M' \xrightarrow{t}$$

Deadlock freedom, formally

$$(P, T, F, M_0)$$

$$\forall M \in [\, M_0 \,\rangle, \quad \exists t \in T, \quad M \xrightarrow{t}$$

Liveness implies deadlock freedom

Lemma If (P, T, F, M_0) is live, then it is deadlock-free

By contradiction, let $M \in [\, M_0 \,\rangle$, with $M \not\rightarrow$

Let $t \in T$ (T cannot be empty).
By liveness, $\exists M' \in [\, M \,\rangle$ with $M' \xrightarrow{t}$.

Since M is dead, $[\, M \,\rangle = \{\, M \,\}$.
Therefore $M = M' \xrightarrow{t}$, which is absurd.

Boundedness, formally

$$(P, T, F, M_0)$$

$$\exists k \in \mathbb{N}, \quad \forall M \in [\, M_0 \,\rangle, \quad \forall p \in P, \quad M(p) \leq k$$

A puzzle about reachability

Theorem: If a system is... then its reachability graph is finite

Theorem: A system is... iff its reachability graph is finite

(fill the dots and the proof)