Business Processes Modelling

MPB (6 cfu, 295AA)

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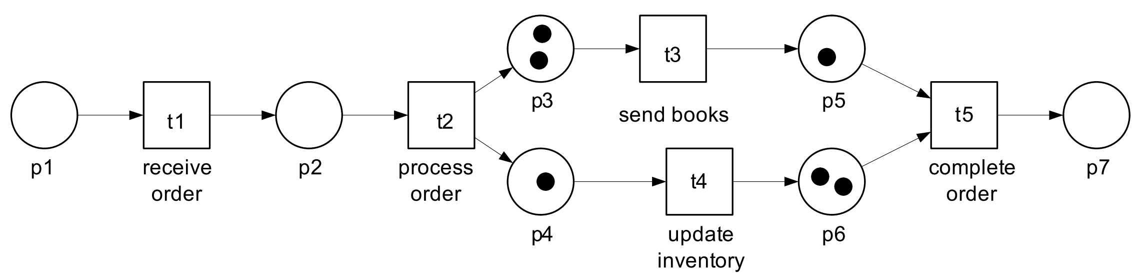
08 - From automata to nets

Object



Overview of the basic concepts of Petri nets

Free Choice Nets (book, optional reading)
https://www7.in.tum.de/~esparza/bookfc.html

Why Petri nets?

Business process analysis:
validation: testing correctness
verification: proving correctness
performance: planning and optimization

Use of Petri nets (or alike)
visual + formal
tool supported

Approaching Petri nets

Are you familiar with automata / transition systems?
They are fine for sequential protocols / systems
but do not capture concurrent behaviour directly

A Petri net is a mathematical model
of a parallel and concurrent system

in the same way that a finite automaton is a
mathematical model of a sequential system

Approaching Petri nets

Petri net theory can be studied
at several level of details

We study some basics aspects, relevant to the analysis of business processes

Petri nets have a faithful and convenient graphical representation, that we introduce and motivate next

Preliminaries

Set notation

8. Are you familiar with set notation?

Altri dettagli

Yes	46
No	5

Set notation

$\varnothing$	$A \cap B$	$A \cup B$	$A \setminus B$ $A - B$	$\overline{A}$
$a \in A$	$A = B$	$A \subseteq B$	$A \subset B$	$A \times B$
$a \notin A$	$A \neq B$	$A \nsubseteq B$	$\wp(A)$	$A \cap B = \emptyset$

$\mathbb{N}$ $\quad \mathbb{Z}$ $\quad \mathbb{Q}$ $\quad \mathbb{R}$ $\quad \mathbb{B}$

$N \subseteq \mathbb{N}$ $\quad N \in \wp(\mathbb{N})$ $\quad S \subseteq \wp(\mathbb{N})$

Functions, relations

9. Are you familiar with functions (f:A->B) and relations?

Altri dettagli | Dati analitici

Yes	44
No	7

10. Do you agree that a subset S of A can be seen as a function from A to the set of Booleans?

Altri dettagli | Dati analitici

Yes	33
No	4
I do not understand the question	14

Functions, relations

$$f : A \to B$$

$$R \subseteq A \times B$$

functions as relations

$$R_f \triangleq \{(a, f(a)) \mid a \in A\}$$

sets as functions
(characteristic function)

$$f_N : \mathbb{N} \to \mathbb{B}$$

$$f_N(n) \triangleq \begin{cases} 1 & n \in N \\ 0 & \text{otherwise} \end{cases}$$

$$N = \{n \mid f_N(n) = 1\}$$

First order logic

12. Are you familiar with propositional logic?

Altri dettagli

Dati analitici

●	Yes	38
●	No	13

First order logic

ff false 0 F tt true 1 T $P \wedge Q$ $P \vee Q$ $\neg P$

$\exists x.\ P(x)$ $\forall x.\ P(x)$ $P \Rightarrow Q$ $P \Leftrightarrow Q$

meaning of implication!

$$P \Rightarrow Q$$

$$Q \vee \neg P$$

$$\neg Q \Rightarrow \neg P$$

order of quantifiers matters!

$$\forall n \in \mathbb{N}.\ \exists m \in \mathbb{N}.\ n < m$$

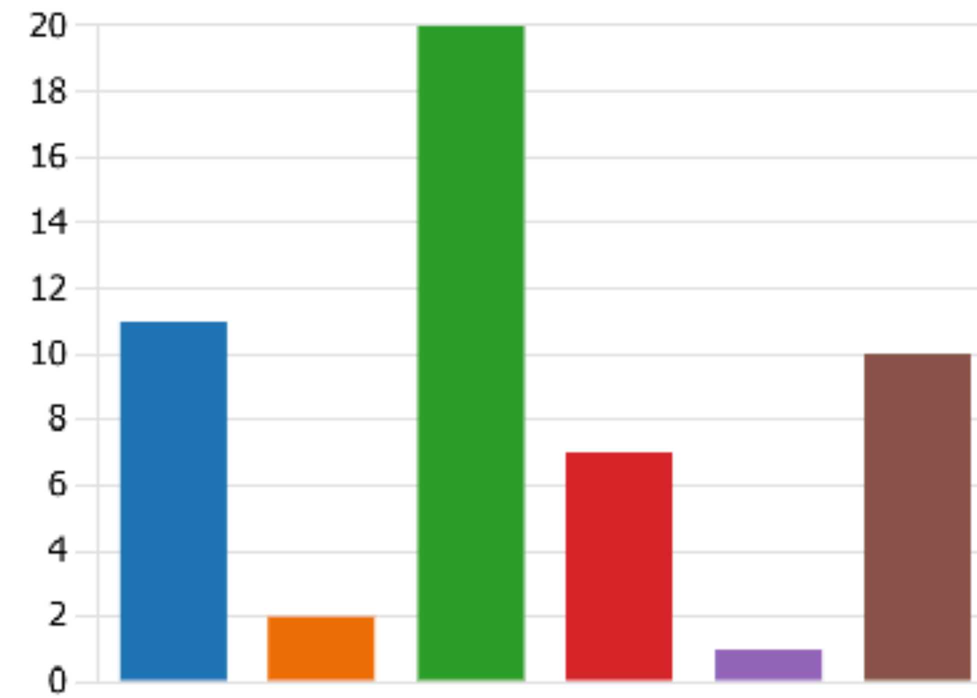
$$\exists m \in \mathbb{N}.\ \forall n \in \mathbb{N}.\ n < m$$

First order logic

13. Logical implication "P implies Q" (also written "P => Q") is equivalent to:

Altri dettagli Dati analitici

●	P and Q	11
●	P or Q	2
●	(not P) or Q	20
●	P or (not Q)	7
●	(not P) or (not Q)	1
●	none of the above	10



14. Do you agree that "P implies Q" is equivalent to "(not Q) implies (not P)?

Altri dettagli Dati analitici

●	Yes	35
●	No	16

First order logic

15. Do you remember De Morgan's law about negation, conjunction and disjunction?

Altri dettagli Dati analitici

Yes	26
No	25
I do not understand the question	0

16. Do you know what are the universal and existential quantifiers in predicate logic?

Altri dettagli Dati analitici

Yes	30
No	21

Kleene-star notation A*

Given a set A we denote by A^*
the set of finite sequences of elements in A, i.e.:
$A^* = \{ a_1 \cdots a_n \mid n \geq 0 \wedge a_1, ..., a_n \in A \}$
We denote the empty sequence by $\epsilon \in A^*$

For example:
$A = \{ a, b \}$ $\qquad$ $A^* = \{ \epsilon, a, b, aa, ab, ba, bb, aaa, aab, ... \}$

Strings

Alphabet A $\quad A^n \triangleq \underbrace{A \times \cdots \times A}_{n} \quad A^* \triangleq \bigcup_{n\in\mathbb{N}} A^n$

$$
\begin{aligned}
\mathbb{B} &= \{0, 1\} \\
\mathbb{B}^0 &= \{\epsilon\} \\
\mathbb{B}^1 &= \{0, 1\} \\
\mathbb{B}^2 &= \{00, 01, 10, 11\} \\
\mathbb{B}^3 &= \{000, 001, 010, 011, 100, 101, 110, 111\} \\
&\dots \\
\mathbb{B}^* &= \{\epsilon, 0, 1, 00, 01, 10, 11, 000, \dots\}
\end{aligned}
$$

Inductive definitions

17. Do you know what is an inductive definition?

Altri dettagli | Dati analitici

Yes	37
No	14

18. Do you know what is a recursive definition?

Altri dettagli | Dati analitici

Yes	40
No	11

Inductive definitions

A natural number is either:

- 0
- or the successor $n+1$ of a natural number n

A sequence over the alphabet A is either:

- the empty sequence ε
- or the juxtaposition wa of a sequence w with an element a of A

Inductively defined functions

Let us define the exponential function k^n

base case: for any $k>0$ we set
$exp(k,0) = 1$

inductive case: for any $k>0$, $n\geq 0$ we set
$exp(k,n+1) = exp(k,n) \times k$

Inductively defined functions

Let us define the exponential function k^n

base case: for any $k>0$ we set
$exp(k,0) \triangleq 1$

inductive case: for any $k>0,\ n\geq 0$ we set
$exp(k,n+1) \triangleq \boxed{exp(k,n)} \times k$

Recursive definition

Inductively defined functions

Let us define the exponential function k^n

base case: for any $k>0$ we set
$exp(k,0) \triangleq 1$

inductive case: for any $k>0,\ n\geq 0$ we set
$exp(k,\boxed{n+1}) \triangleq exp(k,\boxed{n})$ x k

More complex case **Simpler case**

Recursive definitions

$$\binom{n}{k} \triangleq \frac{n!}{k!\,(n-k)!}$$

$$\binom{n}{0} \triangleq 1$$

$$\binom{n+1}{k+1} \triangleq \frac{(n+1)\binom{n}{k}}{k+1}$$

$$f(n) \triangleq \begin{cases} 1 & \text{if } n \leq 1 \\ f(n/2) & \text{if } n > 1 \land n\%2 = 0 \\ f(3n+1) & \text{otherwise} \end{cases}$$

$$f(12) = f(6) = f(3) = f(10) = f(5) = f(16) = f(8) = f(4) = f(2) = f(1) = 1$$

Inductive definitions

$$
\begin{aligned}
0! &\triangleq 1 \\
(n+1)! &\triangleq n! \cdot (n+1)
\end{aligned}
$$

$$
\begin{aligned}
A^0 &\triangleq \{\epsilon\} \\
A^{(n+1)} &\triangleq A \times A^n
\end{aligned}
$$

$$
\begin{aligned}
|\epsilon| &\triangleq 0 \\
|w\ a| &\triangleq |w| + 1
\end{aligned}
$$

Finite automata examples

Finite state automaton

21. Do you know what is a Finite State Automata?

Altri dettagli   Dati analitici

●	Yes	24
●	No	27

22. Do you know what is the language recognized by an automata?

Altri dettagli

●	Yes	15
●	No	31
●	I do not understand the question	5

Applications

Finite automata are widely used, e.g., in
protocol analysis,
text parsing,
video game character behavior,
security analysis,
CPU control units,
natural language processing,
speech recognition,
mechanical devices
(like elevators, vending machines, traffic lights)
and many more …

How to define an automaton

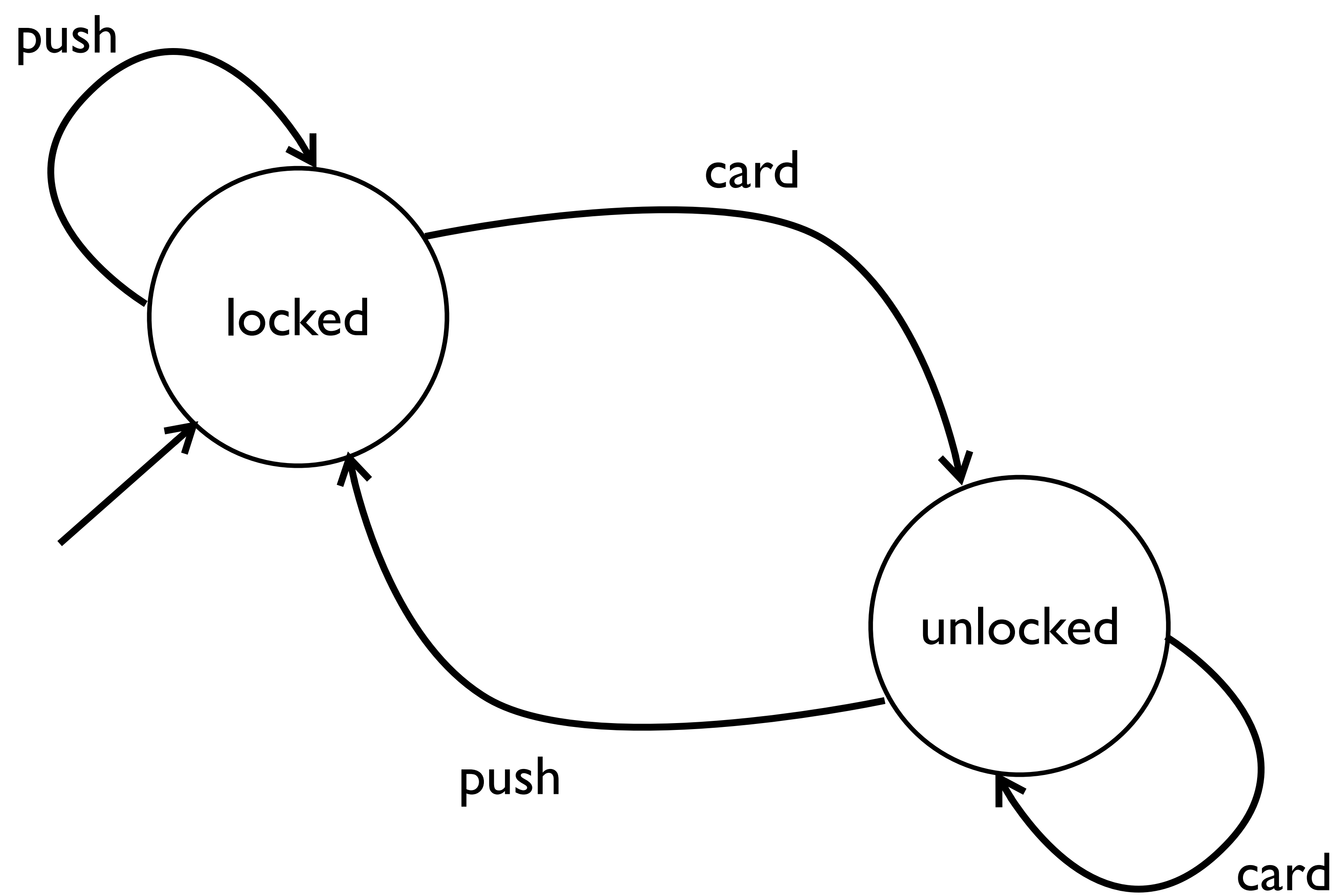
1. Identify the admissible **states** of the system
(Optional: mark some states as error states)

2. **Add transitions**
to move from one state to another
(no transition to recover from error states)

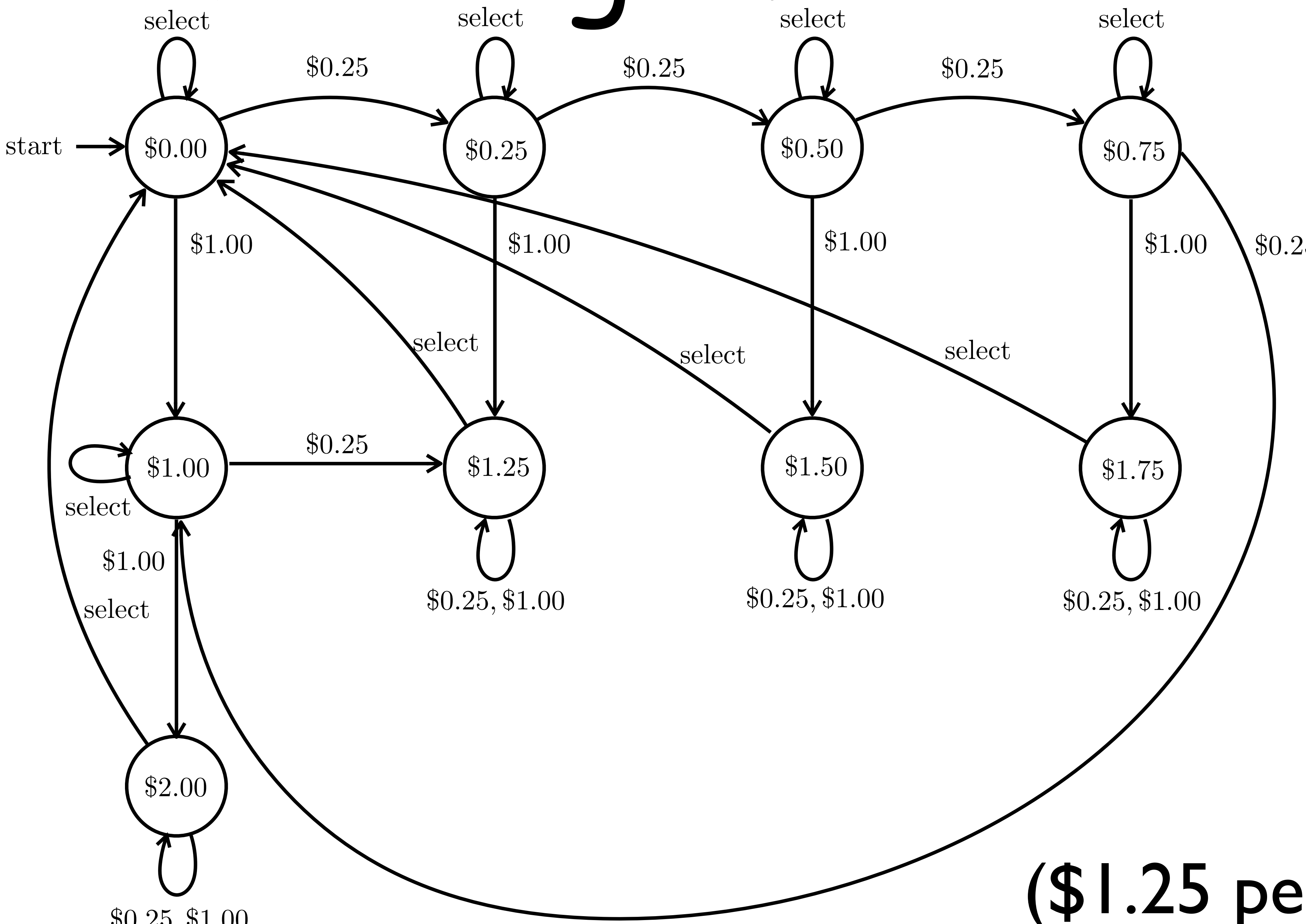
3. Set the **initial state**

4. *(Optional: mark some states as **final states**)*

Example: Turnstile

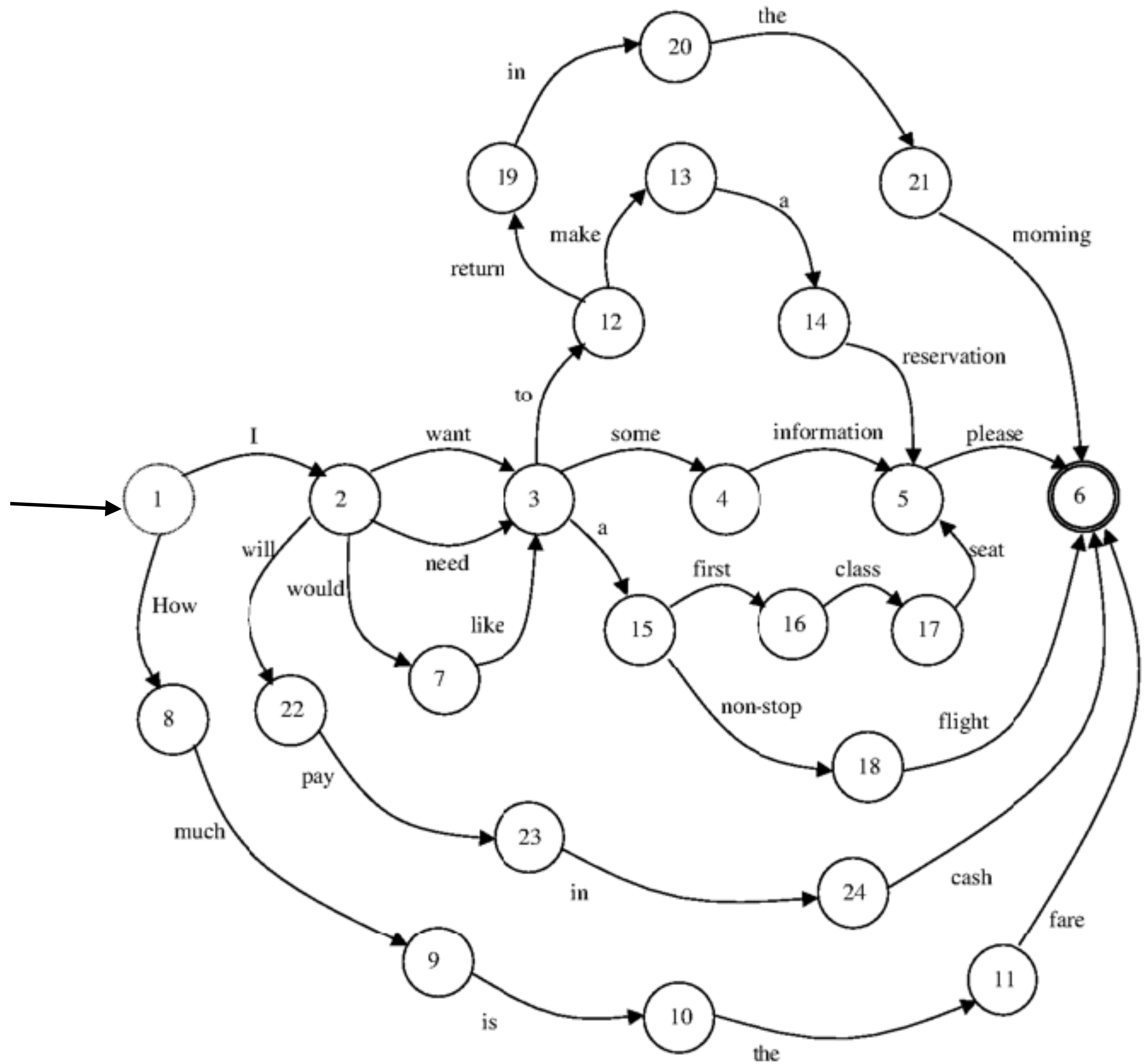


Example: Vending Machine

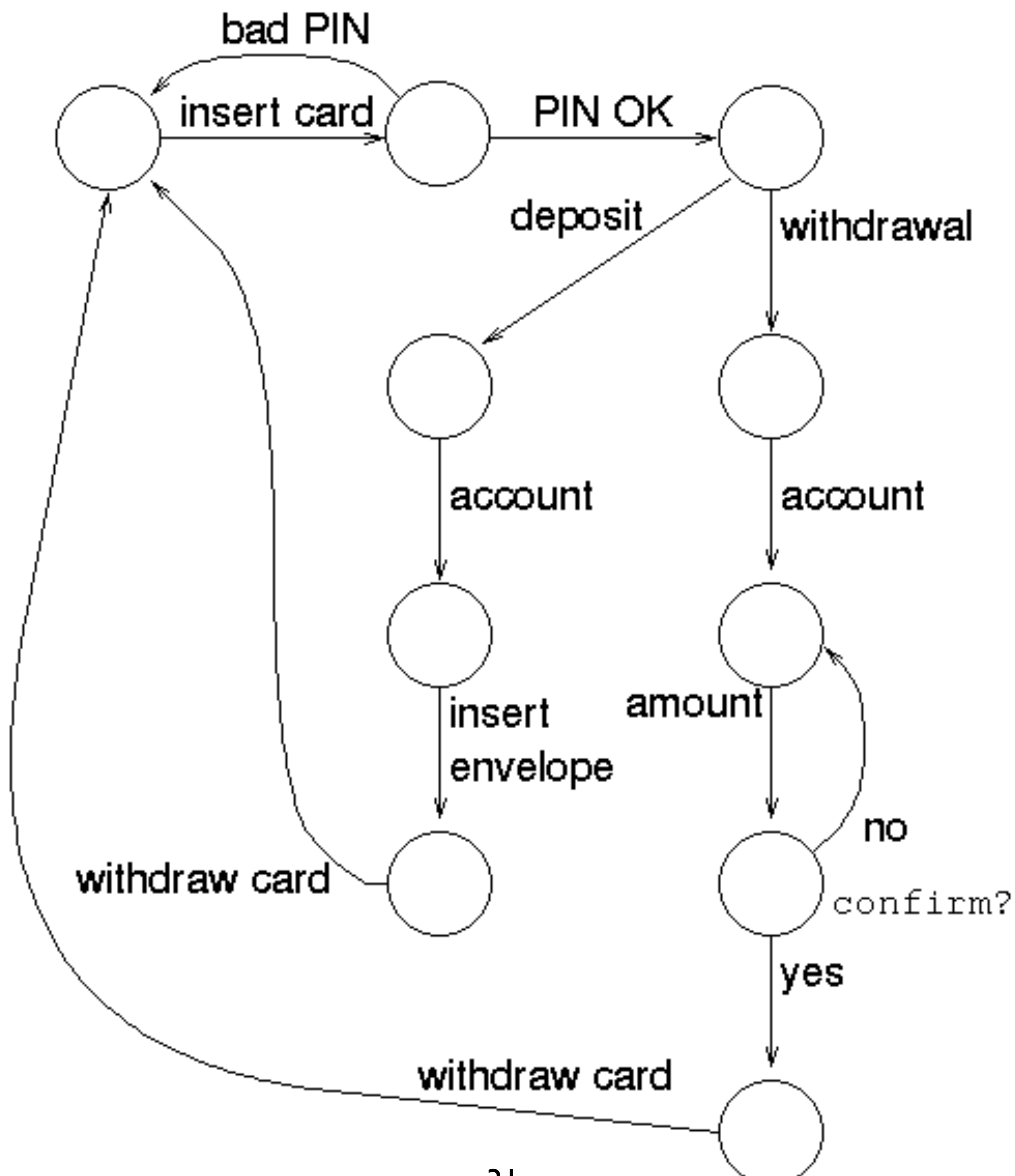


($1.25 per soda)

