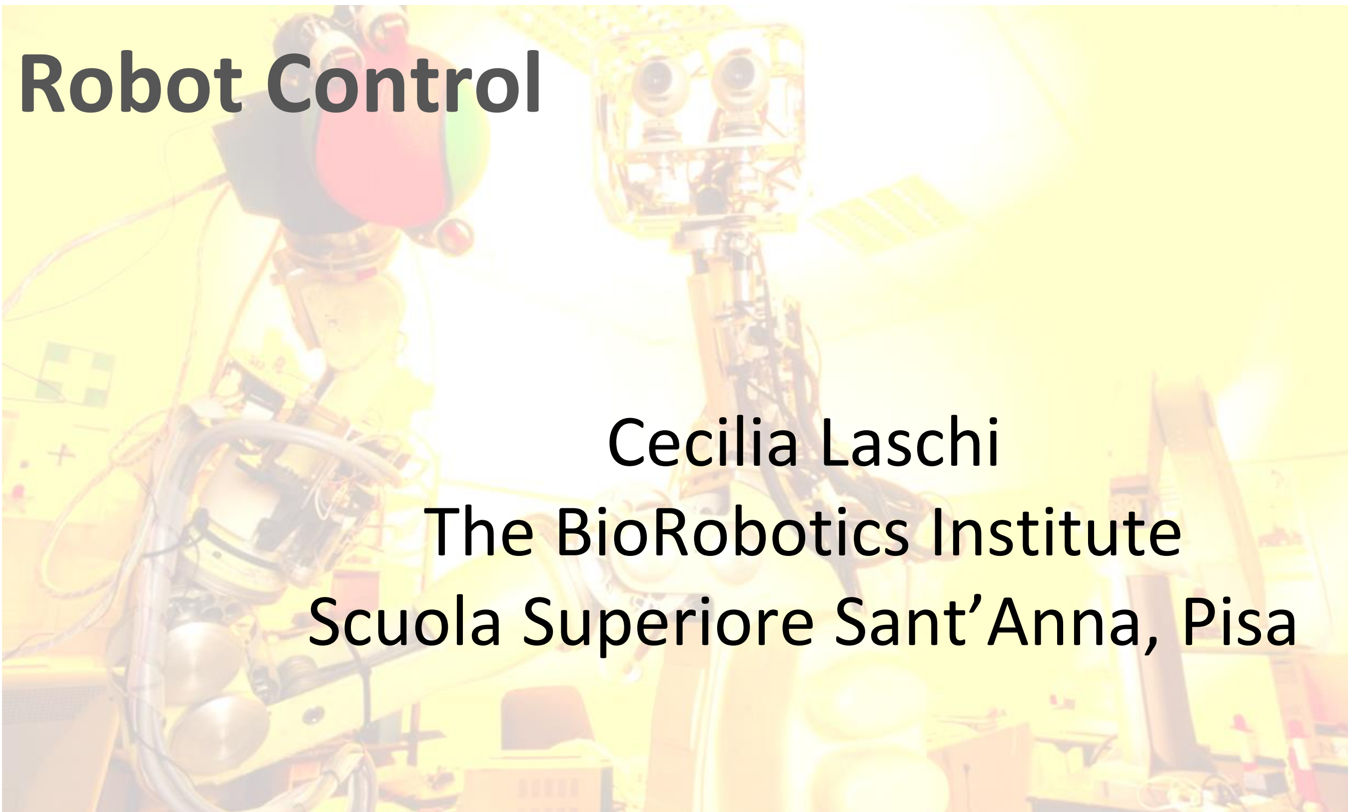
University of Pisa
Master of Science in Computer Science
Course of Robotics (ROB)
A.Y. 2019/20



Robot Control

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http://didawiki.cli.di.unipi.it/doku.php/magistraleinformatica/rob/start

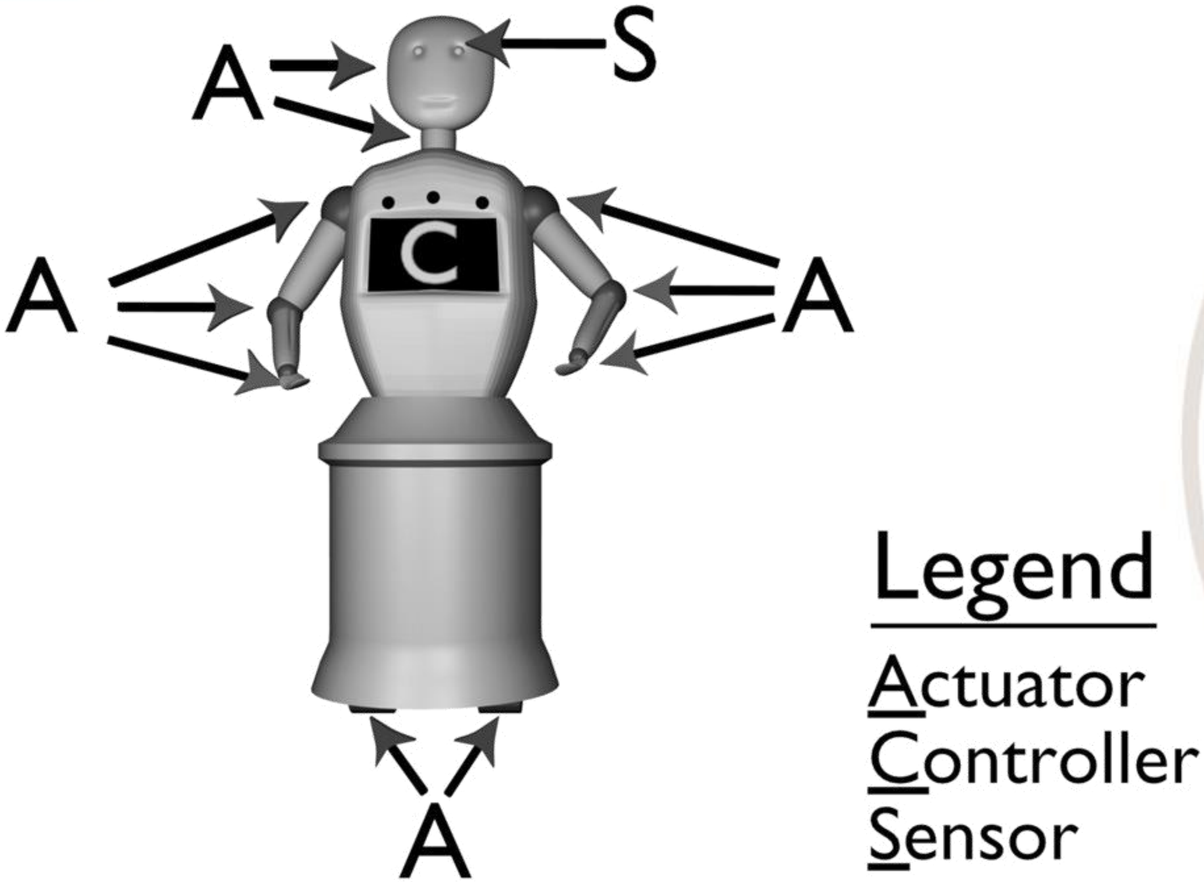
Robot definition

A *robot* is an autonomous system
which exists in the physical world,
can sense its environment,
and can act on it to achieve some goals

Maja J Mataric, *The Robotics Primer*, The MIT Press, 2007

What's in a robot?
Robot components

Scuola Superiore Sant'Anna



Maja J Mataric, *The Robotics Primer*, The MIT Press, 2007

Robot Control

- Control of one joint motion:
  - PID controller
- Control of the manipulator motion:
  - Trajectory planning
  - Motion control in joint space
  - Motion control in operational space

Robot kinematics and differential kinematics

Kinematics

$$x = k(q)$$
$$q = k^{-1}(x)$$

$$x = \begin{bmatrix} x \\ y \\ z \\ \varphi \\ \vartheta \\ \psi \end{bmatrix} \qquad q = \begin{bmatrix} q_o \\ q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \end{bmatrix}$$

$k(\cdot) =$ equations of direct kinematics

Differential kinematics

$$\dot{x} = J(q)\dot{q}$$
$$\dot{q} = J^{-1}(q)\dot{x}$$

Velocity space

$J(q) =$ Jacobian matrix

Robot Control

- **Control of one joint motion**:
  - PID controller
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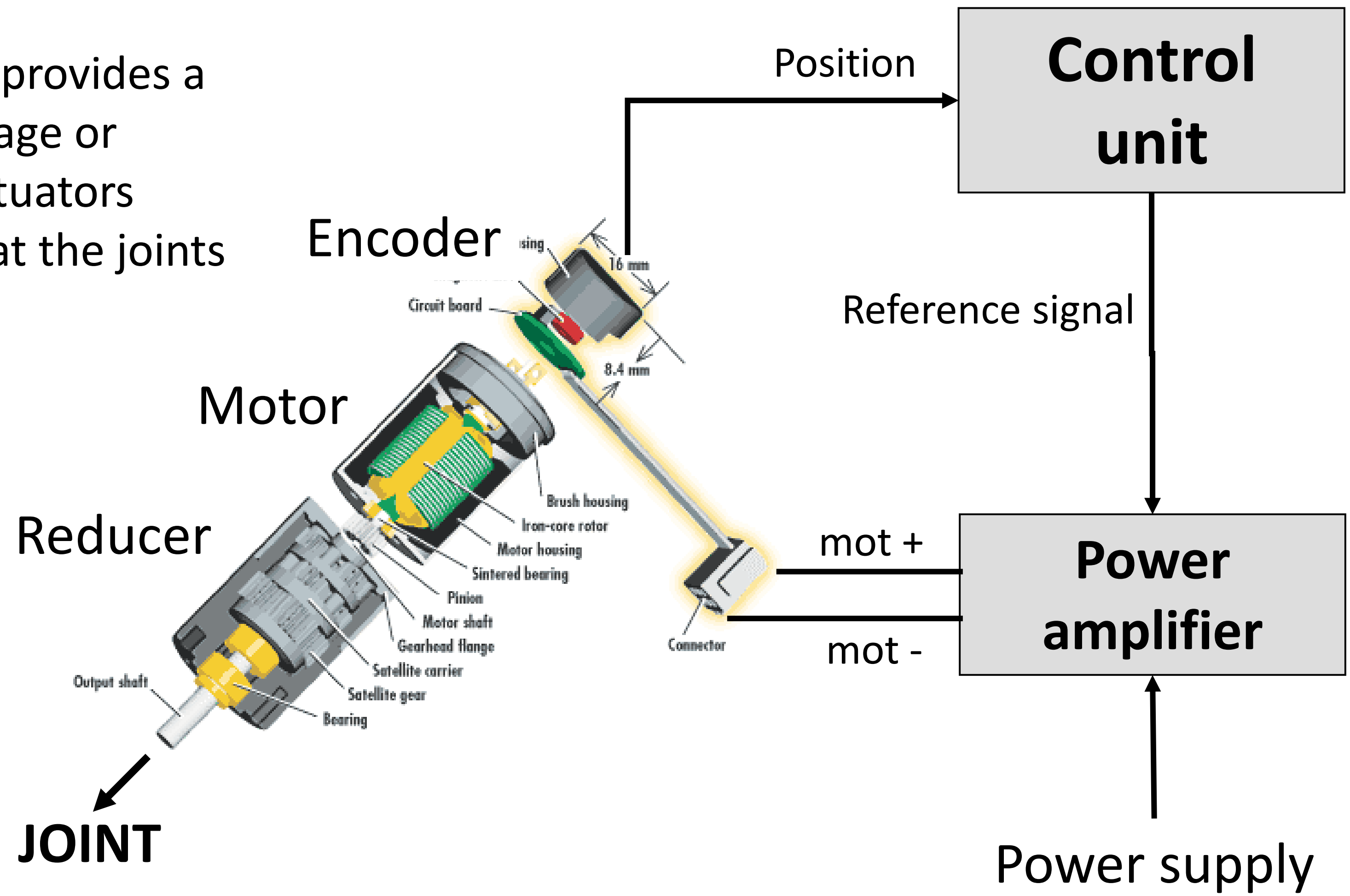
T. Bajd et al., *Robotics*, Springer 2010

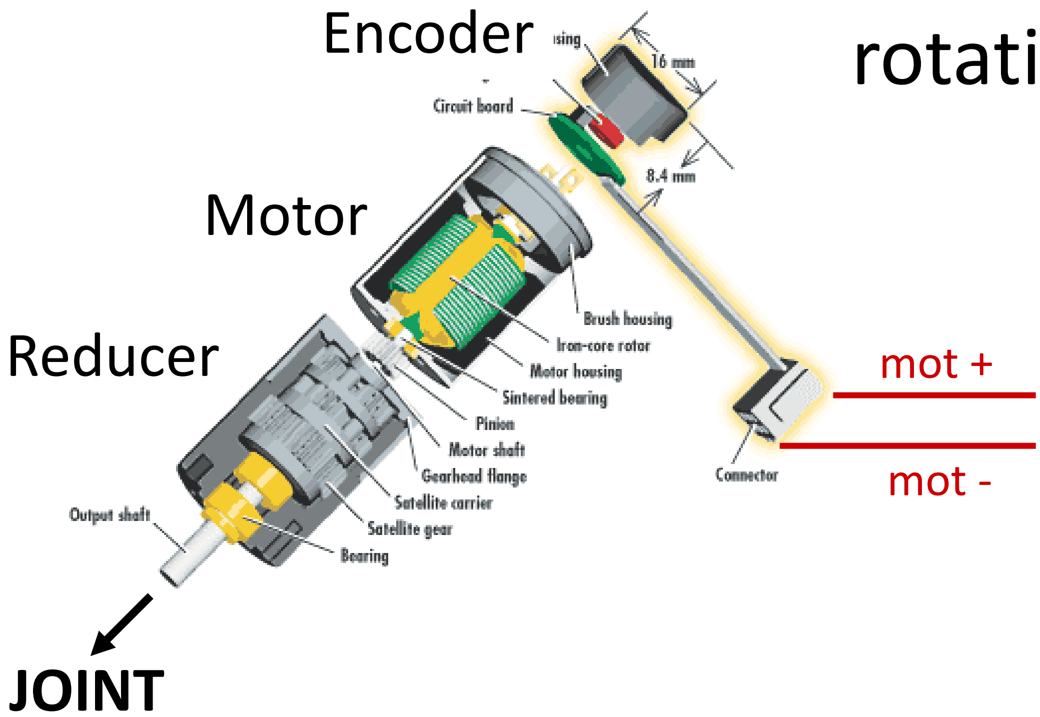
Scheme of an actuator and control system

