NATURAL AND ARTIFICIAL VISION MODULE

387AA – ROBOTICS [WIF-LM]

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Scuola Superiore Sant’Anna

Overall computer vision process and scope of the lessons

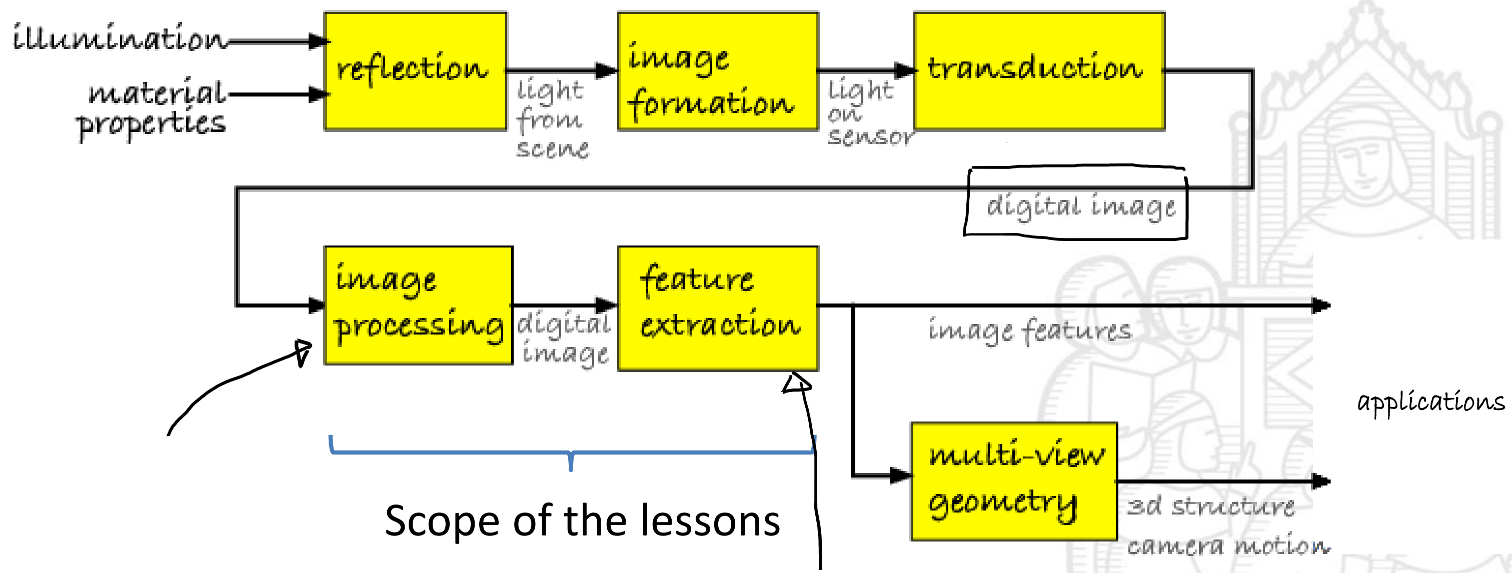
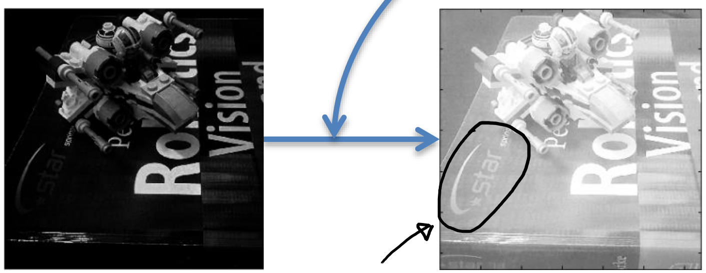
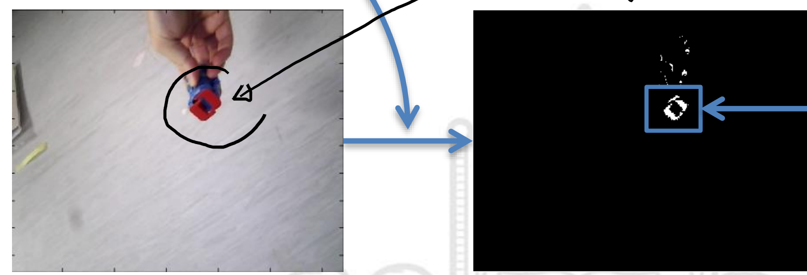


Image processing

Transform one or more input images into an output image.

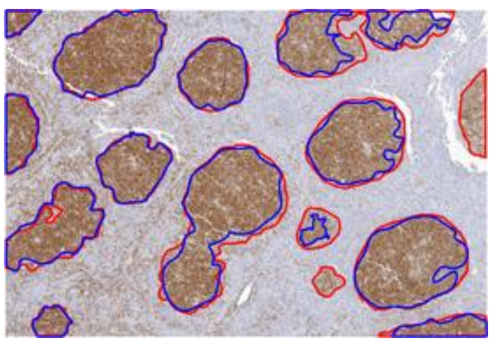
To enhance the image





Human interpretation

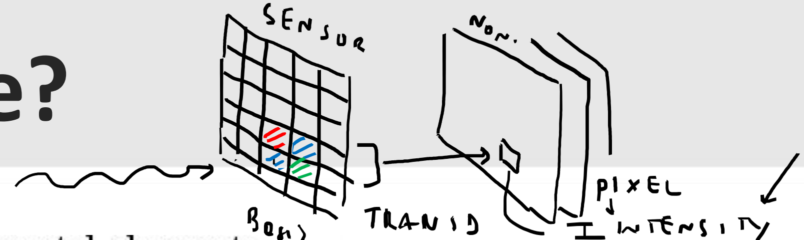
Features extraction



Computer aided diagnosis
Metin Gurcan, Ph.D – Ohio State

Objects recogniton
HERB robot butler – Carnegie Mellon

What's a digital image?



fundamental element:
the pixel

I[600,516]=213

horizontal resolution

image depth:
8-bit = [0, 255]

Digital images are **mosaics** made of **pixels**

Image resolution is the number of pieces (pixel) used to build the mosaic (image)

WHITE → ALL LIGHT

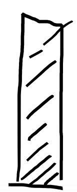


Image depth is the number of colours (levels) of mosaic pieces

← BLACK → NO LIGHT

$Im(u,v) = 200$

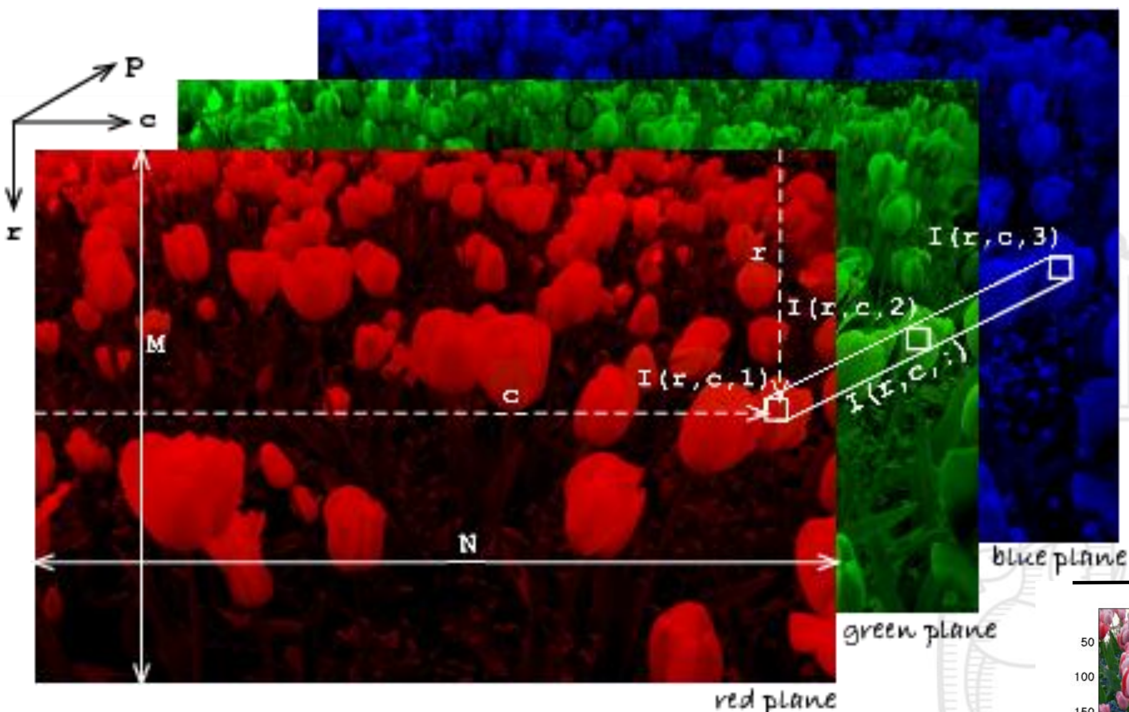
Colour images

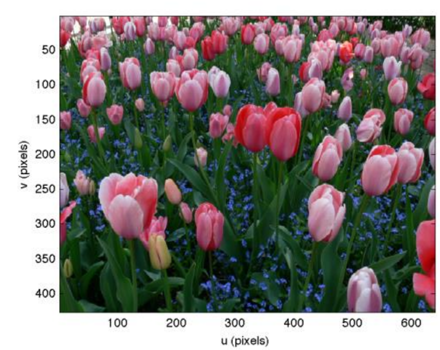
MONOCHROMATIC → (800, 600, 1)

COLOR → (800, 600, 3)

Colour images have three channels: the most common triplet is the R-G-B

RED GREEN BLUE





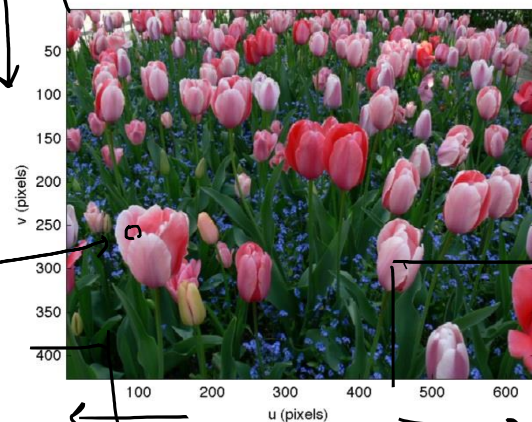
There are other very useful common space:
HSV, XYZ, CIE, YUY, …

Colour images

(0,0)

u

v



460

640x~~854~~x3 uint8

640x460x1

I(450,300) = [250,220,235]

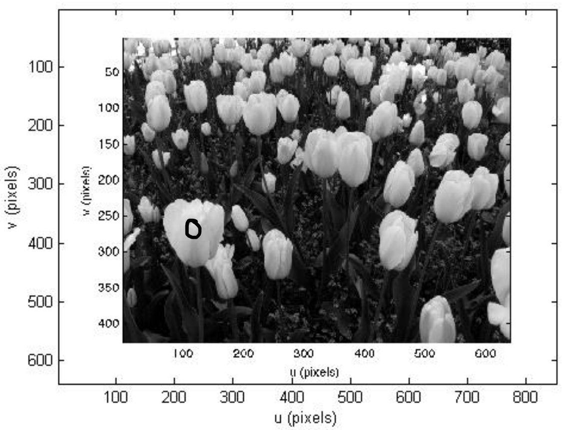
MONOCHR.

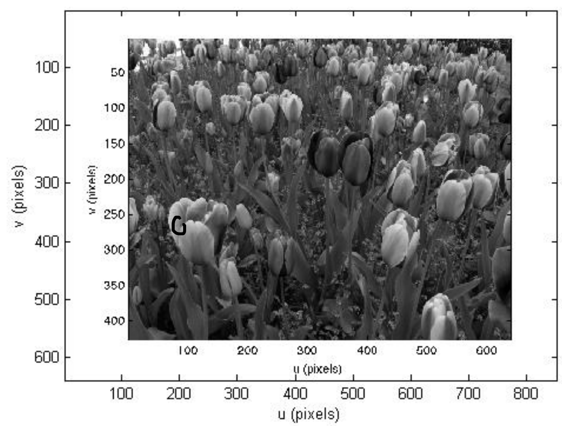
I(50,400) = [10,5,10]

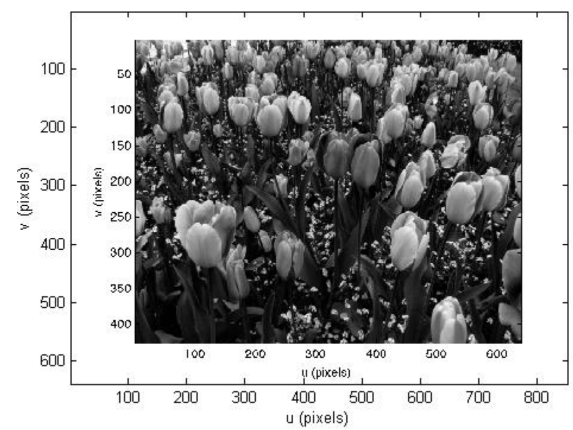
Red channel

Green channel

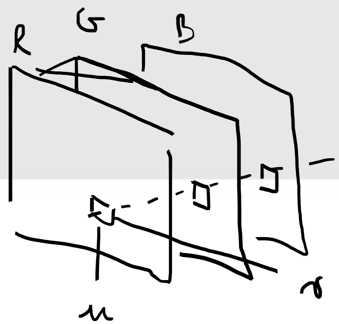
Blue channel







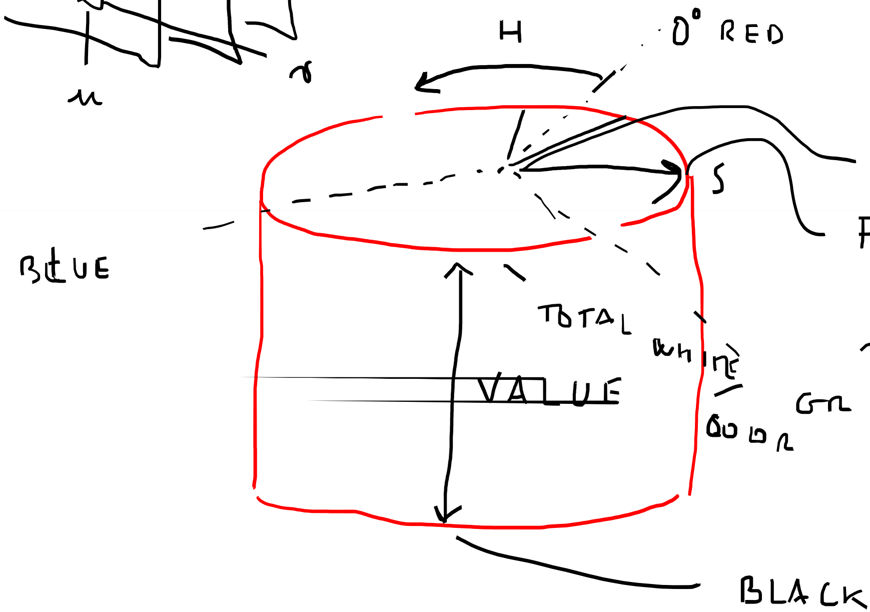
640x854x1 uint8



$I(u,v) = [R, G, B] \longrightarrow$

COLOR SPACE

HUE SATURATION VALUE (HSV)



Cylindrical colorspace

$$I(u,v) = [\underset{0-360^\circ}{H}, \underset{0-1}{S}, \underset{0-1}{V}]$$

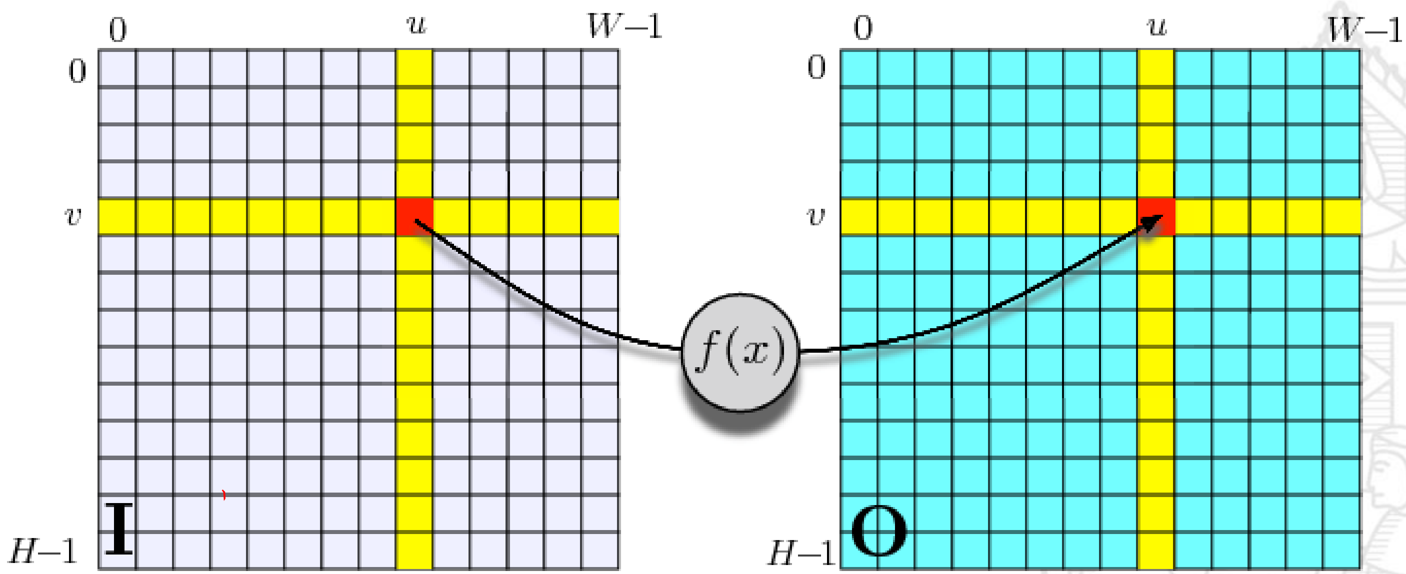
Monadic operations (pixel operators)

$O[u,v] = f(I[u,v]) = f(220) = \sqrt{x} = \sqrt{220}$

$I(u,v) = 220$

$f(x) = \sqrt{x}$

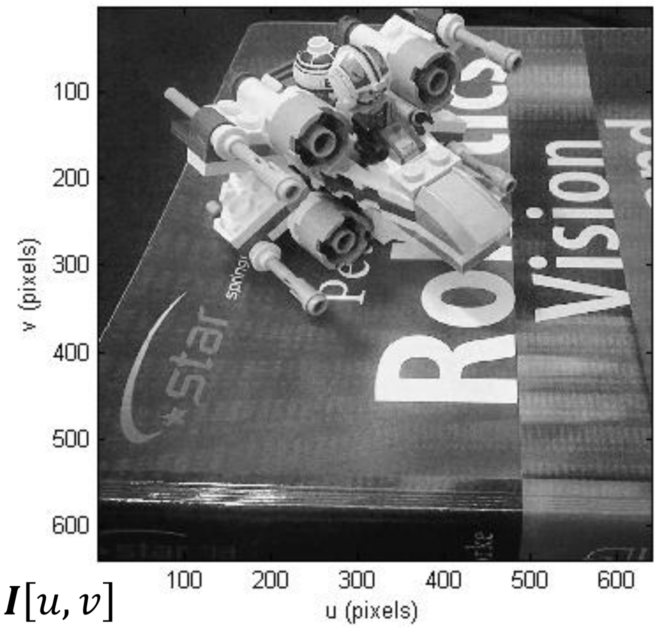
$$\boldsymbol{O}[u,v] = f(\boldsymbol{I}[u,v]), \qquad \forall (u,v) \in \boldsymbol{I}$$