Example: Language Processing



Example: ATM

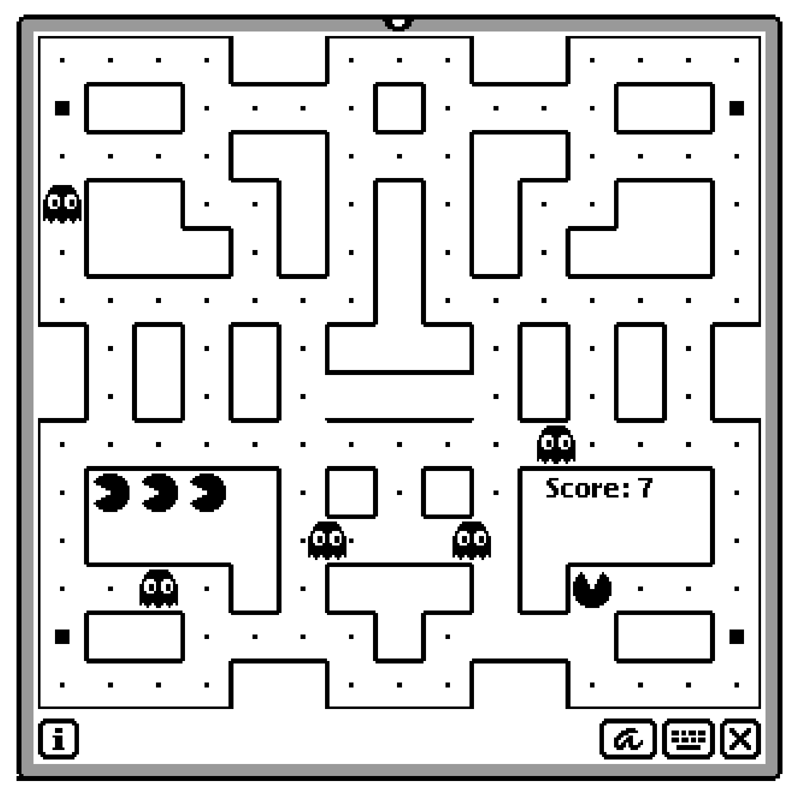


bad PIN
insert card
PIN OK
deposit
withdrawal
account
account
insert
envelope
amount
no
confirm?
withdraw card
yes
withdraw card

Computer controlled characters for games

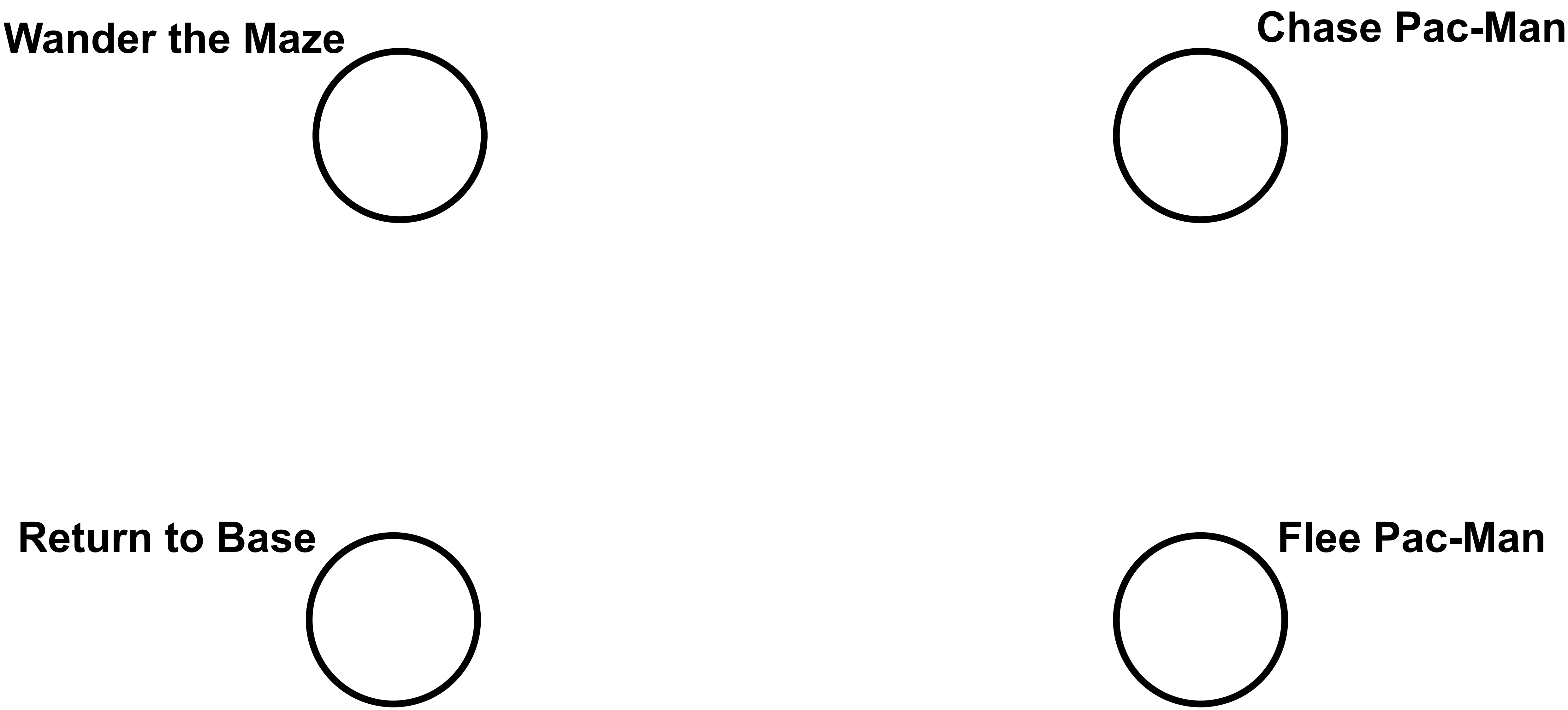
States = characters behaviours

Transitions = events that cause a change in behaviour

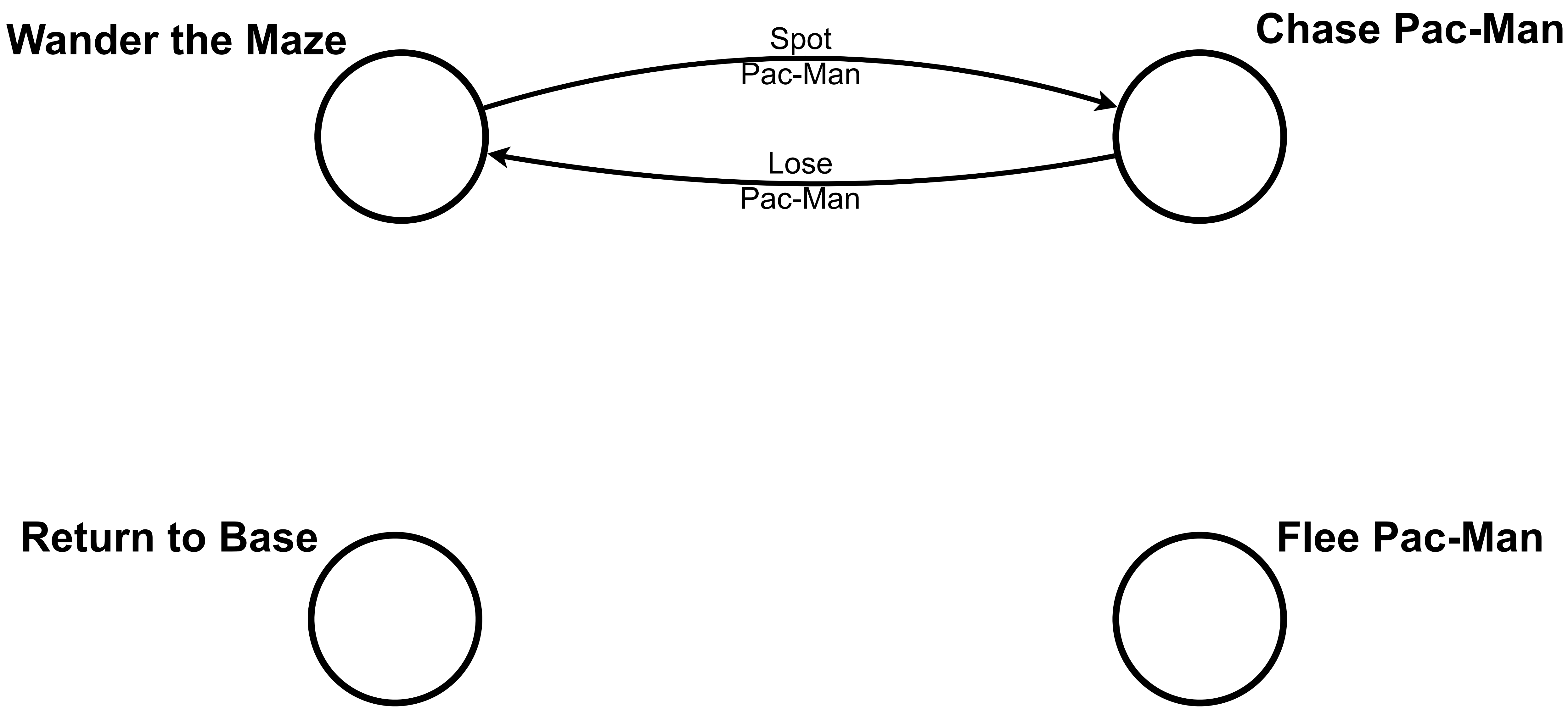


Example:
Pac-man moves in a maze
wants to eat pills
is chased by ghosts
by eating power pills, pac-man can defeat ghosts