A control system provides a command in voltage or current to the actuators (motors) such that the joints reach a desired configuration



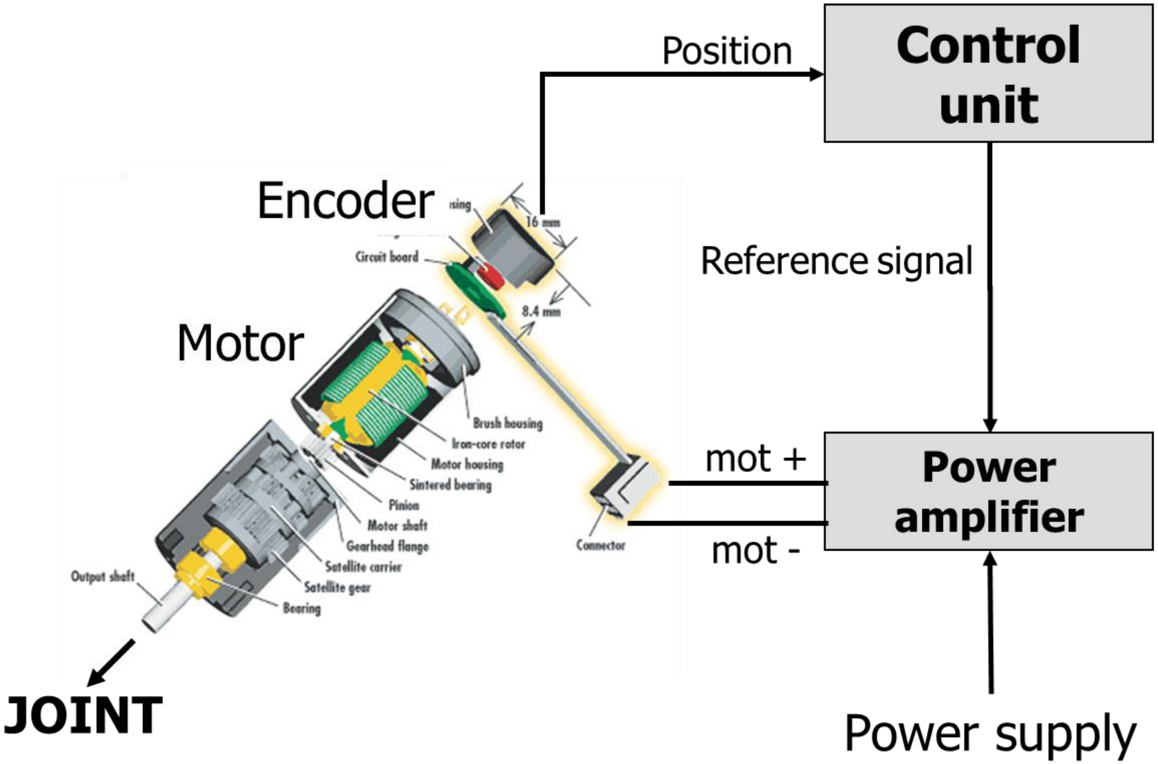
Scheme of an actuator and control system

Opposite-sign voltages produce opposite rotations of the motor



Scheme of an actuator and control system

- **Encoder**: sensor measuring joint rotations, either as an absolute or a relative value. The measurement is given in "*encoder steps*"
- **Reducer:** mechanism reducing the motor rotations with respect to the rotations of the axis mounted on the motor (ex. 1:k reduction)
- **Power amplifier:** it amplifies a reference signal into a power signal for moving the joint
- **Control unit**: unit producing the reference signal for the motor



Relations between joint position and encoder position

- q: joint angular position (in degrees)
- θ: joint position in encoder steps
- k: motor reduction ratio
- R: encoder resolution (number of steps per turn)

$$q = \frac{\theta \times 360°}{R \times k}$$

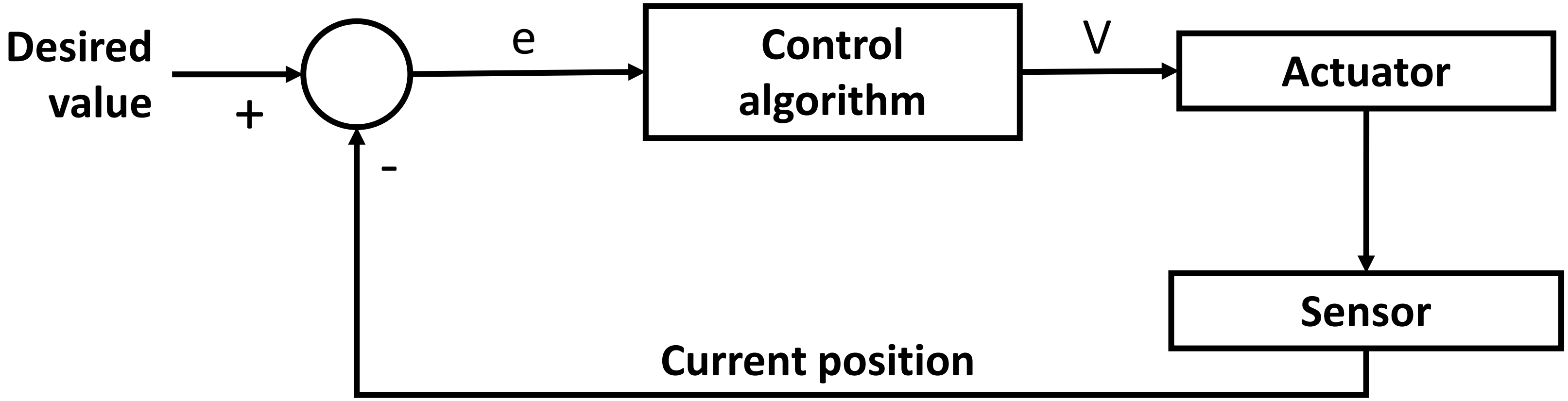
Control of one joint motion

- Objective: move the joint from the current position q_i (in degrees) to the desired position q_f, in a time interval t :

$$q_i \Rightarrow q_f$$

Closed-loop (feedback) control

- The variable to control is measured and compared with the desired value
- The difference, or error, is processed by an algorithm
- The result of processing is the input value for the actuator

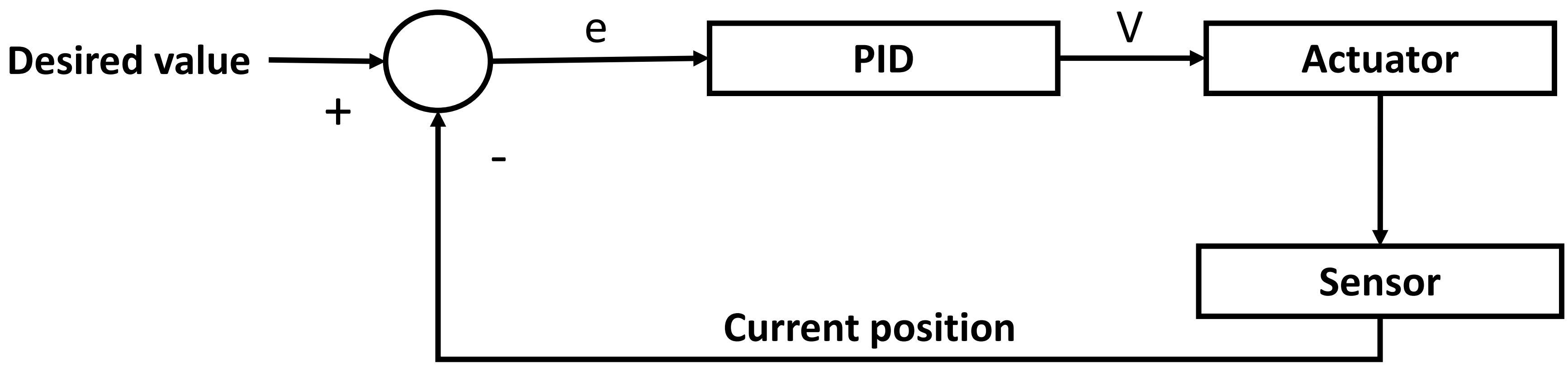


PID control
(Proportional, Integral, Derivative)

- It is a closed-loop control in which the error is processed with an algorithm including **Proportional, Integral and Derivative** components.
- The algorithm processes the error and provides an input to the actuator, with 3 components:
  - **Proportional**, producing a correction proportional to the error;
  - **Integral**, producing a correction given by the error integral in time;
  - **Derivative**, producing a correction which is a function of the error first derivative.
- Not all closed-loop control systems use a PID algorithm

PID control
(Proportional, Integral, Derivative)

- In a PID control system, the error is given to the control algorithm, which calculates the derivative and integral terms and the output signal V



PID control
(Proportional, Integral, Derivative)

$$V = K_p e_q + K_d \dot{e_q} + K_i \int e_q(t) dt$$

$$e_q = q_d - q_a$$

$$\dot{e_q} = \frac{de_q}{dt}$$

K_p is the *proportional* gain or constant

K_i is the *integral* gain or constant

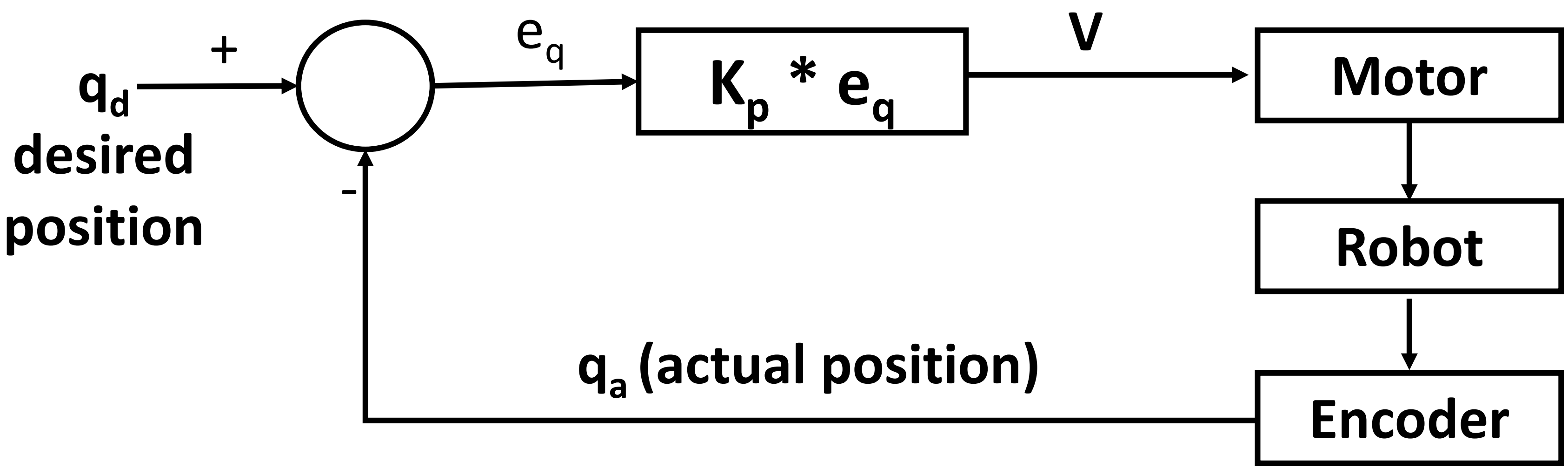
K_d is the *derivative* gain or constant

e_q is the error, i.e. the difference between the desired position and the current (or actual) position

PID control

Proportional term

- The voltage V given to the motor is proportional to the difference between the actual position measured by the sensor and the desired position



PID control

Proportional term:

- The voltage V given to the motor is proportional to the difference between the actual position measured by the sensor and the desired position

$$V = K_p e_q$$

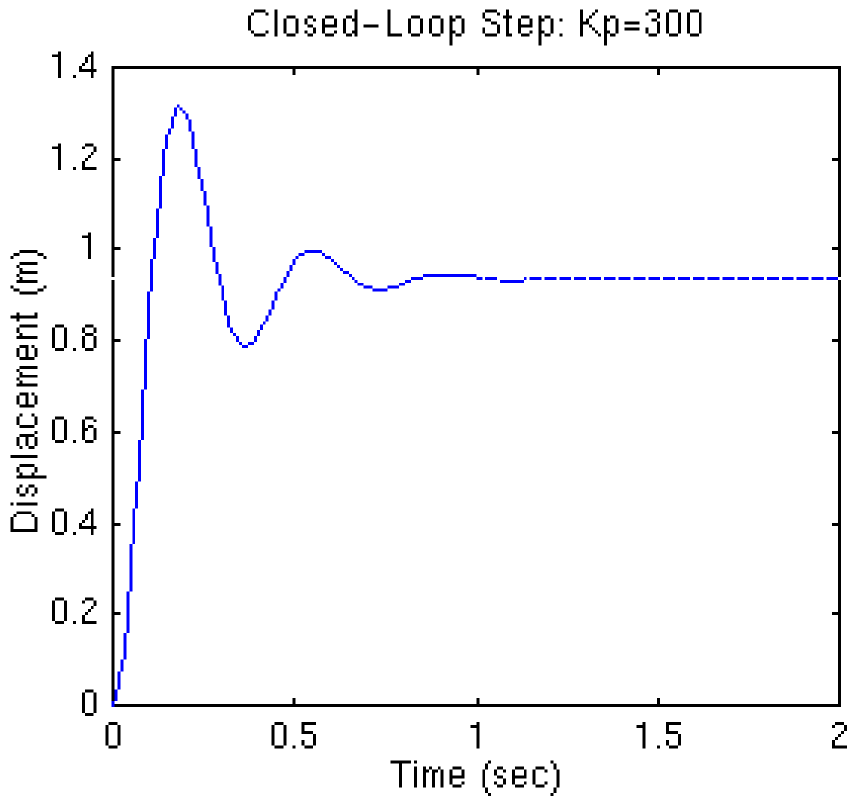
$$e_q = q_d - q_a$$

K_P : proportional constant

PID control

Proportional term: system behaviour

Desired position: 1



- The motor oscillates before converging towards the desired position
- The system may settle without cancelling the error