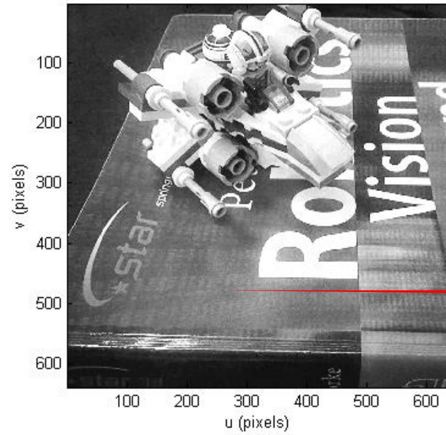
input image

output image

Lightening and darkening



$\boldsymbol{I}[u,v]$

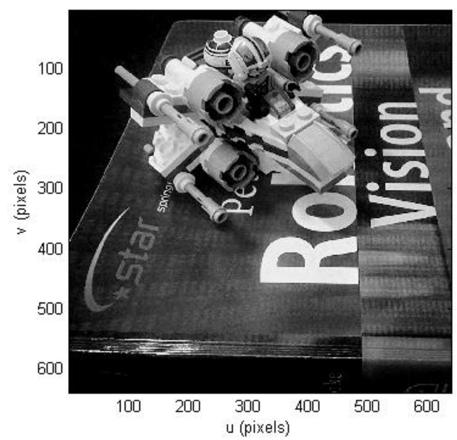


$$\boldsymbol{O}[u,v] = \boldsymbol{I}[u,v] + 50$$

255 - WHITE

8-bit

BLACK

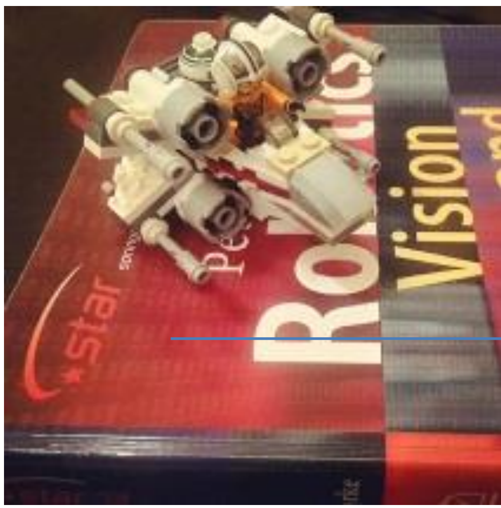


$$\boldsymbol{O}[u,v] = \boldsymbol{I}[u,v] - 50$$

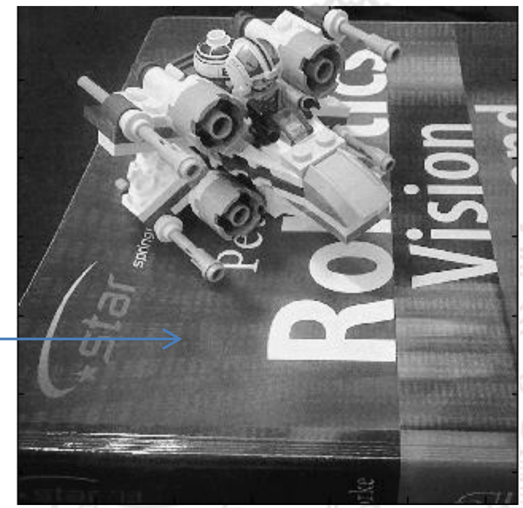
Monadic operations change the distribution of grey levels on images

Simple monadic operation (more channel):

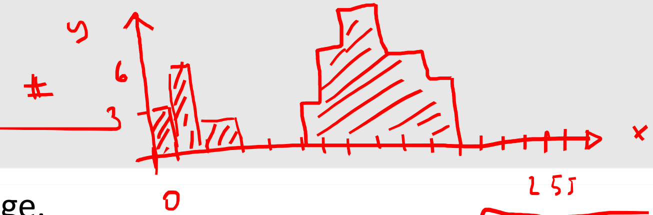
Gray-scale conversion with International Telecommunication Unit (ITU) recommendation 709



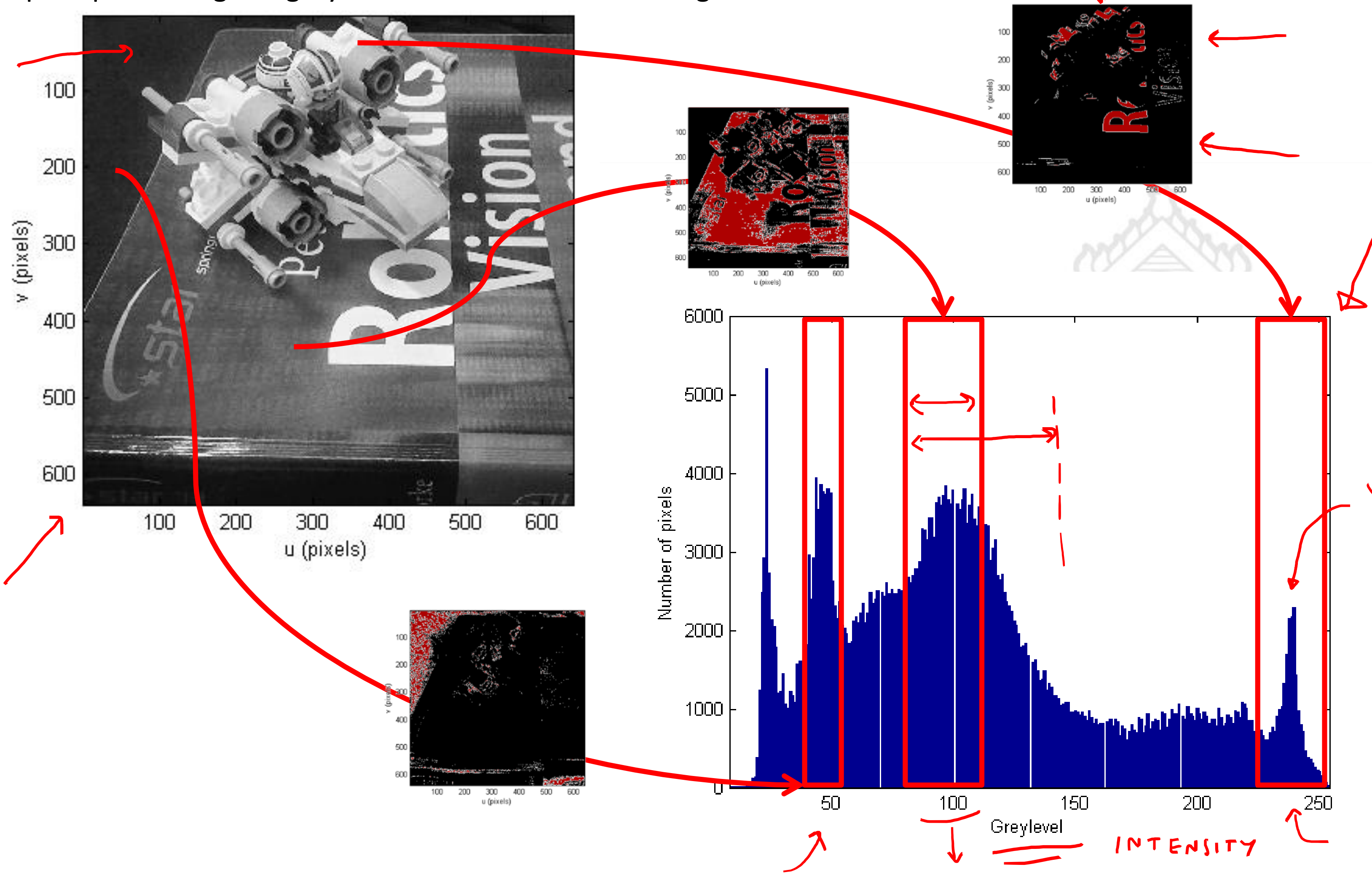
$Y=0{,}212R+0{,}7152G+0{,}0722B$



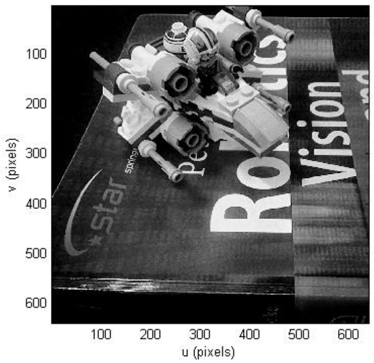
Histogram

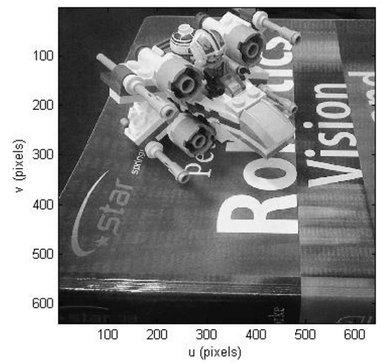


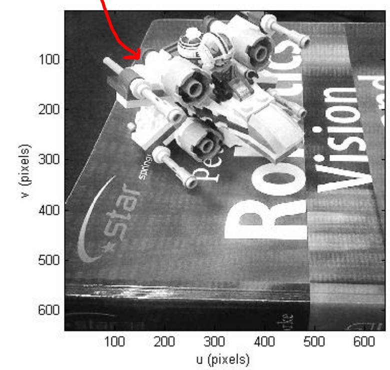
Is a graph representing the grey level occurrences of an image.



Histograms and monadic operations

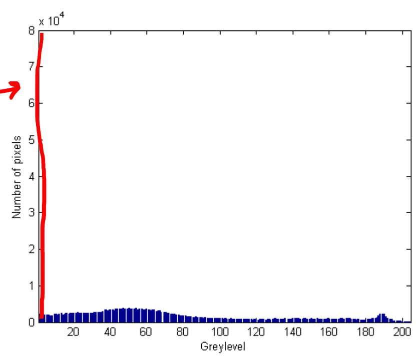






$\boldsymbol{O}[u,v] = \underbrace{\boldsymbol{I}[u,v] - 50}$ $\leftarrow$ $\underline{\boldsymbol{I}[u,v]}$

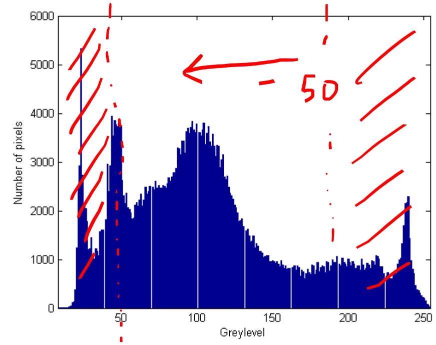
$\boldsymbol{O}[u,v] = \boldsymbol{I}[u,v] \underline{+ 50}$



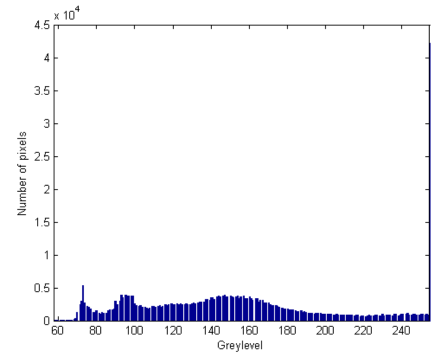
0

205 →

- 50



0 - 50 → 0



255

Common operations

$$s(r) = c \cdot r^{\gamma}$$

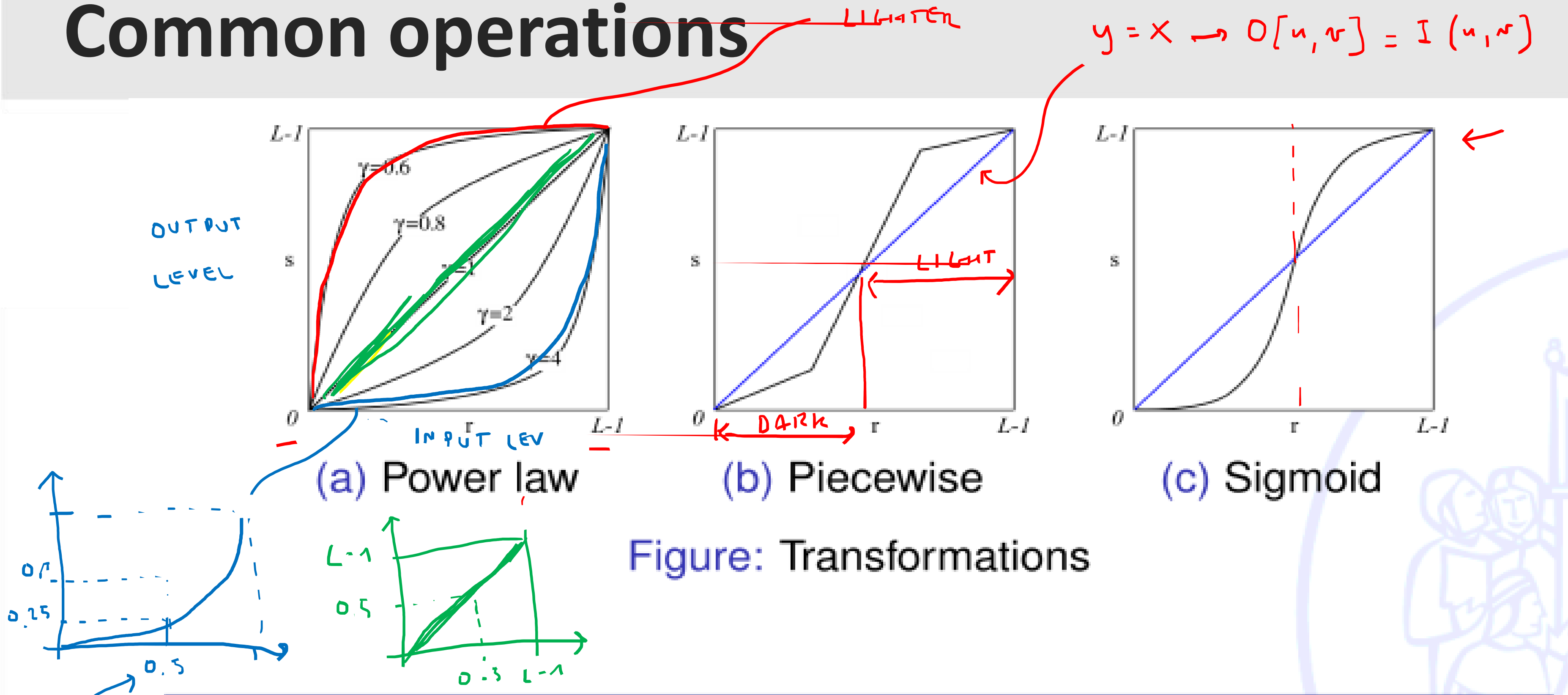
$$s(r) = \begin{cases} c_1 \cdot r & 0 \leq r < r_{min} \\ c_2 \cdot r & r_{min} \leq r < r_{max} \\ c_3 \cdot r & r_{max} \leq r < L - 1 \end{cases}$$

$$s(r) = \frac{c_1}{1 + e^{-r}}$$

- power law
- piecewise
- sigmoid

Each function requires parameters definition

Common operations

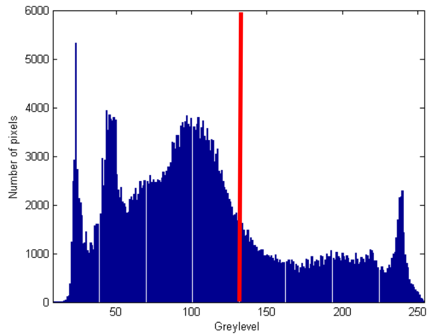


(a) Power law (b) Piecewise (c) Sigmoid

Figure: Transformations

Power law	lighten or darken
Piecewise	flexible
Sigmoid	enhance the contrast

Contrast enhancement

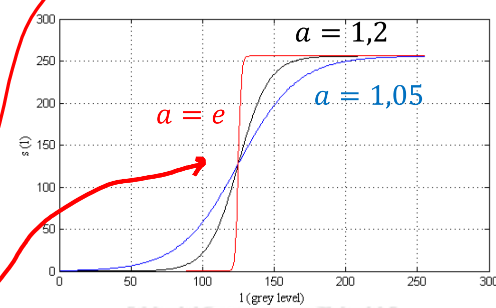


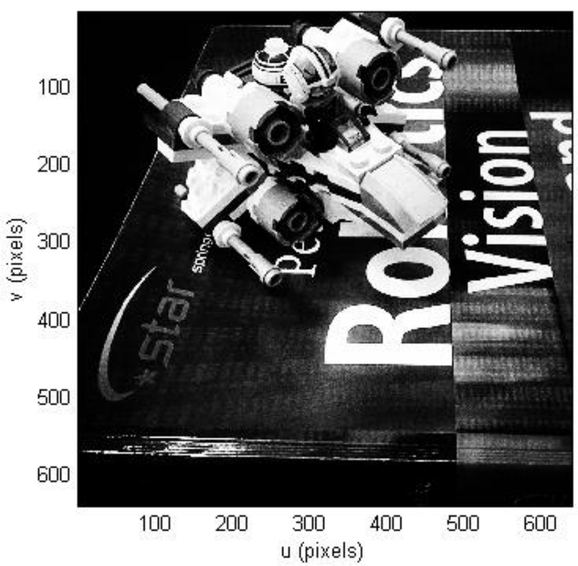
Sigmoid function

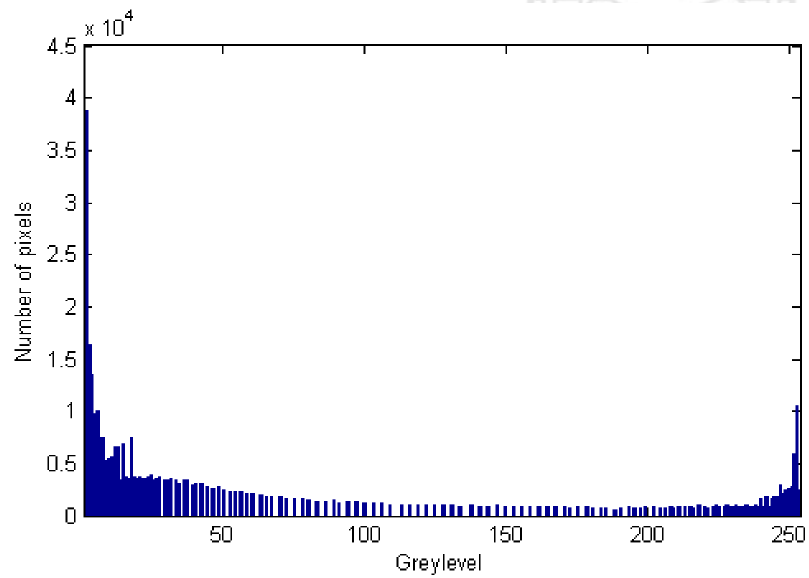
$$s(l) = \frac{256}{1 + a^{-(l-125)}}$$

ST

CUT







Pay attention

$$\begin{aligned} L &= 21 \\ c &= 1 \\ \gamma &= 2 \\ s(r) &= c \cdot r^{\gamma} \end{aligned}$$

$$\begin{aligned} s(1) &= 1 \\ s(2) &= 2^2 = 4 \\ s(3) &= 3^2 = 9 \\ s(4) &= 4^2 = 16 \\ s(5) &= 5^2 = 25 \\ \ldots &= \ldots \\ s(20) &= 20^2 = 400 \end{aligned}$$

$I(u,v) + 50 \rightarrow 300 \overset{?}{\in} [0, 255]$

$300 \rightarrow 255$

???