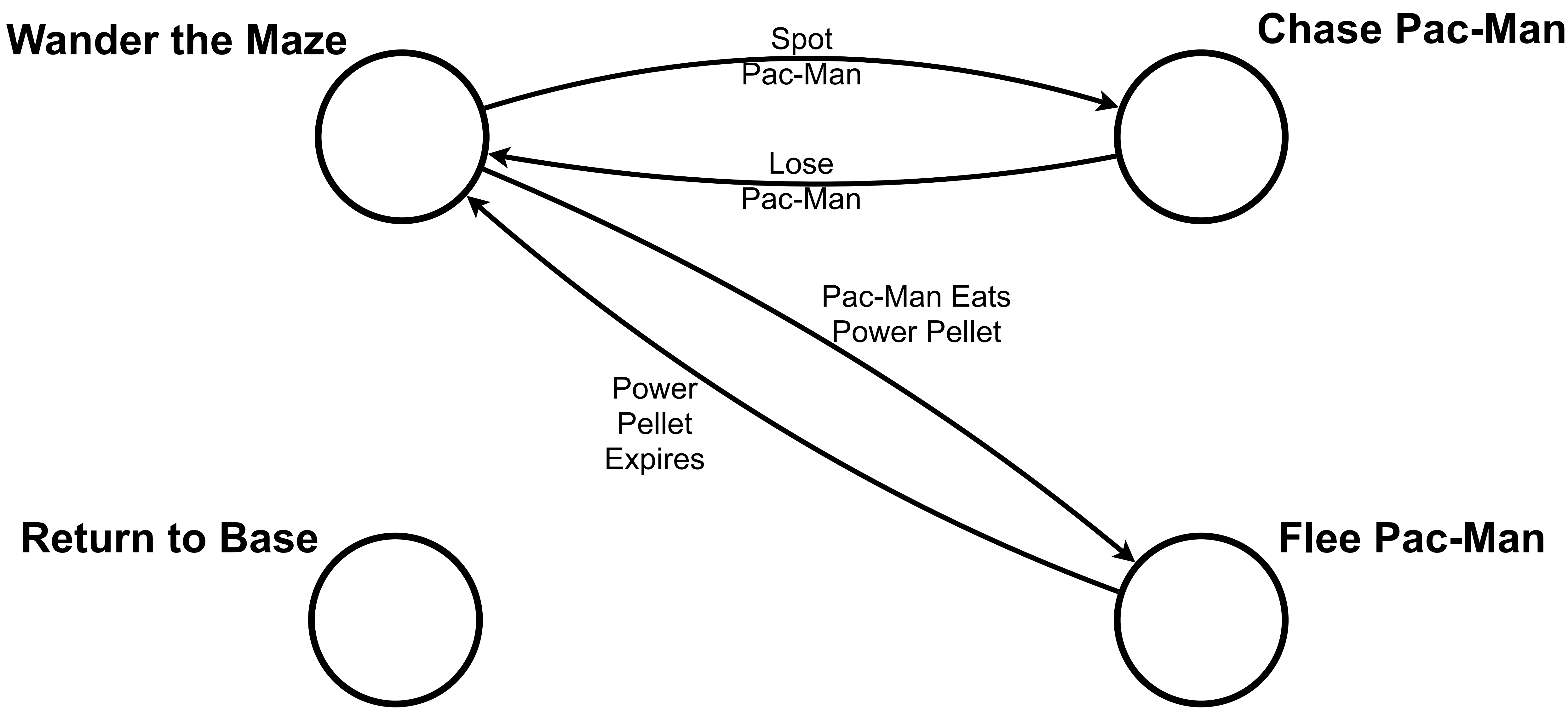
Example: Pac-Man Ghosts



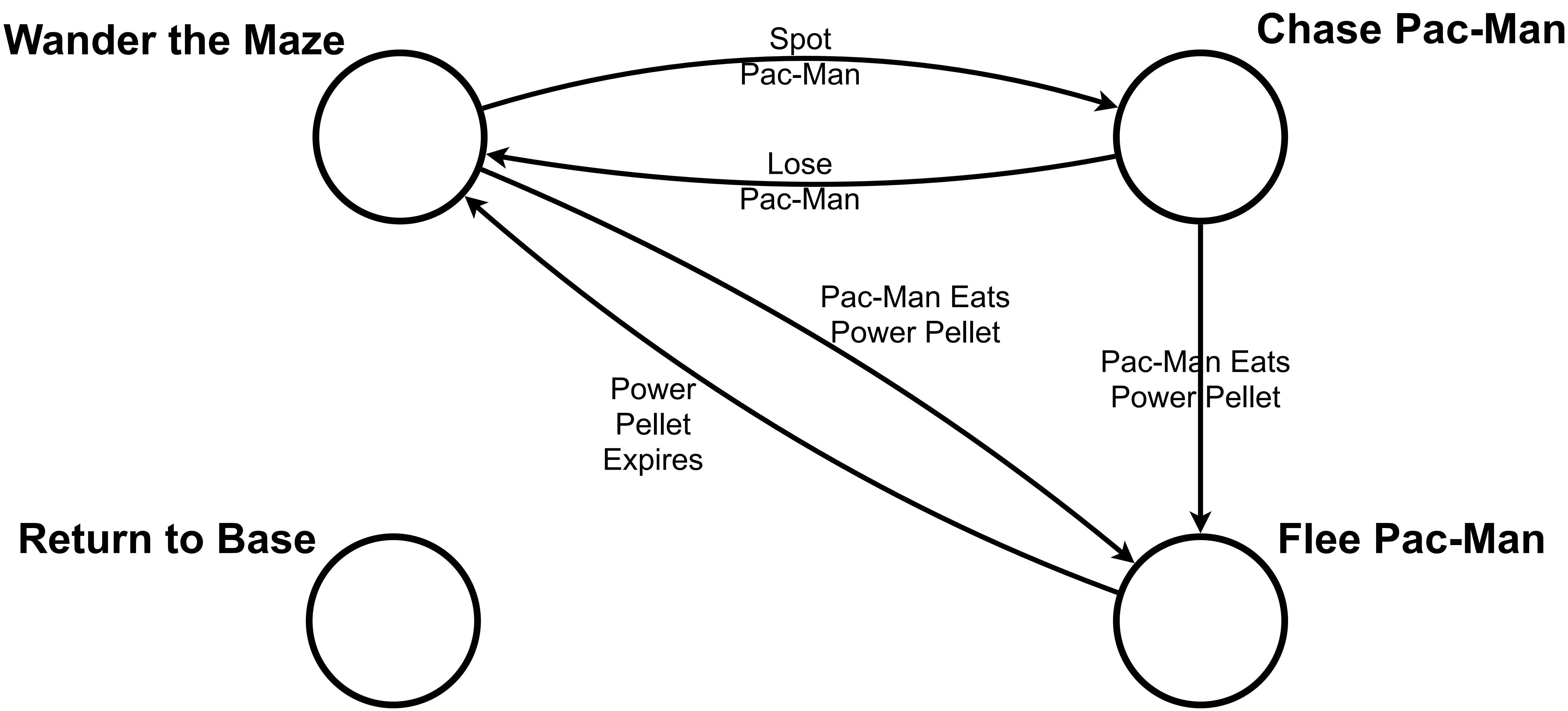
Example: Pac-Man Ghosts



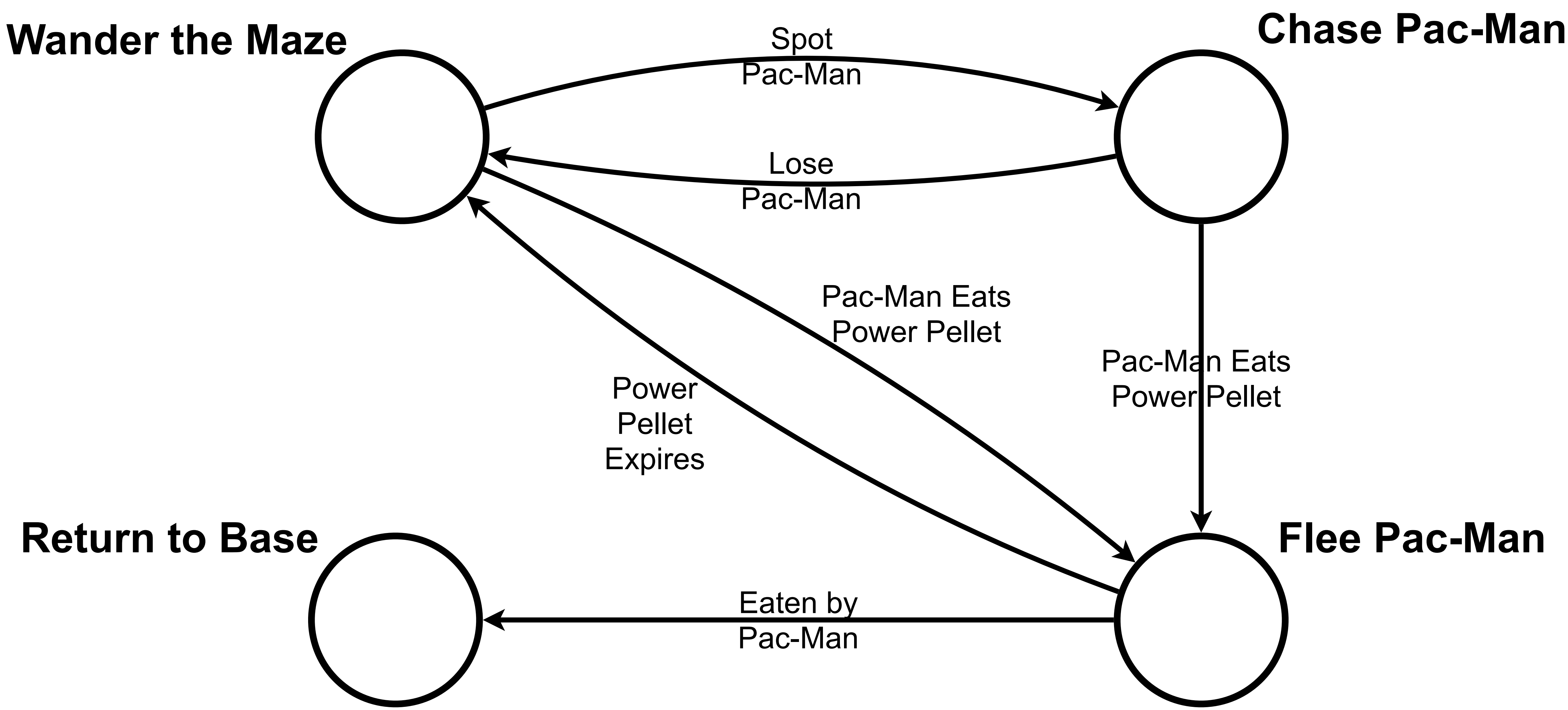
Example: Pac-Man Ghosts



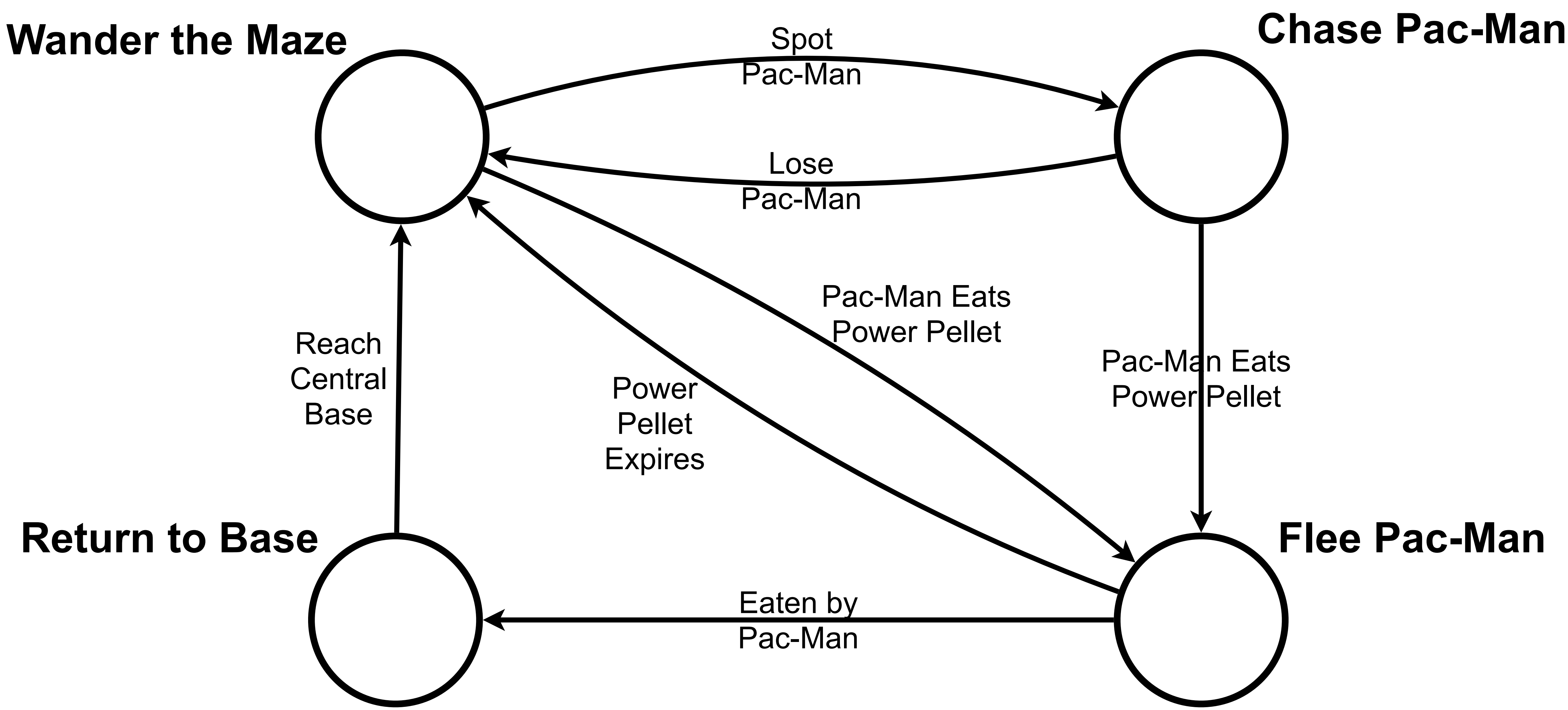
Example: Pac-Man Ghosts



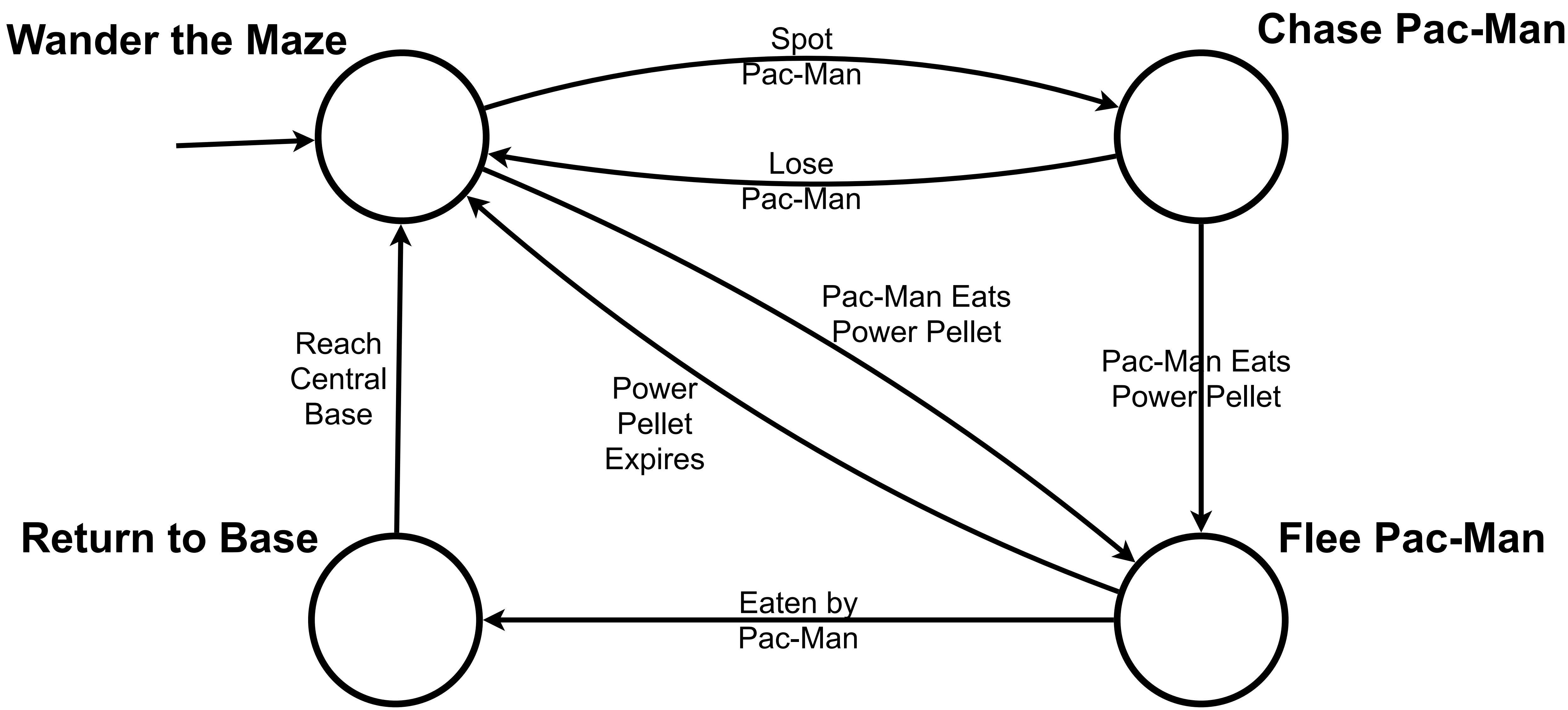
Example: Pac-Man Ghosts



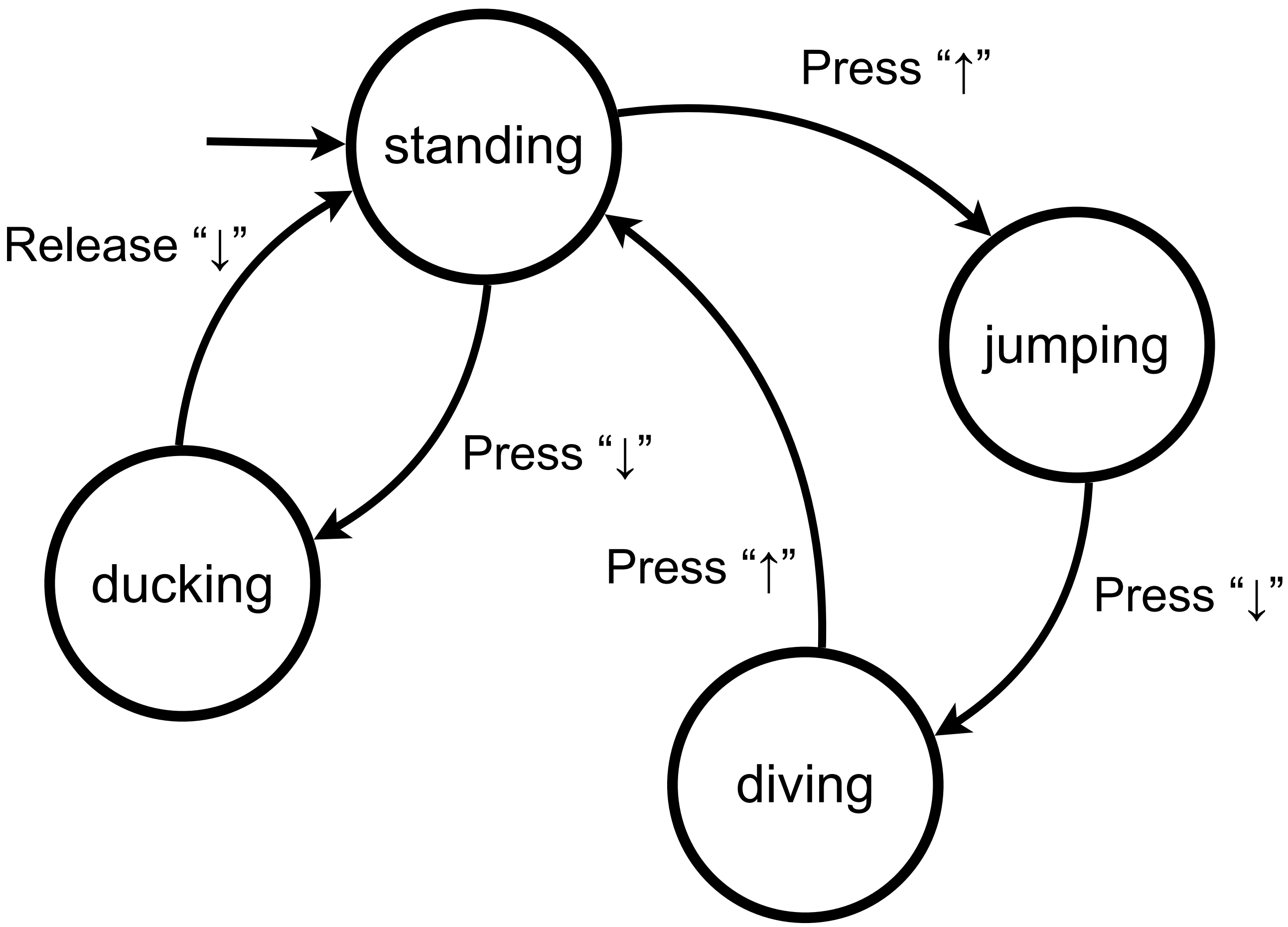
Example: Pac-Man Ghosts

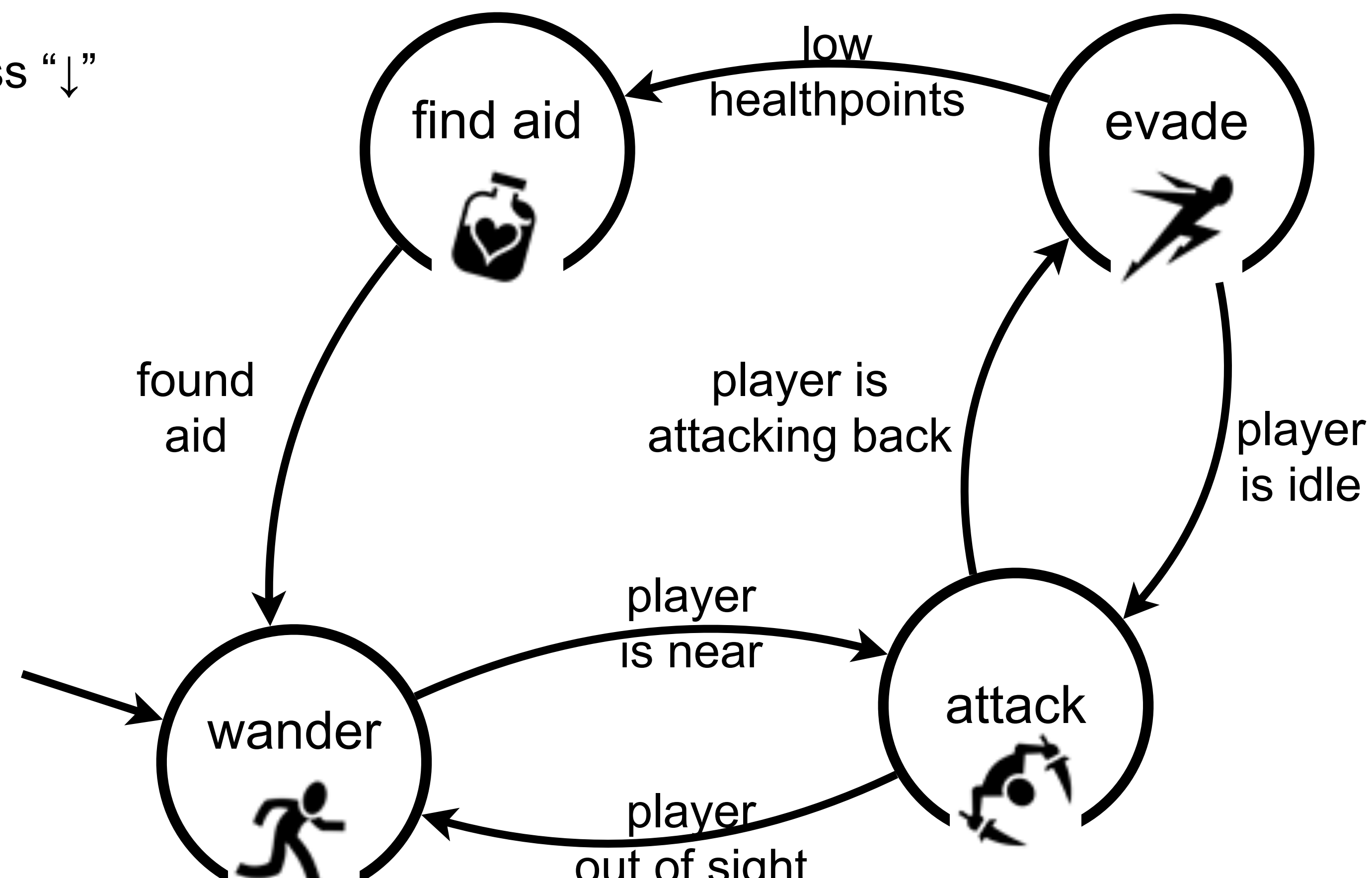


Example: Pac-Man Ghosts



Other examples





Exercises

Choose your favourite (video) game, and draw the finite state automaton for one of the characters in that game

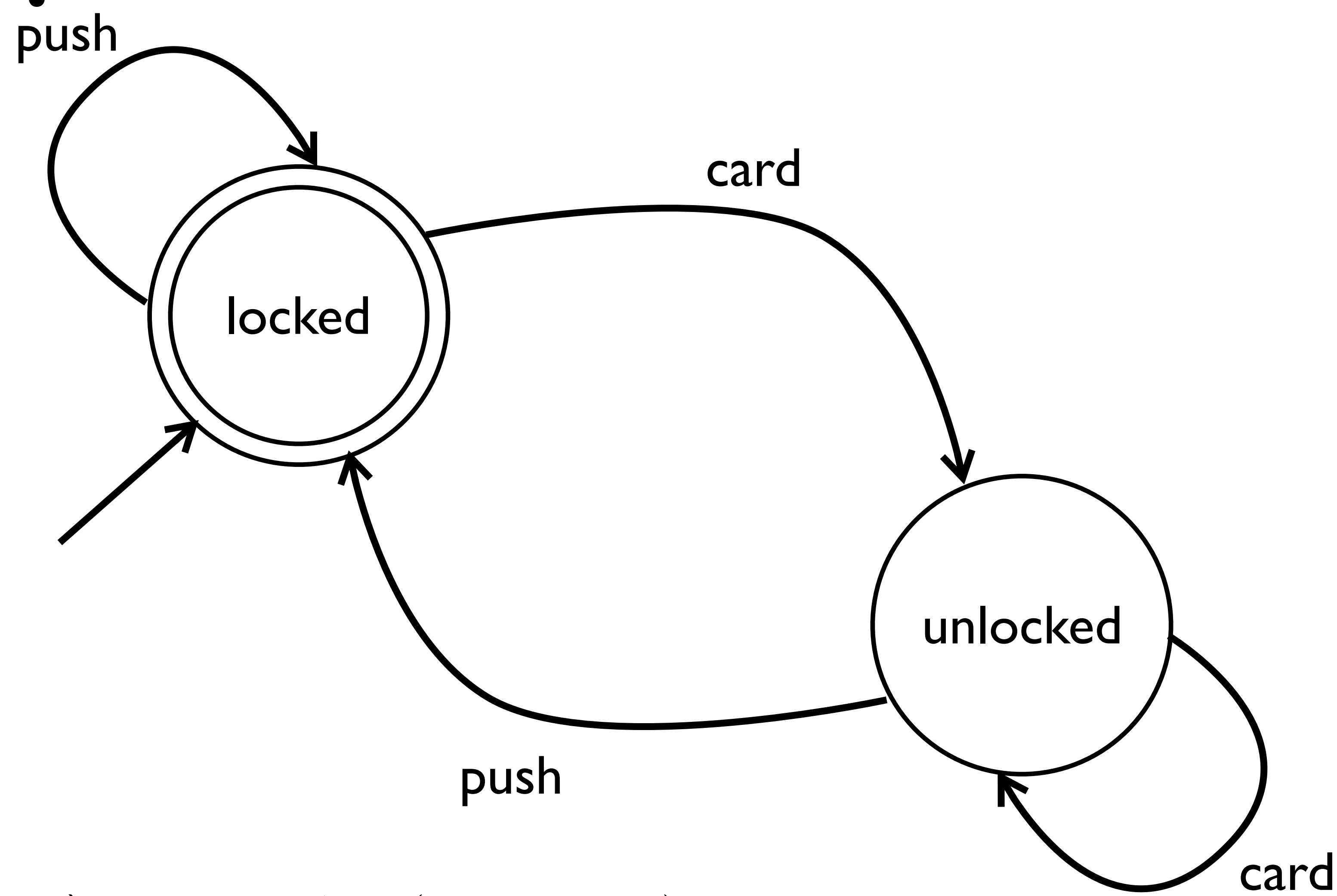
Finite state automata, formally

DFA

A **Deterministic Finite Automaton** (**DFA**) is a tuple $A = (Q, \Sigma, \delta, q_0, F)$, where

- Q is a finite set of states;
- Σ is a finite set of input symbols;
- $\delta : Q \times \Sigma \to Q$ is the transition function;
- $q_0 \in Q$ is the initial state (also called start state);
- $F \subseteq Q$ is the set of final states (also accepting states)

Example: Turnstile



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- $\delta : Q \times \Sigma \to Q$ is the transition function;
- $q_0 \in Q$ is the initial state (also called start state);
- $F \subseteq Q$ is the set of final states (also accepting states)

$$Q = \{\text{locked}, \text{unlocked}\}$$

$$\Sigma = \{\text{push}, \text{card}\}$$

$$\delta(\text{locked}, \text{card}) = \text{unlocked}$$

$$q_0 = \text{locked}$$

$$F = \{\text{locked}\}$$

Extended transition function (destination function)

Given $A = (Q, \Sigma, \delta, q_0, F)$, we define $\widehat{\delta} : Q \times \Sigma^* \to Q$ by induction:

base case: For any $q \in Q$ we let

$$\widehat{\delta}(q, \epsilon) \triangleq q$$

inductive case: For any $q \in Q, a \in \Sigma, w \in \Sigma^*$ we let

$$\widehat{\delta}(q, wa) \triangleq \delta(\ \widehat{\delta}(q, w)\ ,\ a\ )$$

($\widehat{\delta}(q, w)$ returns the state reached from q by observing w)

Extended transition function (destination function)

Given $A = (Q, \Sigma, \delta, q_0, F)$, we define $\widehat{\delta} : Q \times \Sigma^* \to Q$ by induction:

base case: For any $q \in Q$ we let

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inductive case: For any $q \in Q, a \in \Sigma, w \in \Sigma^*$ we let

$$\widehat{\delta}(q, wa) \triangleq \delta(\boxed{\widehat{\delta}(q, w)}\,,\; a\;)$$

Recursive definition

($\widehat{\delta}(q, w)$ returns the state reached from q by observing w)

Extended transition function (destination function)

Given $A = (Q, \Sigma, \delta, q_0, F)$, we define $\widehat{\delta} : Q \times \Sigma^* \to Q$ by induction:

base case: For any $q \in Q$ we let

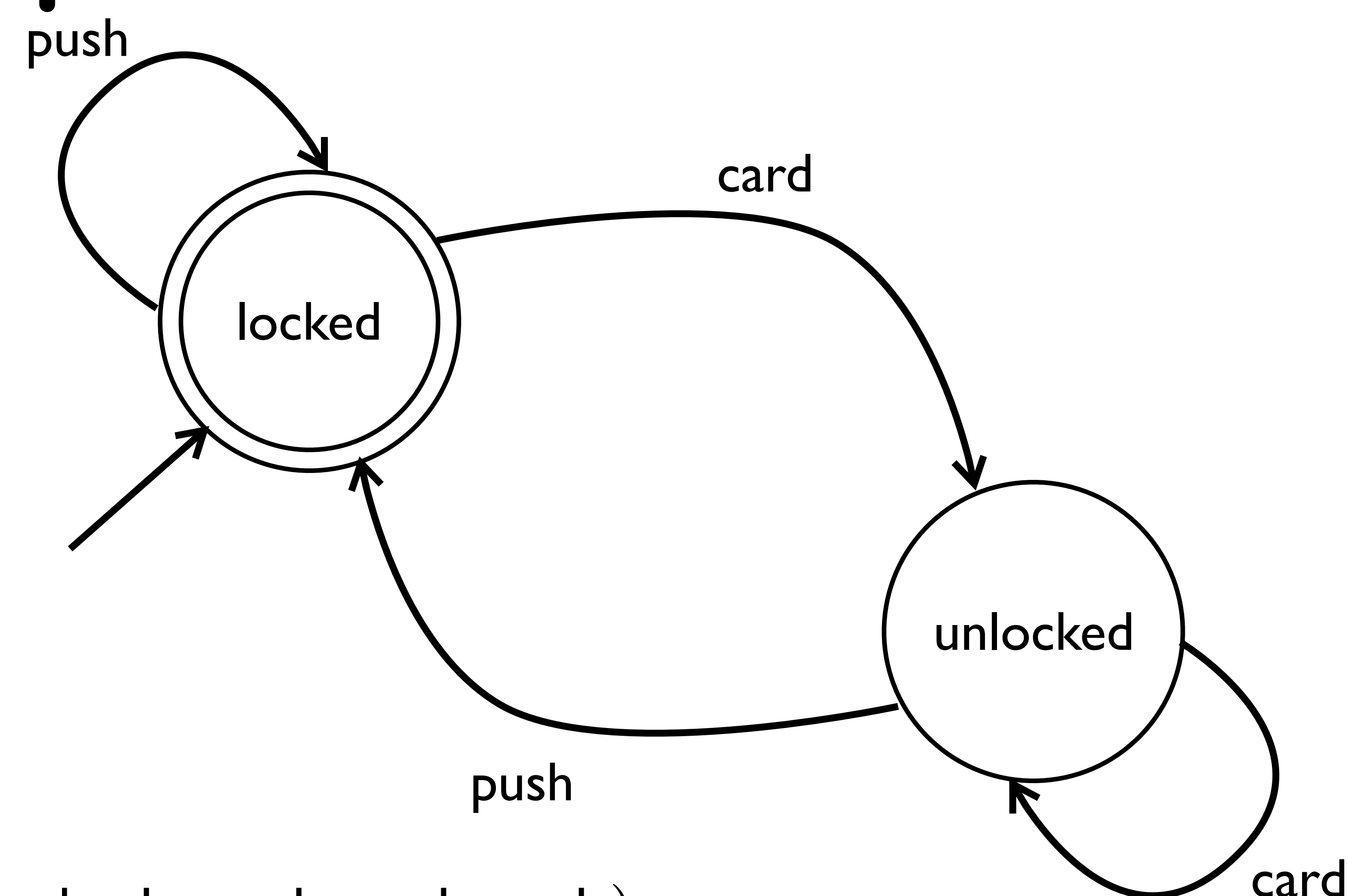
$$\widehat{\delta}(q, \epsilon) \triangleq q$$

inductive case: For any $q \in Q, a \in \Sigma, w \in \Sigma^*$ we let

$$\widehat{\delta}(q, \underbrace{\boxed{wa}}_{\text{More complex case}}) \triangleq \delta(\ \widehat{\delta}(q, \underbrace{\boxed{w}}_{\text{Simpler case}})\ ,\ a\ )$$