PID control

Derivative term:

$$V = K_p e_q + K_d \dot{e}_q$$

$$\dot{e}_q = \frac{de_q}{dt}$$ Error derivative in time

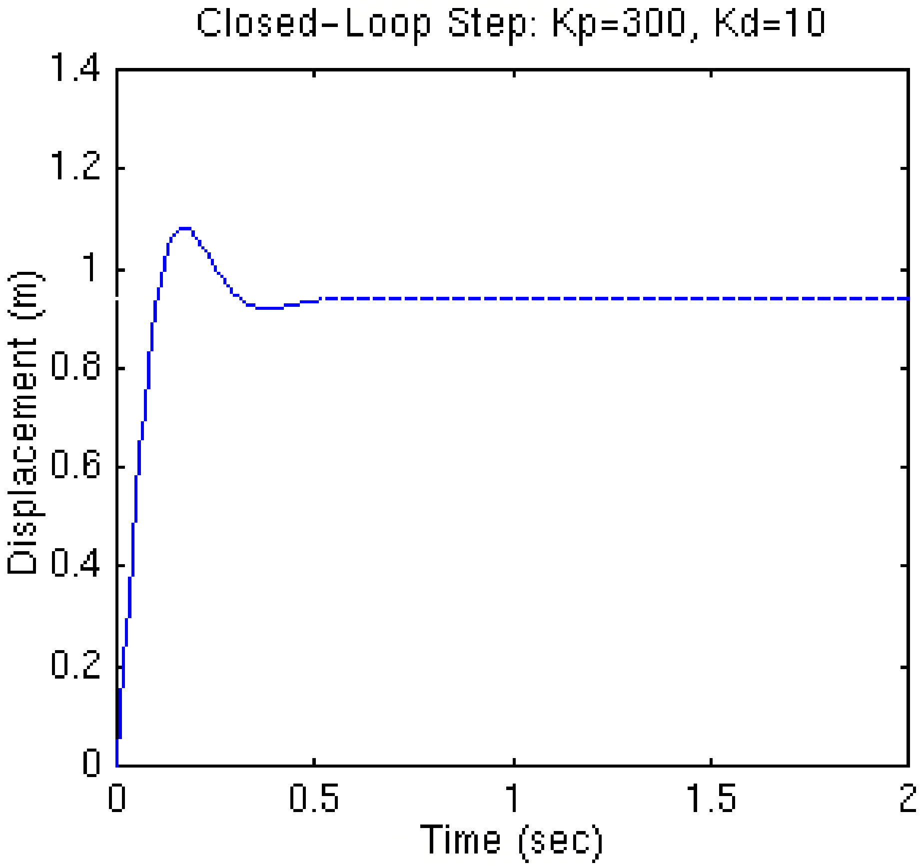
$$e_q = q_d - q_a$$

K_d : derivative constant

PID control

Proportional and derivative terms:

Desired position: 1



- Oscillation reductions
- Reduction of settlement time
- The system may settle without cancelling the error

PID control

Integral terms:

$$K_i \int e_q(t)\,dt$$ Error integral in time

$$V = K_p e_q + K_i \int e_q(t)\,dt$$

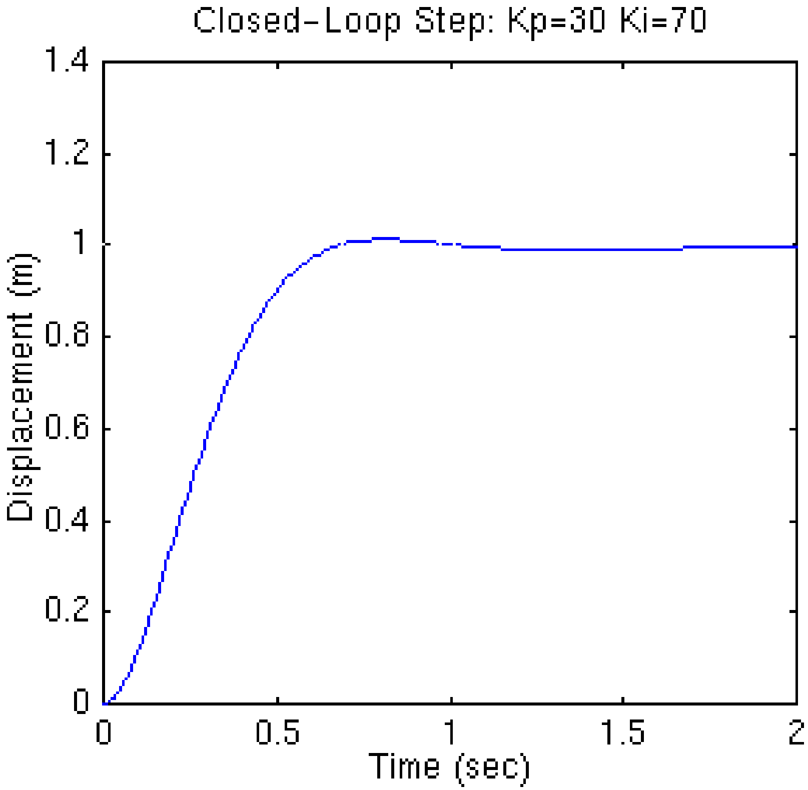
$$e_q = q_d - q_a$$

K_i : integral constant

PID control

Proportional and integral terms:

Desired position: 1



The system settles and cancels the error

PID control

Proportional, Integral and Derivative terms:

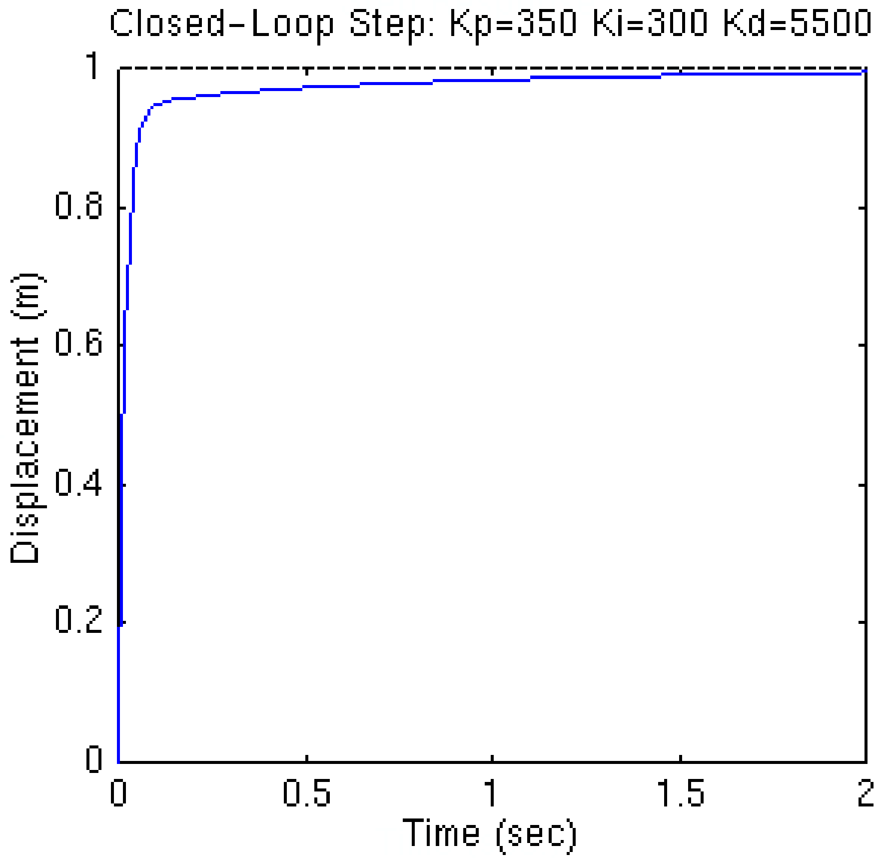
$$V = K_p e_q + K_d \dot{e_q} + K_i \int e_q(t)dt$$

$$e_q = q_d - q_a$$

$$\dot{e_q} = \frac{de_q}{dt}$$

PID control

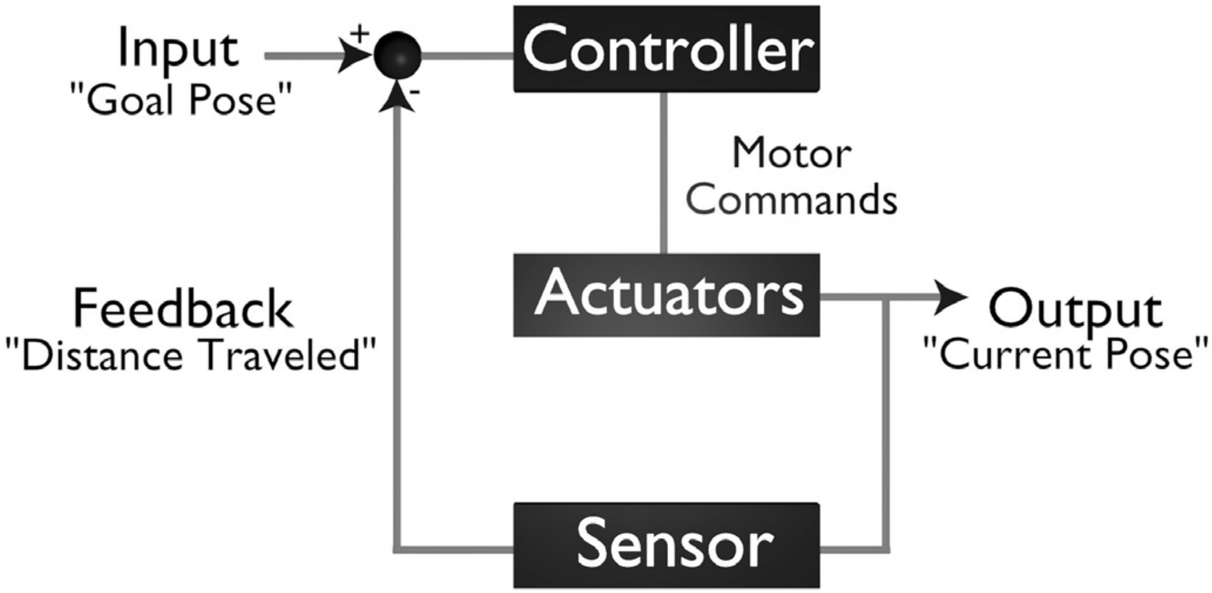
Proportional, Integral and Derivative terms:



K_p, K_d, K_i constants are set empirically or with specific methods

Video: https://www.youtube.com/watch?v=wkfEZmsQqiA

PID controller for a wall-following robot



$$V = K_p e_q + K_d \dot{e}_q + K_i \int e_q(t)dt$$

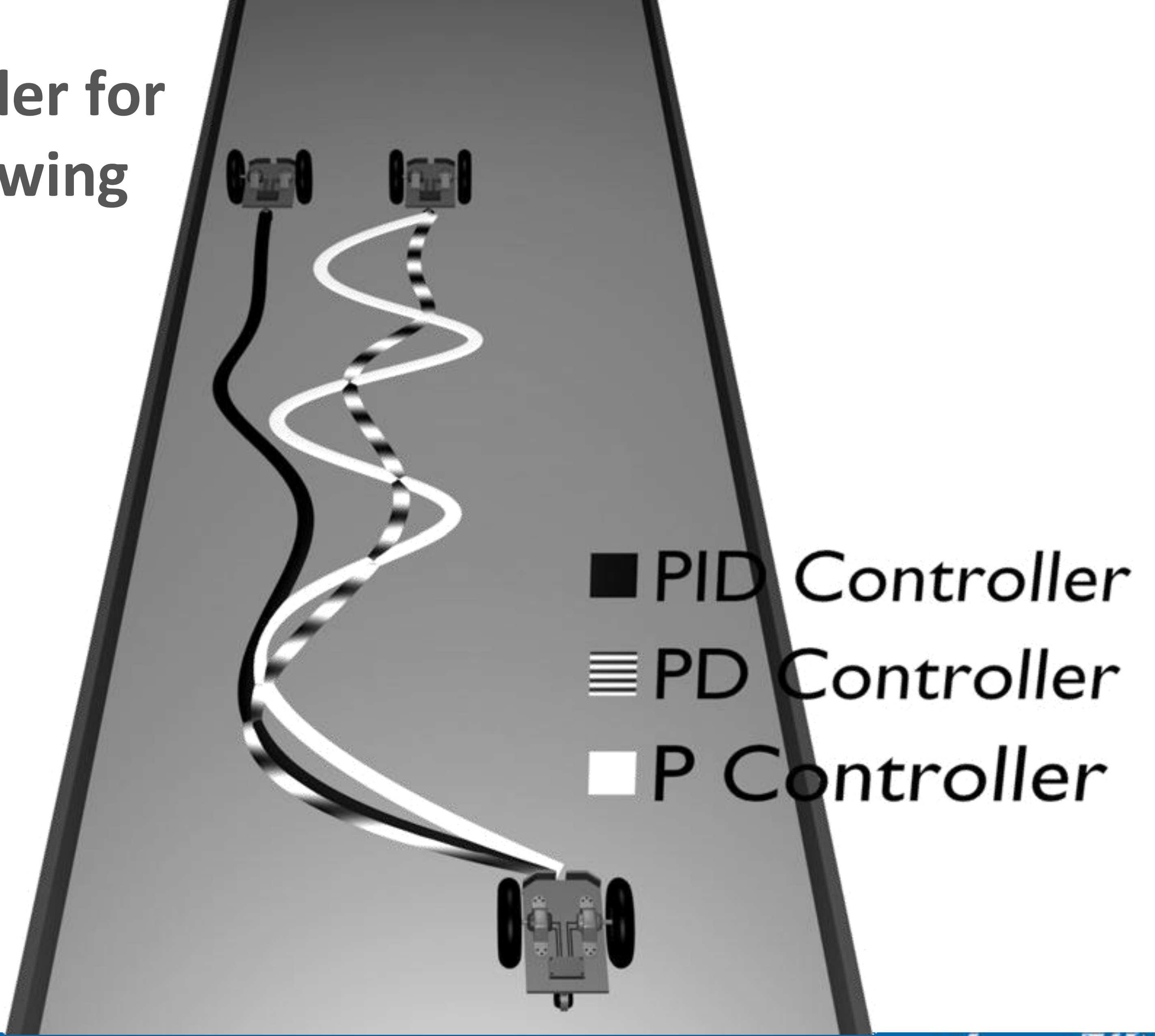
$$e_q = q_d - q_a$$

$$\dot{e}_q = \frac{de_q}{dt}$$

- e_q is the error, i.e. the difference between the desired position and the current position
- K_p is the *proportional* gain or constant
- K_i is the *integral* gain or constant
- K_d is the *derivative* gain or constant