We have only 21 levels, but:

$$\underbrace{s(5) = 5^2 = 25}$$

Monadic operations

Code sample >

```
% lightening/darkening
xwing_light=xwing_grey+50;
idisp(xwing_light);
xwing_dark=xwing_grey-50;
idisp(xwing_dark);
% select areas by levels
level48 = (xwing_grey>=40) & (xwing_grey<=50) ;
idisp(level48);
level225 = (xwing_grey>=225) &
(xwing_grey<=255) ;
idisp(level225);
% contrast enanch
xwing_contrast=zeros(r,c);
    for i=1:r
       for j=1:c
           xwing_contrast(i,j)=256./(1+1.05.^-(
double(xwing_grey(i,j))-150));    % Sigmoid
       end
    end
 idisp(xwing_contrast)
```

Pay attention

We need to remap the output between $[0, L-1]$:

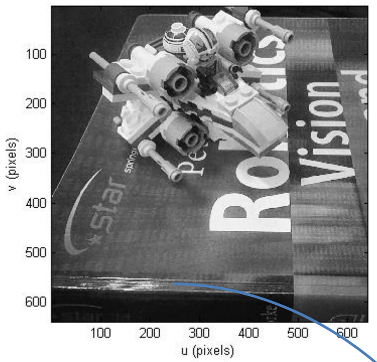
$$\frac{s'}{s} = \frac{20}{400}$$

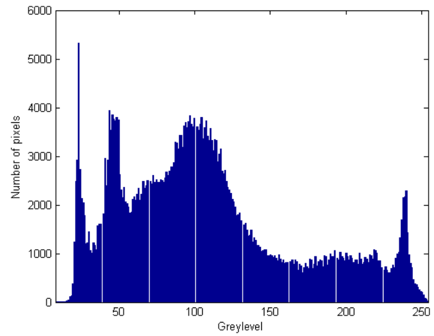
$$\frac{20}{400} = \frac{L-1}{(L-1)^{\gamma}} = (L-1)^{1-\gamma}$$

$$s' = (L-1)^{1-\gamma} s = c \cdot s = c \cdot r^{\gamma}$$

Thus c is related to L and γ.

Histogram equalization





$$c(l) = \frac{1}{N}\sum_{i=0}^{l} h(l) = c(l-1) + \frac{h(l)}{N}$$

c(l)	Cumulative distribution
h(l)	histogram
l	Grey level

$f(l)=c(l)$

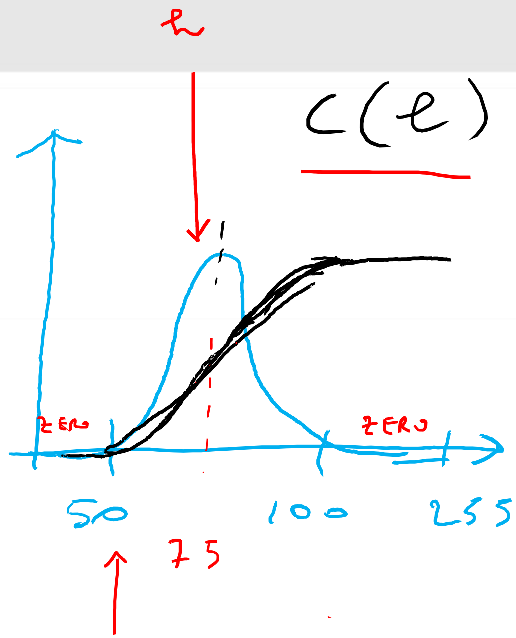
Monadic operation

$$c(l) = \frac{1}{N}\sum_{i=0}^{l} h(l) = c(l-1) + \frac{h(l)}{N}$$

N: total number of pixel

$$\boldsymbol{O}[u,v] = c(\boldsymbol{I}[u,v]), \forall (u,v) \in \boldsymbol{I}$$

N: total pixel



$$\rightarrow c(l) = \frac{1}{N}\sum_{i=0}^{l} h(i) = c(l-1) + \frac{h(l)}{N}$$

1°) $l=0$

$$c(0) = \frac{1}{N}\,\overbrace{h(0)}^{0} = 0$$

$c(l)=0 \quad 0<l<49$

$l=49$

$h(49) = 0$

$l=50$

$$c(50) = \frac{1}{N}\,h(50) = \frac{1}{N} \qquad c(50) = \frac{1}{N}$$

$l=51$

$$c(51) = \frac{1}{N}\big(\overbrace{h(50)}^{c(50)} + \overbrace{h(51)}^{3}\big) = $$

$$= c(l-1) + \frac{h(l)}{N}$$

$$\frac{1}{N} + \frac{3}{N} = \frac{4}{N}$$

$$c(l) = c(l-1) + \frac{h(l)}{N}$$

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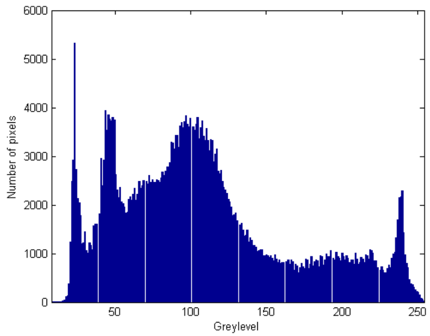
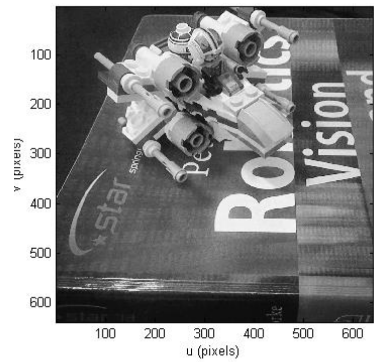
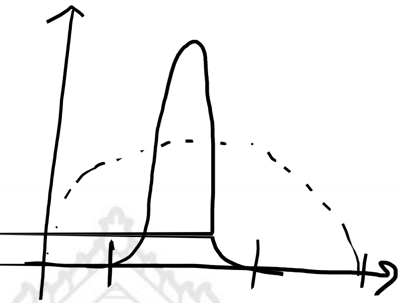
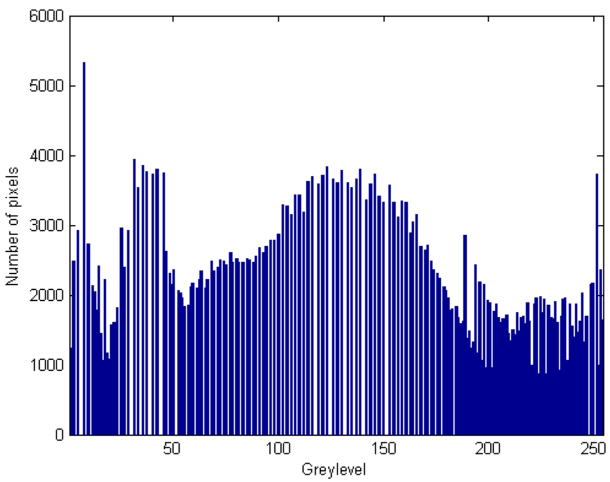
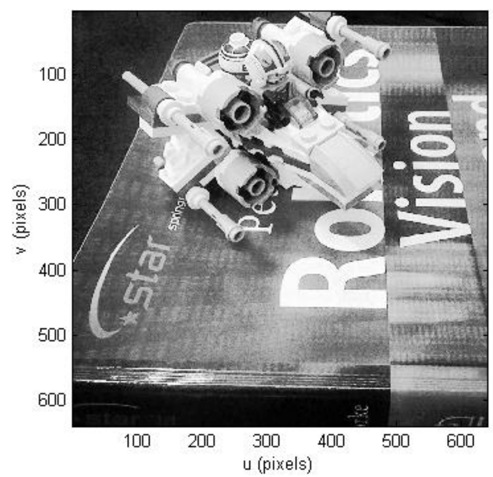
Histogram equalization





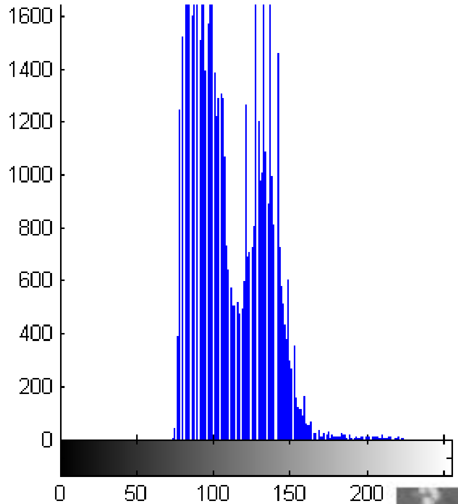






After equalization

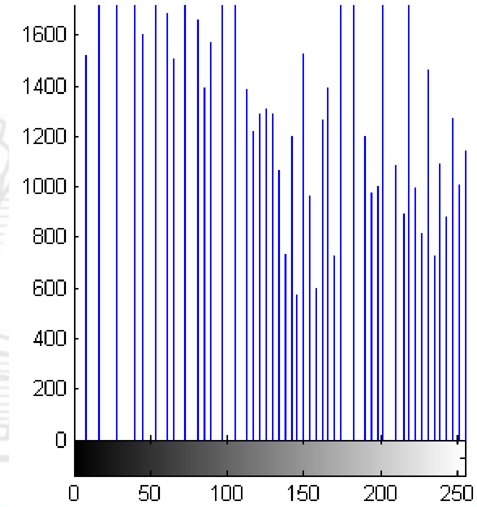
Histogram equalization



75 175

before

after

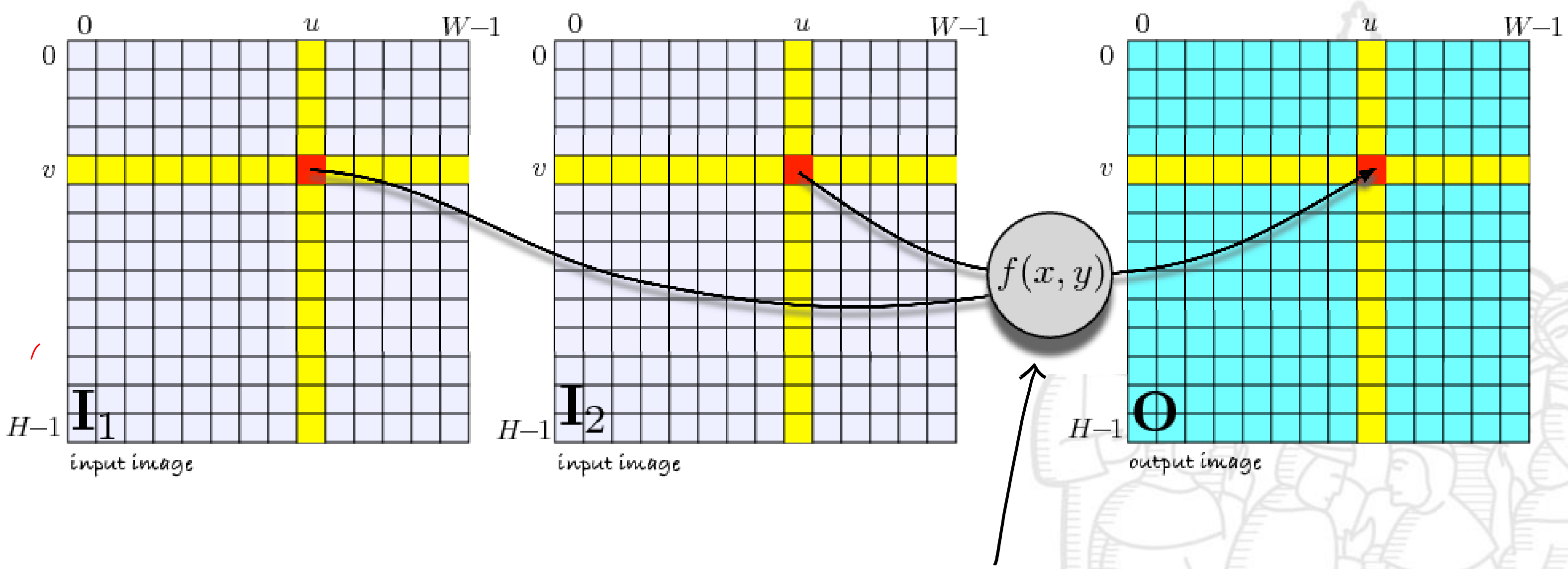


Code sample >

```
 %hist equalization
[n,v]=ihist(xwing_grey);
plot(v,n)
cd=zeros(length(v),1);
cd(1)=v(1)/(r*c);
    for l=2:length(v)
       cd(l)=cd(l-1)+1/(r*c)*n(l); % cumulative distribution
    end
xwing_equalized=zeros(r,c);
    for i=1:r
       for j=1:c
           xwing_equalized(i,j)=255*cd(xwing_grey(i,j)+1);    % Equalization
       end
    end
 idisp(xwing_equalized)
```

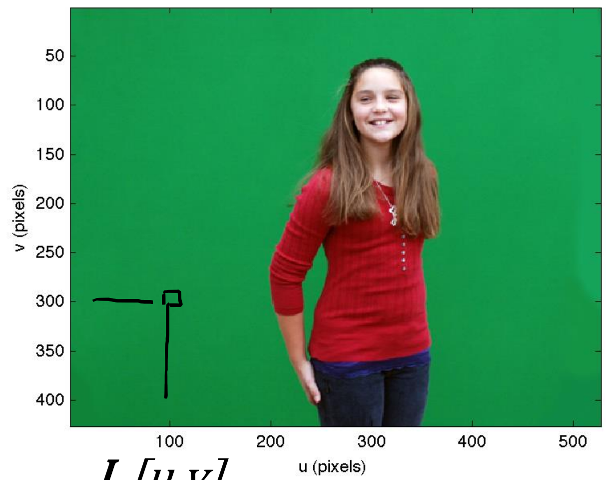
Diadic operations

$$\boldsymbol{O}[u,v] = f(\boldsymbol{I}_1[u,v], \boldsymbol{I}_2[u,v]), \qquad \forall (u,v) \in \boldsymbol{I}_1$$



Green screen

FOREGROUND IMAGE



$I_1[u,v]$

BACKGROUND IMAGE

$I_2[u,v]$

RGB → [·· , G > t , –]

HSV → [$t_1 < H < t_2$, ·· , ··]

If $\boldsymbol{I}_1[u,v]$ isGreen

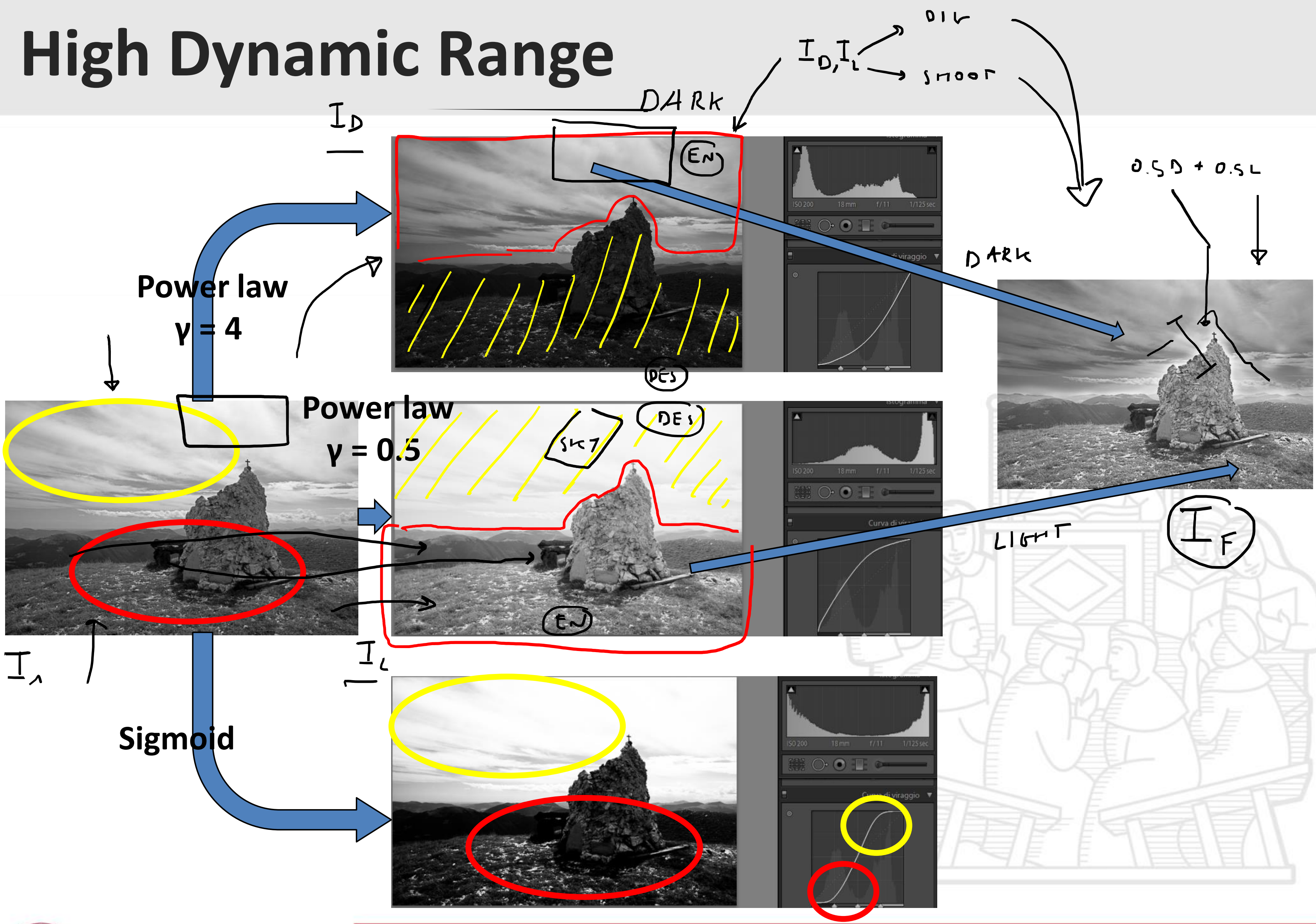
$$\boldsymbol{O}[u,v] = \boldsymbol{I}_2[u,v]$$

Else

$$\boldsymbol{O}[u,v] = \boldsymbol{I}_1[u,v]$$

$O[u,v]$

High Dynamic Range



Background subtraction

Another important diadic operation is the background subtraction to find novel elements (foreground) of a scene.

~~u, v ∈ R → depth~~

$$\boldsymbol{O}[u,v] = \underbrace{\boldsymbol{I}_1[u,v]}_{\text{CURRENT IMAGE}} - \underbrace{\boldsymbol{I}_2[u,v]}_{\text{REF. IMAGE}} = \boldsymbol{I}_1[u,v] - \boldsymbol{B}[u,v]$$

background

How we estimate the background $\boldsymbol{B}[u,v]$?