($\widehat{\delta}(q, w)$ returns the state reached from q by observing w)

Example: Turnstile



$\delta(\text{locked}, \text{card}) = \text{unlocked}$

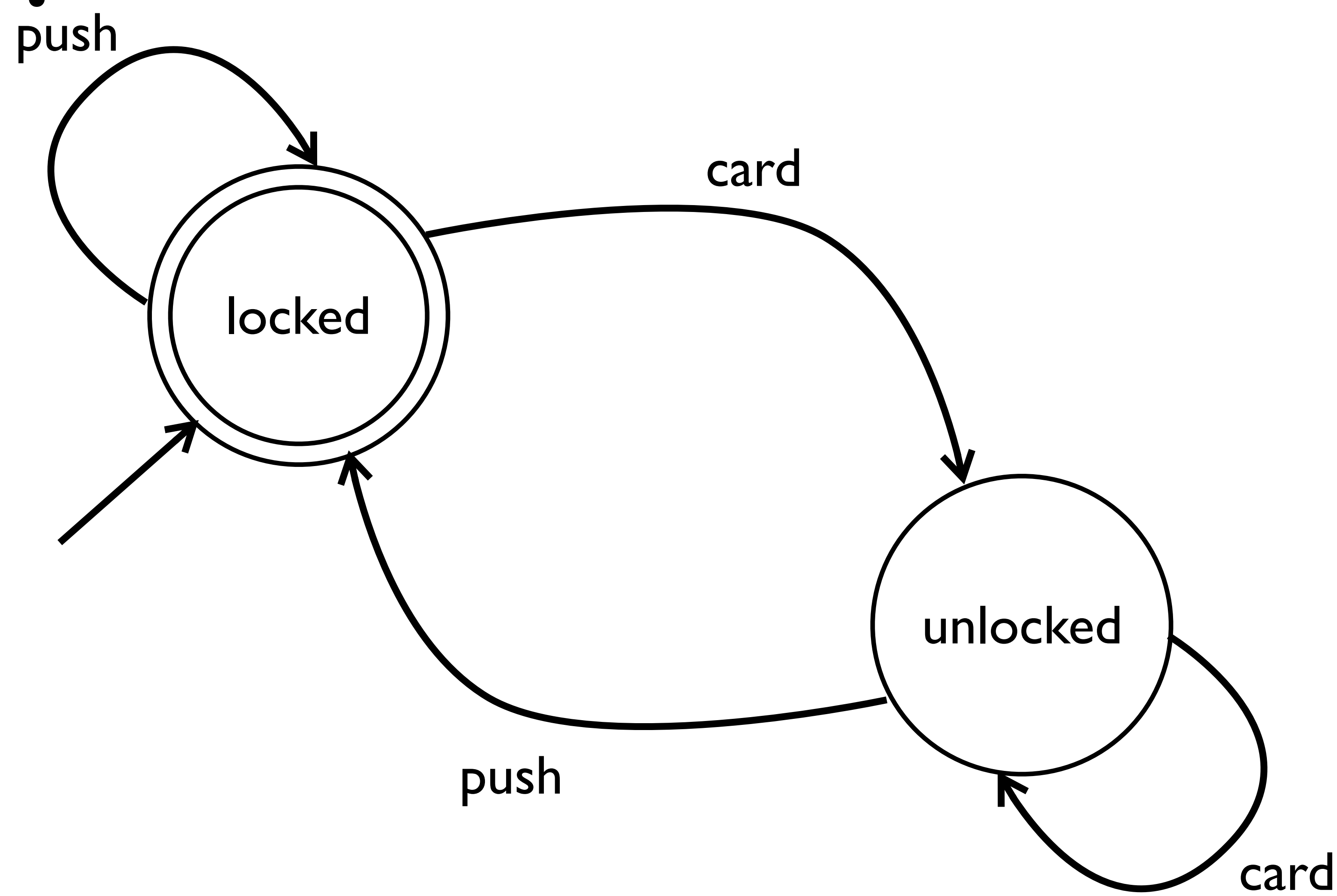
$\delta(\text{locked}, \text{push}) = \text{locked}$

$\delta(\text{unlocked}, \text{card}) = \text{unlocked}$

$\delta(\text{unlocked}, \text{push}) = \text{locked}$

$$
\begin{aligned}
&\widehat{\delta}(\text{locked}, \text{card card push}) \\
&= \delta(\widehat{\delta}(\text{locked}, \text{card card}),\ \text{push}) \\
&= \delta(\delta(\widehat{\delta}(\text{locked}, \text{card}),\ \text{card}),\ \text{push}) \\
&= \delta(\delta(\delta(\widehat{\delta}(\text{locked}, \epsilon),\ \text{card}),\ \text{card}),\ \text{push}) \\
&= \delta(\delta(\delta(\text{locked},\ \text{card}),\ \text{card}),\ \text{push}) \\
&= \delta(\delta(\text{unlocked},\ \text{card}),\ \text{push}) \\
&= \delta(\text{unlocked},\ \text{push}) = \text{locked}
\end{aligned}
$$

Example: Turnstile



$$\delta(\text{locked}, \text{card}) = \text{unlocked} \qquad \widehat{\delta}(\text{locked}, \epsilon) = \text{locked}$$

$$\delta(\text{locked}, \text{push}) = \text{locked} \qquad \widehat{\delta}(\text{locked}, \text{card}) = \text{unlocked}$$

$$\delta(\text{unlocked}, \text{card}) = \text{unlocked} \qquad \widehat{\delta}(\text{locked}, \text{card card}) = \text{unlocked}$$

$$\delta(\text{unlocked}, \text{push}) = \text{locked} \qquad \widehat{\delta}(\text{locked}, \text{card card push}) = \text{locked}$$

String processing

Given $A = (Q, \Sigma, \delta, q_0, F)$ and $w \in \Sigma^*$ we say that A **accept** w iff

$$\widehat{\delta}(q_0, w) \in F$$

The **language** of $A = (Q, \Sigma, \delta, q_0, F)$ is

$$L(A) = \{ \ w \ \mid \ \widehat{\delta}(q_0, w) \in F \ \}$$

Transition diagram

We represent $A = (Q, \Sigma, \delta, q_0, F)$ as a graph s.t.

- Q is the set of nodes;
- $\{ q \xrightarrow{a} q' \mid q' = \delta(q, a) \}$ is the set of arcs.

Plus some graphical conventions:

- there is one special arrow $Start$ with $\xrightarrow{Start} q_0$
- nodes in F are marked by double circles;
- nodes in $Q \setminus F$ are marked by single circles.

String processing as paths

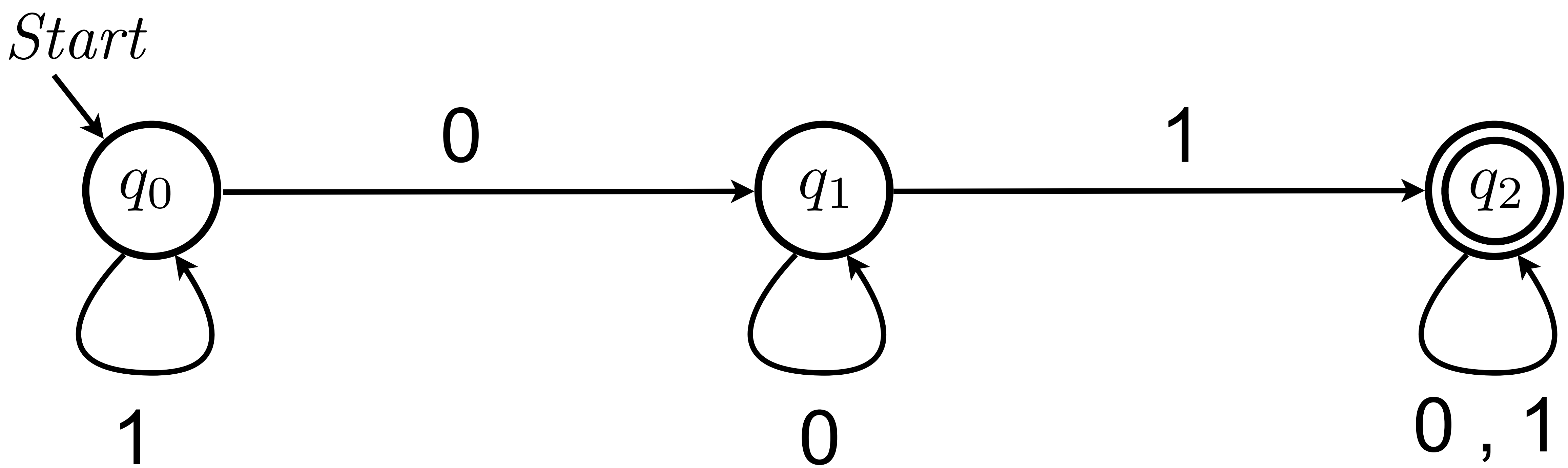
A DFA accepts a string w, if there is a path in its transition diagram such that:

it starts from the initial state

it ends in one final state

the sequence of labels in the path is exactly w

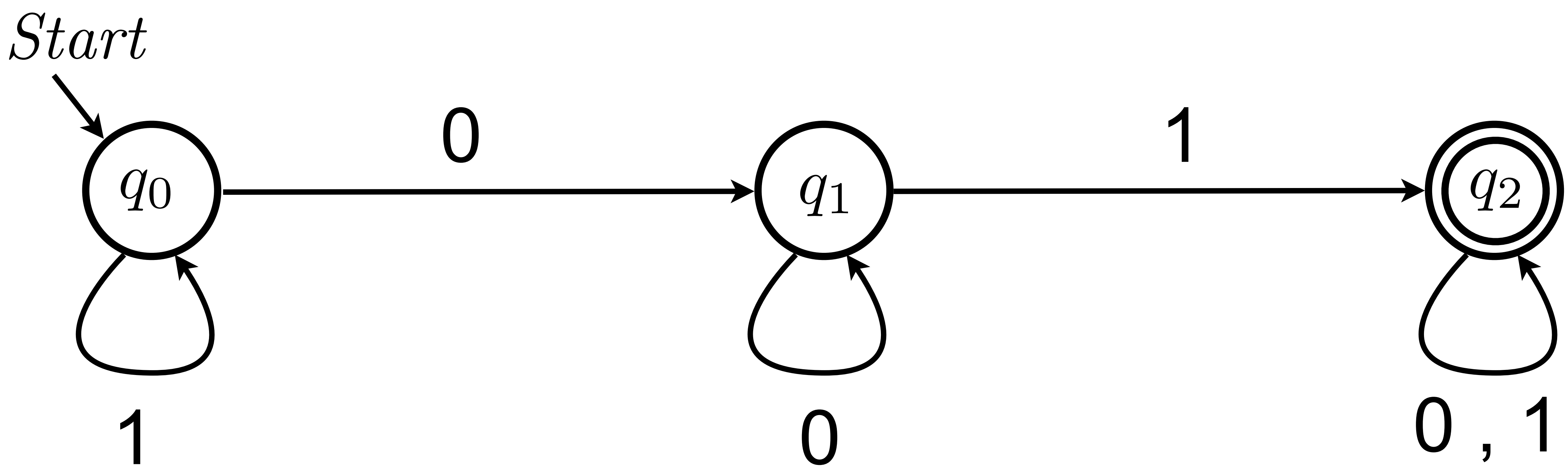
DFA: example



$$q_0 \; 1 \; q_0 \; 1 \; q_0 \; 1 \; q_0 \; 0 \; q_1 \; 0 \; q_1 \; 0 \; q_1 \notin F$$

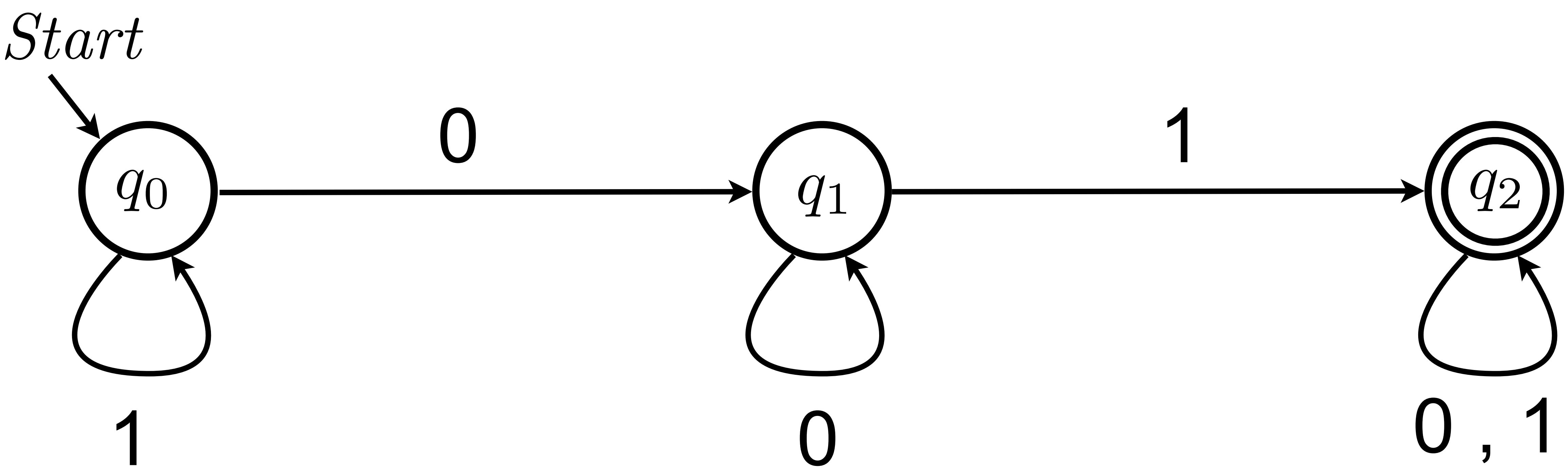
$$q_0 \; 1 \; q_0 \; 0 \; q_1 \; 0 \; q_1 \; 1 \; q_2 \; 1 \; q_2 \; 0 \; q_2 \in F$$

DFA: question time



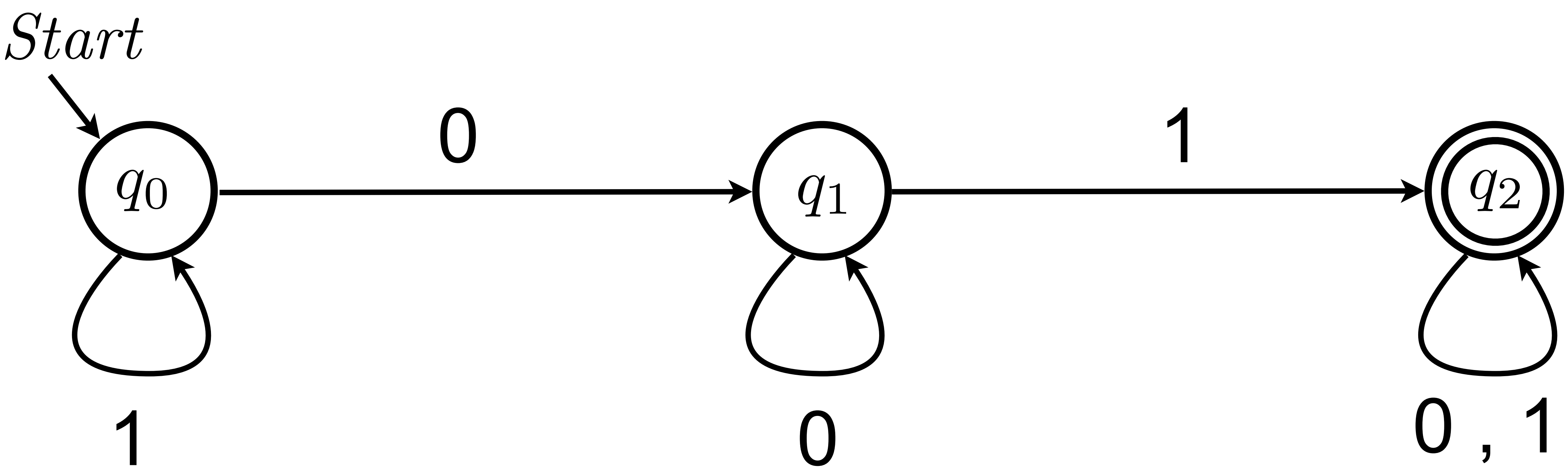
Does it accept 100 ?
Does it accept 011 ?
Does it accept 1010010 ?
What is L(A) ?

DFA: question time



Does it accept 100 ? NO
Does it accept 011 ?
Does it accept 1010010 ?
What is L(A) ?

DFA: question time



Does it accept 100 ? NO
Does it accept 011 ? YES
Does it accept 1010010 ?
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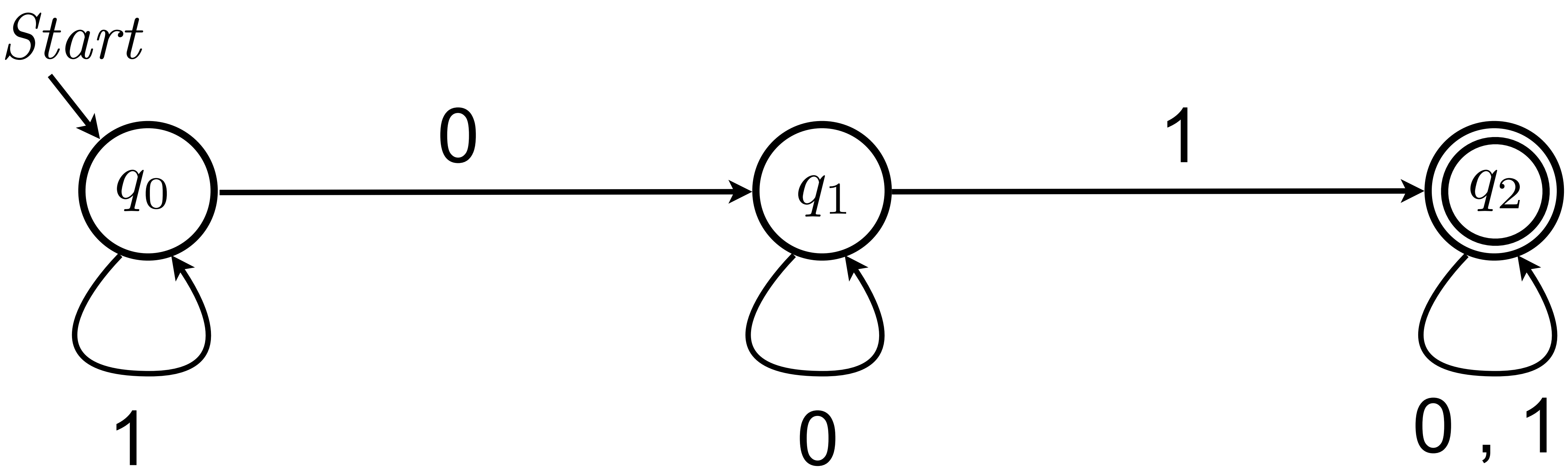
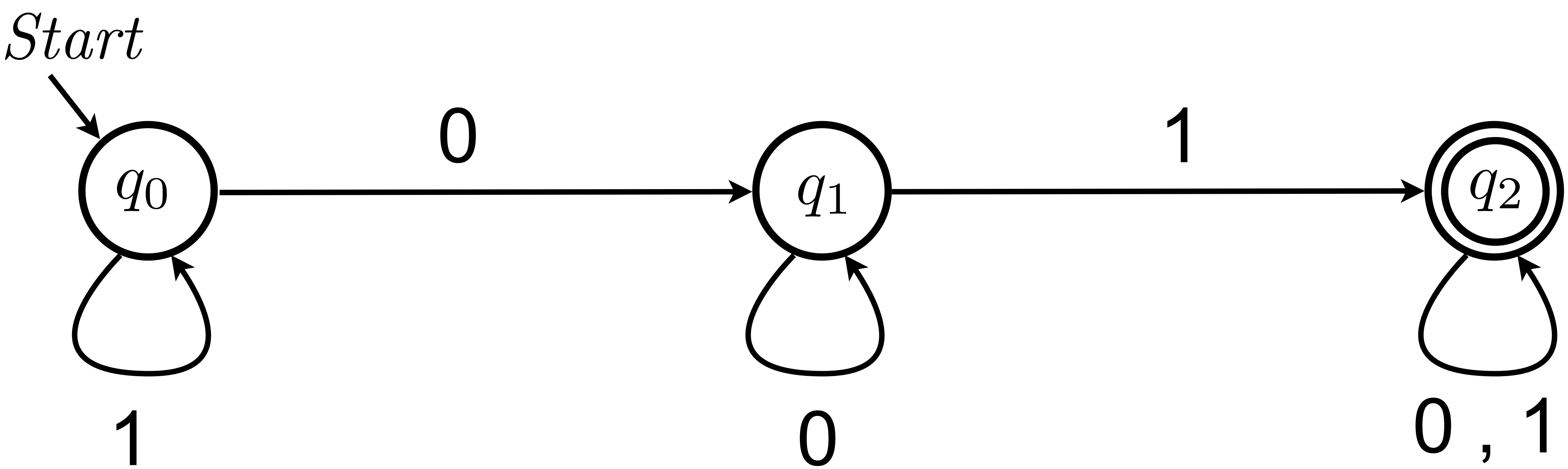
DFA: question time



Does it accept 100 ? NO
Does it accept 011 ? YES
Does it accept 1010010 ? YES
What is L(A) ?