Maja J Mataric, *The Robotics Primer*, The MIT Press, 2007

PID controller for a wall-following robot



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Robot Control

- Control of one joint motion:
  - PID controller
- **Control of the manipulator motion:**
  - Trajectory planning
  - Motion control in joint space
  - Motion control in operational space

T. Bajd et al., *Robotics*, Springer 2010

Control of robot manipulator motion

- Objective: to have the robot arm moving from a starting position to a final position, both expressed in operational space coordinates
- In general, the control problem consists in finding the torques that the actuators have to give to the joints, so that the resulting arm motion follows a planned trajectory

Robot Control

- Control of one joint motion:
  - PID controller
- Control of the manipulator motion:
  - **Trajectory planning**
  - Motion control in joint space
  - Motion control in operational space

T. Bajd et al., *Robotics*, Springer 2010

Trajectory planning

Objective: generate the reference inputs to the robot control system, which will ensure that the robot end effector will follow a desired trajectory when moving from x_{start} to x_f

- PATH: set of points, in joint space or operational space, that the robot has to reach in order to perform the desired movement

- TRAJECTORY: path with a specified time course (velocity and acceleration at each point)

Trajectory planning

Objective: generate the reference inputs to the robot control system, which will ensure that the robot end effector will follow a desired trajectory when moving from x_{start} to x_f

- INPUT DATA:
  - Path definition
  - Path constraints
  - Constraints given by the robot dynamics
- OUTPUT DATA:
  - **in joint space**: joint trajectories $\{\mathbf{q}(t),\ \dot{\mathbf{q}}(t),\ \ddot{\mathbf{q}}(t)\}$
  - **in operational space**: end-effector trajectory $\{\mathbf{p}(t),\ \mathbf{\Phi}(t),\ \mathbf{v}(t),\ \mathbf{\Omega}(t)\}$

Trajectories in joint space

- Between two points: the robot manipulator must be displaced from the initial to the final joint configuration, in a given time interval t.
- In order to give the time course of motion for each joint variable, we can choose a trapezoidal velocity profile or polynomial functions:
  - Cubic polynomial: it allows to set
    - the initial and final values of joint variables q_i and q_d
    - the initial and final velocities (usually null).
  - Fifth-degree polynomial: it allows to set
    - the initial and final values of joint variables q_i and q_d
    - the initial and final velocities
    - the initial and final accelerations.

Trajectories in joint space

Trapezoidal velocity profile:

- Constant acceleration in the starting phase
- Constant cruise velocity
- Constant deceleration in the arrival phase.

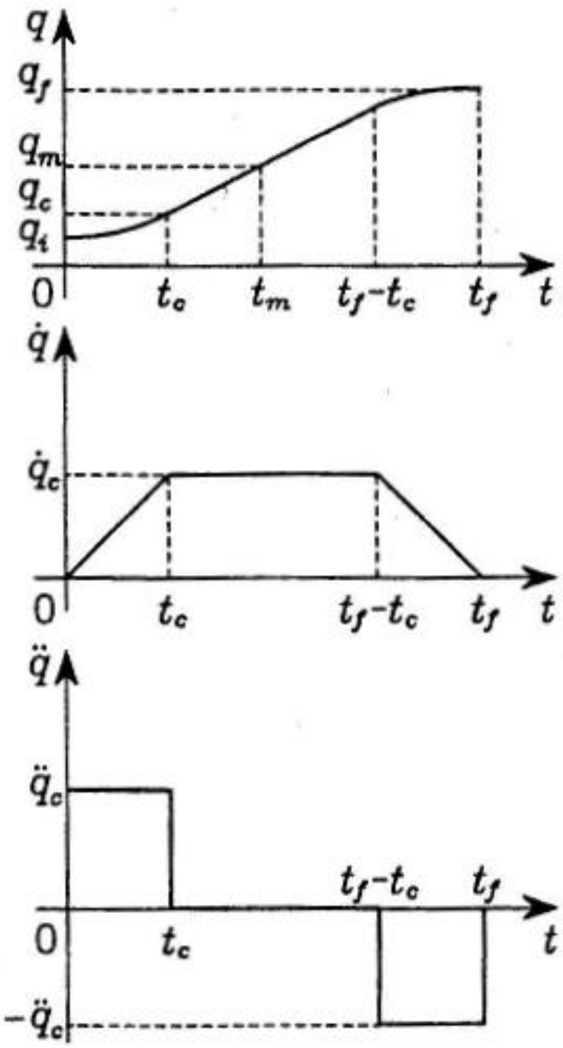
The corresponding trajectory is mixed polynomial: a linear segment connected to parabolic parts in the neighbourhood of the initial and final positions.

Trapezoidal velocity profile

Position

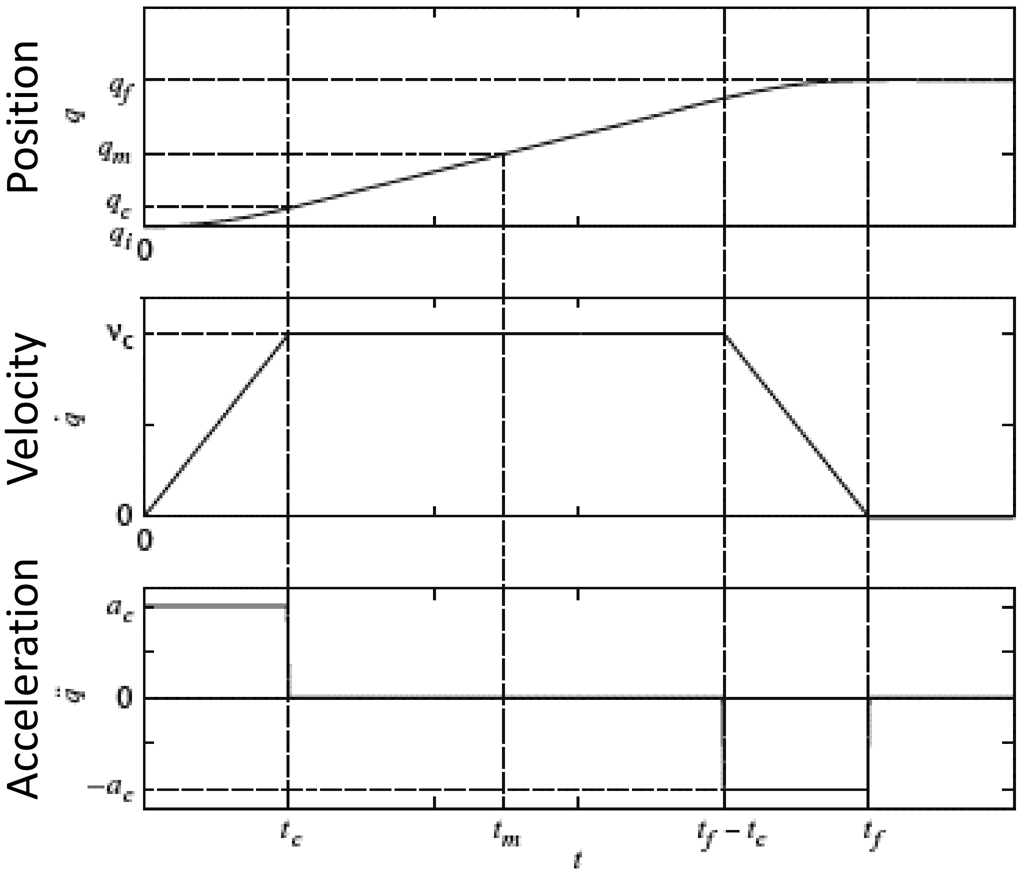
Velocity

Acceleration



Note: velocities and accelerations at the initial and final times can be different from zero

Trapezoidal velocity profile



First phase:

$$q_1(t) = q_i + \frac{1}{2}a_c t^2 \qquad 0 \le t \le t_c$$

Second phase:

$$q_2(t) = q_i + a_c t_c (t - \frac{t_c}{2}) \qquad t_c < t \le (t_f - t_c)$$

Third phase:

$$q_3 = q_f - \frac{1}{2}a_c (t - t_f)^2 \qquad (t_f - t_c) < t \le t_f$$

Trajectory interpolation

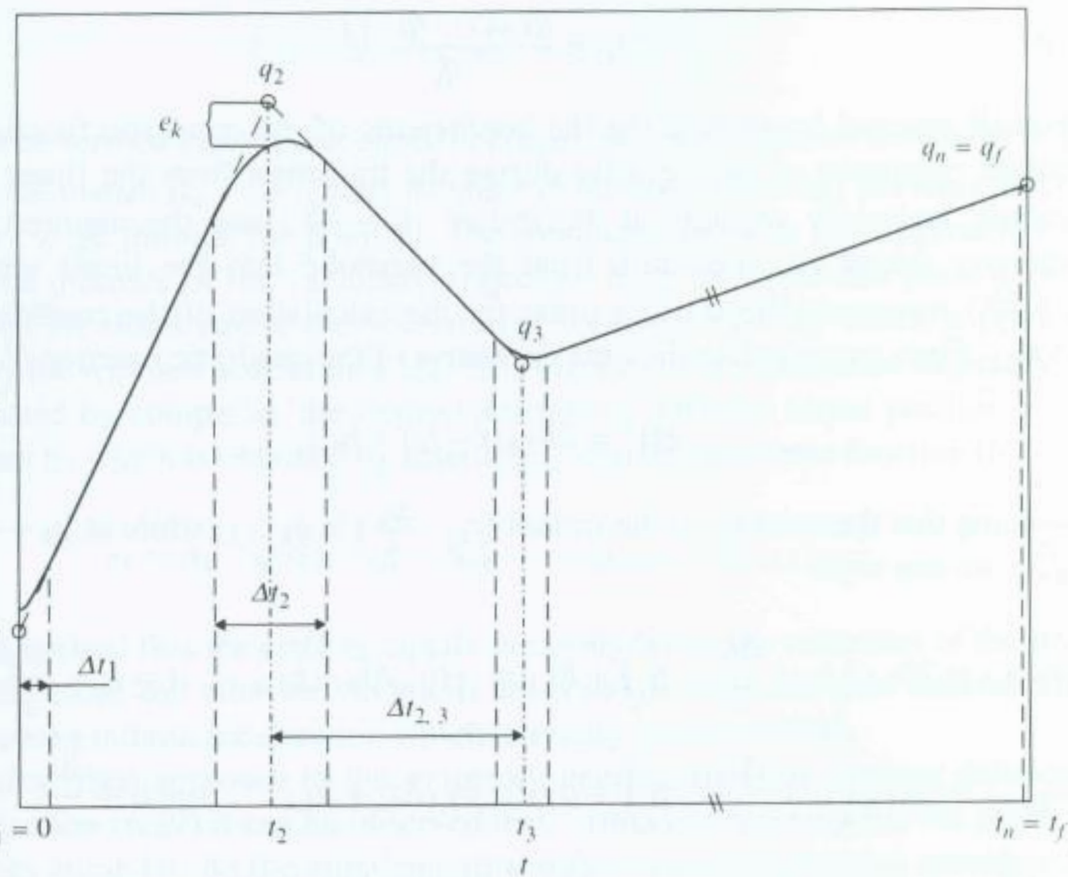


Fig. 6.2 Trajectory interpolation through n via points – linear segments with parabolic transitions are used

Trajectories in operational space

- The trajectory planning algorithm generates the time course of motion of the end effector, according to a path of geometric characteristics defined in the operational space.