We can take a shoot when we know that only background is visible

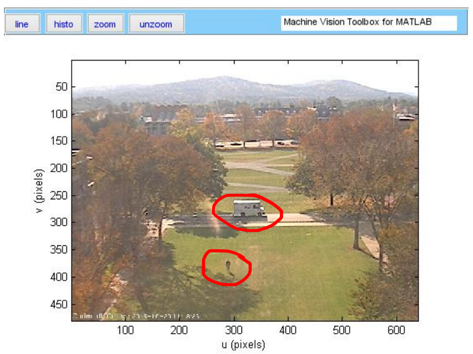
'http://wc2.dartmouth.edu', 05:19 p.m., Rome time

Background subtraction

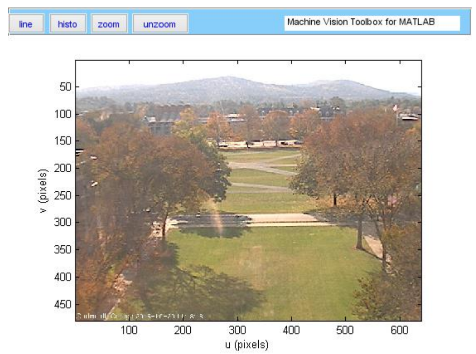
'http://wc2.dartmouth.edu', 05:19 p.m., Rome time

negative values which
scale within the available
range

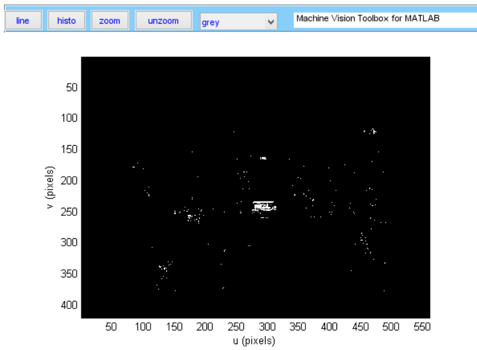
$$\boldsymbol{I}_1[u,v] - \boldsymbol{B}[u,v] = \boldsymbol{O}[u,v]$$



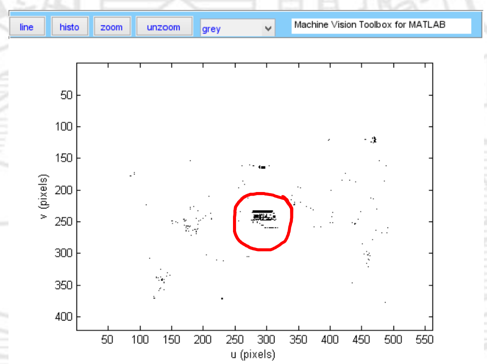
−



=



foreground

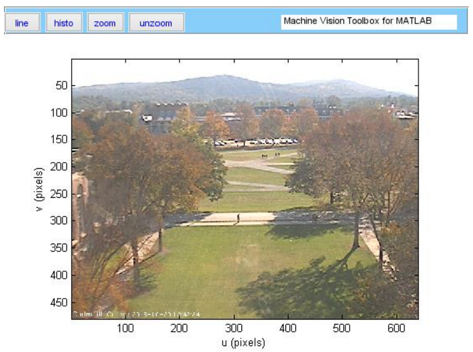


background

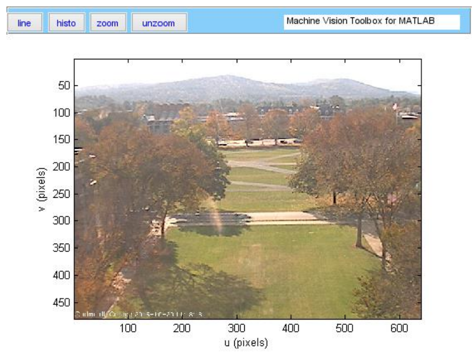
Background subtraction

'http://wc2.dartmouth.edu', 07:48 p.m., Rome time

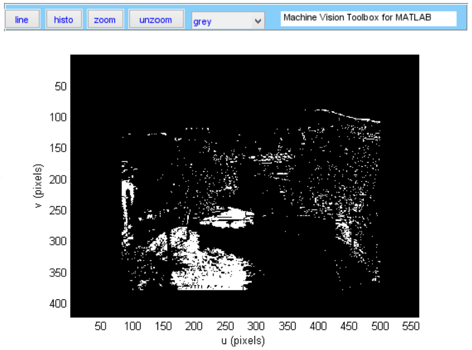
$$\boldsymbol{I}_1[u,v] - \boldsymbol{B}[u,v] = \boldsymbol{O}[u,v]$$



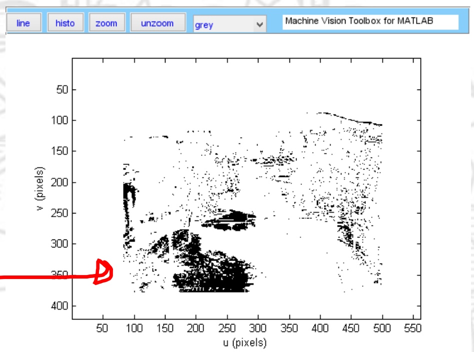
$-$



$=$



foreground



background

What went wrong?

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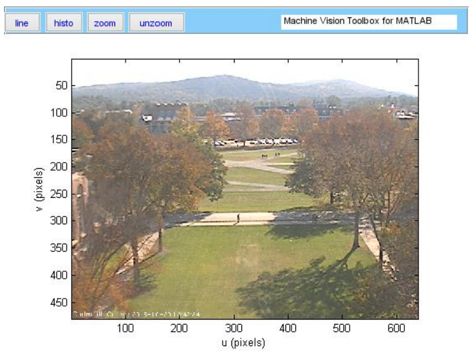
Background subtraction

'http://wc2.dartmouth.edu‘, 10:55 p.m., Rome time

$$\boldsymbol{I}_1[u,v] - \boldsymbol{B}[u,v] = \boldsymbol{O}[u,v]$$



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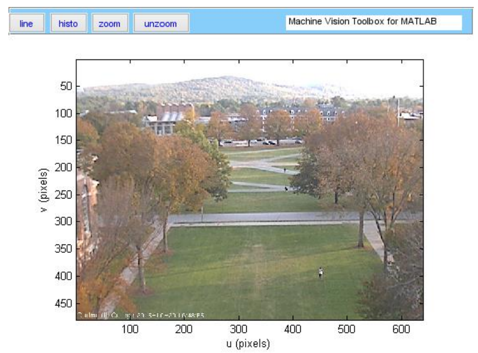

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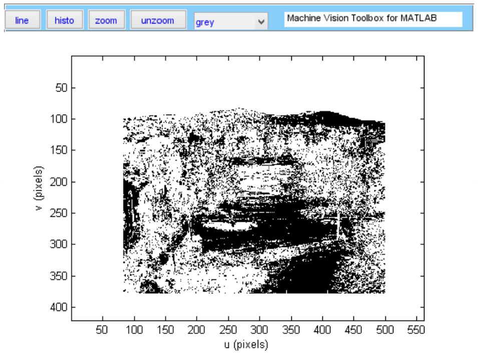
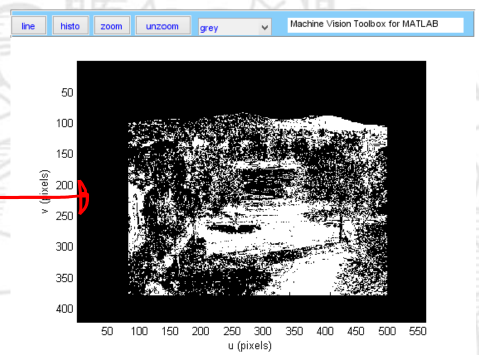


foreground



background

What went wrong?

Background estimation

We require a progressive adaptation to small, persistent changes in the background.

Rather than take a static image as background, we estimated it as follow:

$$\boldsymbol{B}\langle k+1\rangle = \boldsymbol{B}\langle k\rangle + c(\boldsymbol{I}\langle k\rangle - \boldsymbol{B}\langle k\rangle)$$

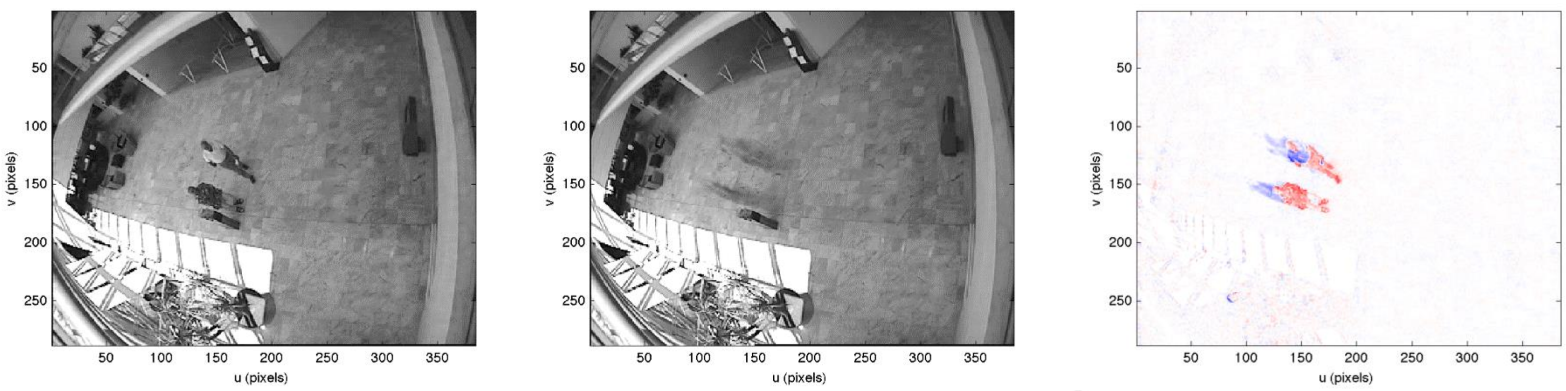
$$c(x) = \begin{cases} \sigma, & x > \sigma \\ x, & -\sigma \le x \le \sigma \\ -\sigma, & x < -\sigma \end{cases}$$

$I\langle k\rangle - B\langle k\rangle > \sigma$

$\| I\langle k\rangle - B\langle k\rangle \| < \sigma$

$I\langle k\rangle - B\langle k\rangle < -\sigma$

$B\langle k+1\rangle = B\langle k\rangle - \sigma$



Background subtraction

Code sample >

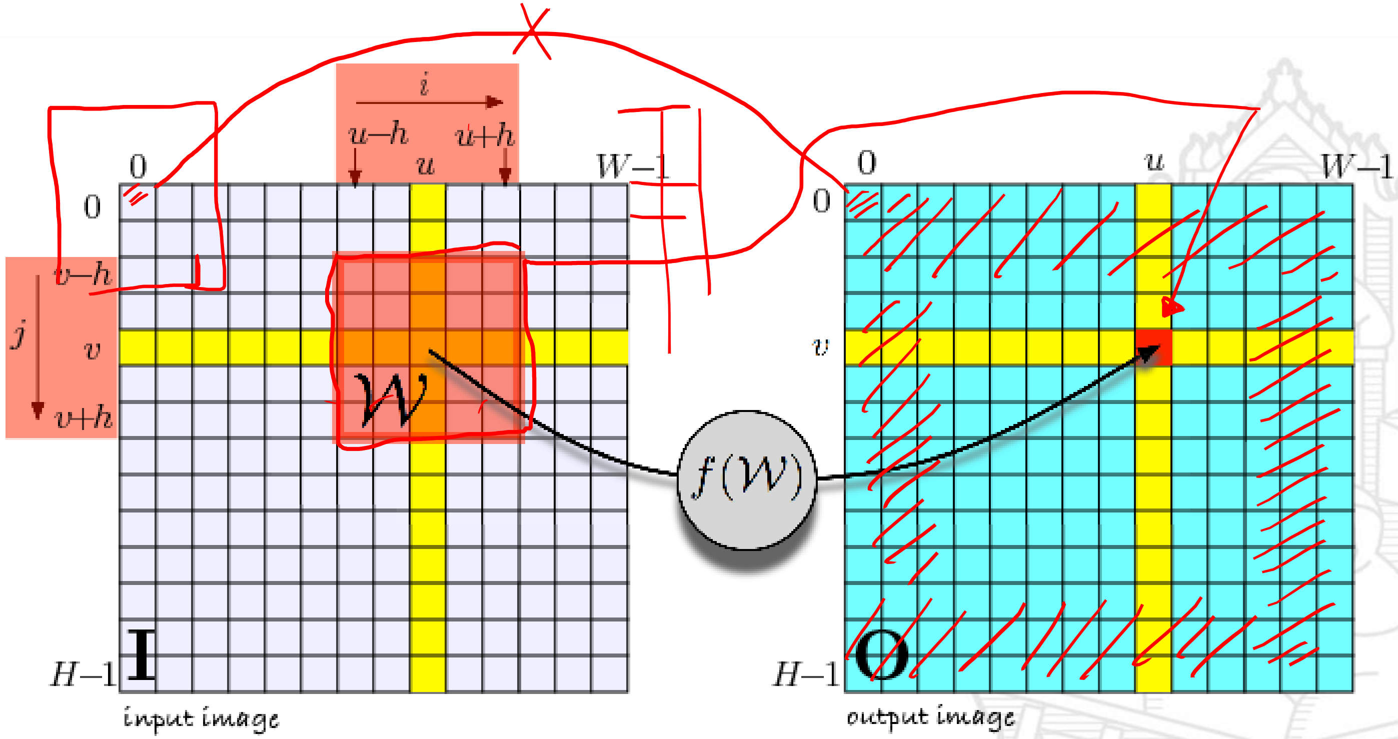
$\sigma = 0.01$ fps = 25 → small changes

$\sigma = 1$ fps = 25 → quick changes

```
% backgorund estimation
sigma=0.01;
vid = videoinput('winvideo', 1);
bg=getsnapshot(vid);
bg_small=idouble(imono(bg));
while 1
      img=getsnapshot(vid);
      img_small=idouble(imono(img));
      if isempty(img), break; end
      d=img_small-bg_small;
      d=max(min(d,sigma), -sigma);
      bg_small=bg_small+d;
      idisp(bg_small); drawnow
end
```

Spatial operation (local operators)

$$\boldsymbol{O}[u,v] = f(\boldsymbol{I}[u+i, v+j]), \qquad \forall (i,j) \in \boldsymbol{W}, \forall (u,v) \in \boldsymbol{I}$$



Smaller O $\frac{(W+1)}{2}$

Enlarge I ⟶ black, white, repetition, interpolation

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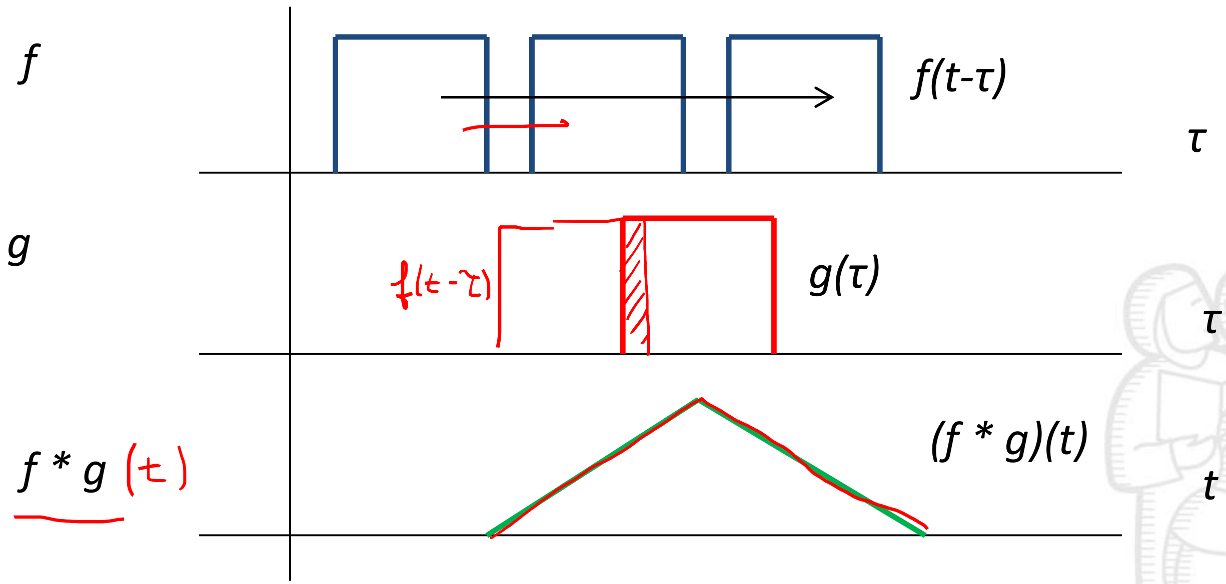
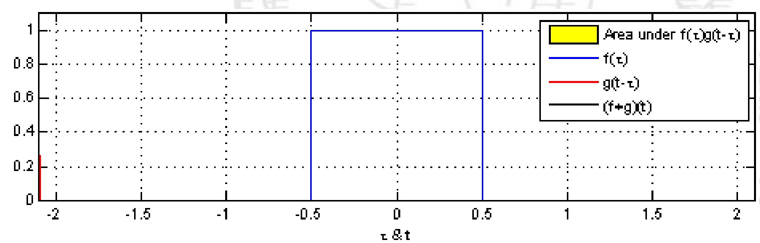
1D Convolution

One important local operator is the convolution:

$$(f * g)(t) := \int_{-\infty}^{\infty} f(t-\tau)g(\tau)d\tau$$





wikipedia

2D Convolution

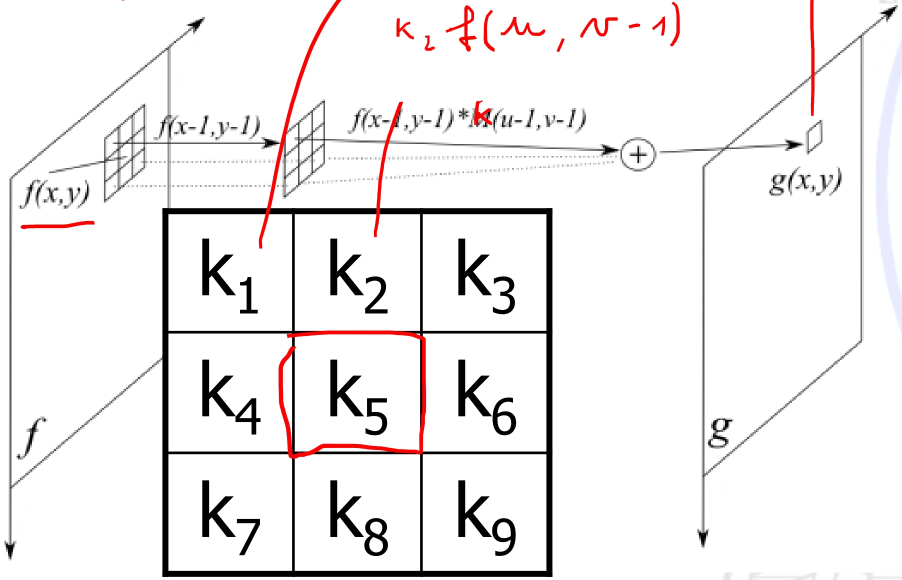
$$\boldsymbol{O}[u,v] = \sum_{i,j \in \boldsymbol{K}} \underbrace{\boldsymbol{K}[i,j]}_{3\times 3} \boldsymbol{I}[u-i, v-j], \qquad \forall (u,v) \in \boldsymbol{I}$$

$$\boldsymbol{O} = \boldsymbol{K} \otimes \boldsymbol{I}$$

$k_1 f(u-1, f(v-1))$

$g(x,y)$

$k_2 f(u, v-1)$



Convolution mask

2D Convolution

kernel

·1+	·1+	·1+
·1+	·1+	·1+
0 ·1+	1 ·1+	·1 2

Input image

0	1	2	0	12	5	0	1
5	2	6	0	0	1	1	1
5	0	0	4	5	6	1	0
12	25	0	24	56	8	2	3
1	2	6	0	0	1	5	2
1	2	0	2	1	2	1	0
12	0	12	25	3	5	0	1
1	1	1	35	57	5	3	1

Output image

	21						

Convolution

3×3

$\frac{3-1}{2} = 1$ border

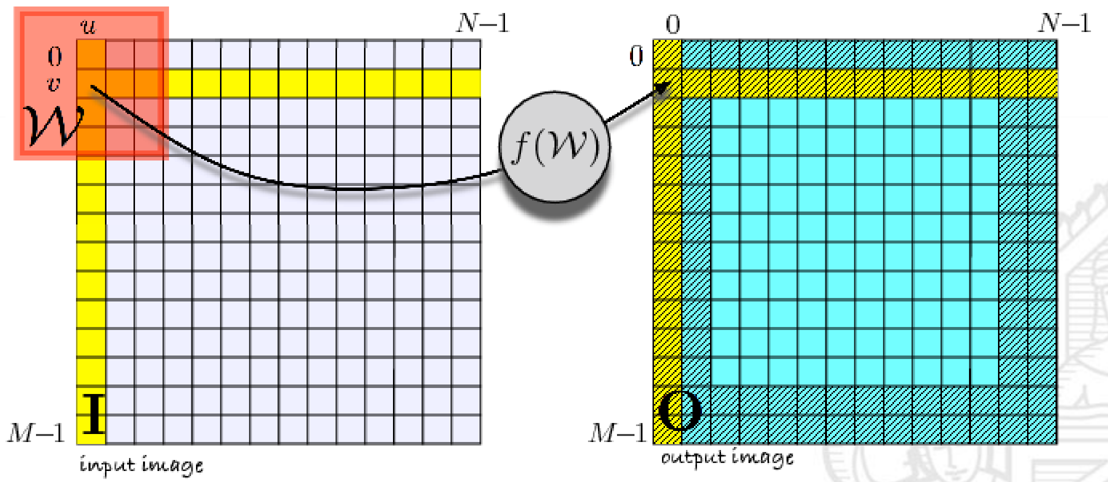
Input image

0·1+	1·1+	2·1+	0	12	5	0	1
5·1+	2·1+	6·1+	0	0	1	1	1
5·1+	0·1+	0·1	4	5	6	1	0
12	25	0	24	56	8	2	3
1	2	6	0	0	1	5	2
1	2	0	2	1	2	1	0
12	0	12	25	3	5	0	1
1	1	1	35	57	5	3	1