DFA: question time



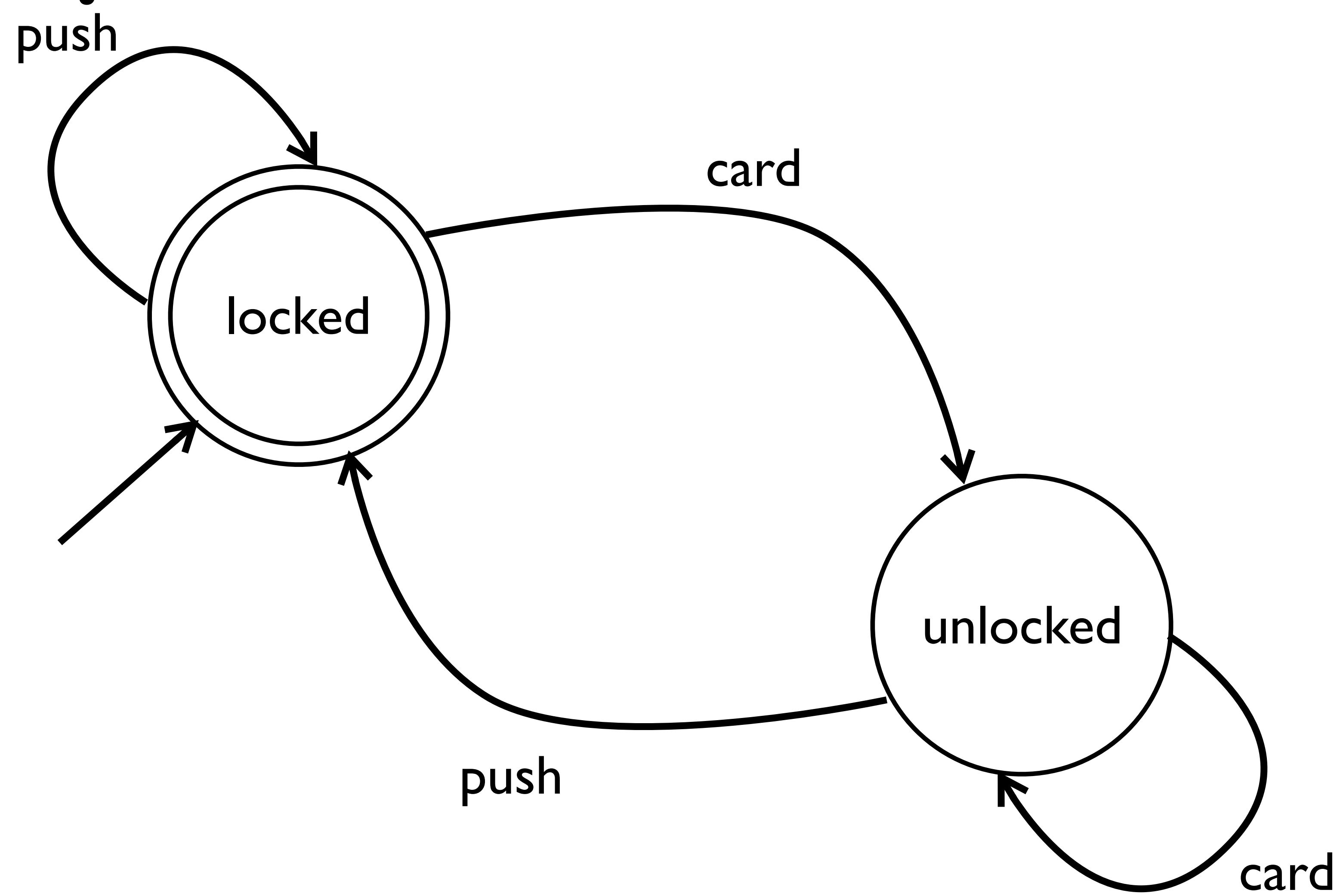
Does it accept 100 ? NO

Does it accept 011 ? YES

Does it accept 1010010 ? YES

What is L(A) ? $\{ \; x01y \;|\; x, y \in \{0,1\}^* \; \}$

DFA: question time



What is L(A) ?

Transition table

Conventional tabular representation

its rows are in correspondence with states

its columns are in correspondence with input symbols

its entries are the states reached after the transition

Plus some decoration

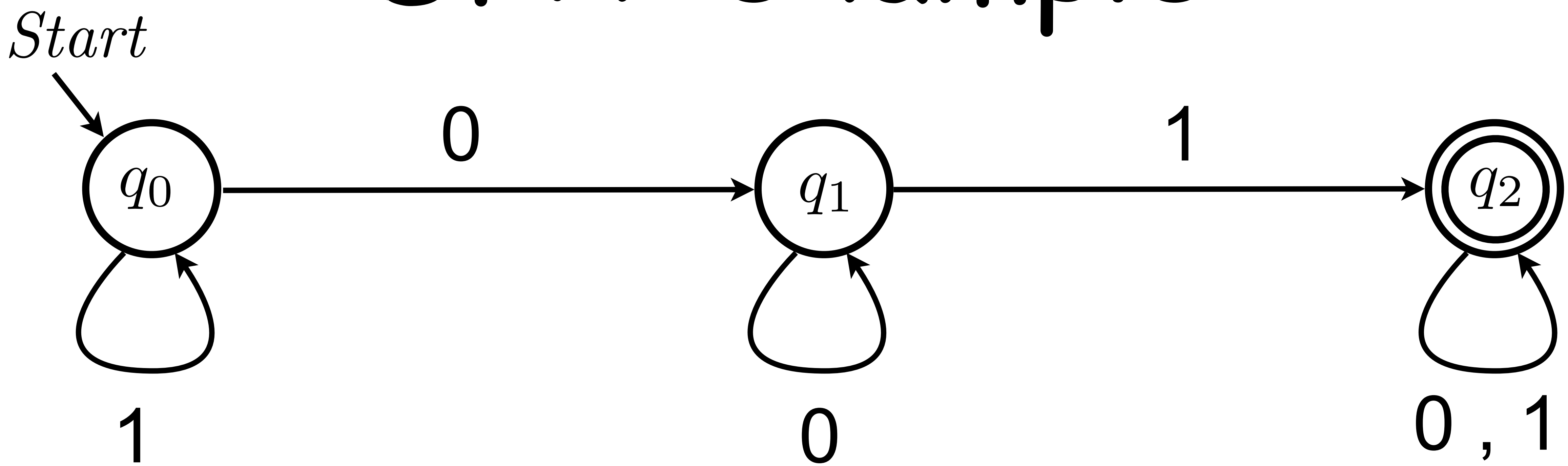
start state decorated with an arrow

all final states decorated with *

Transition table

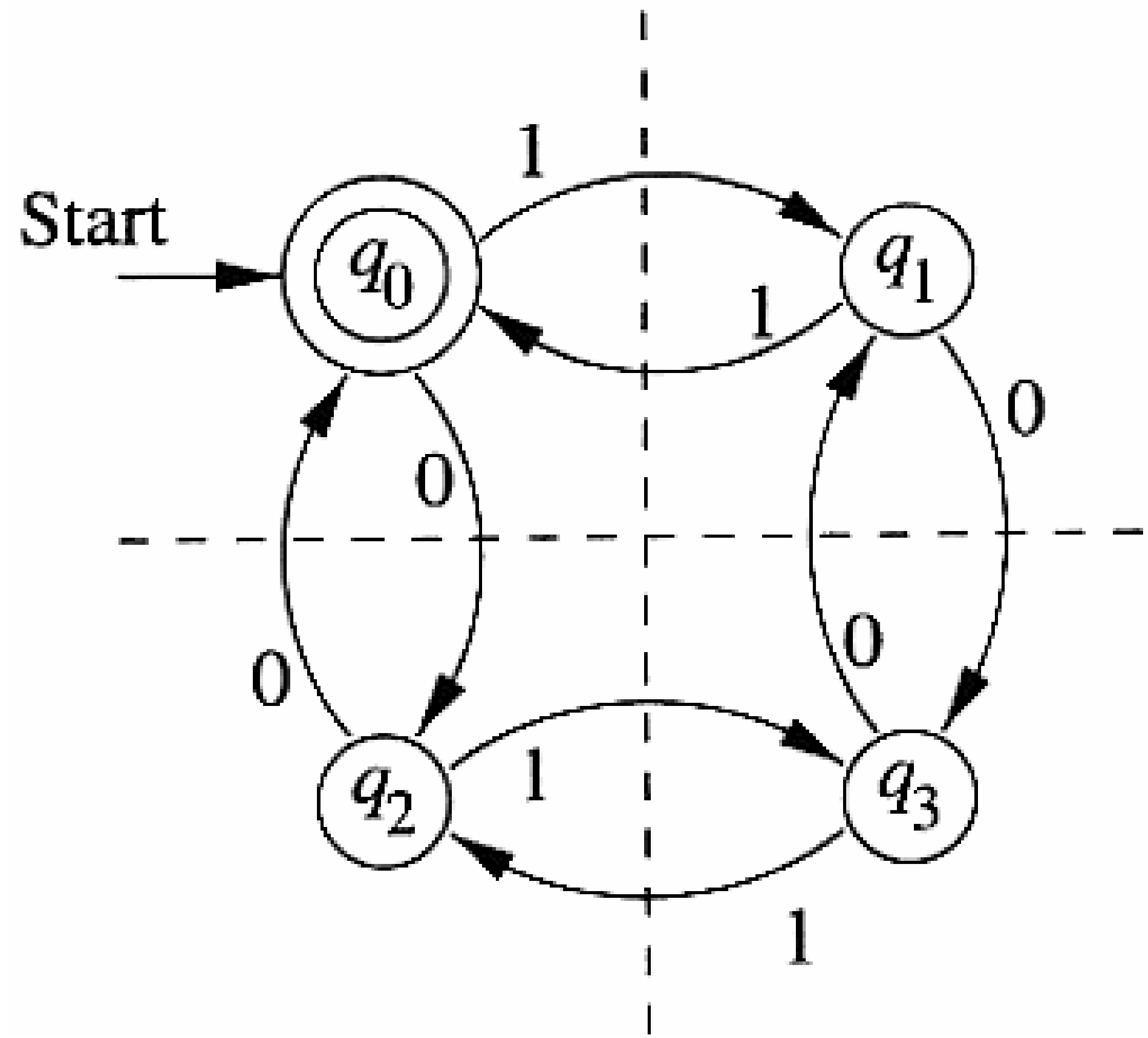
					$\mathbf{a}$			
$\rightarrow$								
	$\mathbf{q}$				$\delta(q, a)$			
$*$								
$*$								

DFA: example



	0	1
$\rightarrow \mathbf{q_0}$		
$\mathbf{q_1}$		
$\star\ \mathbf{q_2}$		

DFA: exercise



Does it accept 100 ?

Write its transition table.

Does it accept 1010 ?

What is L(A) ?

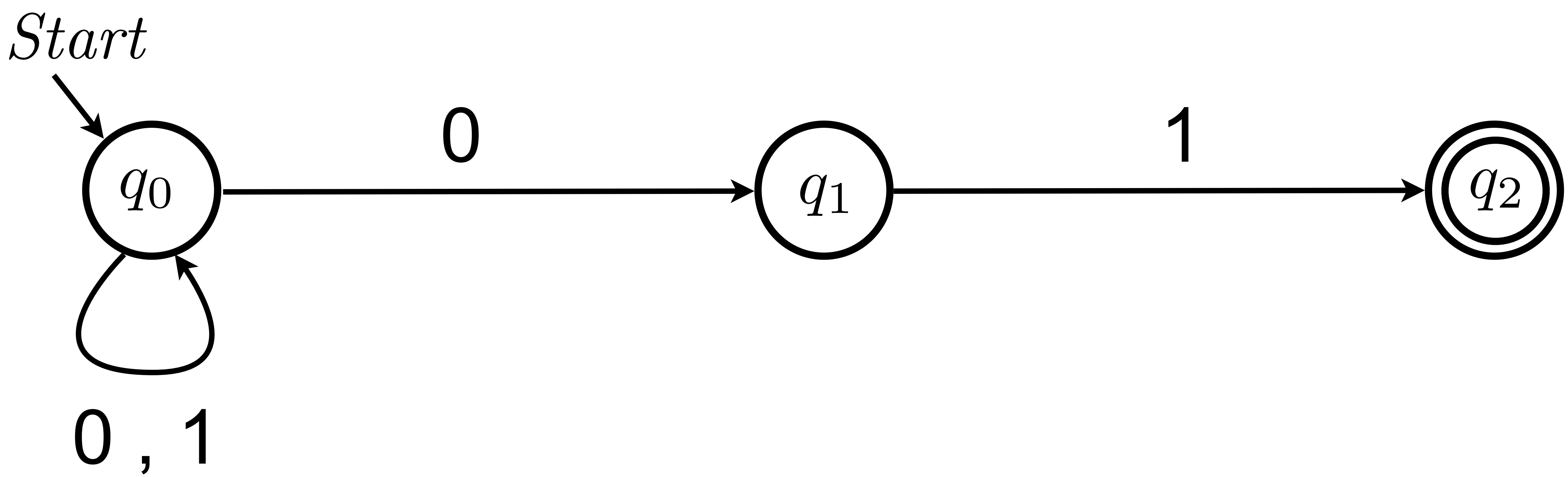
NFA

A **Non-deterministic Finite Automaton** (**NFA**) is a tuple $A = (Q, \Sigma, \delta, q_0, F)$, where

powerset of Q = set of sets over Q

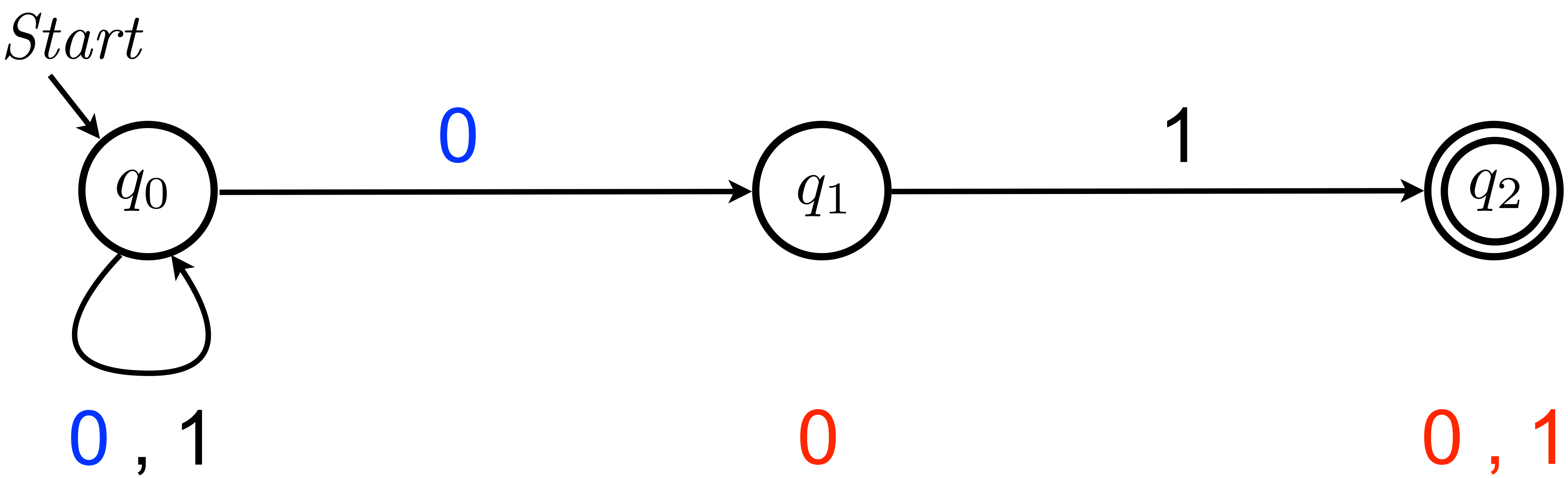
- Q is a finite set of states;
- Σ is a finite set of input symbols;
- $\delta : Q \times \Sigma \to \wp(Q)$ is the transition function;
- $q_0 \in Q$ is the initial state (also called start state);
- $F \subseteq Q$ is the set of final states (also accepting states)

NFA: example



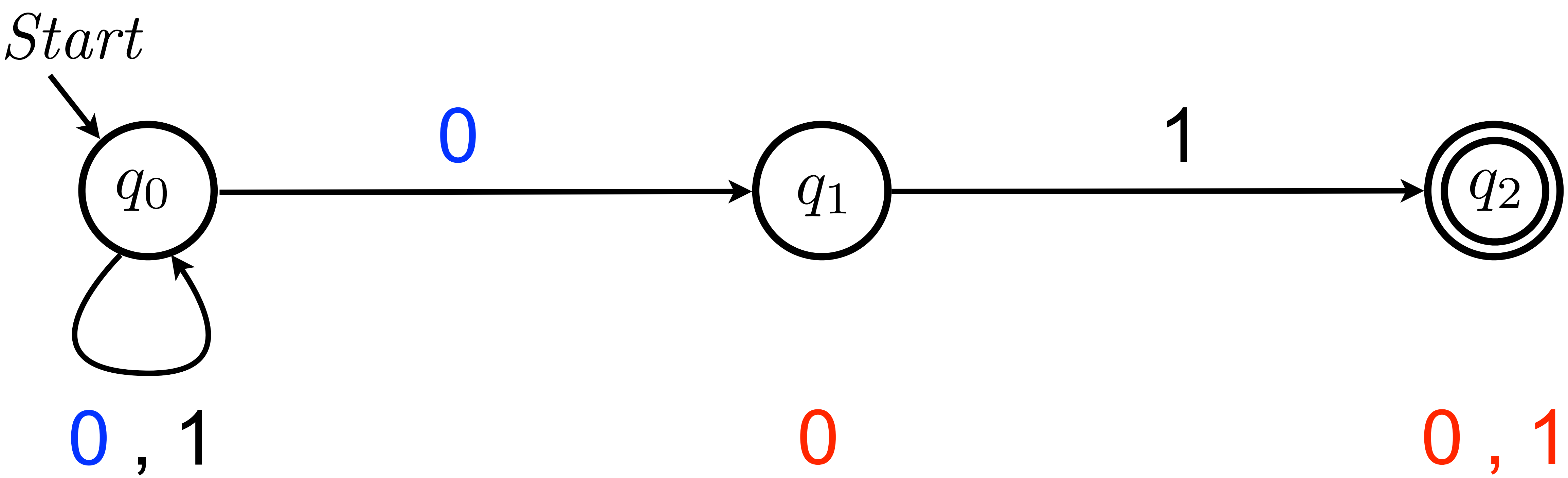
Can you explain why it is not a DFA?

NFA: example



Can you explain why it is not a DFA?

NFA: example



$$\delta(q_0, 0) = \{q_0, q_1\}$$
$$\delta(q_0, 1) = \{q_0\}$$

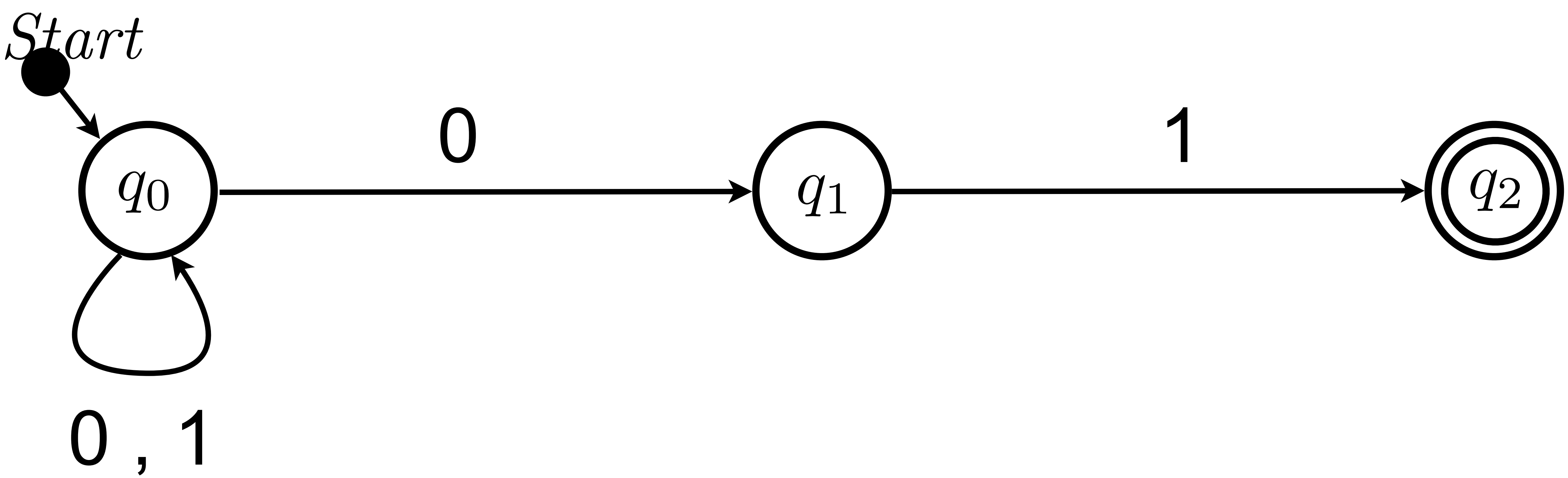
$$\delta(q_1, 0) = \emptyset$$
$$\delta(q_1, 1) = \{q_2\}$$

$$\delta(q_2, 0) = \delta(q_2, 1) = \emptyset$$

Can you explain why it is not a DFA?