- The result of planning is a sequence of n-uples: $(p(t), \Phi(t), v(t), \omega(t))$

Robot Control

- Control of one joint motion:
  - PID controller
- Control of the manipulator motion:
  - Trajectory planning
  - **Motion control in joint space**
  - Motion control in operational space

T. Bajd et al., *Robotics*, Springer 2010

Motion control in joint space

- It can be used for moving the end-effector from x_i to x_d expressed in the operational space, without taking into account the trajectory followed by the end effector
- The final position x_d is transformed in the corresponding final position in joint space q_d, by using the inverse kinematics transformation

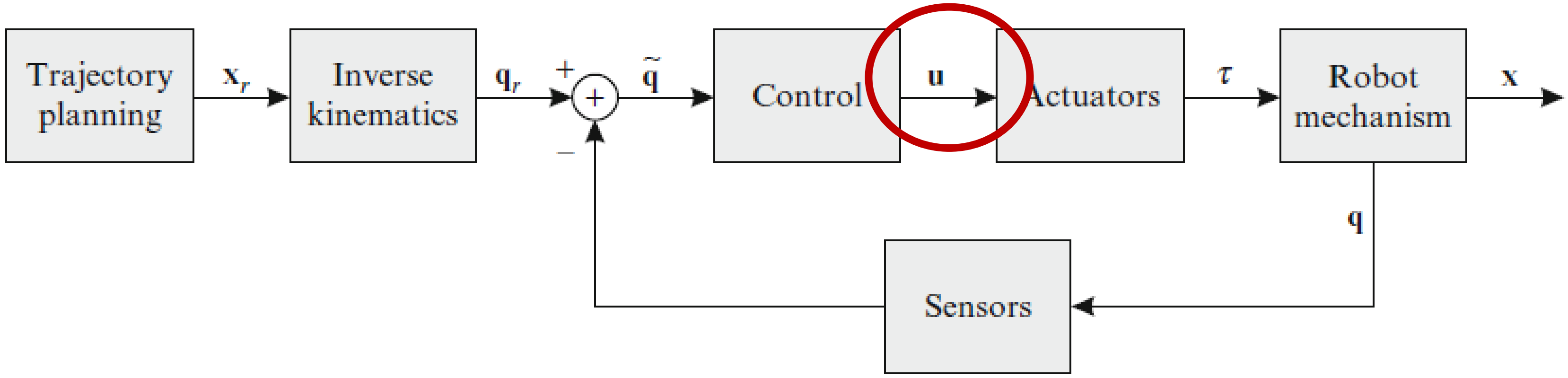
$$q_d = K^{-1} (x_d)$$

- All joints are moved from the current position q_i to the desired position q_d

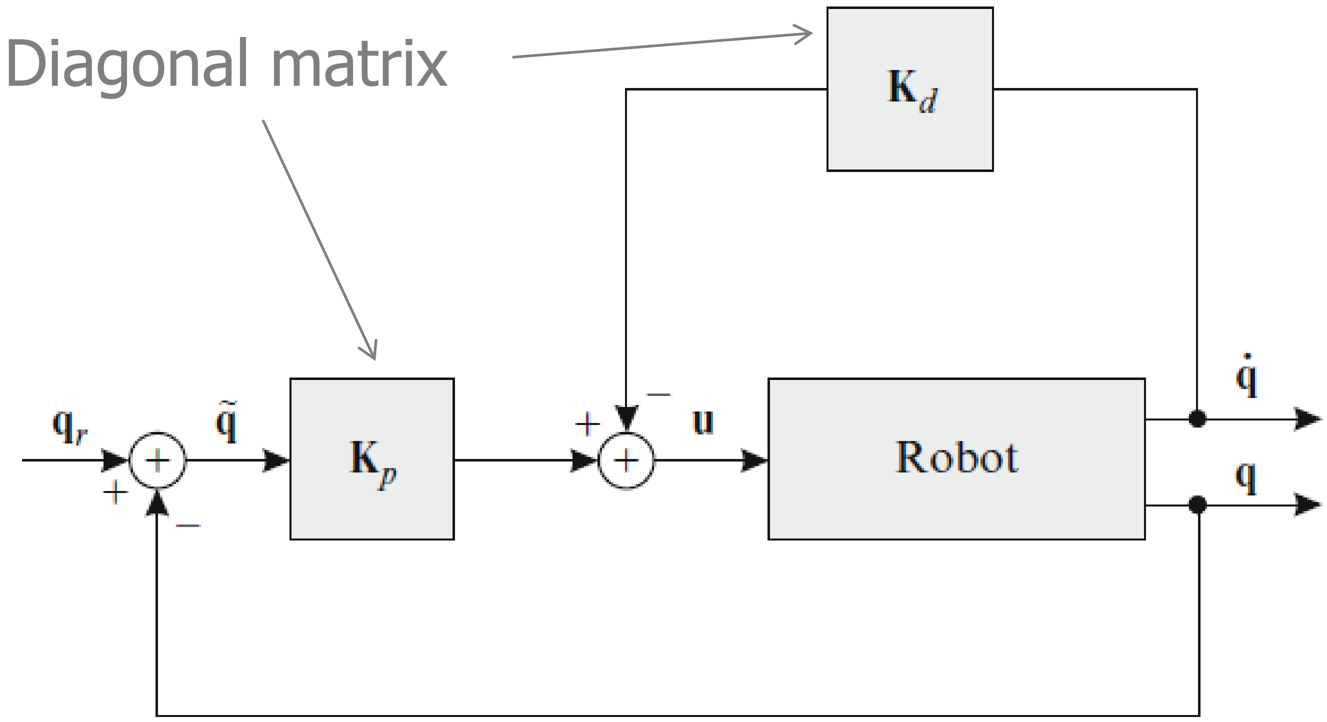
Motion control in joint space

- The trajectory of the end effector in the operational space is not controlled and it is not predictable, due to the non-linear effects of direct kinematics

General scheme of robot control in joint space

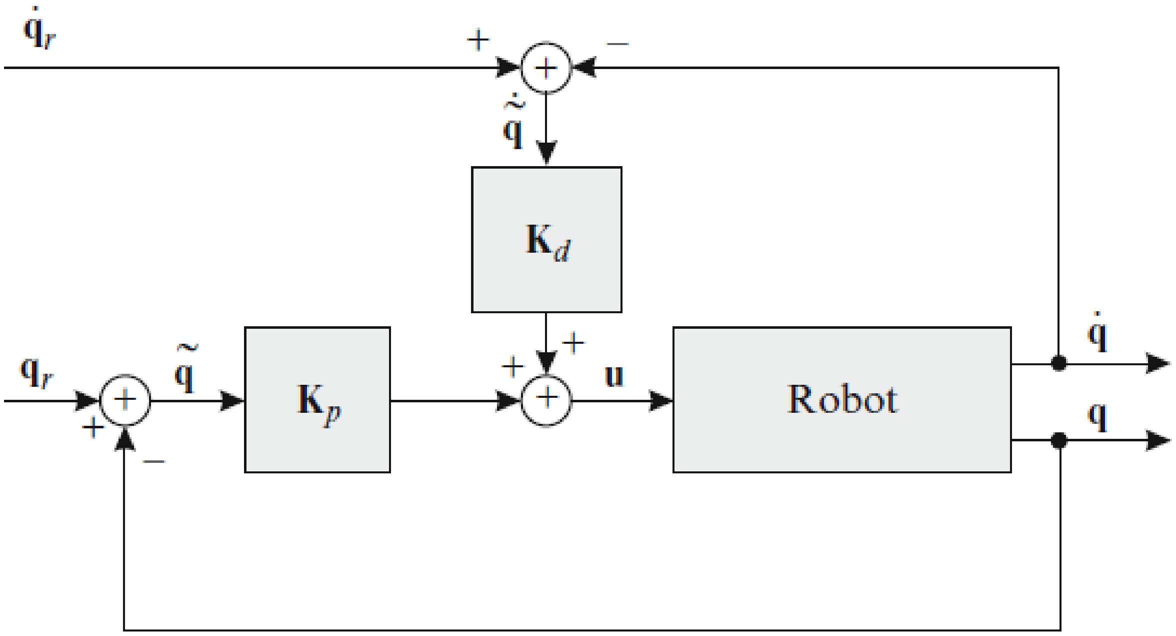


Motion control in joint space
PD position control



$$\mathbf{u} = \mathbf{K}_p(\mathbf{q}_r - \mathbf{q}) - \mathbf{K}_d\dot{\mathbf{q}},$$

Motion control in joint space
PD position control



$$\mathbf{u} = \mathbf{K}_p(\mathbf{q}_r - \mathbf{q}) + \mathbf{K}_d(\dot{\mathbf{q}}_r - \dot{\mathbf{q}})$$

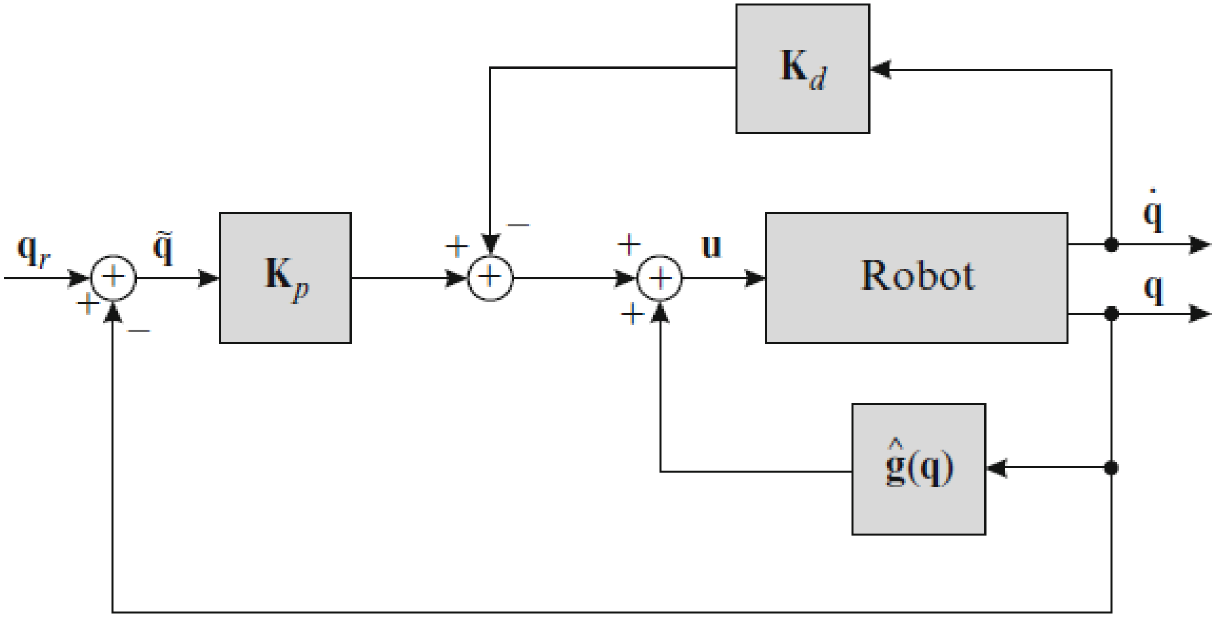
Motion control in joint space
PD position control

Setting the K_p e K_d parameter matrices:

- Fast response: high K_p
- K_d sets the best damping and guarantees a fast response without oscillations
- The K parameters needs to be set independently for each joint

Motion control in joint space
PD position control with gravity compensation

$$\mathbf{u} = \mathbf{K}_p(\mathbf{q}_r - \mathbf{q}) - \mathbf{K}_d\dot{\mathbf{q}} + \hat{\mathbf{g}}(\mathbf{q}).$$



Robot dynamic model

$\tau = B(q)\ddot{q} + C(q,\dot{q})\dot{q} + F_v\dot{q} + g(q)$ In quasi static conditions: $\tau = g(q)$

Robot dynamic model

$$\tau = B(q)\ddot{q} + C(q,\dot{q})\dot{q} + Fv\dot{q} + g(q)$$

$\tau =$ torque

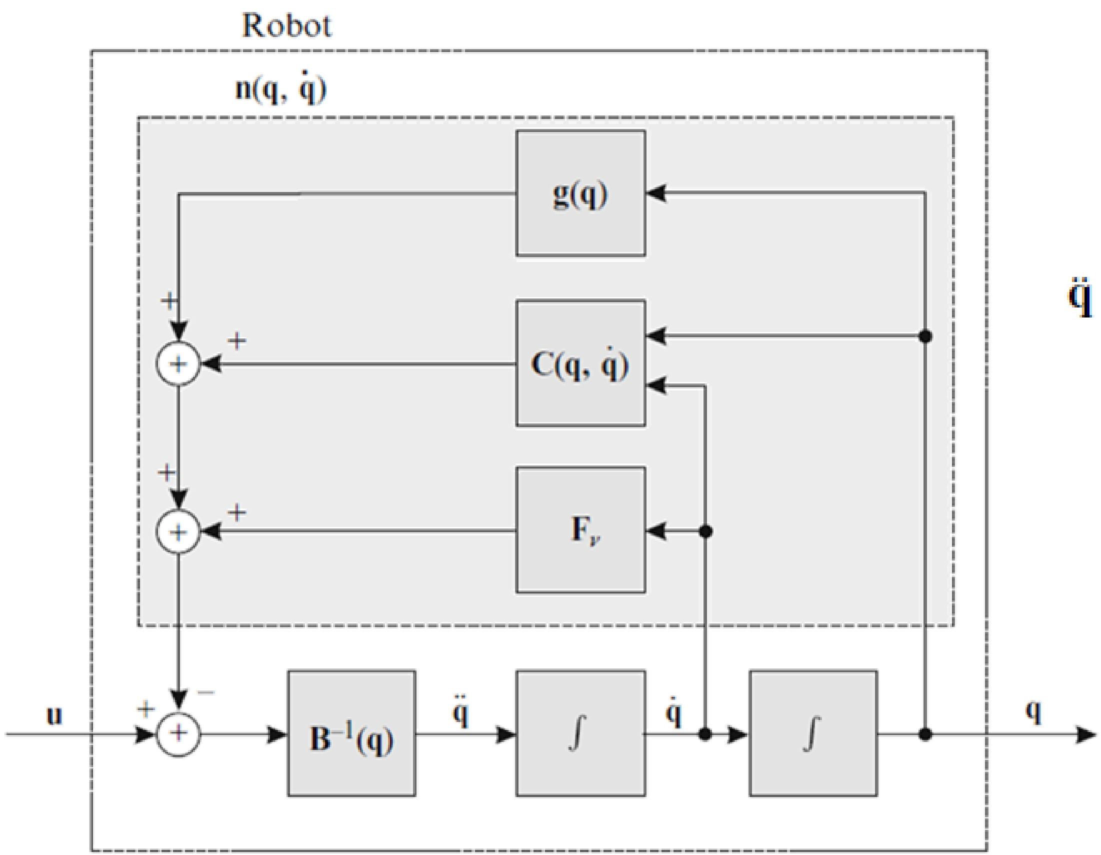
$B =$ inertia term

$C =$ Coriolis term

$F_v =$ friction coefficients

$g =$ gravity terms

Motion control in joint space
based on inverse dynamics



The direct dynamic model of a robot mechanism

$$\mathbf{B}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{F}_v\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{u}.$$