Output image

	21	15					

Boundary effect



ORIGINAL INPUT IMAGE

- Duplicate
- All black
- Reduce size
- …

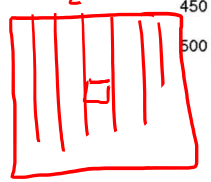
Smoothing

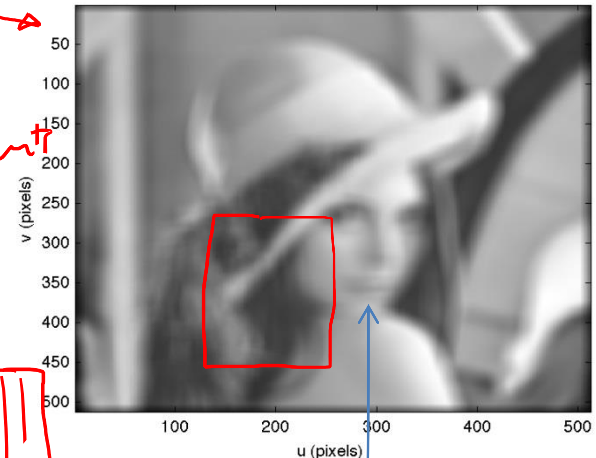
$$\begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$$

$$I[u-1, v-1]\cdot\frac{1}{9} + I[u, v-1]\cdot\frac{1}{9} + I[u+1, v-1]\cdot\frac{1}{9} + \dots = \frac{1}{9}\sum_{i,j=-1}^{1} I[u+i, v+j]$$

9 elements

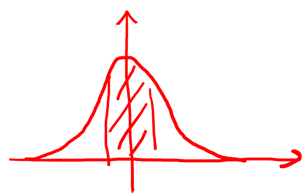
21



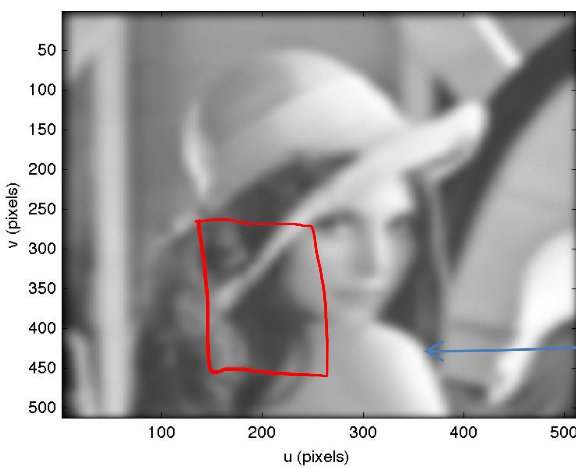


I

$$K = ones(21, 21)/21^2$$



$$O = K \otimes I$$



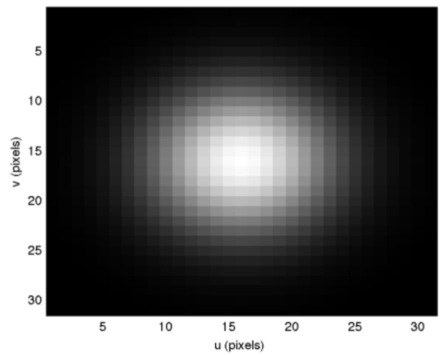
$$G[u, v] = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$

$$\begin{bmatrix} 0 & 2 & 0 \\ 2 & 5 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

Kernel examples

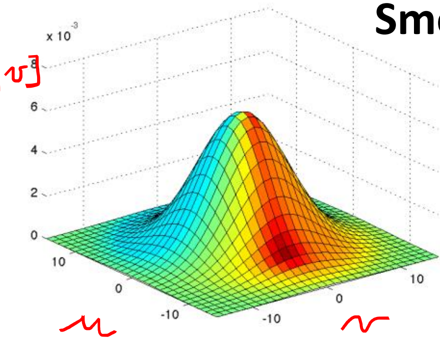
3D when Z-AXIS INTENSITY

2D



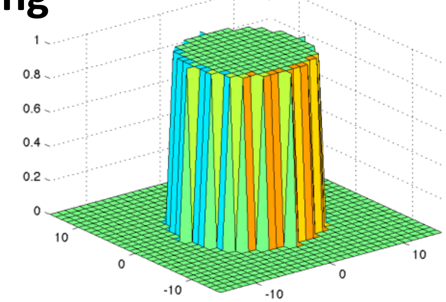
I[u,v]

Smoothing



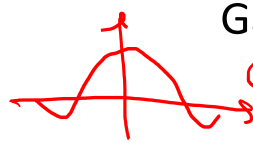
u v

Gaussian



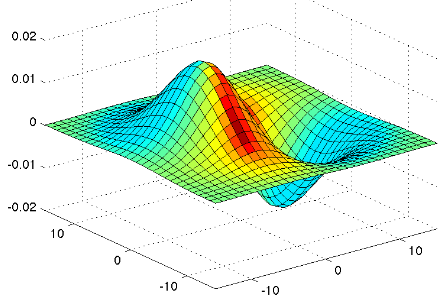
Top hat

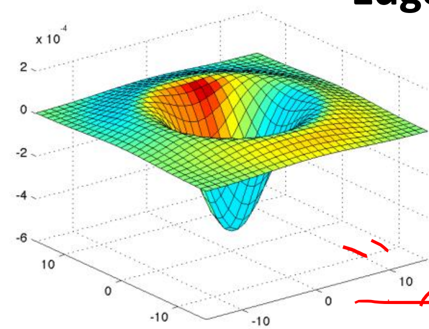
C-ON

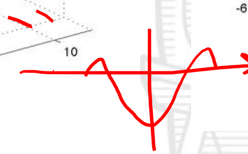


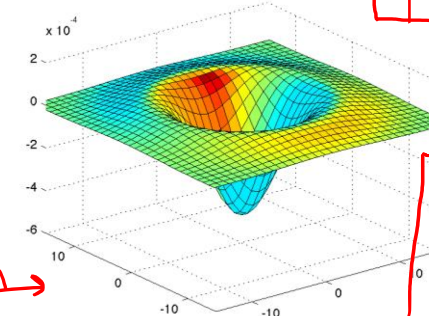
Gradient

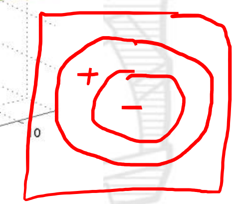
Edge detection











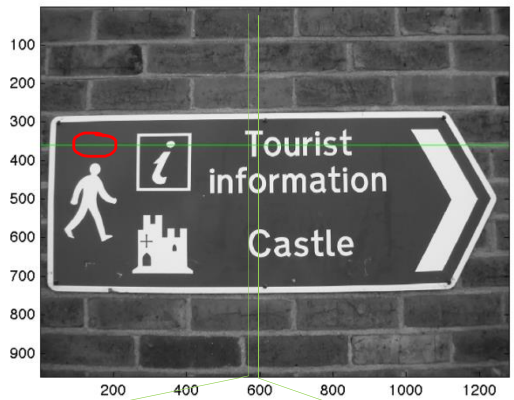
C-OFF

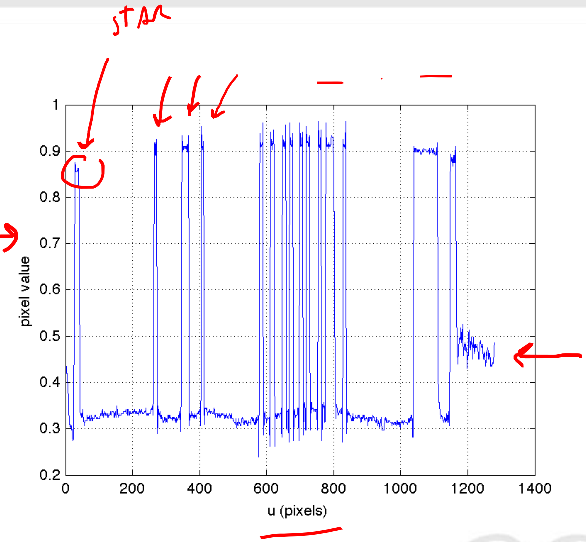
Derivative of Gaussian (DoG)

Laplacian of Gaussian (LoG)

Difference of Gaussian (DiffG)

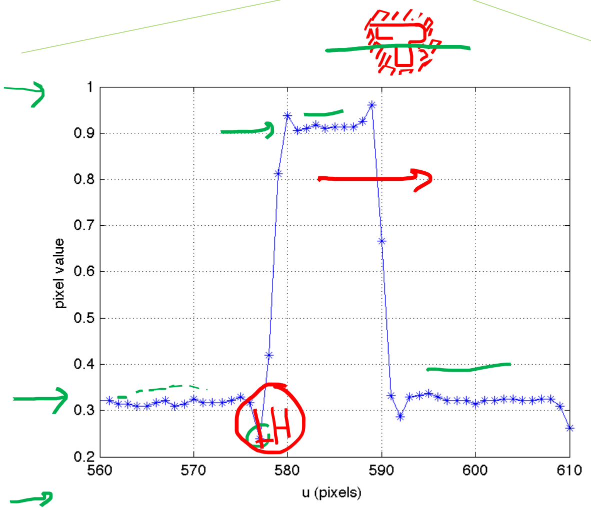
Edge detection





Horizontal profile of the image at v=360

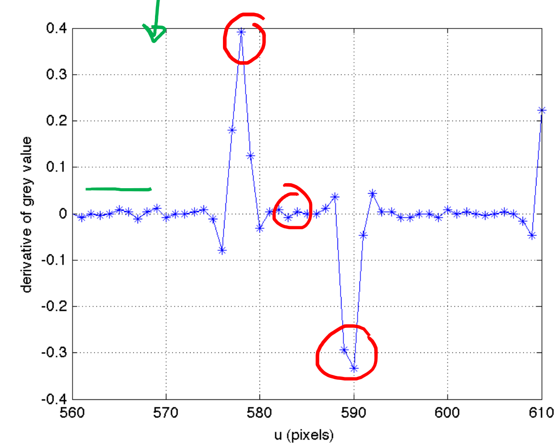
$$\frac{\partial I}{\partial u} = \frac{I(u+1) - I(u)}{\partial u}$$



$$p'[u] = p[u] - p[u-1]$$

$$p'[u] = \frac{1}{2}(p[u+1] - p[u-1])$$

$$\boldsymbol{K} = \left[-\frac{1}{2} \;\; 0 \;\; \frac{1}{2}\right]$$



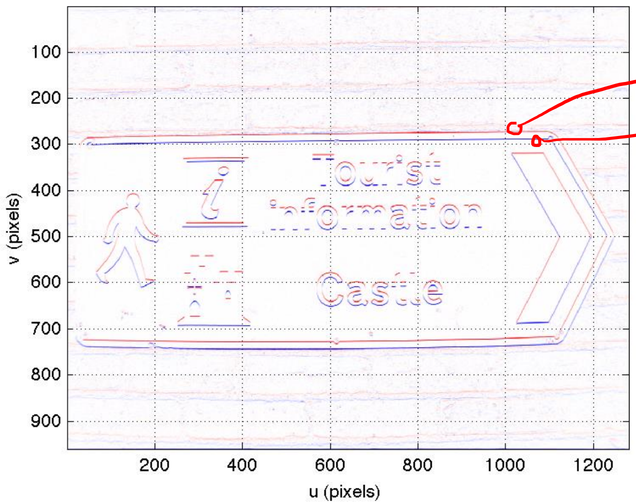
Gradient computation

$I[u,v] = \frac{\partial I}{\partial u}, \frac{\partial I}{\partial v}$

Common convolution kernel: Sobel, Prewitt, Roberts, …

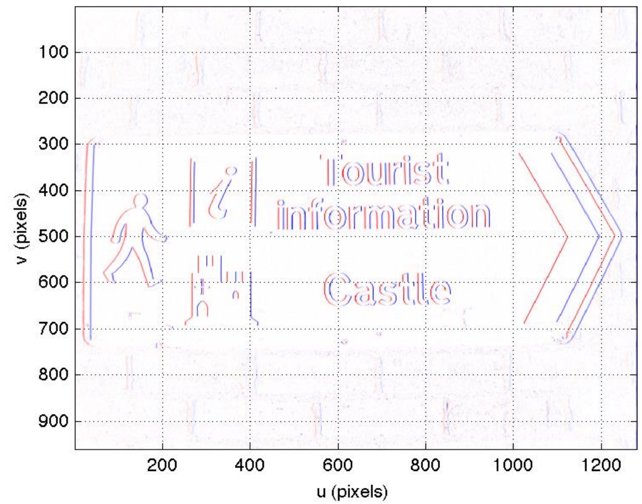
$\frac{\partial I}{\partial v}$

Sobel $\quad \boldsymbol{D}_v = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \qquad \boldsymbol{D}_u = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$



+4

-4



$\boldsymbol{I}_v = \boldsymbol{D}_v \otimes \boldsymbol{I} \qquad \boldsymbol{I}_u = \boldsymbol{D}_u \otimes \boldsymbol{I}$

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Direction and magnitude

$m = I_u + I_v$

I_u

$\mathrm{tg}\,\theta = \dfrac{I_v}{I_u} \rightarrow \theta = \mathrm{arctg}\left(\dfrac{I_v}{I_u}\right)$

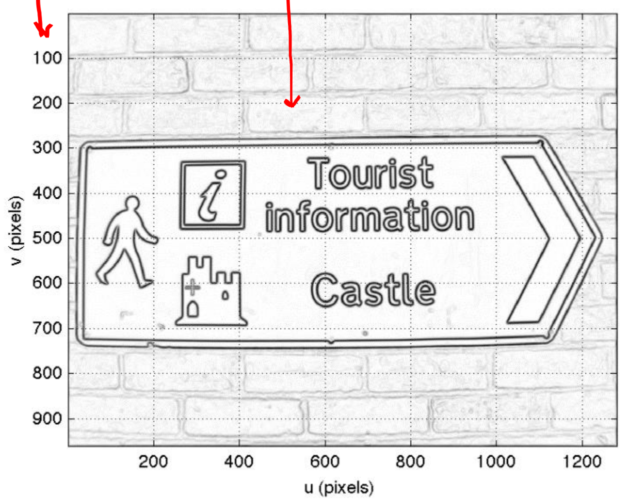
m

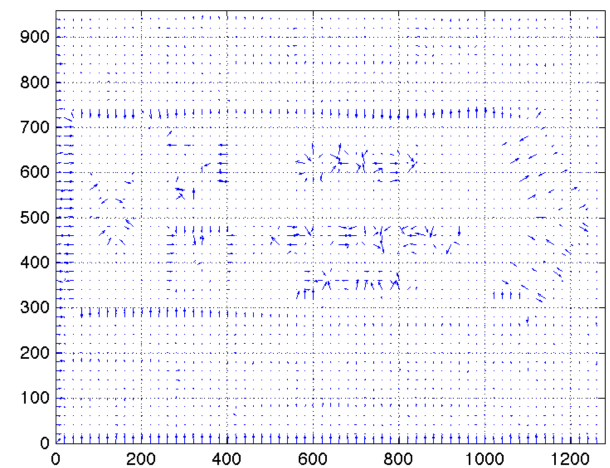
I_v

$\|m\| = \sqrt{I_u^2 + I_v^2}$

$$m = \sqrt{\boldsymbol{I}_v^2 + \boldsymbol{I}_u^2}$$

$$\theta = \mathrm{atan}(\boldsymbol{I}_v, \boldsymbol{I}_u)$$





Noise amplification

(w × w mult.) + w × w (sum.) (all the pixel)

Derivative amplifies high-frequency noise. So, firstly we can smooth the image, after that we can take the derivative:

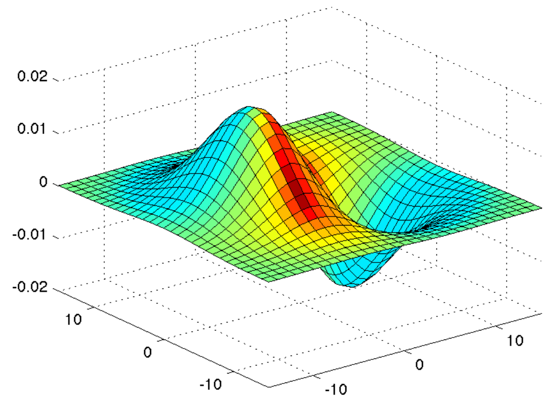
$$\boldsymbol{I}_u = \boldsymbol{D}_u \otimes (\boldsymbol{G} \otimes \boldsymbol{I})$$

Associative property:

$$\boldsymbol{I}_u = \underbrace{(\boldsymbol{D}_u \otimes \boldsymbol{G})} \otimes \boldsymbol{I}$$

Derivative of Gaussian (DoG)

$$\boldsymbol{G}_u = -\frac{u}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



Derivative of Gaussian (DoG)

<<DoG acts as a bandpass filter!>>

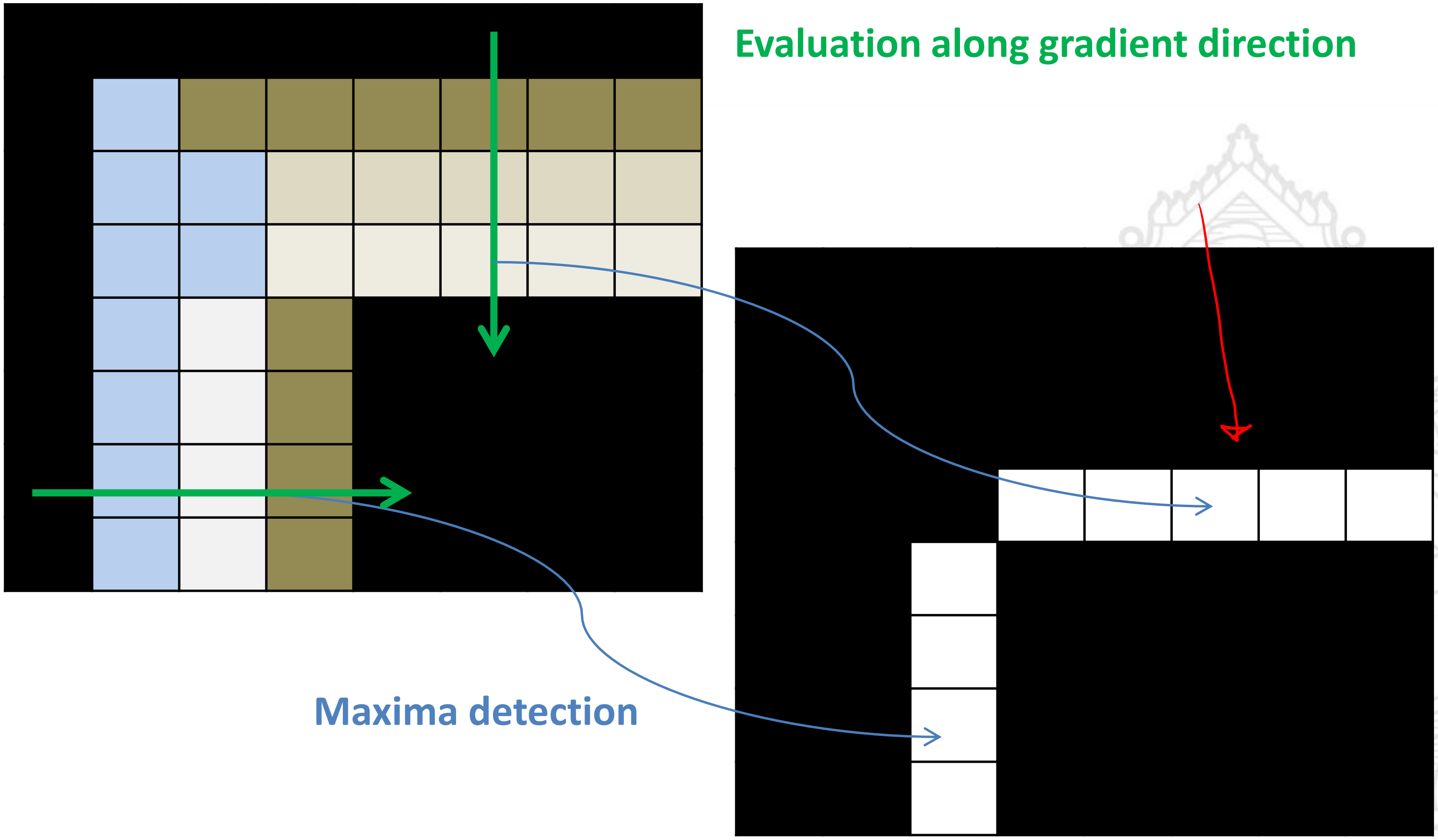
Canny edge detection

The algorithm is based on a few steps:

1. Gaussian filtering
2. Gradient intensity and direction
3. non-maxima suppression (edge thinning)
4. hysteresis threshold

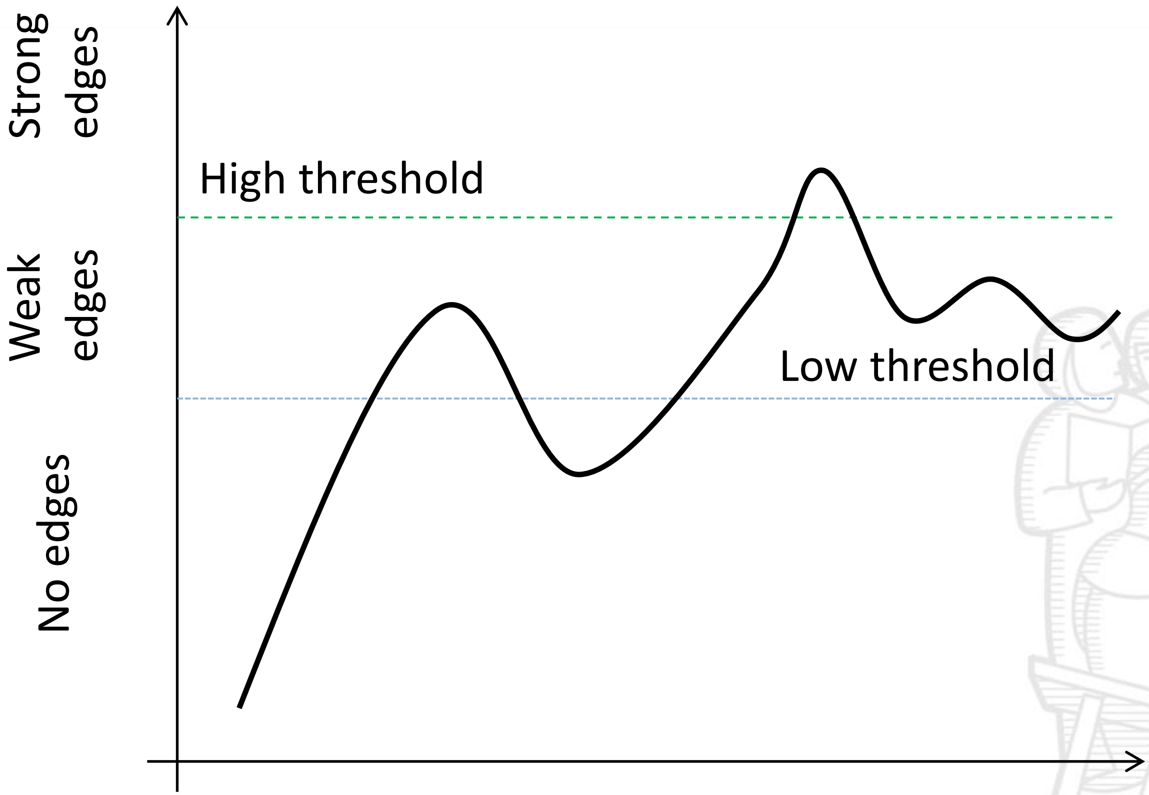
Canny edge detection

3. Non local maxima suppression

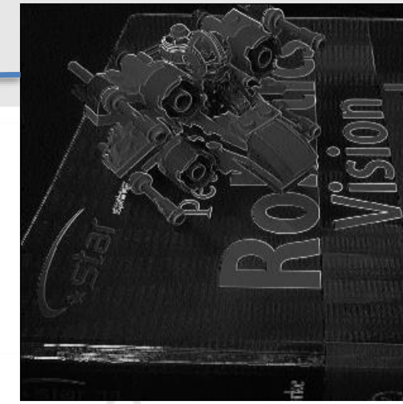


Canny edge detection

4. hysteresis threshold

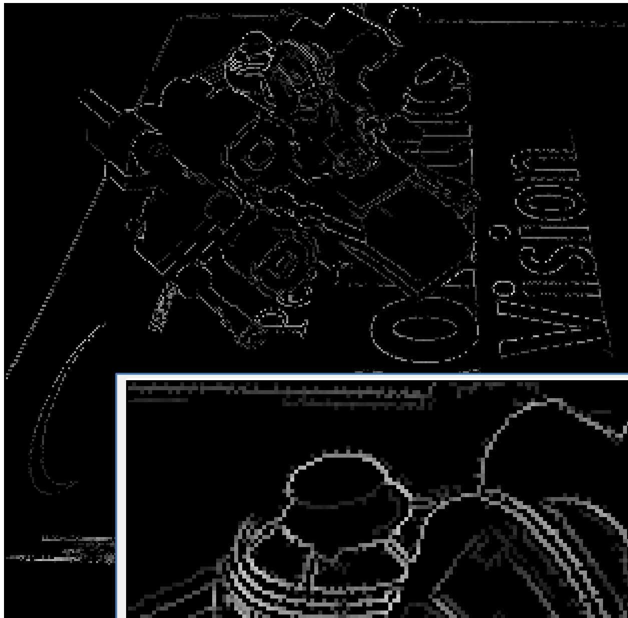


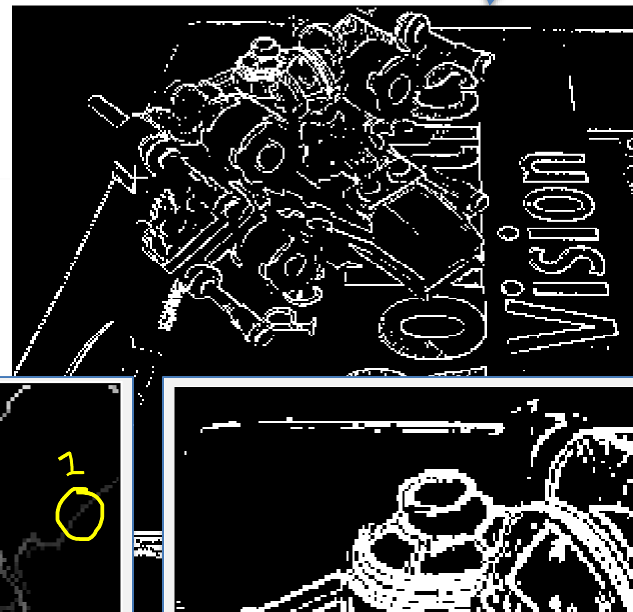
Magnitude of the gradient

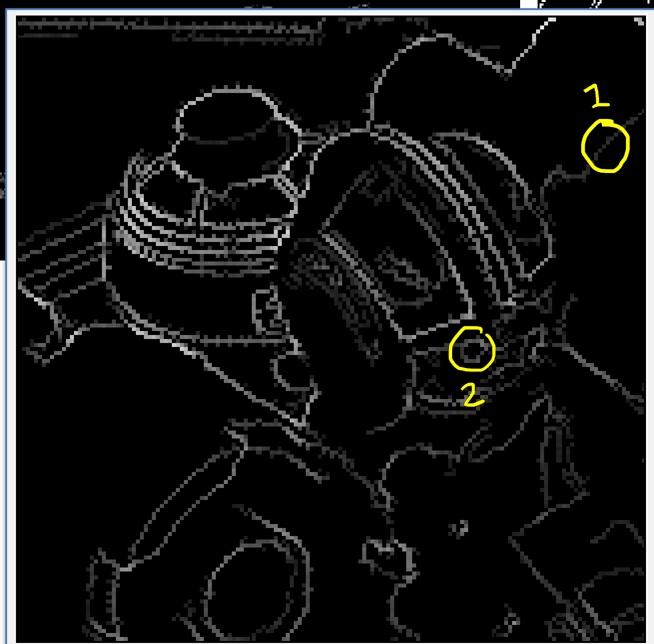


Thresholding

canny







Edge detection

$\boxed{\begin{array}{l} D_u \\ D_v \end{array}}$

thresh

High gradient → Local maxima

Alternative approach is to use second derivative and to find where there is a zero

Laplacian operator

$$\nabla \boldsymbol{I}^2 = \frac{\partial^2 \boldsymbol{I}}{\partial u^2} + \frac{\partial^2 \boldsymbol{I}}{\partial v^2} = \boldsymbol{I}_{uu} + \boldsymbol{I}_{vv} = \boldsymbol{L} \otimes \boldsymbol{I}$$

$$\boldsymbol{L} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

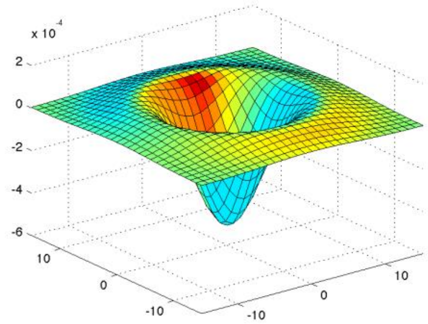
Noise sensitivity

Again, derivative amplifies high-frequency noise. So firstly we can smooth the image, after that we take the derivative:

$$\boldsymbol{L} \otimes (\boldsymbol{G} \otimes \boldsymbol{I}) = \underbrace{(\boldsymbol{L} \otimes \boldsymbol{G})} \otimes \boldsymbol{I}$$

Laplacian of Gaussian
(LoG)

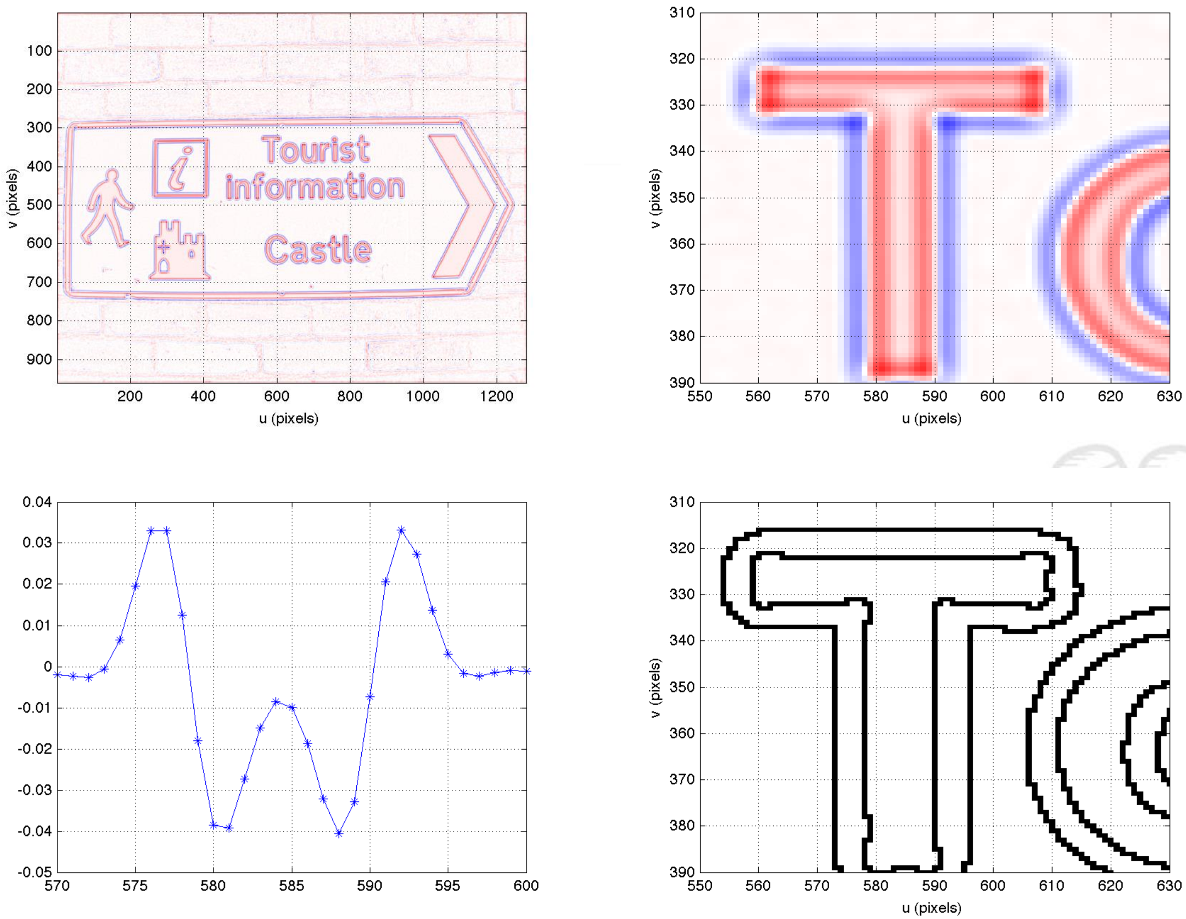
$$\boldsymbol{LoG}(u,v) = \frac{1}{\pi\sigma^4}\left(\frac{u^2+v^2}{2\sigma^2} - 1\right) e^{-\frac{u^2+v^2}{2\sigma^2}}$$



Laplacian of Gaussian
(LoG)

Marr-Hildreth operator or the Mexican hat kernel

Edge detection



Gradient and Laplacian

Example

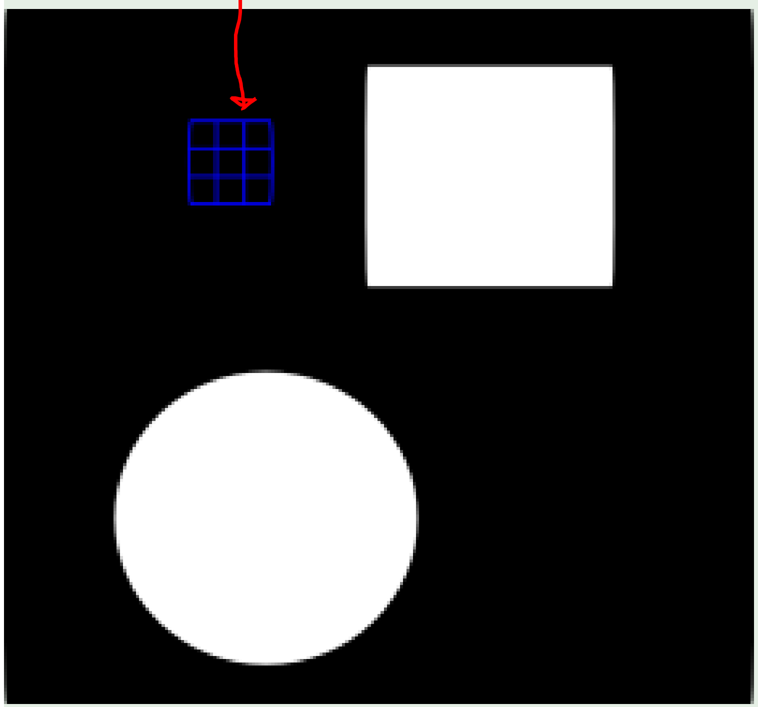


Image window:

$$f(x,y) = \begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline\end{array}$$

CENT - ON

$$\nabla^2 = \begin{array}{|c|c|c|}\hline 0 & -1 & 0 \\ \hline -1 & 4 & -1 \\ \hline 0 & -1 & 0 \\ \hline\end{array}$$

D_x

$$G_x = \begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline\end{array}$$

Products are:

$$\nabla^2 \otimes f(x,y) = 0$$

$$G_x \otimes f(x,y) = 0$$

Gradient and Laplacian

Example

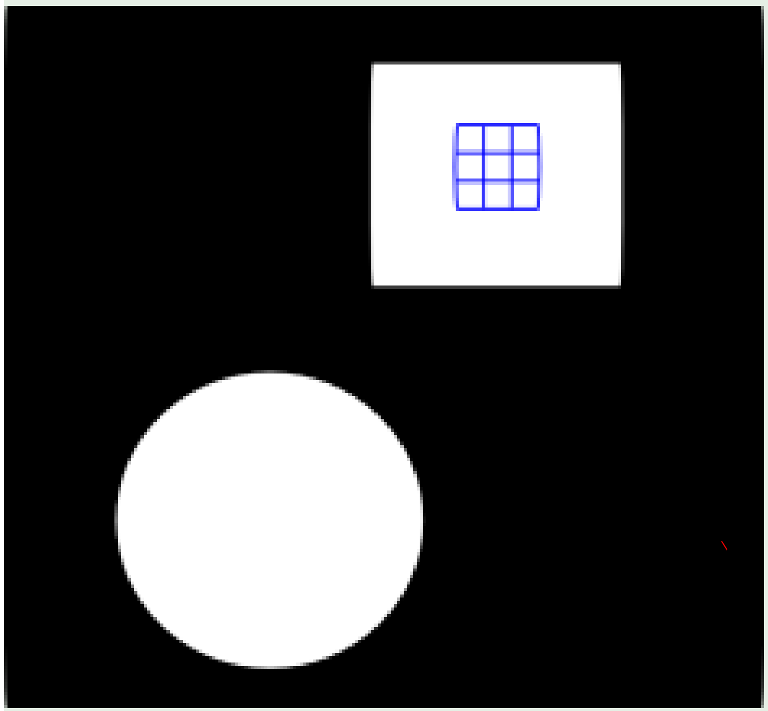


Image window:

$$f(x, y) = \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

$$\nabla^2 = \begin{array}{|c|c|c|} \hline 0 & -1 & 0 \\ \hline -1 & 4 & -1 \\ \hline 0 & -1 & 0 \\ \hline \end{array}$$

$$G_x = \begin{array}{|c|c|c|} \hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array}$$