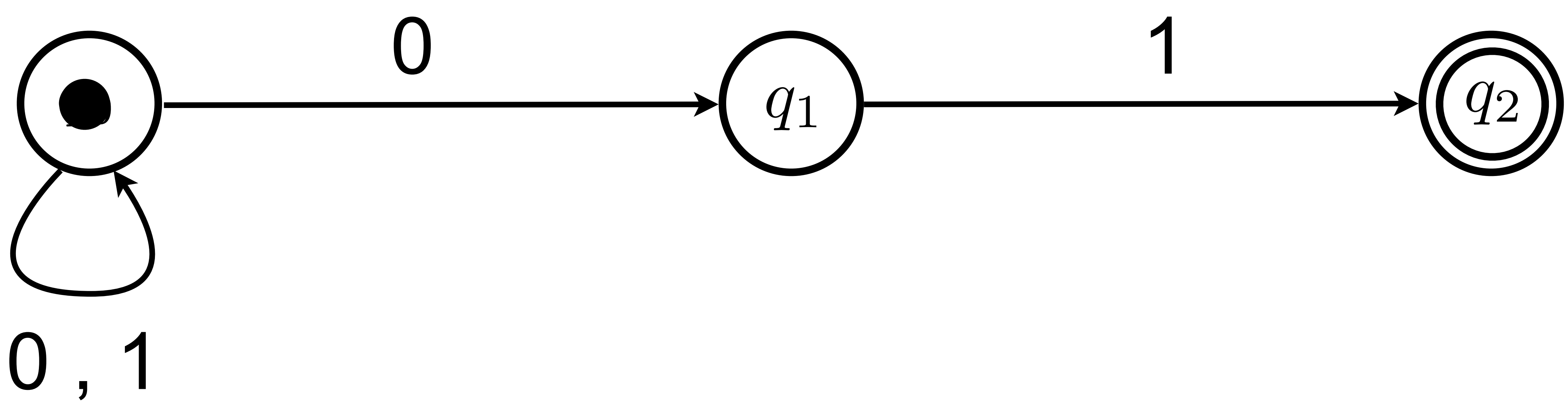
Reshaping

Step 1: get a token

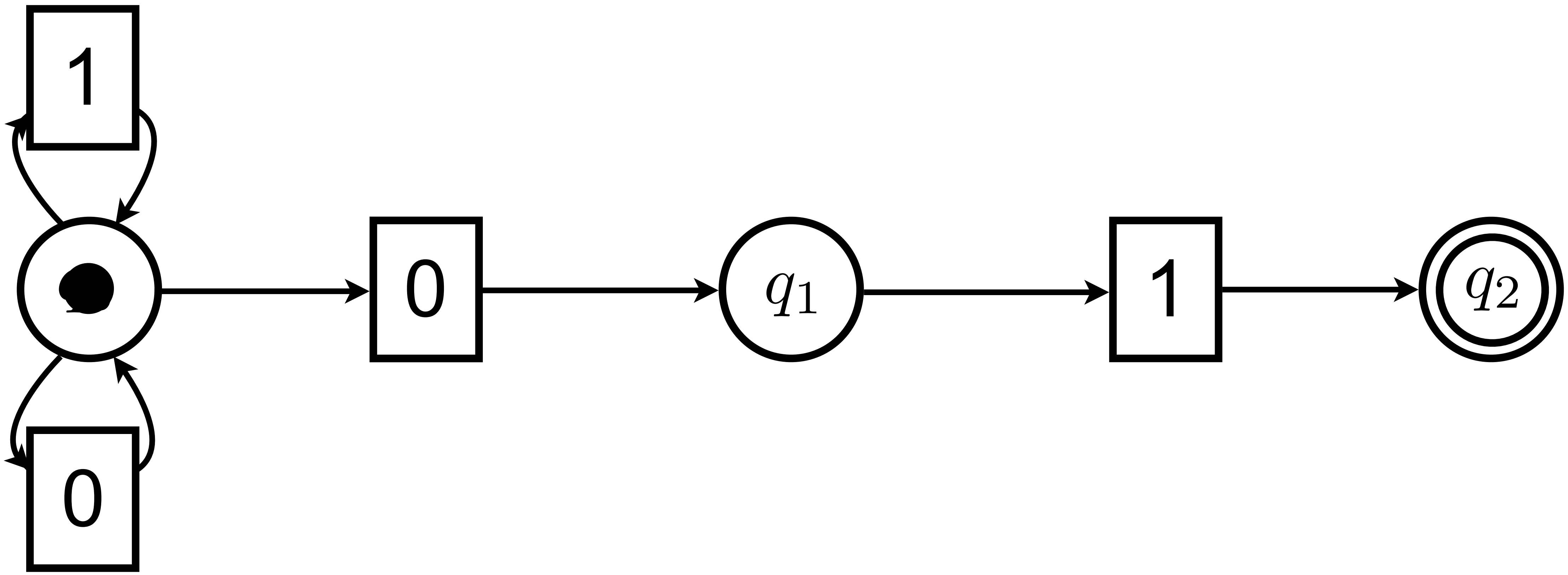


0	1	0	0	1

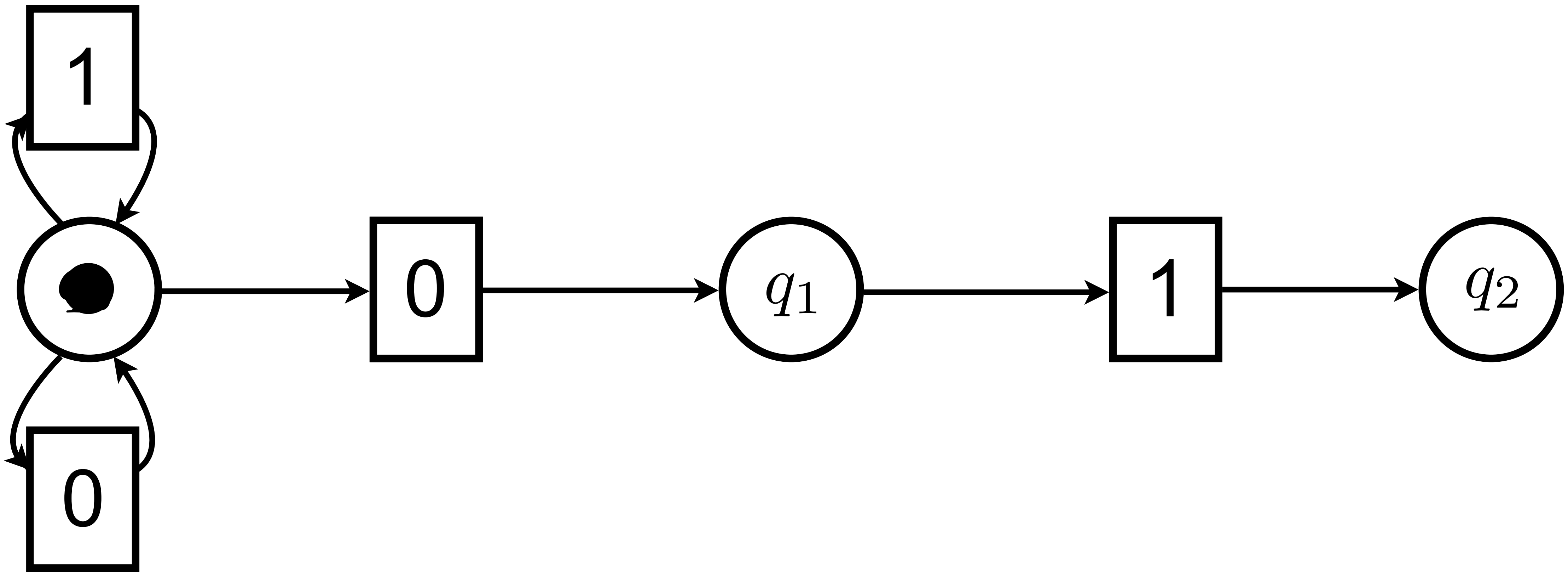
Step 2: forget initial state decoration



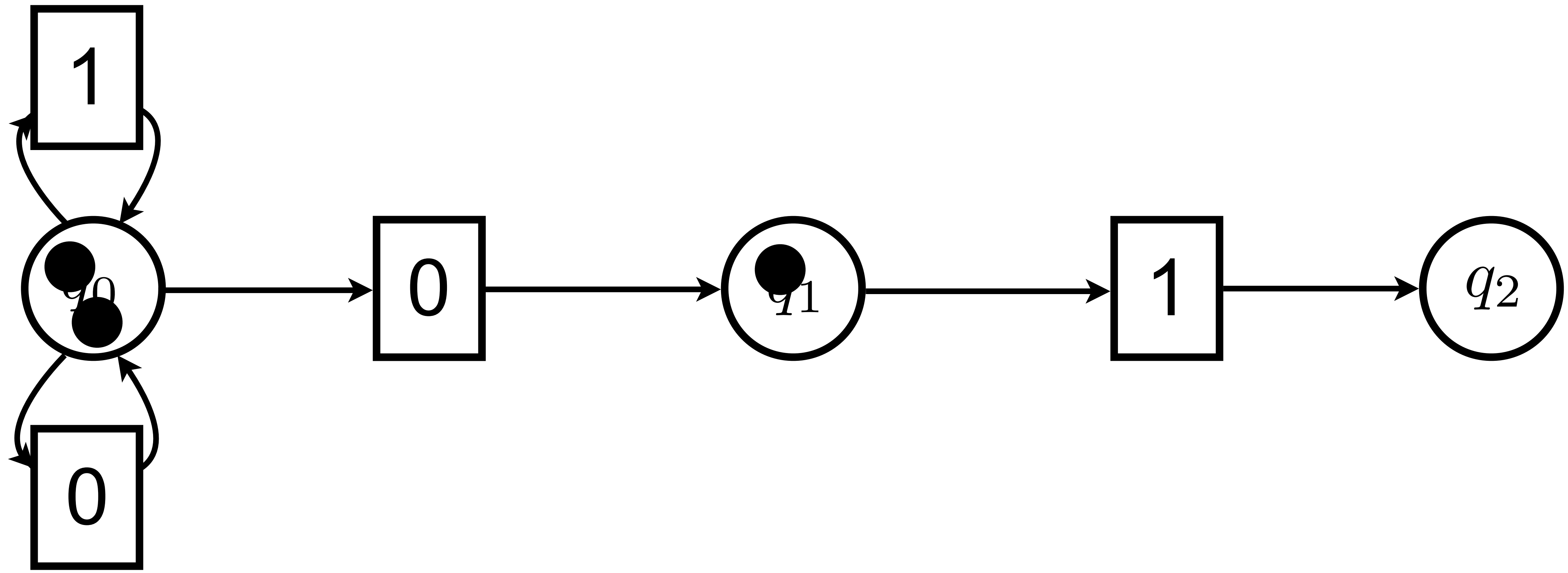
Step 3: transitions as boxes



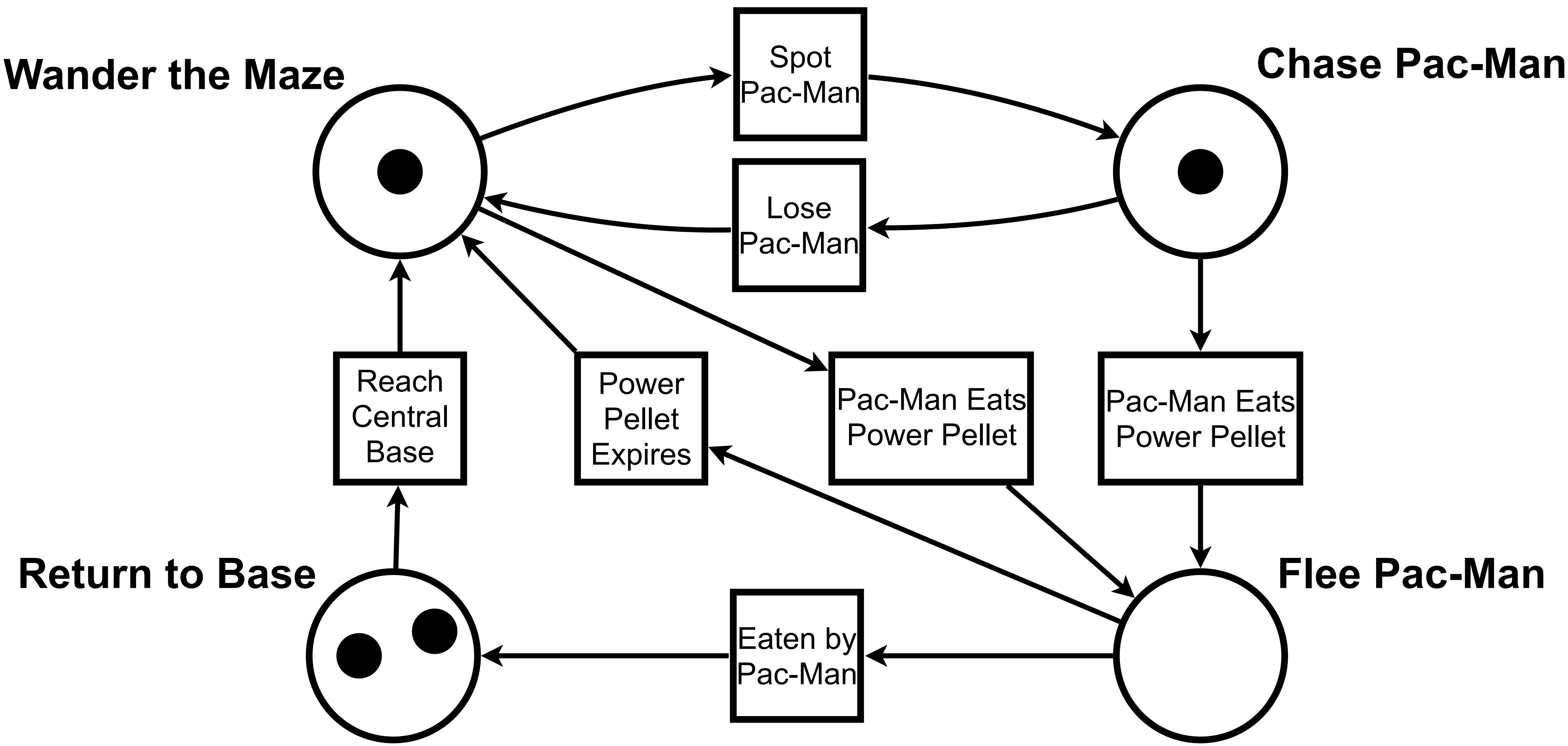
Step 4: forget final states



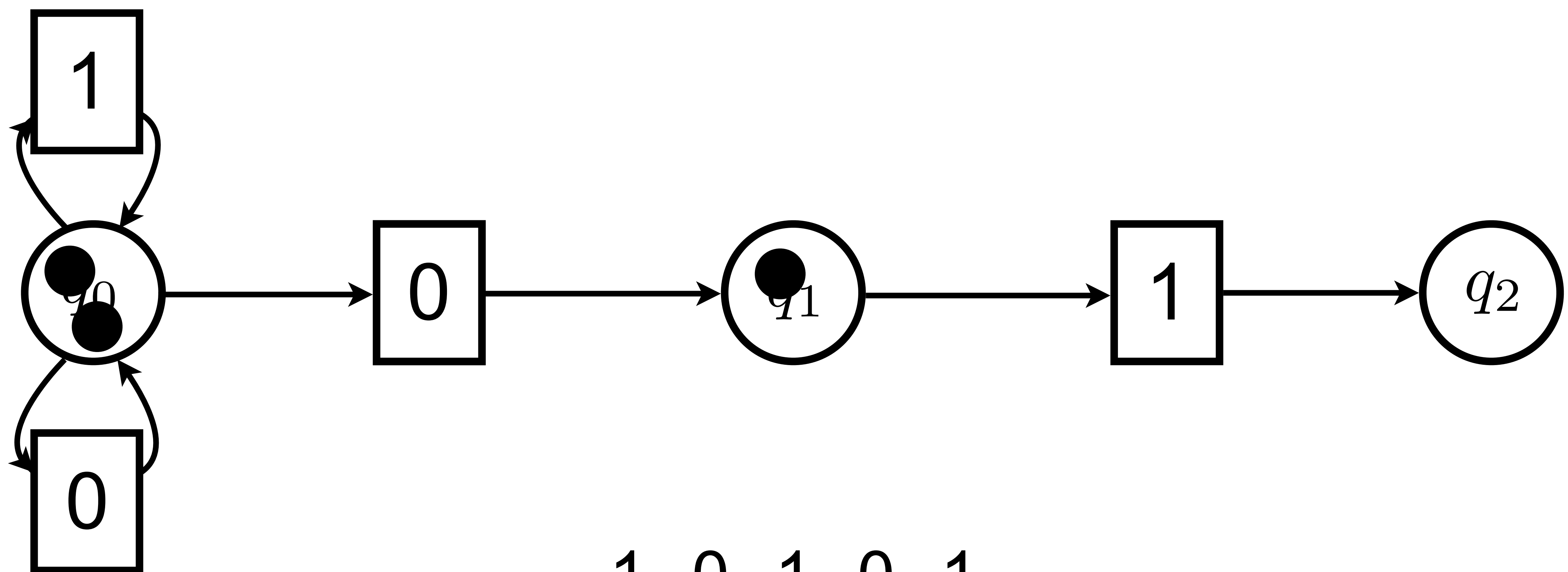
Step 5: allow for more tokens



Example: Four Pac-Man Ghosts

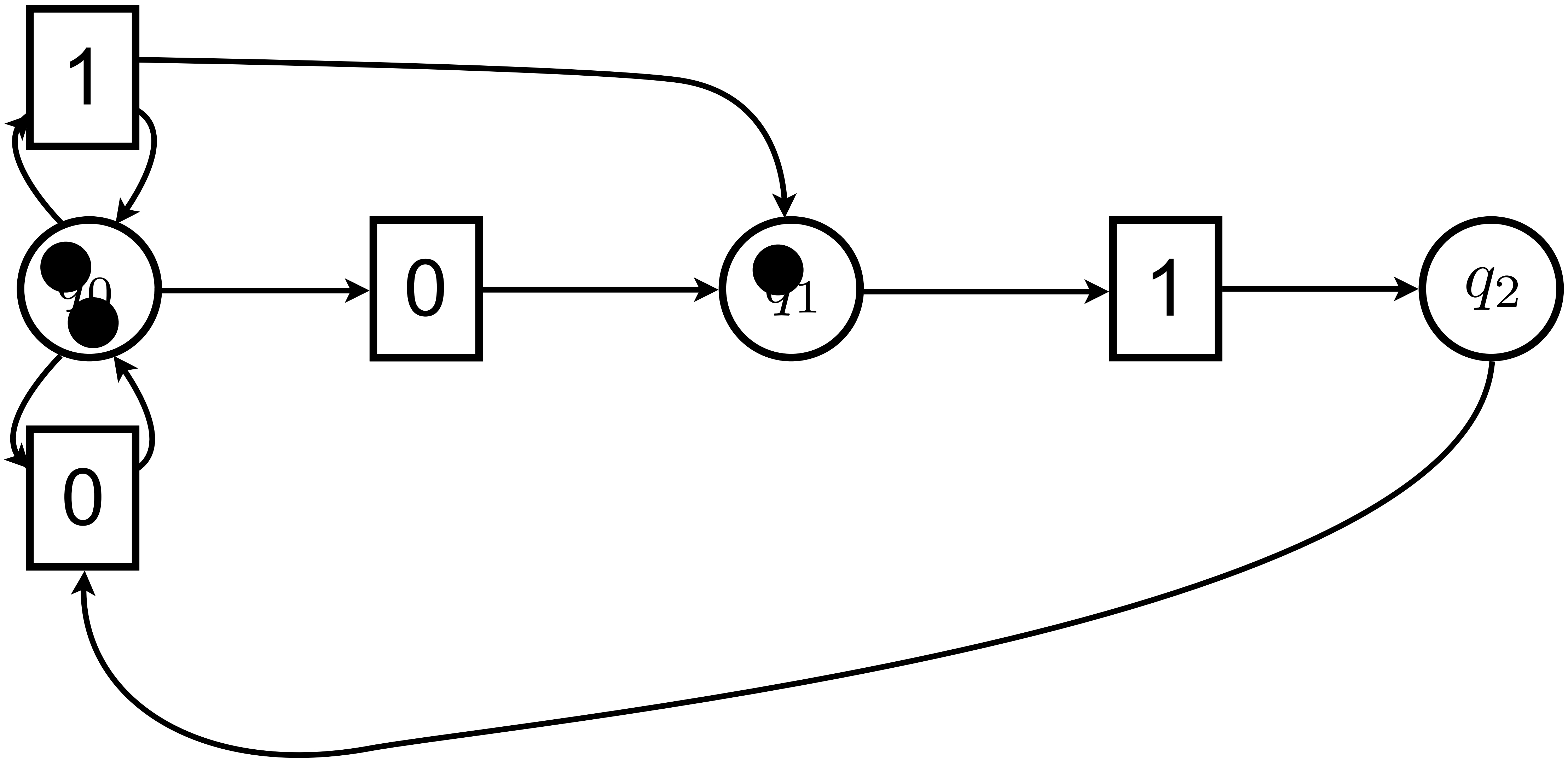


Example: token game

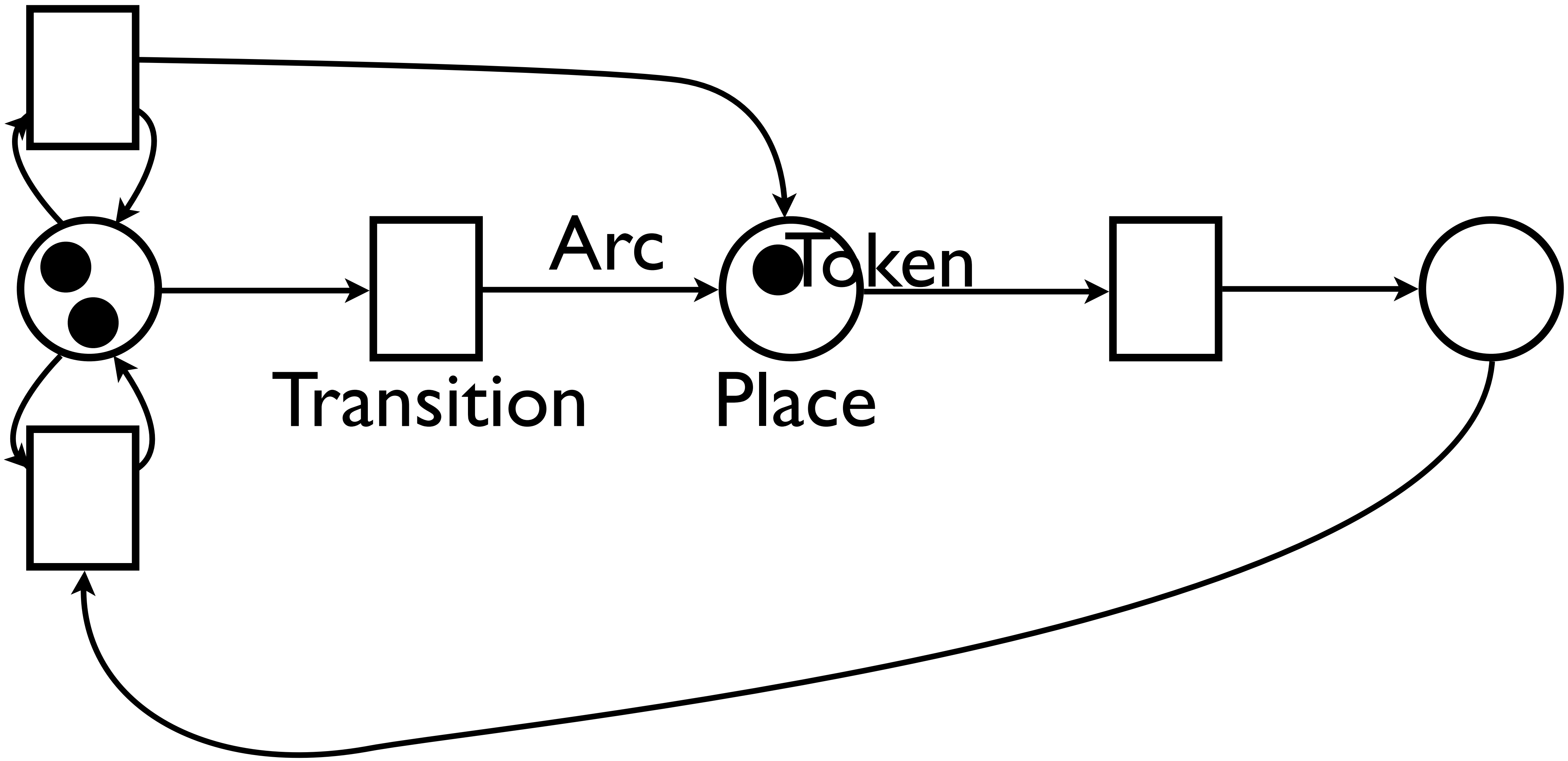


1 0 1 0 1

Step 6: allow for more arcs



Terminology



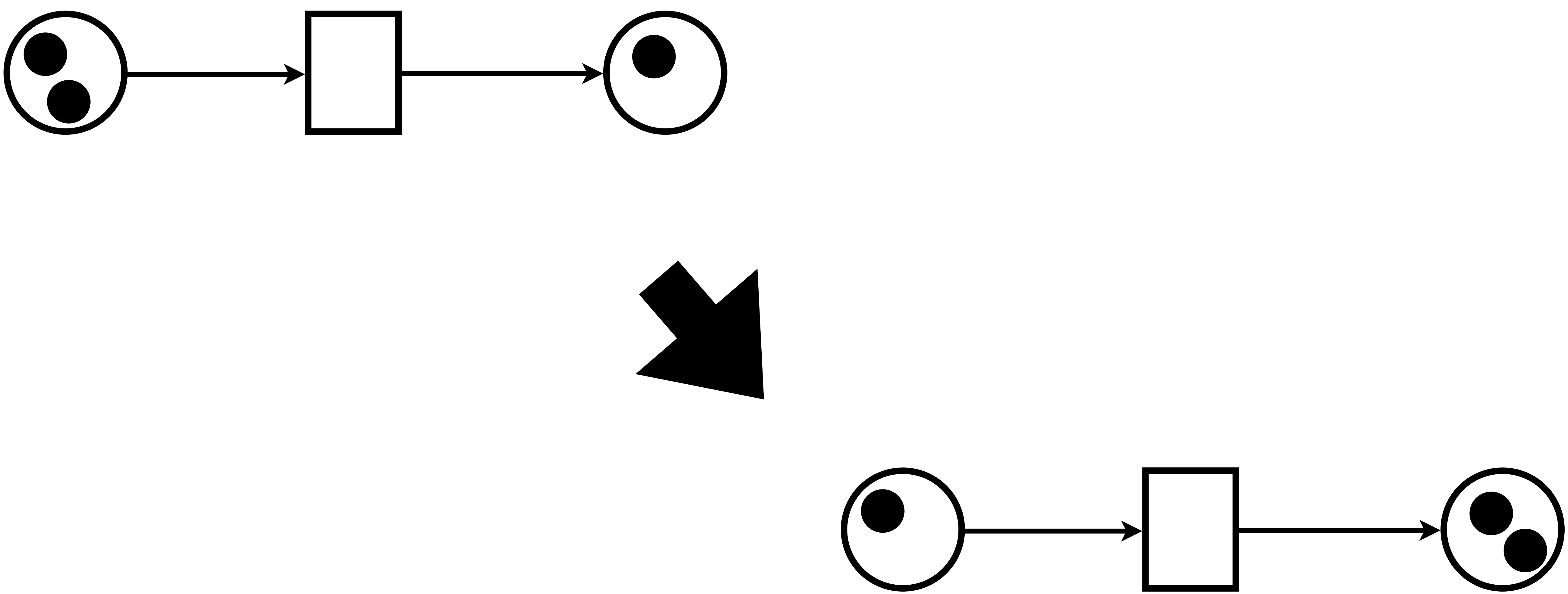
Some facts

Nets are **bipartite graphs**:
arcs never connect two places
arcs never connect two transitions