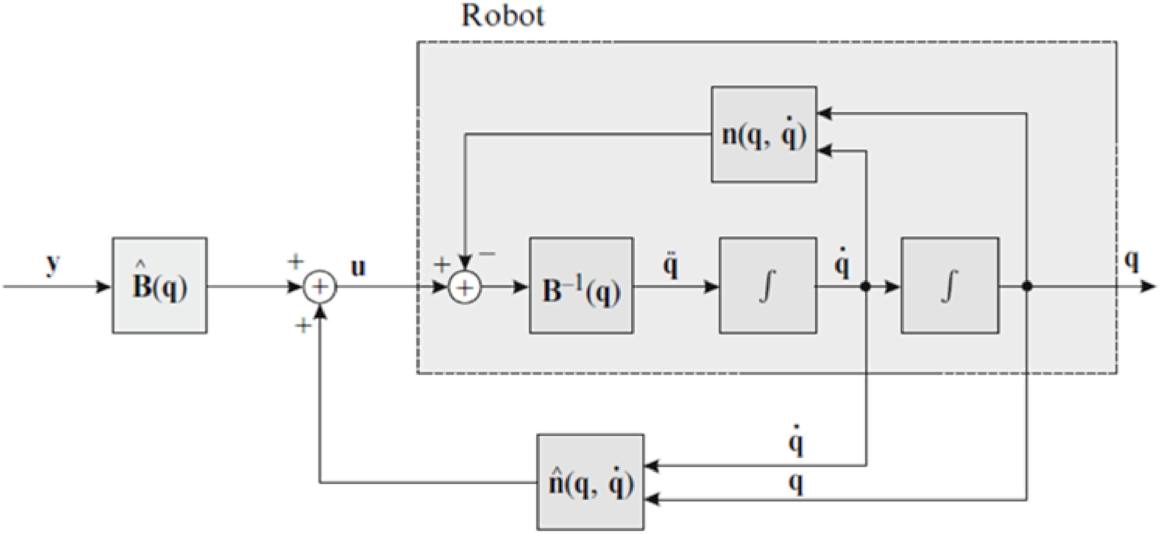
$$\ddot{\mathbf{q}} = \mathbf{B}^{-1}(\mathbf{q})\left(\mathbf{u} - \left(\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{F}_v\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q})\right)\right).$$

$$\mathbf{n}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{F}_v\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}).$$

$$\mathbf{B}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{n}(\mathbf{q},\dot{\mathbf{q}}) = \boldsymbol{\tau}.$$

$$\ddot{\mathbf{q}} = \mathbf{B}^{-1}(\mathbf{q})\left(\mathbf{u} - \mathbf{n}(\mathbf{q},\dot{\mathbf{q}})\right).$$

Motion control in joint space
based on inverse dynamics



. 7.7 Linearization of the control system by implementing the inverse dynamic model

$$\mathbf{B}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{F}_v\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{u}.$$

$$\ddot{\mathbf{q}} = \mathbf{B}^{-1}(\mathbf{q})\left(\mathbf{u} - \left(\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{F}_v\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q})\right)\right).$$

$$\mathbf{n}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{F}_v\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}).$$

Let us assume that the robot dynamic model is known. The inertial matrix $\hat{\mathbf{B}}(\mathbf{q})$ is an approximation of the real values $\mathbf{B}(\mathbf{q})$, while $\hat{\mathbf{n}}(\mathbf{q},\dot{\mathbf{q}})$ represents an approximation of $\mathbf{n}(\mathbf{q},\dot{\mathbf{q}})$ as follows

$$\hat{\mathbf{n}}(\mathbf{q},\dot{\mathbf{q}}) = \hat{\mathbf{C}}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \hat{\mathbf{F}}_v\dot{\mathbf{q}} + \hat{\mathbf{g}}(\mathbf{q}).$$

$$\mathbf{B}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{n}(\mathbf{q},\dot{\mathbf{q}}) = \boldsymbol{\tau}.$$

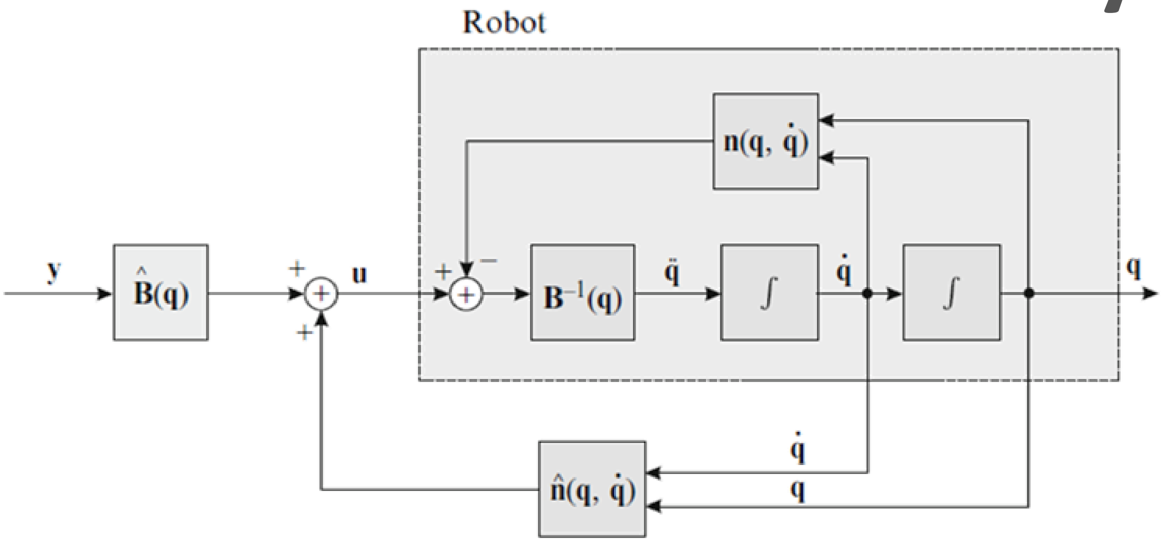
The controller output $\mathbf{u}$ is determined by the following equation

$$\mathbf{u} = \hat{\mathbf{B}}(\mathbf{q})\mathbf{y} + \hat{\mathbf{n}}(\mathbf{q},\dot{\mathbf{q}}),$$

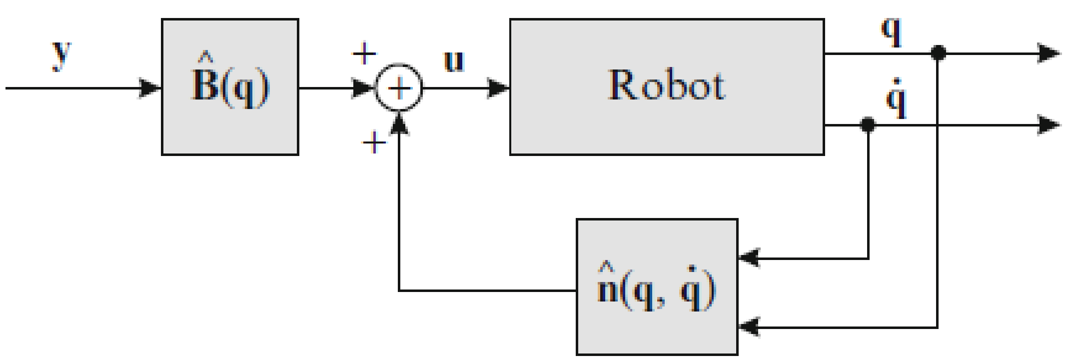
where the approximate inverse dynamic model of the robot was used.

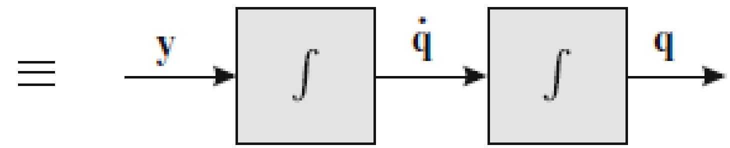
$$\ddot{\mathbf{q}} = \mathbf{B}^{-1}(\mathbf{q})\left(\mathbf{u} - \mathbf{n}(\mathbf{q},\dot{\mathbf{q}})\right).$$

Motion control in joint space
based on inverse dynamics



7.7 Linearization of the control system by implementing the inverse dynamic model





$$\mathbf{y} = \ddot{\mathbf{q}}_r + \mathbf{K}_p(\mathbf{q}_r - \mathbf{q}) + \mathbf{K}_d(\dot{\mathbf{q}}_r - \dot{\mathbf{q}}).$$

. 7.8 The linearized system

Motion control in joint space based on inverse dynamics

$$\mathbf{y} = \ddot{\mathbf{q}}_r + \mathbf{K}_P(\mathbf{q}_r - \mathbf{q}) + \mathbf{K}_d(\dot{\mathbf{q}}_r - \dot{\mathbf{q}}).$$

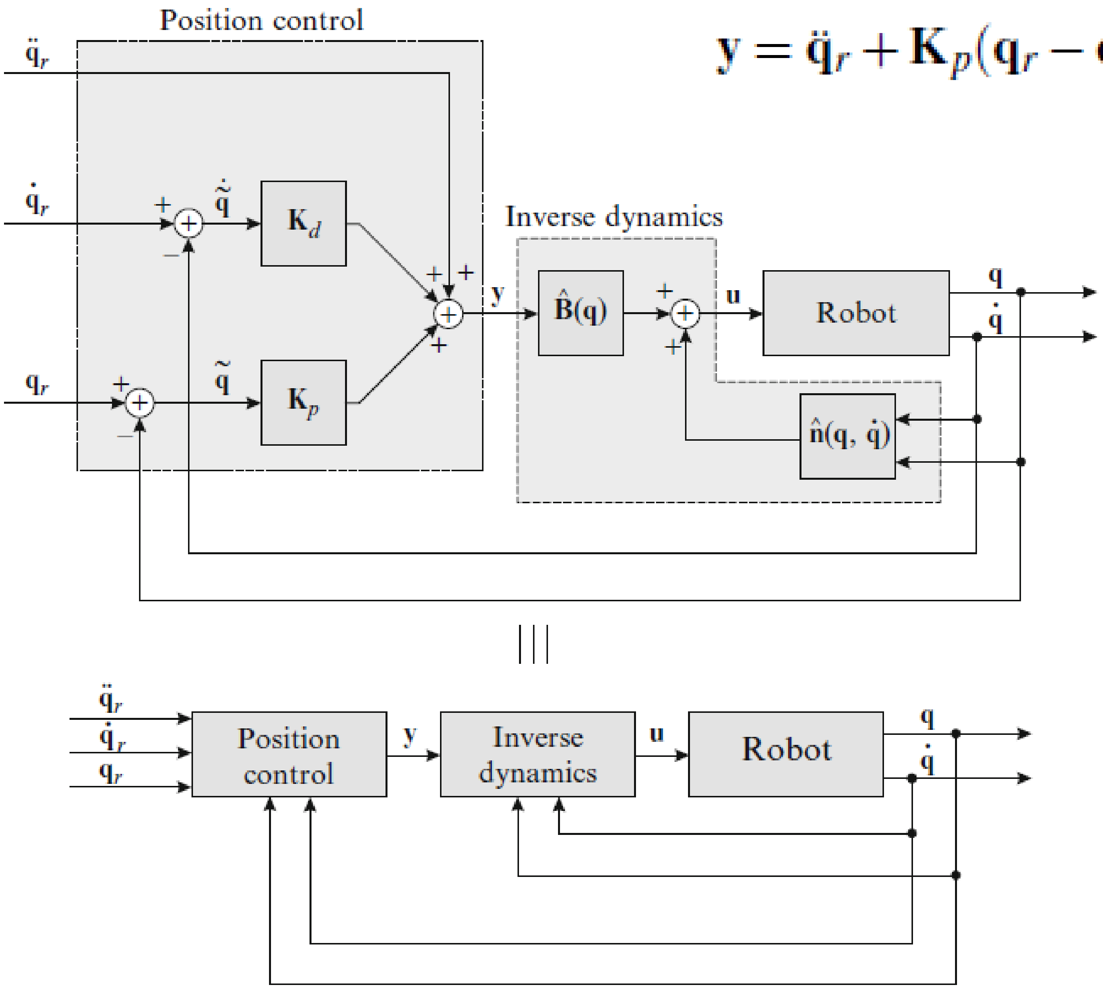


Fig. 7.9 Control of the robot based on inverse dynamics

Robot Control

- Control of one joint motion:
  - PID controller
- Control of the manipulator motion:
  - Trajectory planning
  - Motion control in joint space
  - **Motion control in operational space**