Products are:

$$\nabla^2 \otimes f(x, y) = 0$$

$$G_x \otimes f(x, y) = 0$$

Gradient and Laplacian

Example

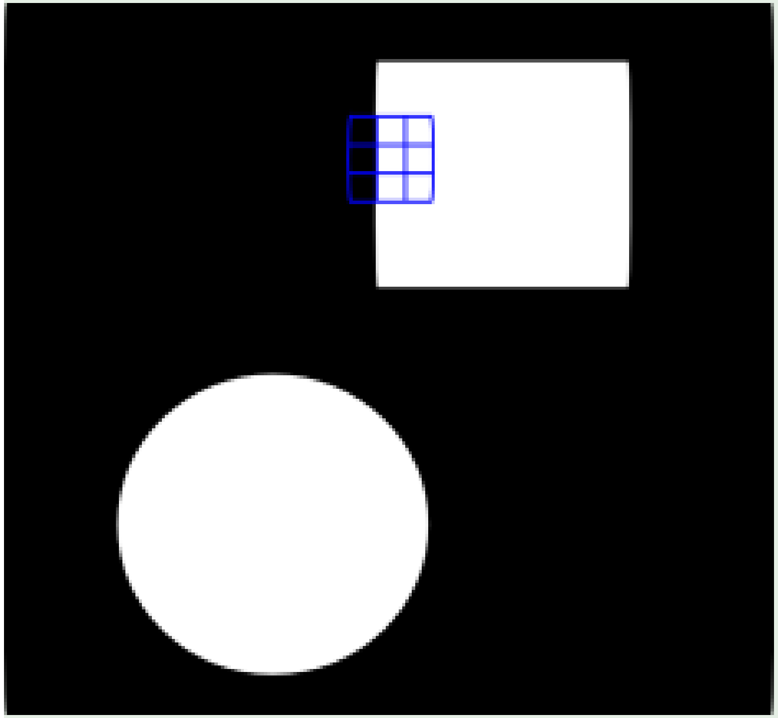


Image window:

$$f(x,y) = \begin{array}{|c|c|c|}\hline 0 & 1 & 1 \\ \hline 0 & 1 & 1 \\ \hline 0 & 1 & 1 \\ \hline\end{array}$$

$$\nabla^2 = \begin{array}{|c|c|c|}\hline 0 & -1 & 0 \\ \hline -1 & 4 & -1 \\ \hline 0 & -1 & 0 \\ \hline\end{array}$$

$$G_x = \begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline\end{array}$$

Products are:

$$\nabla^2 \otimes f(x,y) = 1$$

$$G_x \otimes f(x,y) = 4$$

Gradient and Laplacian

Example

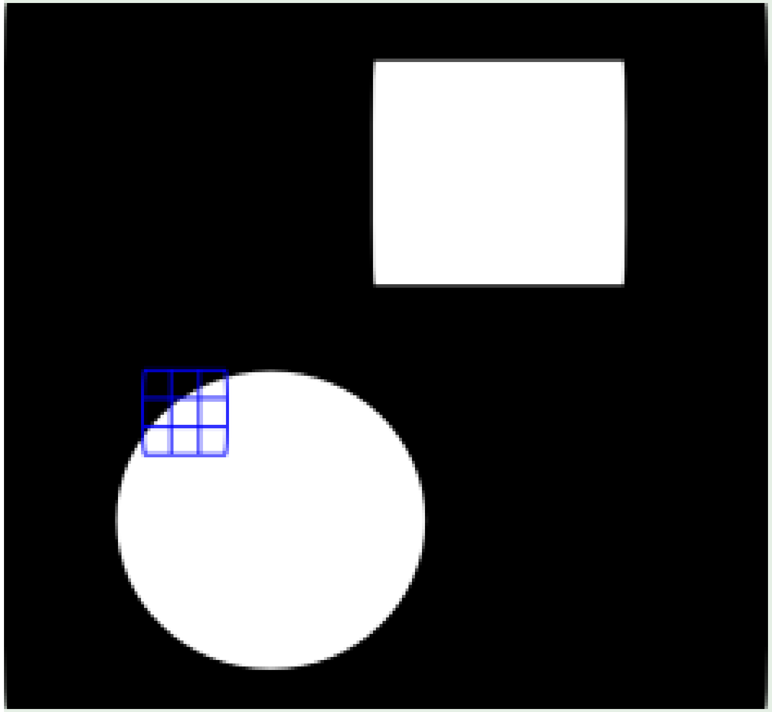


Image window:

$$f(x,y) = \begin{array}{|c|c|c|}\hline 0 & 0 & 1 \\ \hline 0 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

$$\nabla^2 = \begin{array}{|c|c|c|}\hline 0 & -1 & 0 \\ \hline -1 & 4 & -1 \\ \hline 0 & -1 & 0 \\ \hline \end{array}$$

$$G_x = \begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array}$$

Products are:

$$\nabla^2 \otimes f(x,y) = 2$$

$$G_x \otimes f(x,y) = 3$$

Gradient and Laplacian

Example

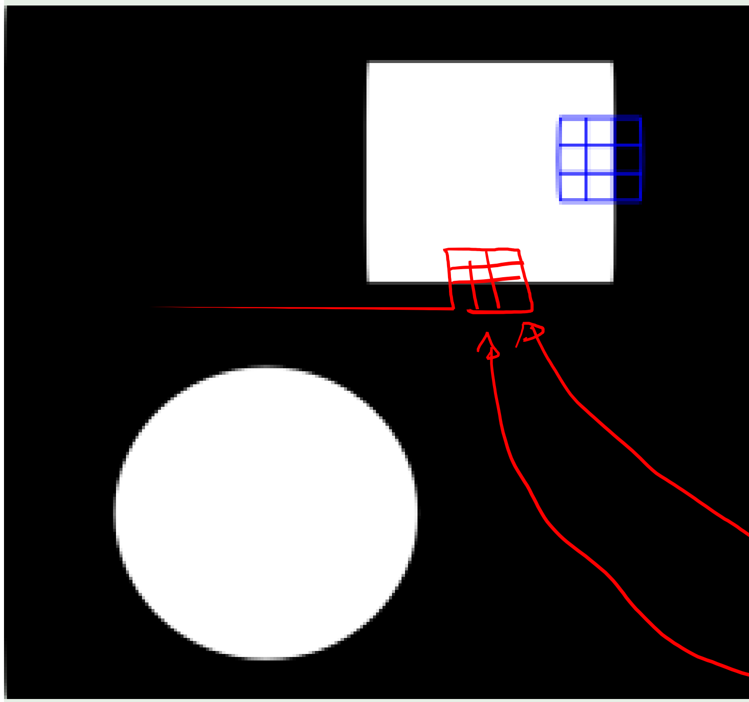


Image window:

$$f(x,y) = \begin{array}{|c|c|c|} \hline 1 & 1 & 0 \\ \hline 1 & 1 & 0 \\ \hline 1 & 1 & 0 \\ \hline \end{array}$$

$$\nabla^2 = \begin{array}{|c|c|c|} \hline 0 & -1 & 0 \\ \hline -1 & 4 & -1 \\ \hline 0 & -1 & 0 \\ \hline \end{array}$$

$$G_x = \begin{array}{|c|c|c|} \hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array}$$

Products are:

$\neq 0$

$$\nabla^2 \otimes f(x,y) = 1$$

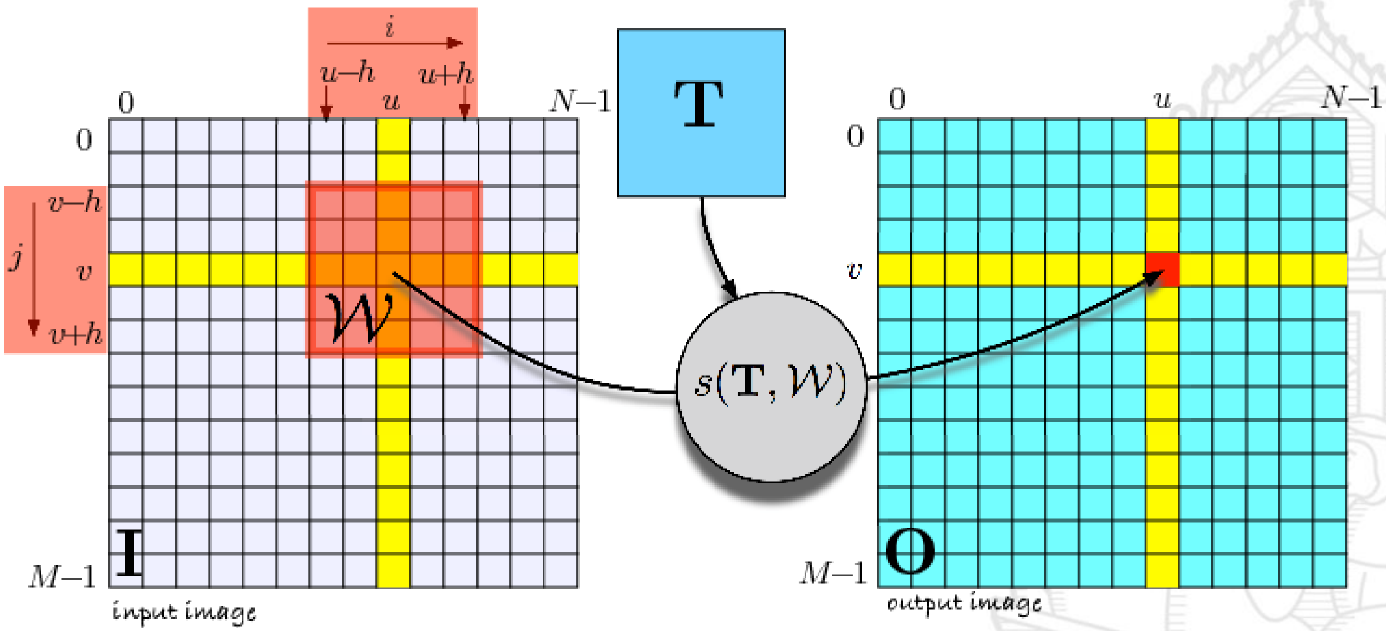
$$= 0 \quad G_x \otimes f(x,y) = -4$$

Code sample >

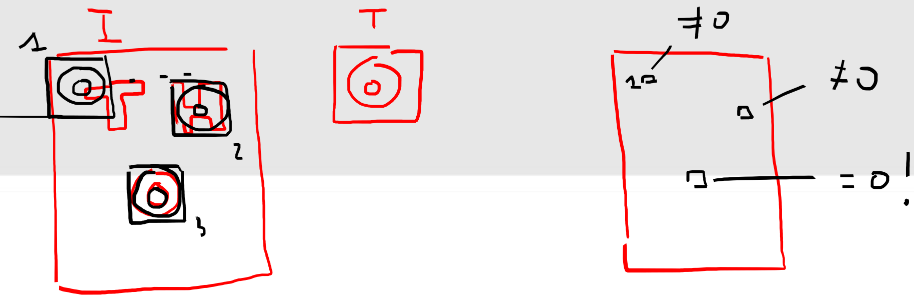
```
% denoising/edge detection
dx=[-1 0 1;-2 0 1; -1 0 1];
dy=[-1 -2 -1;0 0 0;1 2 1];
K=kgauss(3);
K1=ones(19,19).*1/(19*19);
xwingDenoisMean=iconv(K1,xwing_grey);
idisp(xwingDenoisMean)
xwingDenoisGaus=iconv(K,xwing_grey);
idisp(xwingDenois)
xwingIx=iconv(dx,xwing_grey);
idisp(xwingIx)
xwingIy=iconv(dy,xwing_grey);
idisp(xwingIy)
magnGrad=sqrt(xwingIx.^2+xwingIy.^2);
idisp(magnGrad)
edgeGrad=magnGrad>250;

edgeLapl=iconv(klog(2),xwing_grey);
idisp(iint(edgeLapl)>250);

edgeLapl=iconv(klog(1),xwing_grey);
idisp(iint(edgeLapl)>250);

edgeLapl=iconv(klog(3),xwing_grey);
idisp(iint(edgeLapl)>250);
```

Template matching

$$\boldsymbol{O}[u,v] = \underline{s(\boldsymbol{T}, W)}, \qquad \forall (u,v) \in \boldsymbol{I}$$



Template matching

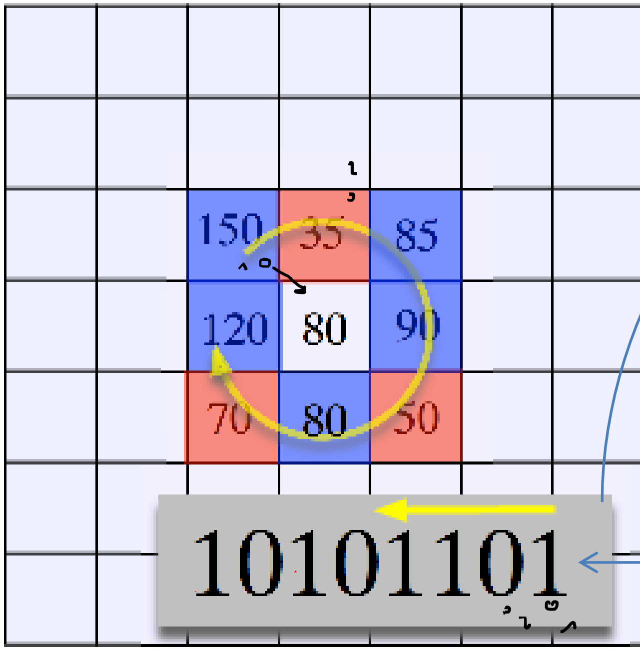


Similarity measures

Sum of absolute differences	
SAD	$s = \sum_{(u,v)\in \boldsymbol{I}} \lvert \boldsymbol{I}_1[u,v] - \boldsymbol{I}_2[u,v] \rvert$
ZSAD	$s = \sum_{(u,v)\in \boldsymbol{I}} \lvert (\boldsymbol{I}_1[u,v] - \bar{\boldsymbol{I}}_1) - (\boldsymbol{I}_2[u,v] - \bar{\boldsymbol{I}}_2) \rvert$
Sum of squared differences	
SSD	$s = \sum_{(u,v)\in \boldsymbol{I}} (\boldsymbol{I}_1[u,v] - \boldsymbol{I}_2[u,v])^2$
ZSSD	$s = \sum_{(u,v)\in \boldsymbol{I}} \left((\boldsymbol{I}_1[u,v] - \bar{\boldsymbol{I}}_1) - (\boldsymbol{I}_2[u,v] - \bar{\boldsymbol{I}}_2) \right)^2$
Cross correlation	
NCC	$s = \dfrac{\sum_{(u,v)\in \boldsymbol{I}} \boldsymbol{I}_1[u,v] \cdot \boldsymbol{I}_2[u,v]}{\sqrt{\sum_{(u,v)\in \boldsymbol{I}} \boldsymbol{I}_1^2[u,v] \cdot \sum_{(u,v)\in \boldsymbol{I}} \boldsymbol{I}_2^2[u,v]}}$
ZNCC	$s = \dfrac{\sum_{(u,v)\in \boldsymbol{I}} (\boldsymbol{I}_1[u,v] - \bar{\boldsymbol{I}}_1) \cdot (\boldsymbol{I}_2[u,v] - \bar{\boldsymbol{I}}_2)}{\sqrt{\sum_{(u,v)\in \boldsymbol{I}} (\boldsymbol{I}_1[u,v] - \bar{\boldsymbol{I}}_1)^2 \cdot \sum_{(u,v)\in \boldsymbol{I}} (\boldsymbol{I}_2[u,v] - \bar{\boldsymbol{I}}_2)^2}}$

Non-parametric similarity measures

Census



$$s(x) = \begin{cases} 1, if\ x > R \\ 0, otherwise \end{cases}$$

Census representation

Hamming distance

Rank transform = 5

Non-parametric similarity measures

Rank transform is more compact but does not encode position information

50	10	205
1	25	2
102	250	240

Census: 01110101

Rank: 5

Hamming distance: 6!

10	26	2
101	25	202
1	250	214

Census: 10111010

Rank: 5

Non-linear operators

- Variance measure (on windows): Edge detection
- Median filter: noise removal

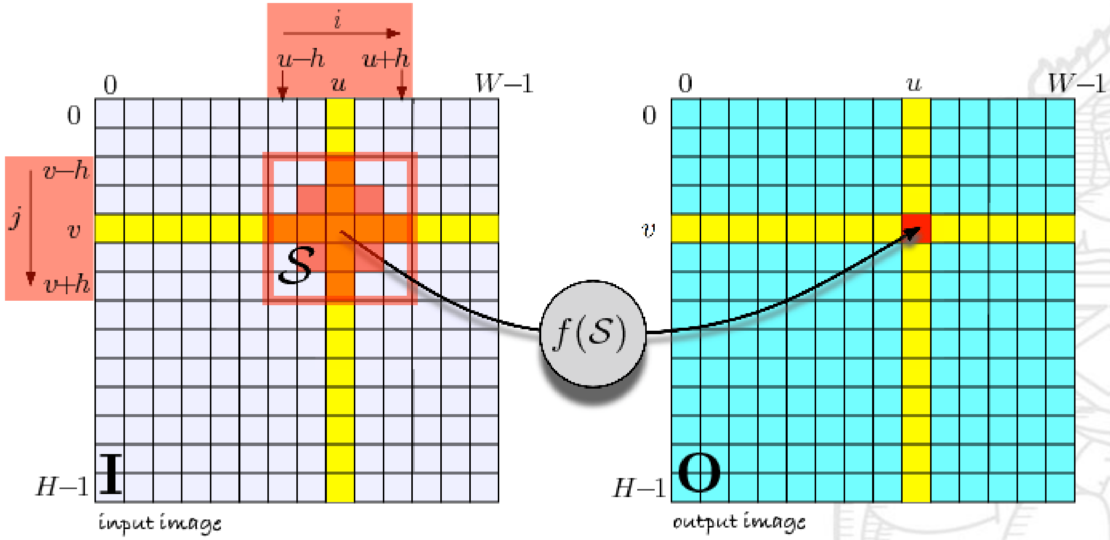
median

- Rank transform: non-local maxima suppression

Mathematical morphology

$I[u,v](t) = I[u(t)], I(v(t)$

$$\boldsymbol{O}[u,v] = f(\boldsymbol{I}[u+i, v+j]), \qquad \forall (i,j) \in S, \forall (u,v) \in \boldsymbol{I}$$

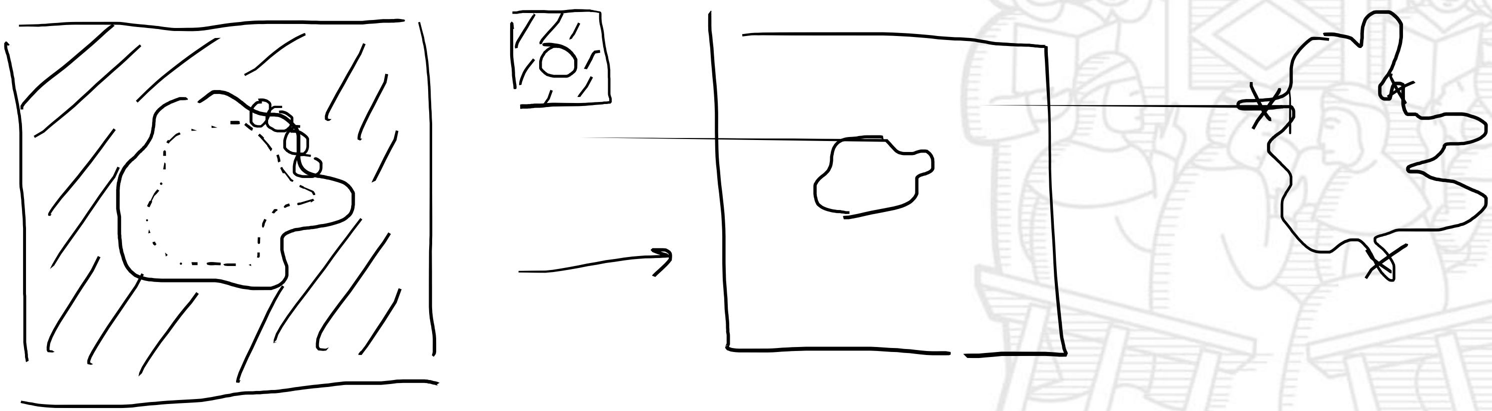


Erosion

Erosion is a specific procedure of the more general Morphological Image Processing techniques.

It belongs to the concept of mathematical morphology and it is strictly related to the set theory.

Here the concept is roughly introduced to understand the basis of erosion.



Notation

Let consider A as a set in Z^2

Definition $a = (a_1, a_2)$ **belongs to** A

We write:

$$a \in A$$

Definition $a = (a_1, a_2)$ **does not belong to** A

We write:

$$a \notin A$$

A set is represented by the parenthesis{·}.