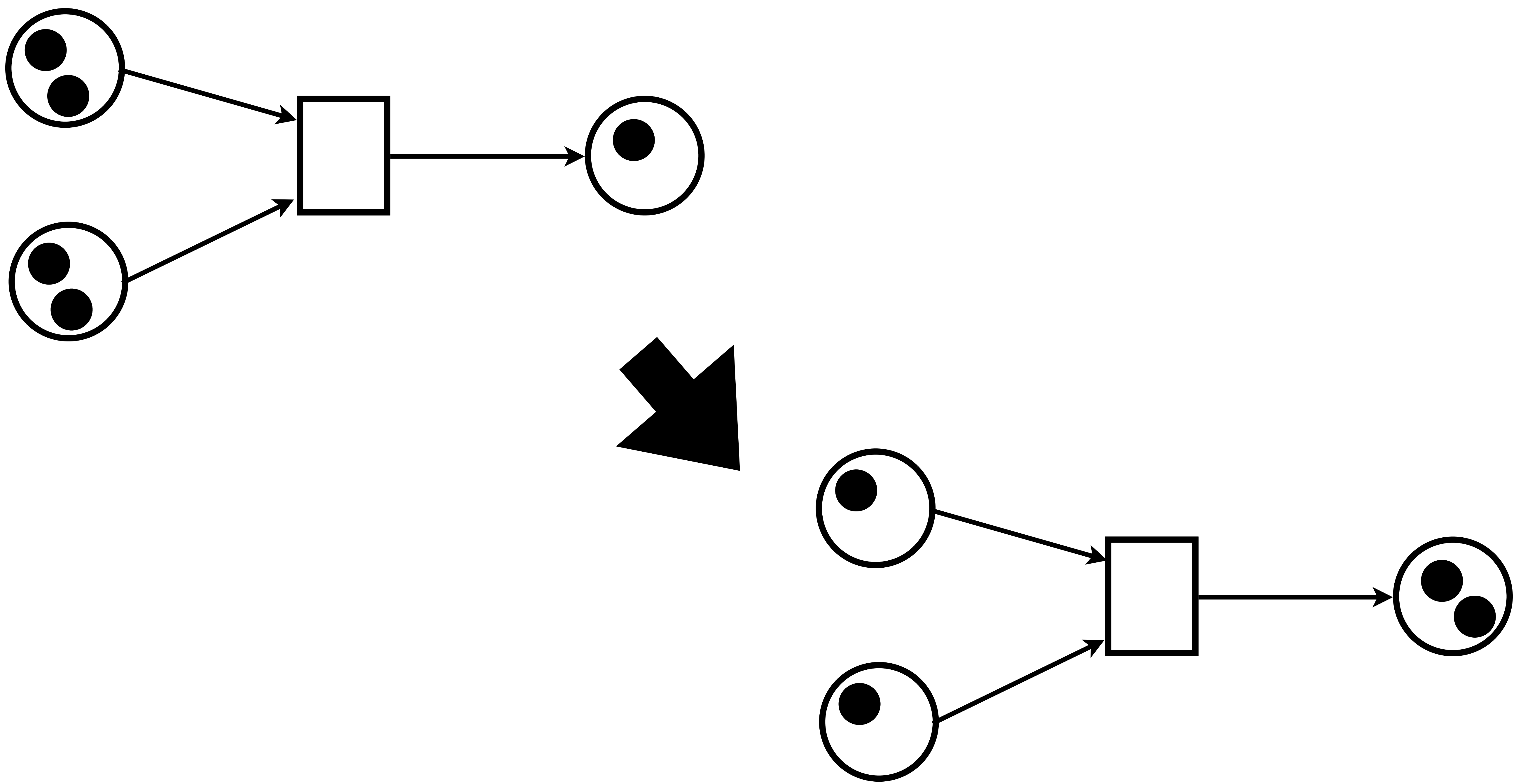
Static structure for dynamic systems:
places, transitions, arcs do not change
tokens move around places

Places are passive components
Transitions are active components:
tokens do not flow!
(they are removed or freshly created)