T. Bajd et al., *Robotics*, Springer 2010

Motion control in operational space

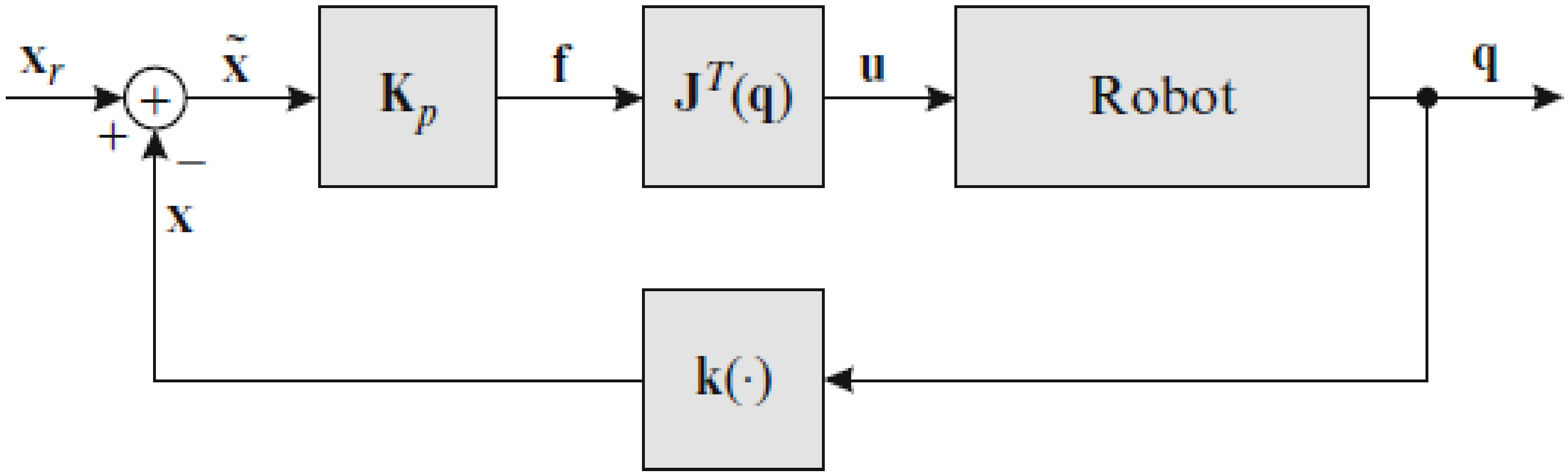
- In the movement from x_i to x_d the robot end effector follows a trajectory in the operational space, according to a planned time law.
- e.g. linear or curvilinear trajectory

Motion control in operational space

- To make the robot follow a trajectory

  $(t, p(t), \Phi(t), \dot{p}(t), \omega(t))$
- To set joint velocities and accelerations in time, in order to reach the final desired position, expressed in Cartesian coordinates (Jacobian)
- To set voltages and currents to give to the motors in order to apply to the joints the velocities and the accelerations calculated with the Jacobian

Motion control in operational space based on the transposed Jacobian matrix



Control based on the transposed Jacobian matrix

$$\mathbf{f} = \mathbf{K}_p \tilde{\mathbf{x}}. \qquad \mathbf{u} = \mathbf{J}^T(\mathbf{q})\mathbf{f}.$$

f = force at end effector

Motion control in operational space based on the inverse Jacobian matrix

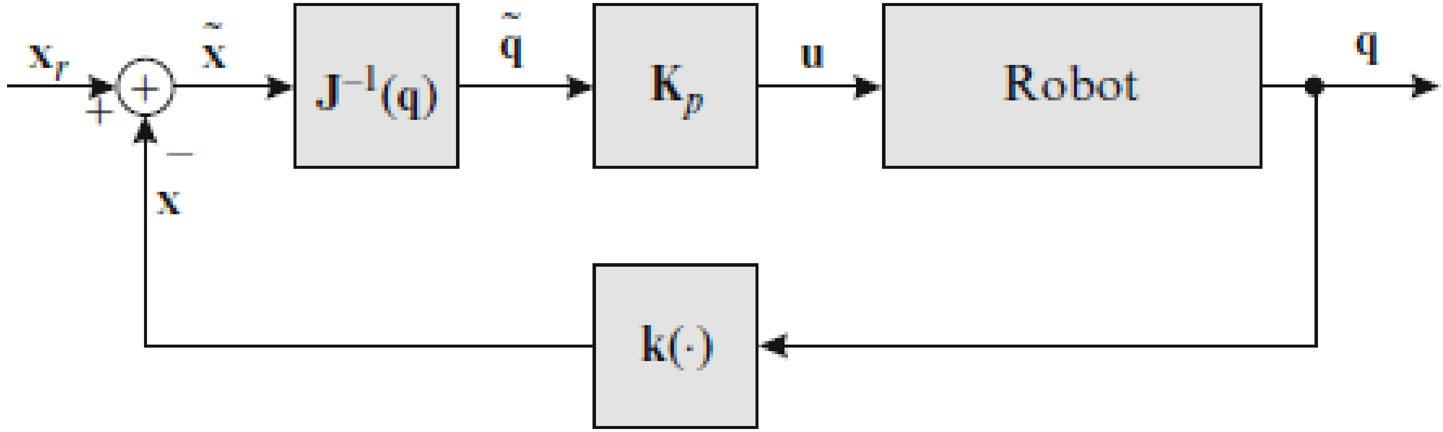


Fig. 7.11 Control based on the inverse Jacobian matrix

$$\dot{\mathbf{x}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} \quad \Leftrightarrow \quad \frac{d\mathbf{x}}{dt} = \mathbf{J}(\mathbf{q})\frac{d\mathbf{q}}{dt}.$$

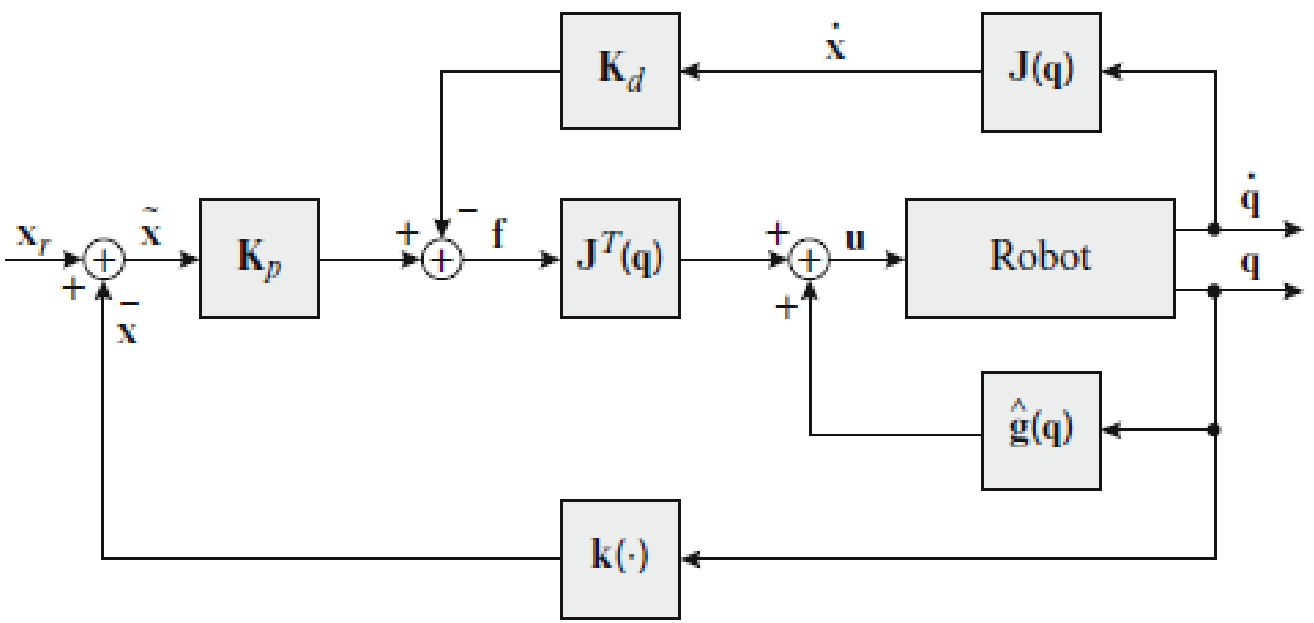
$$d\mathbf{x} = \mathbf{J}(\mathbf{q})d\mathbf{q}.$$

for small displacements

$$\tilde{\mathbf{q}} = \mathbf{J}^{-1}(\mathbf{q})\tilde{\mathbf{x}}.$$

$$\mathbf{u} = \mathbf{K}_p\tilde{\mathbf{q}}.$$

Motion control in operational space
PD control with gravity compensation



PD control with gravity compensation in external coordinates

$$\dot{\mathbf{x}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}.$$

$$\mathbf{f} = \mathbf{K}_p\tilde{\mathbf{x}} - \mathbf{K}_d\dot{\mathbf{x}}.$$

$$\mathbf{u} = \mathbf{J}^T(\mathbf{q})\mathbf{f} + \hat{\mathbf{g}}(\mathbf{q}).$$

Motion control in operational space based on inverse dynamics

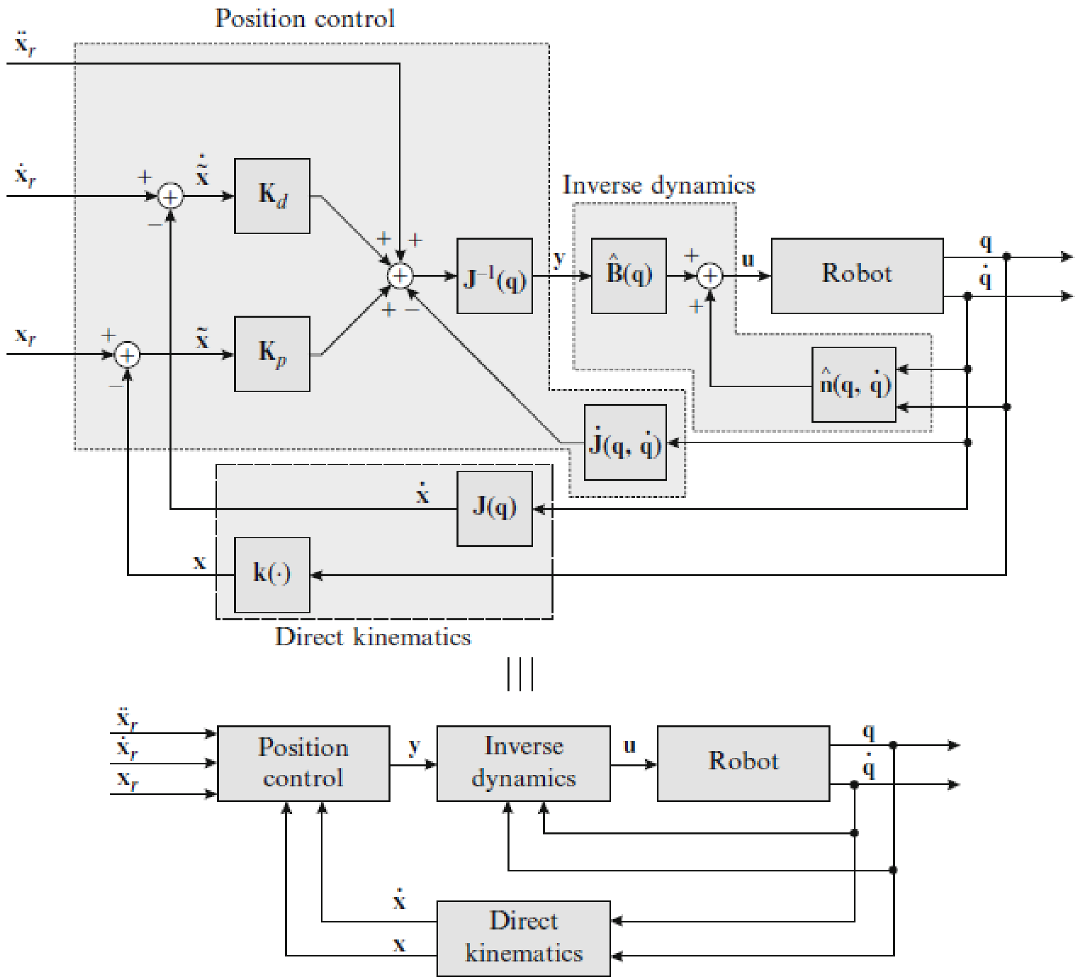


Fig. 7.13 Robot control based on inverse dynamics in external coordinates

$$\dot{\mathbf{x}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}. \tag{7.33}$$

By calculating the time derivative of equation (7.33), we obtain

$$\ddot{\mathbf{x}} = \mathbf{J}(\mathbf{q})\ddot{\mathbf{q}} + \dot{\mathbf{J}}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}. \tag{7.34}$$

The error of the pose of the robot end-effector is determined as the difference between its desired and its actual pose

$$\tilde{\mathbf{x}} = \mathbf{x}_r - \mathbf{x} = \mathbf{x}_r - \mathbf{k}(\mathbf{q}). \tag{7.35}$$

In a similar way the velocity error of the robot end-effector is determined

$$\dot{\tilde{\mathbf{x}}} = \dot{\mathbf{x}}_r - \dot{\mathbf{x}} = \dot{\mathbf{x}}_r - \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}. \tag{7.36}$$

The acceleration error is the difference between the desired and the actual acceleration

$$\ddot{\tilde{\mathbf{x}}} = \ddot{\mathbf{x}}_r - \ddot{\mathbf{x}}. \tag{7.37}$$

When developing the inverse dynamics based controller in the internal coordinates, equation (7.19) was derived describing the dynamics of the control error in the form $\ddot{\tilde{\mathbf{q}}} + \mathbf{K}_d\dot{\tilde{\mathbf{q}}} + \mathbf{K}_p\tilde{\mathbf{q}} = \mathbf{0}$. An analogous equation can be written for the error of the end-effector pose. From this equation the acceleration $\ddot{\mathbf{x}}$ of the robot end-effector can be expressed

$$\ddot{\tilde{\mathbf{x}}} + \mathbf{K}_d\dot{\tilde{\mathbf{x}}} + \mathbf{K}_p\tilde{\mathbf{x}} = \mathbf{0} \quad \Rightarrow \quad \ddot{\mathbf{x}} = \ddot{\mathbf{x}}_r + \mathbf{K}_d\dot{\tilde{\mathbf{x}}} + \mathbf{K}_p\tilde{\mathbf{x}}. \tag{7.38}$$

From equation (7.34) we express $\ddot{\mathbf{q}}$ taking into account the equality $\mathbf{y} = \ddot{\mathbf{q}}$

$$\mathbf{y} = \mathbf{J}^{-1}(\mathbf{q})\left(\ddot{\mathbf{x}} - \dot{\mathbf{J}}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}\right). \tag{7.39}$$

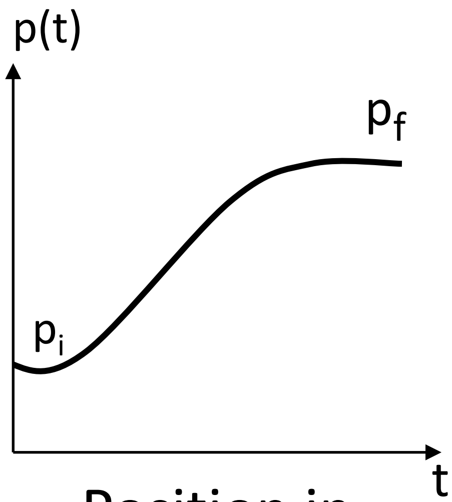
By replacing $\ddot{\mathbf{x}}$ in equation (7.39) with expression (7.38), the control algorithm based on inverse dynamics in the external coordinates is obtained

$$\mathbf{y} = \mathbf{J}^{-1}(\mathbf{q})\left(\ddot{\mathbf{x}}_r + \mathbf{K}_d\dot{\tilde{\mathbf{x}}} + \mathbf{K}_p\tilde{\mathbf{x}} - \dot{\mathbf{J}}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}\right). \tag{7.40}$$

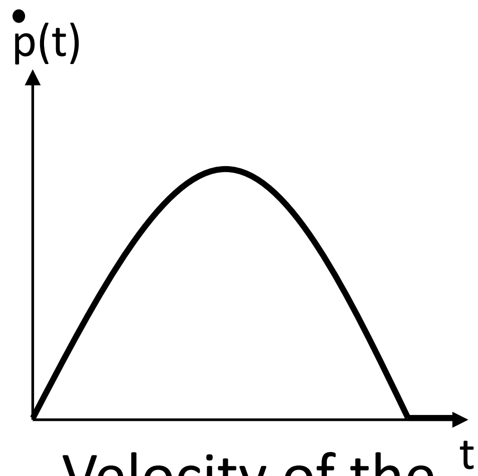
The control scheme encompassing the linearization of the system based on inverse dynamics (7.31) and the closed loop control (7.40) is shown in Figure 7.13.