In our case, the elements of a set are the pixels belonging to a certain area or object of an image. When we write:

Example

$$\downarrow$$

$$C = \{w | w = -d, \ per \ d \in D\}$$

This means that C is composed by all the elements w which are obtained by scalar product of the elements of D and the value -1.

Definition

When all elements of *A* are also elements of *B*, we say that *A* is a subset of *B*.

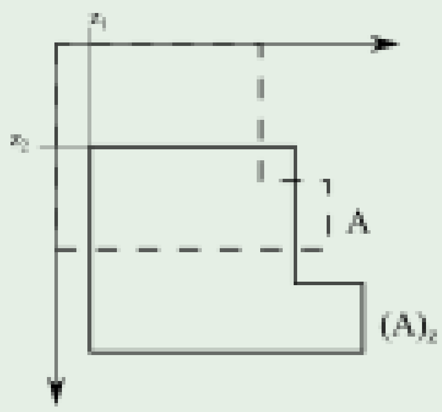
$$A \subseteq B$$

Definition

The translation of a set *A* by an element *z*, is represented as $(A)_z$ and is defined by:

$$(A)_z = \{c | c = a + z, \ per \ a \in A\}$$

Example



Now we can write the morphological operation which interest us, thus:

Definition

$$A \ominus B = \{z | (B)_z \subseteq A\}$$

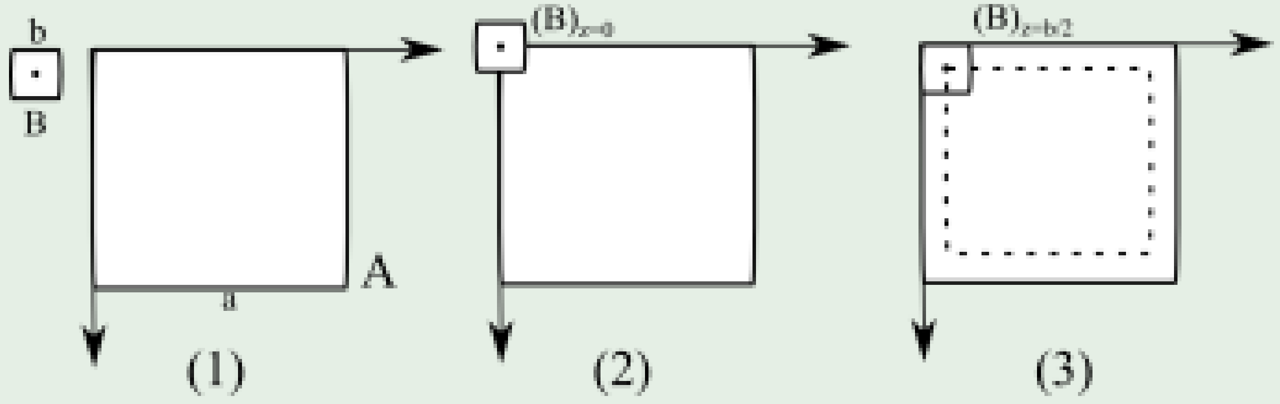
This definition represents an:

erosion

The verbose definition is: <u>the erosion of *A* through *B* is the set of all the points *z* whom the translation of *B* by *z* is a subset of *A*.</u>

It’s simple to see that graphically:

Example



1. I due insiemi A e B.
2. $(B)_0 \subseteq A$? No z=0
3. $(B)_{b/2} \subseteq A$? Sì z=b/2

In this case the eroded set will be;

$$A \ominus B = [\frac{b}{2} : a - \frac{b}{2}; \frac{b}{2} : a - \frac{b}{2}]$$

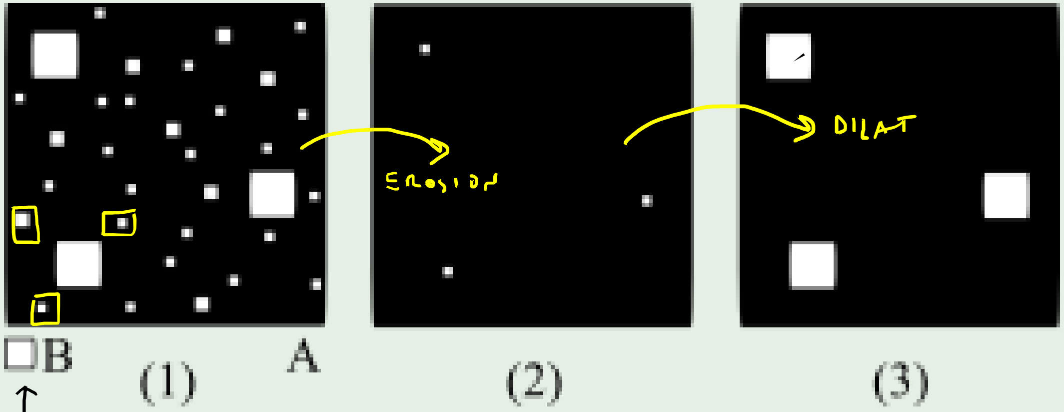
We can figure the erosion as a “shape-cutting” of the most external part of the set.

Dilation is the «opposite» operation, but formally they are related by:

$$A \oplus B = \overline{\bar{A} \ominus B}$$

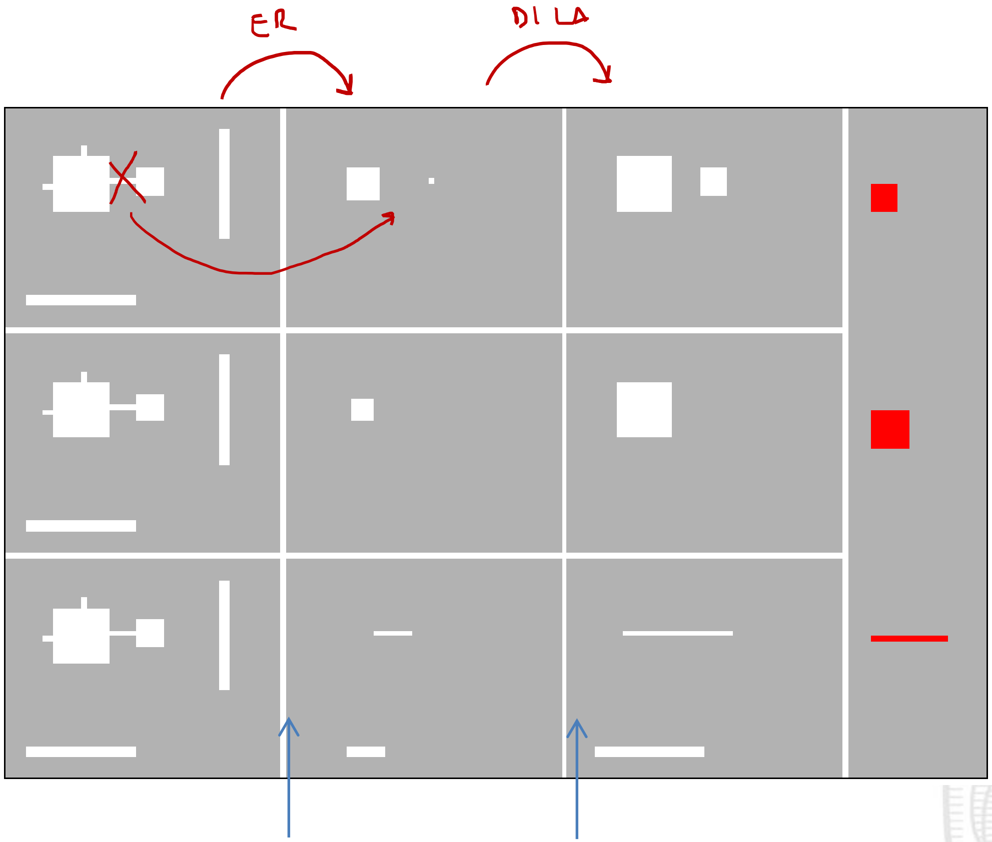
Which means that eroding the white pixels is the same as dilating the dark pixels, and vice versa.

Example

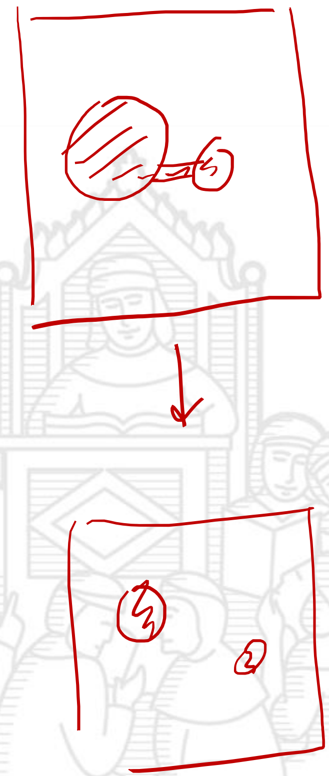


1) Original image
2) Erosion by the element B
3) Dilatation (the opposite procedure of the Erosion)

Example: opening



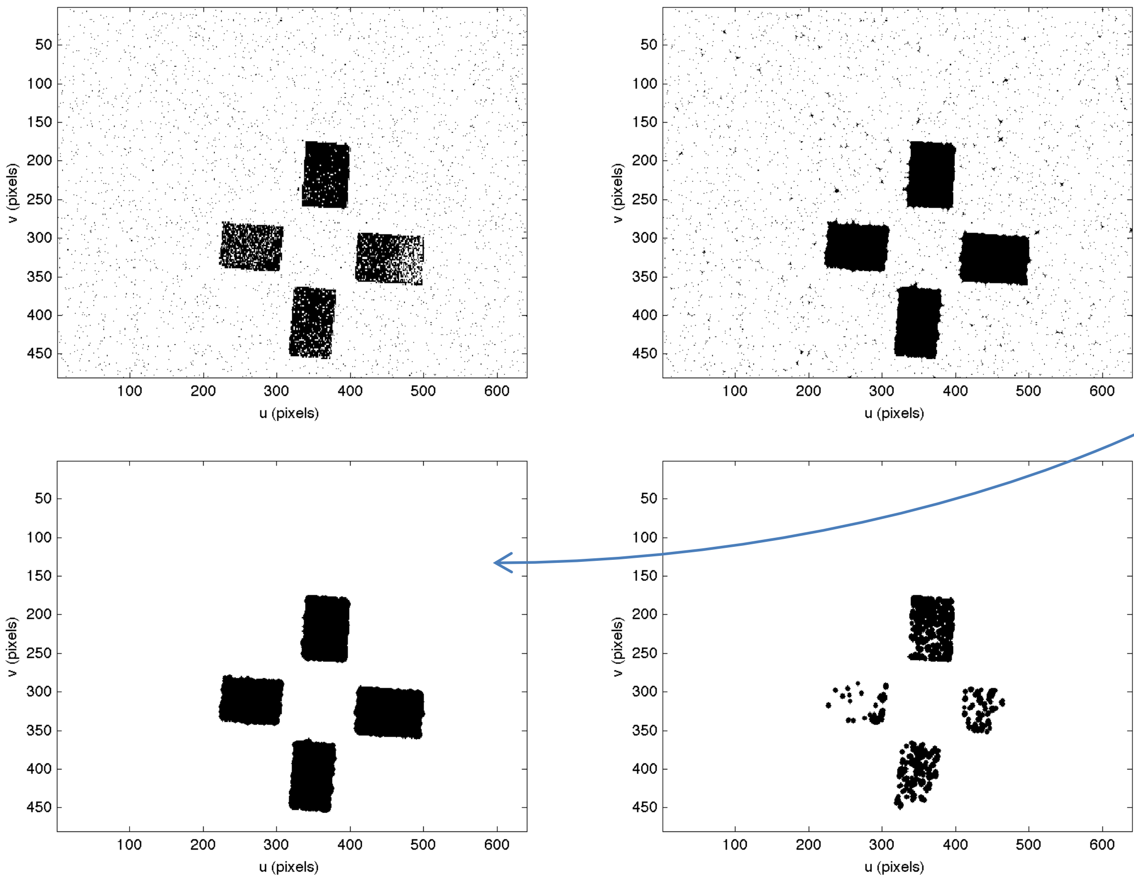
$O = I \ominus S$ $\quad$ $O = I \oplus S$



Example: closing

$$O = I \oplus S \qquad O = I \ominus S$$

Noise removal & boundary detection



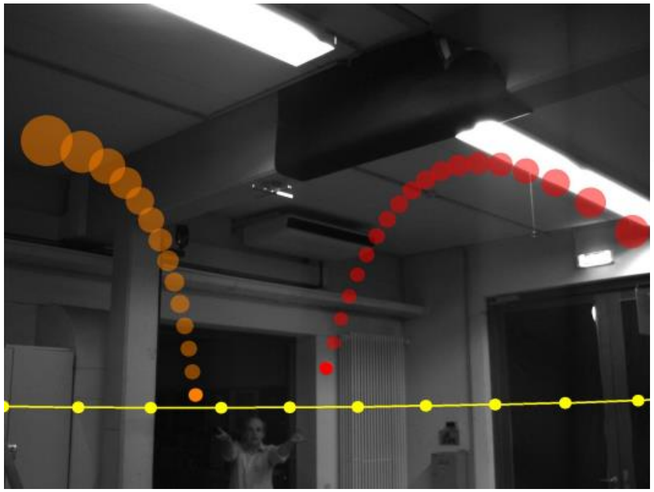
$$O = I \ominus S$$

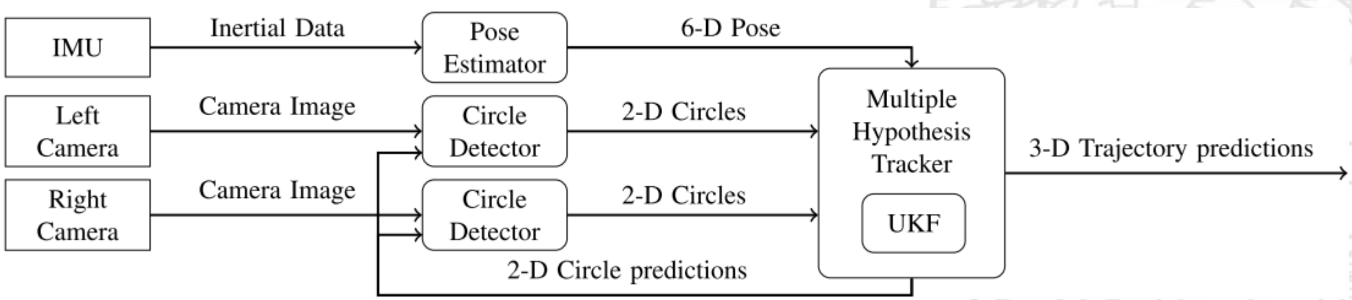
$$O_2 = I - O$$

?

Matlab sample of buondary detection

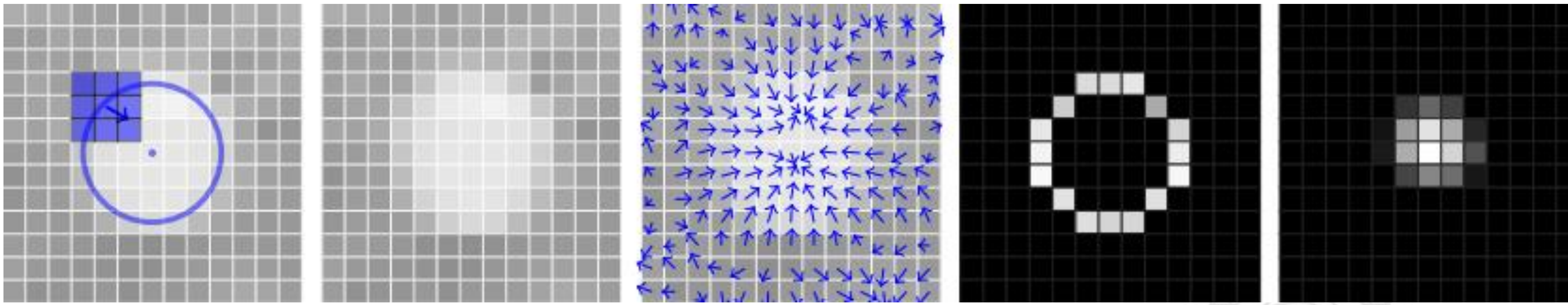
Example DLR





Oliver Birbach, Udo Frese and Berthold Bauml, (2011) ‘Realtime Perception for Catching a Flying Ball with a Mobile Humanoid’

Example DLR



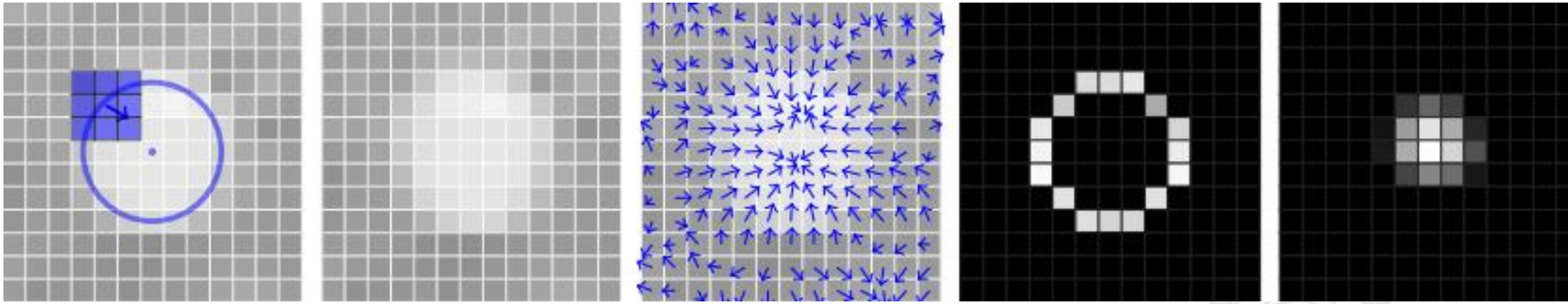
$$C = \frac{\sqrt{2}\left(\left(\begin{pmatrix} +1 & 0 & -1 \\ +2 & 0 & -2 \\ +1 & 0 & -1 \end{pmatrix} * I,\ \begin{pmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{pmatrix} * I\right)^T\right)}{\sqrt{16\begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{pmatrix} * I^2 - \left(\left(\begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{pmatrix}\right) * I\right)^2 + \varepsilon^2}}$$

α

$$R(x,y,\alpha) = \left(\begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} \cdot C(x,y)\right)^2 = |C(x,y)|^2 \cdot \cos^2\delta$$

$$CR(x_c,y_c,r) = \frac{1}{2\pi}\int_{\alpha=0}^{2\pi} R(x_c + r\cos\alpha, y_c + r\sin\alpha, \alpha)\,d\alpha$$

Example DLR



Filtraggio con Sobel

$$C = \frac{\sqrt{2}\left(\begin{pmatrix}+1&0&-1\\+2&0&-2\\+1&0&-1\end{pmatrix}*I,\ \begin{pmatrix}+1&+2&+1\\0&0&0\\-1&-2&-1\end{pmatrix}*I\right)^T}{\sqrt{16\begin{pmatrix}1&2&1\\2&4&2\\1&2&1\end{pmatrix}*I^2-\left(\begin{pmatrix}1&2&1\\2&4&2\\1&2&1\end{pmatrix}*I\right)^2+\varepsilon^2}}$$

Normalizzazione rispetto alla varianza locale

$$R(x,y,\alpha) = \left(\begin{pmatrix}\cos\alpha\\ \sin\alpha\end{pmatrix}\cdot C(x,y)\right)^2 = |C(x,y)|^2\cdot\cos^2\delta$$

$$CR(x_c,y_c,r) = \frac{1}{2\pi}\int_{\alpha=0}^{2\pi} R(x_c+r\cos\alpha, y_c+r\sin\alpha,\alpha)\,d\alpha$$