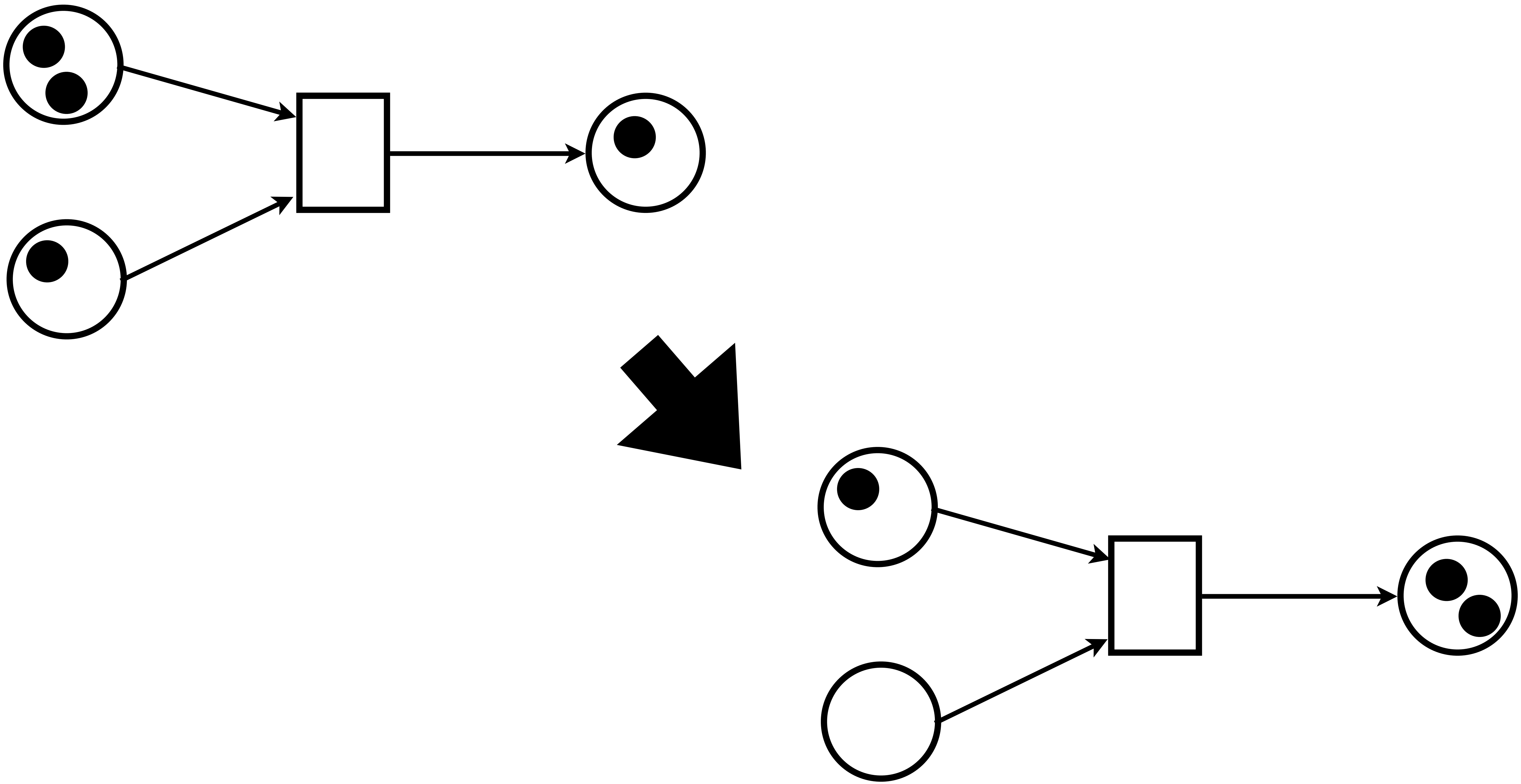
Token game: example



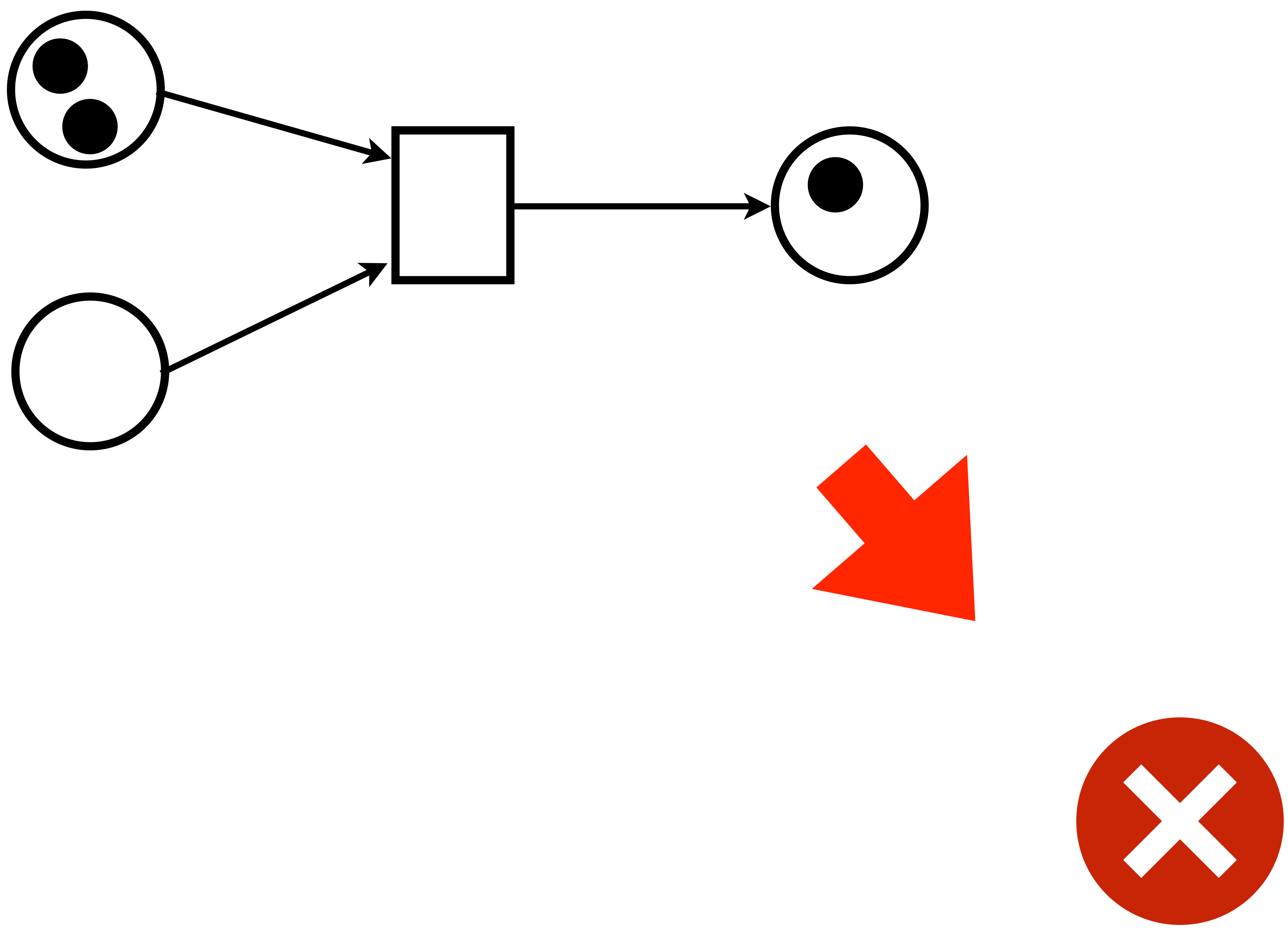
Token game: example



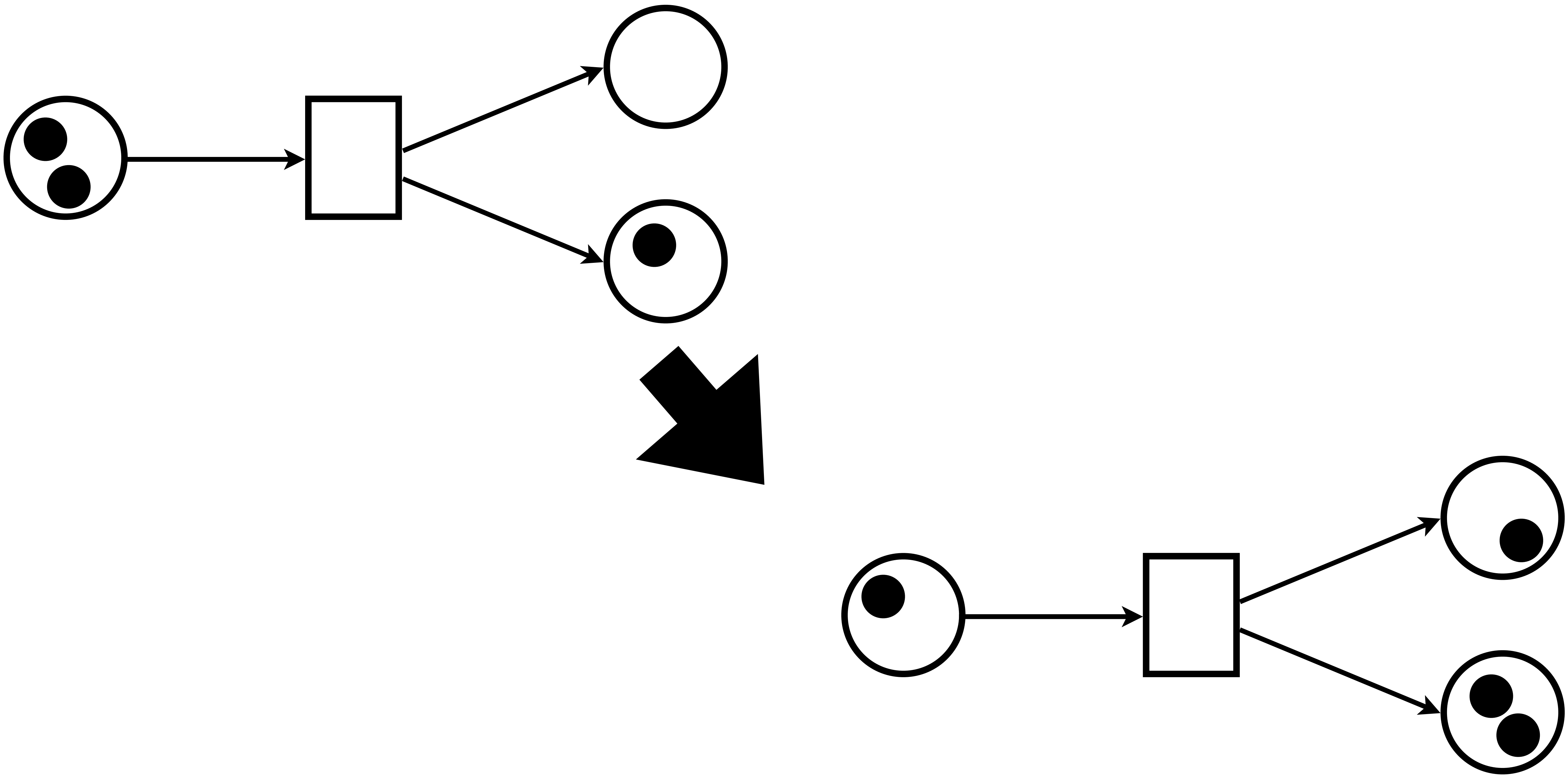
Token game: example



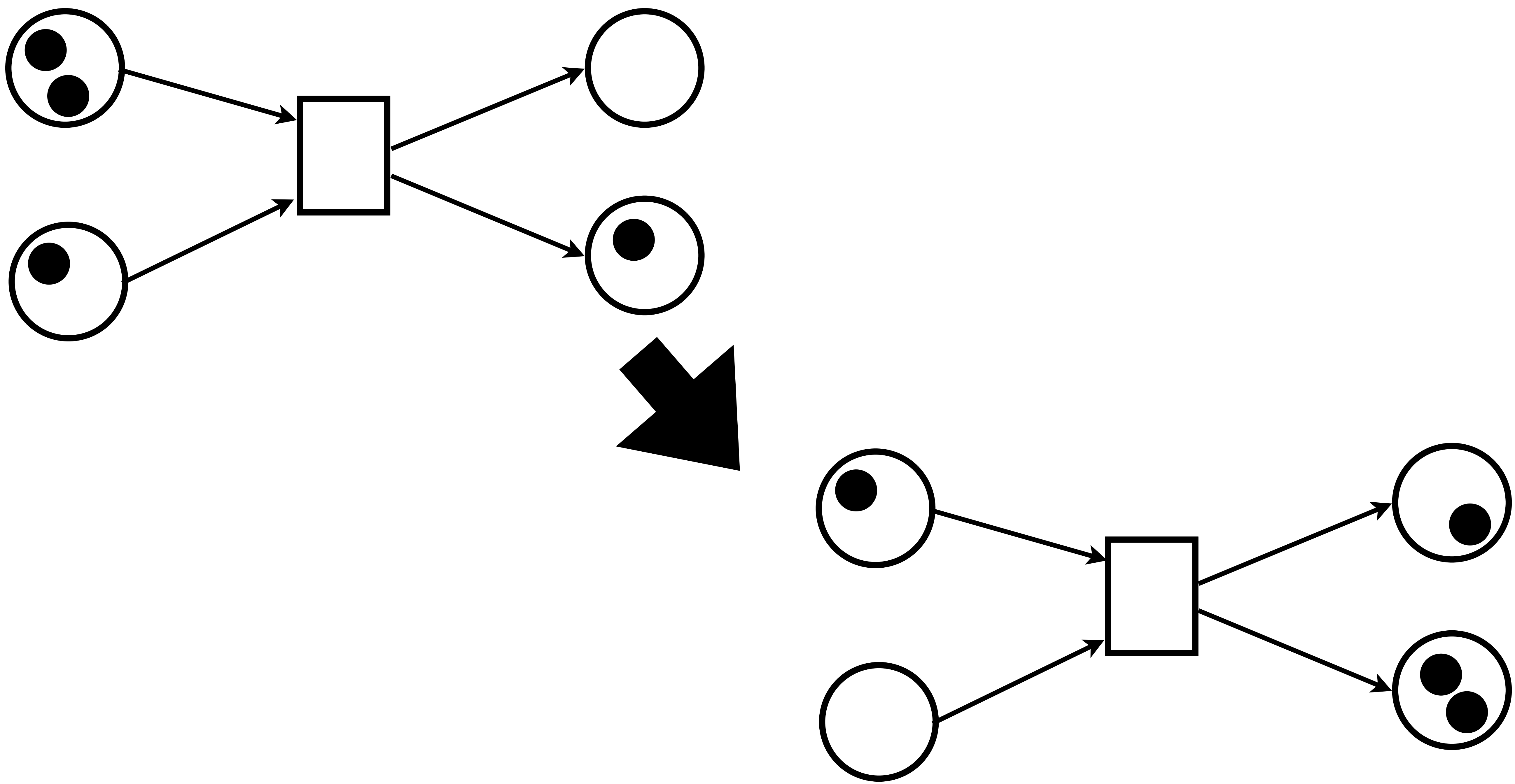
Token game: example



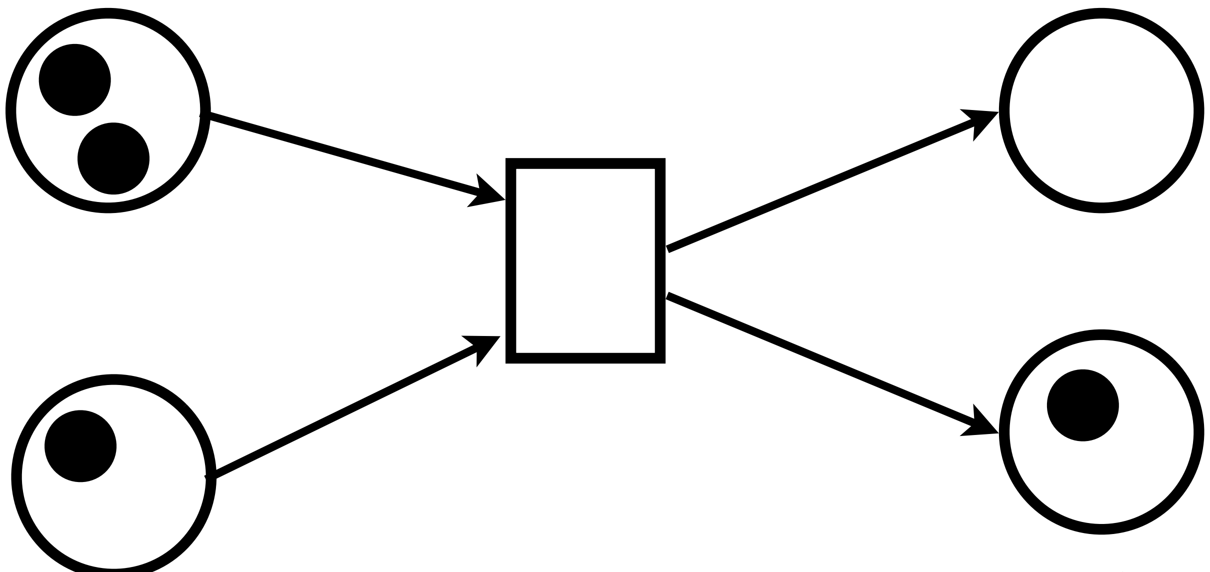
Token game: example



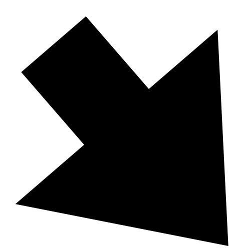
Token game: example

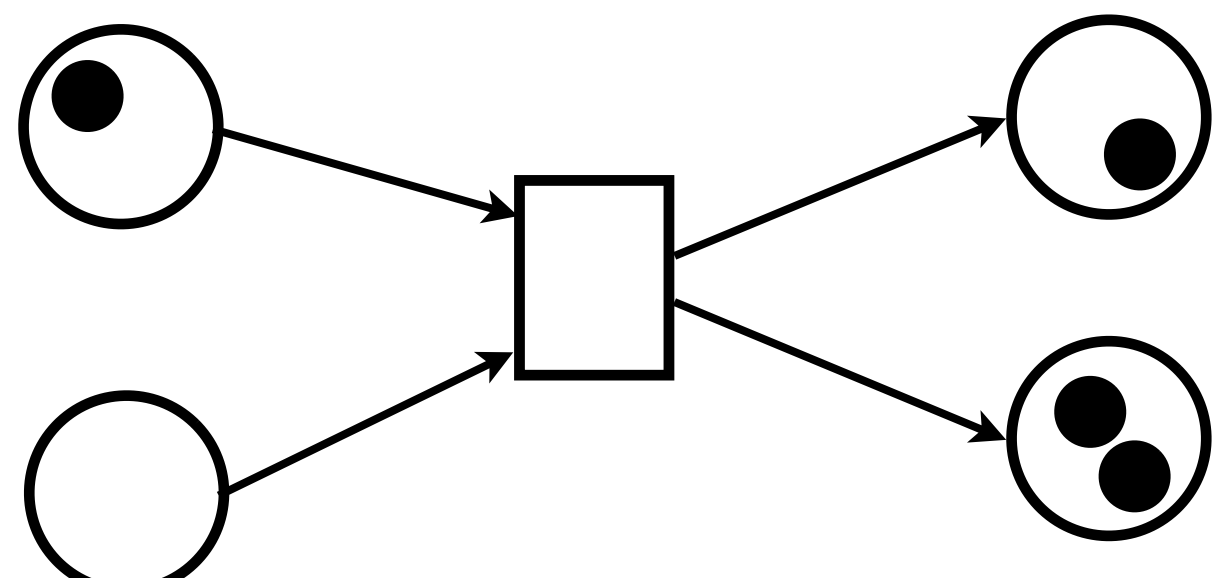


Token game: firing rule



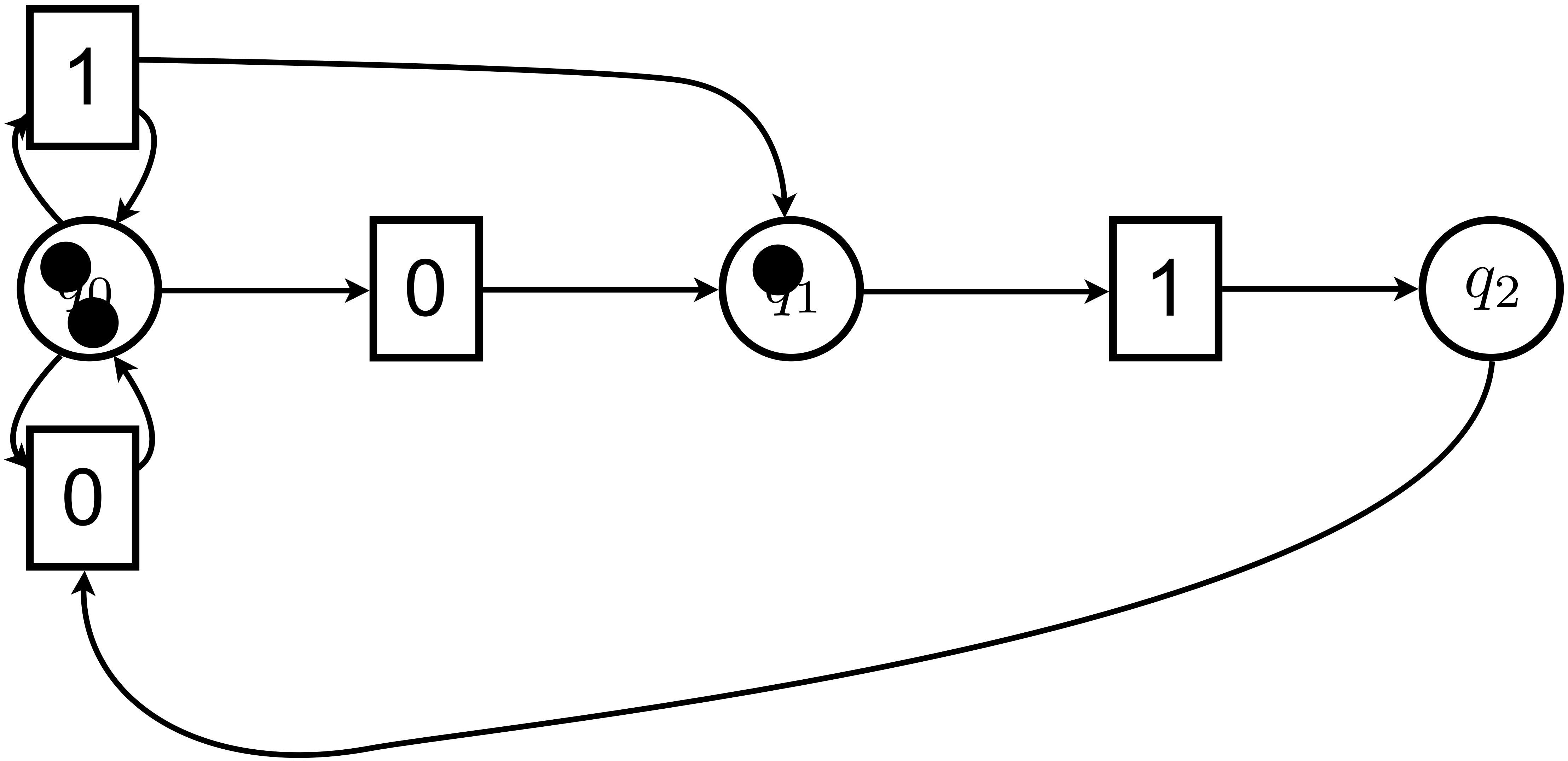
Collect one token from each “input” place



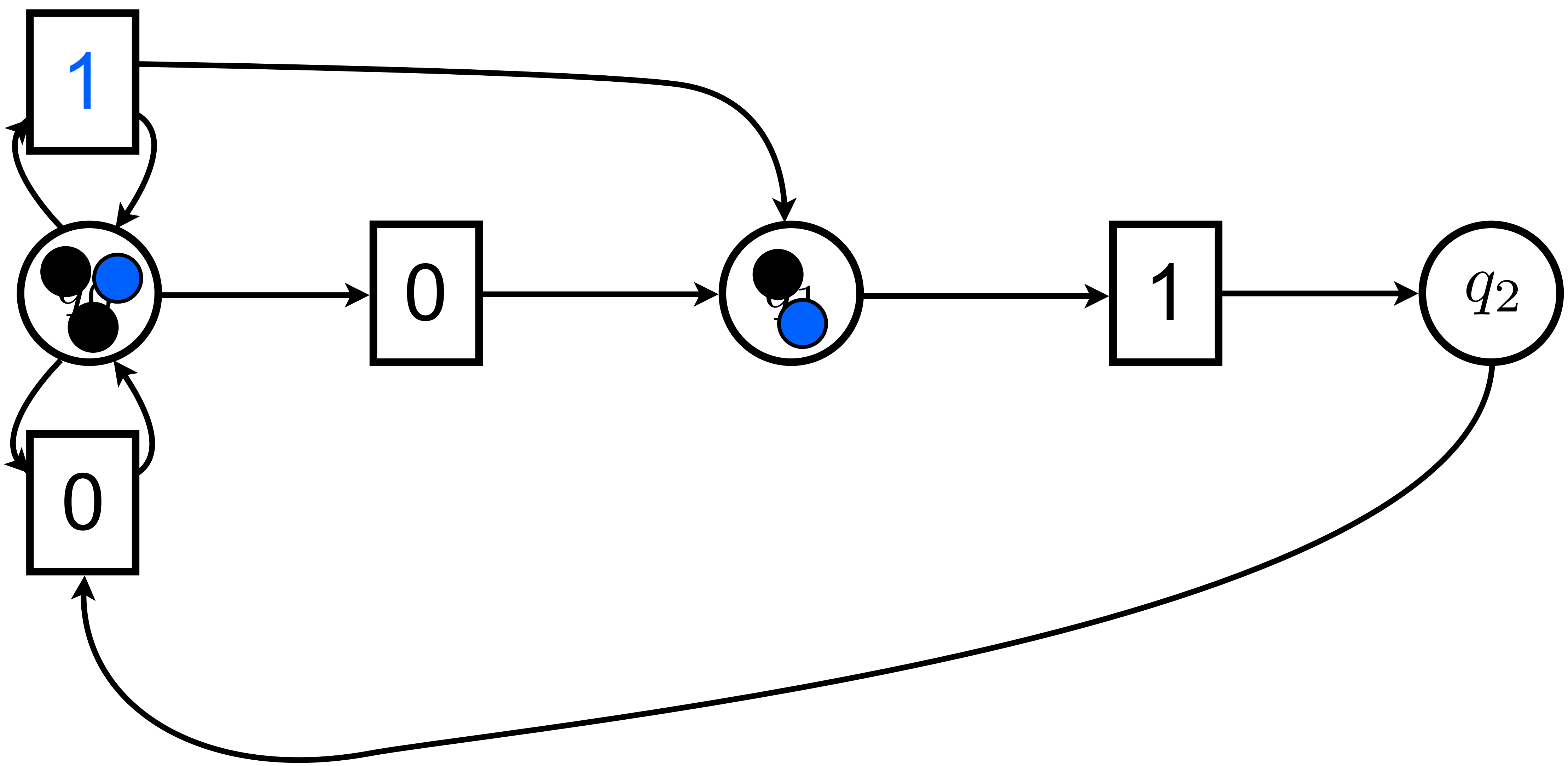


Produce one token into each “output” place

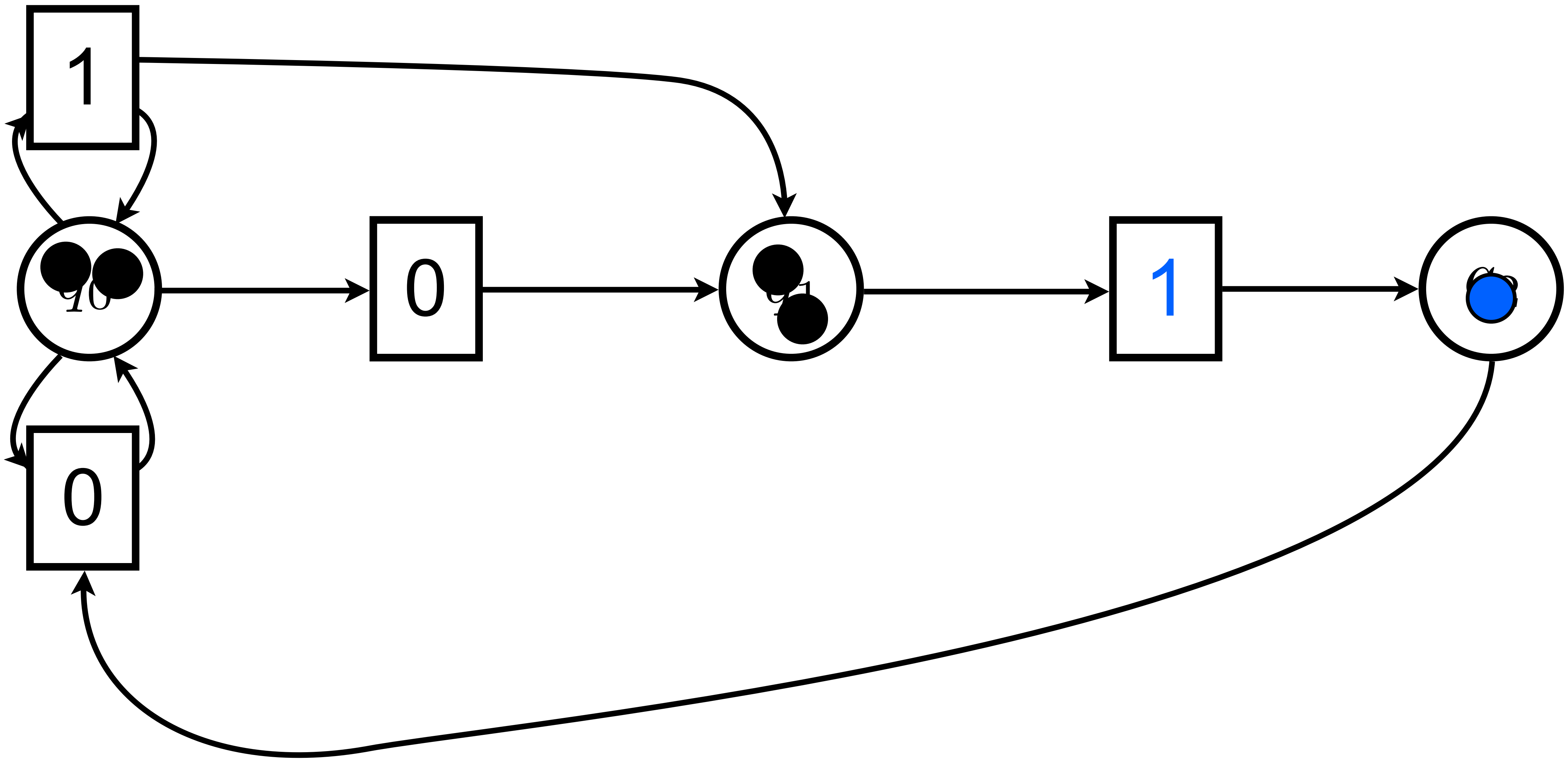
Example: token game



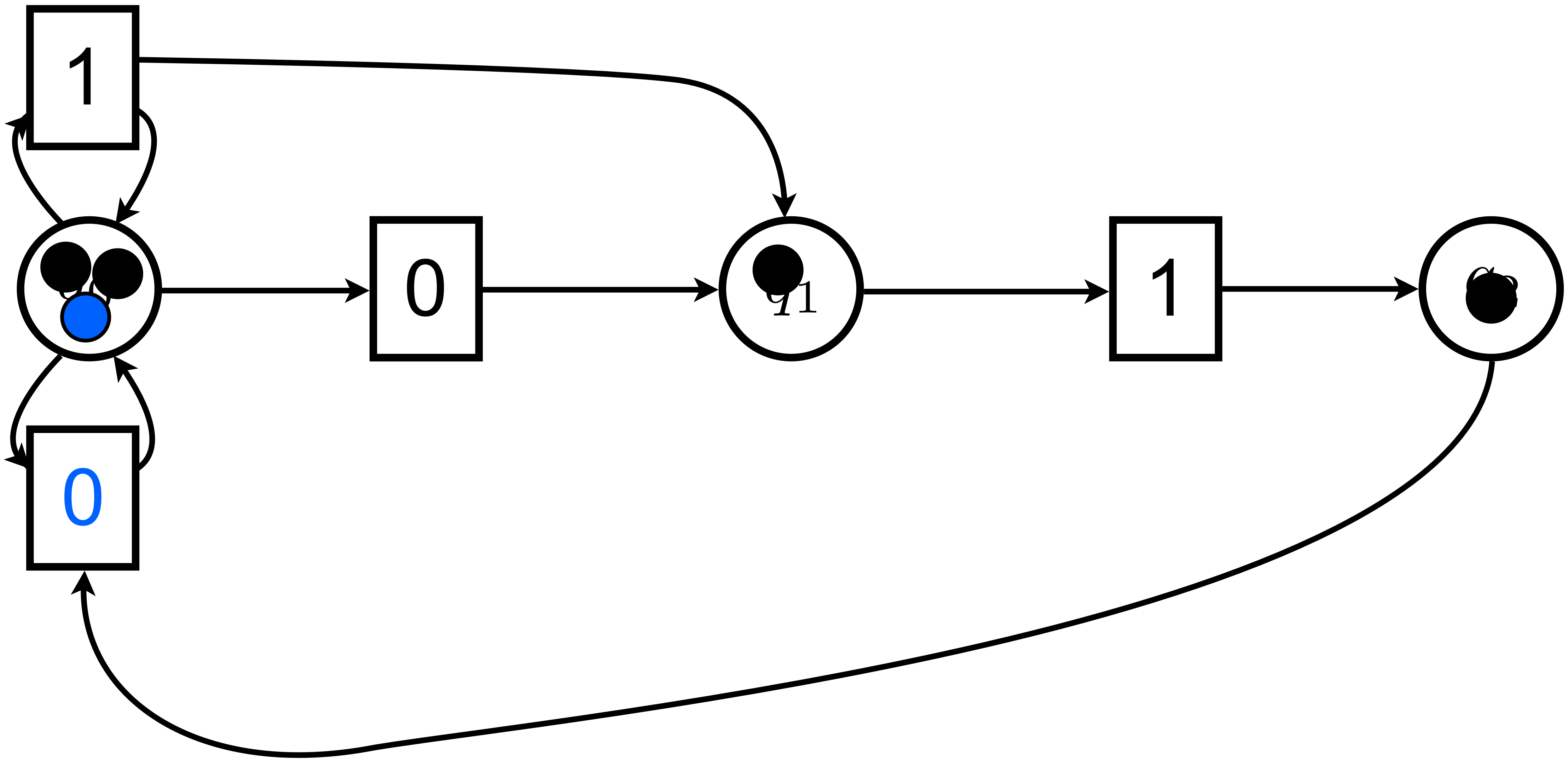
Example: token game



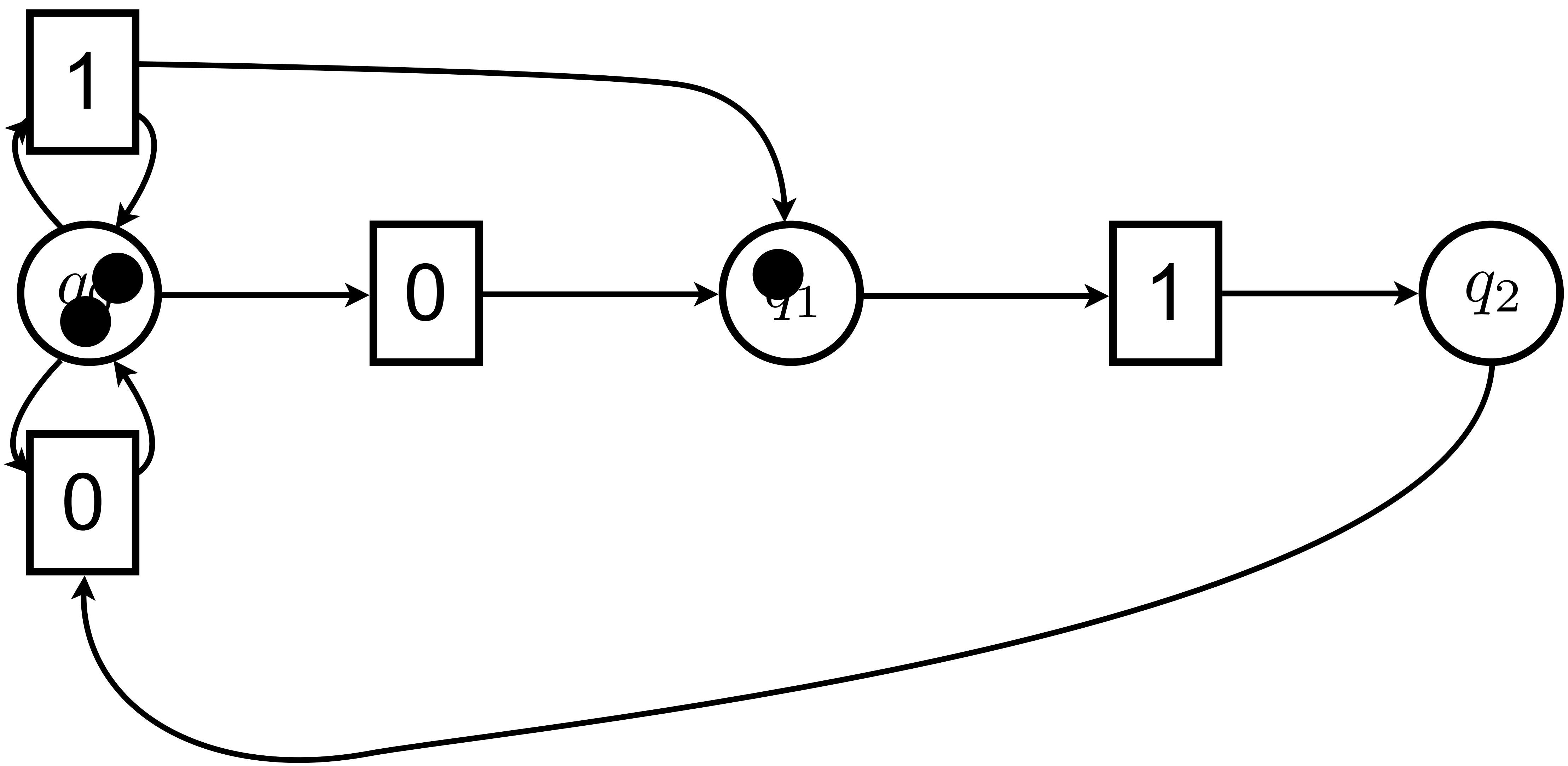
Example: token game



Example: token game



Example: token game



Example: Coin Handling