Motion control in operational space

Trajectory planning | **Joint velocities**

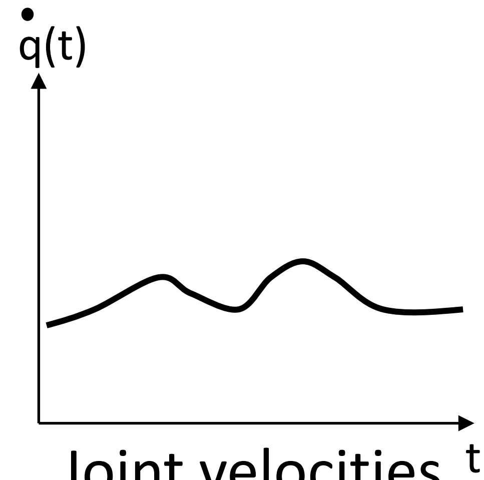


Position in operational space, in time



Velocity of the end effector, in time

$J^{-1}(p(t))$ →



Joint velocities, in time

$(t, p(t), \Phi(t), \dot{p}(t), \omega(t))$ $\xrightarrow{J^{-1}(p(t))}$ $(t, \dot{q}(t))$

Force control

$$\mathbf{B}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{F}_v\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau} - \mathbf{J}^T(\mathbf{q})\mathbf{f}.$$

Effect of external forces

$$\mathbf{n}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{F}\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}),$$

$$\mathbf{B}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{n}(\mathbf{q},\dot{\mathbf{q}}) = \boldsymbol{\tau} - \mathbf{J}^T(\mathbf{q})\mathbf{f}.$$

$$\mathbf{B}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{n}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{J}^T(\mathbf{q})\mathbf{f} = \mathbf{u}.$$

$$\ddot{\mathbf{q}} = \mathbf{B}^{-1}(\mathbf{q})\left(\mathbf{u} - \mathbf{n}(\mathbf{q},\dot{\mathbf{q}}) - \mathbf{J}^T(\mathbf{q})\mathbf{f}\right).$$

$$\mathbf{u} = \hat{\mathbf{B}}(\mathbf{q})\mathbf{y} + \hat{\mathbf{n}}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{J}^T(\mathbf{q})\mathbf{f},$$

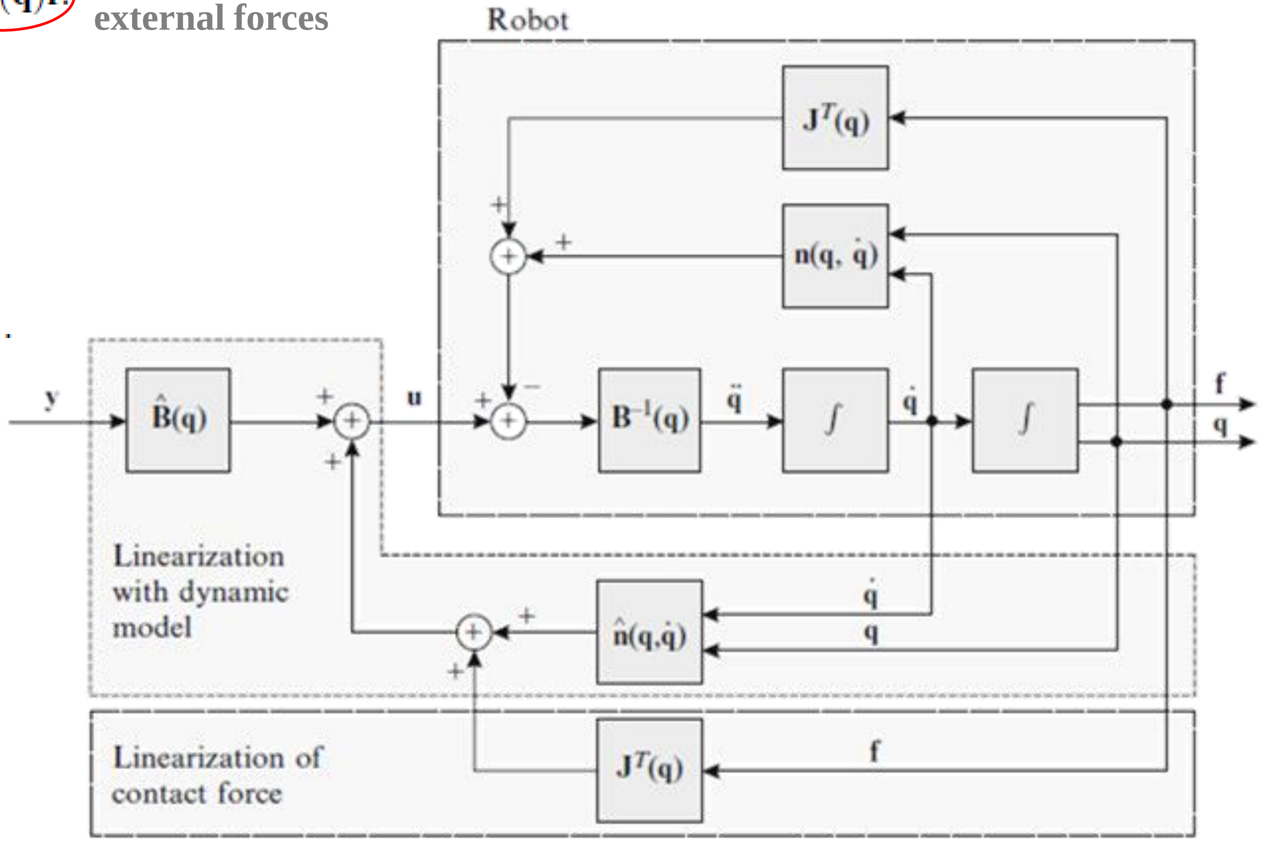


Fig. 7.14 Linearization of the control system by implementing the inverse dynamic model and the measured contact force

Force control

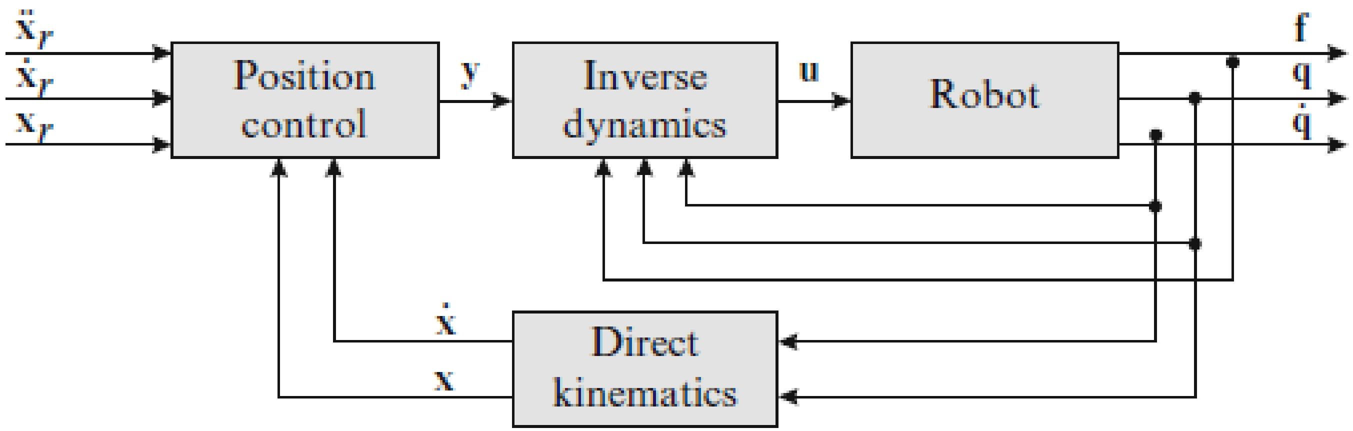


Fig. 7.15 Robot control based on inverse dynamics in external coordinates including the contact force

Force control

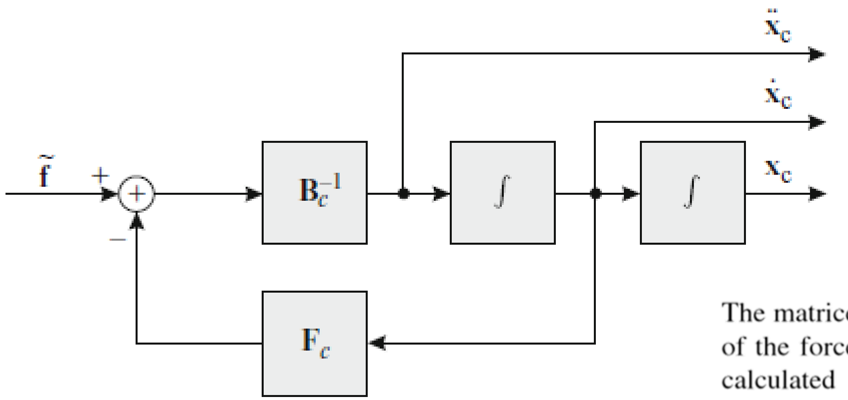


Fig. 7.16 Force control translated into control of the pose of robot end-effector

$$\tilde{\mathbf{f}} = \mathbf{B}_c\ddot{\mathbf{x}}_c + \mathbf{F}_c\dot{\mathbf{x}}_c. \tag{7.48}$$

The matrices $\mathbf{B}_c$ and $\mathbf{F}_c$ determine the movement of the object under the influence of the force $\tilde{\mathbf{f}}$. From equation (7.48) the acceleration of the virtual object can be calculated

$$\ddot{\mathbf{x}}_c = \mathbf{B}_c^{-1}\left(\tilde{\mathbf{f}} - \mathbf{F}_c\dot{\mathbf{x}}_c\right). \tag{7.49}$$

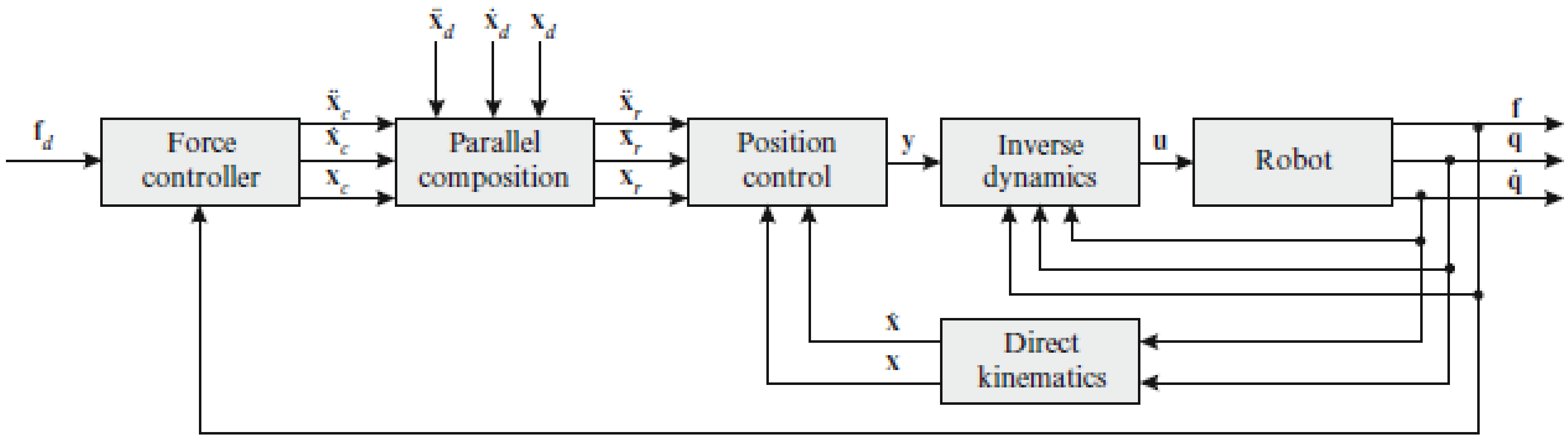


Fig. 7.17 Direct force control in the external coordinates