



Neural Networks and Neurocontrollers





Outline

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- ✓ Introduction to Neural Network
 - ✓ Biological Neuron
 - ✓ Artificial Neural Network

- ✓ Supervised Learning
 - ✓ Perceptron
 - ✓ Multilayer Perceptron
 - ✓ Back Propagation
 - ✓ Recurrent Neural Network

- ✓ Unsupervised Learning

- ✓ Competitive Learning
 - ✓ Kohonen Networks

- ✓ Reinforcement Learning

- ✓ Neurocontrollers





History

- **Artificial Neural Networks** (ANNs) are an abstract simulation of the nervous system, which contains a set of neurons exchanging information through connections (*axons*)
- The ANN model try to mimic axons and dendrites of the nervous system.
- The first neural model was proposed by McCulloch and Pitts (1943). The model was presented as a computational model of the nervous activity. After this, other models were proposed John von Neumann, Marvin Minsky, Frank Rosenblatt, etc.

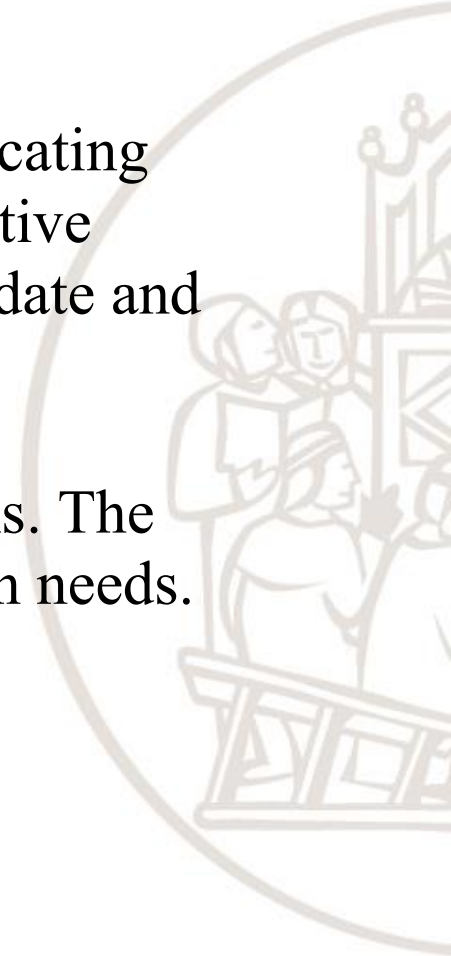




Two types of neuron models...

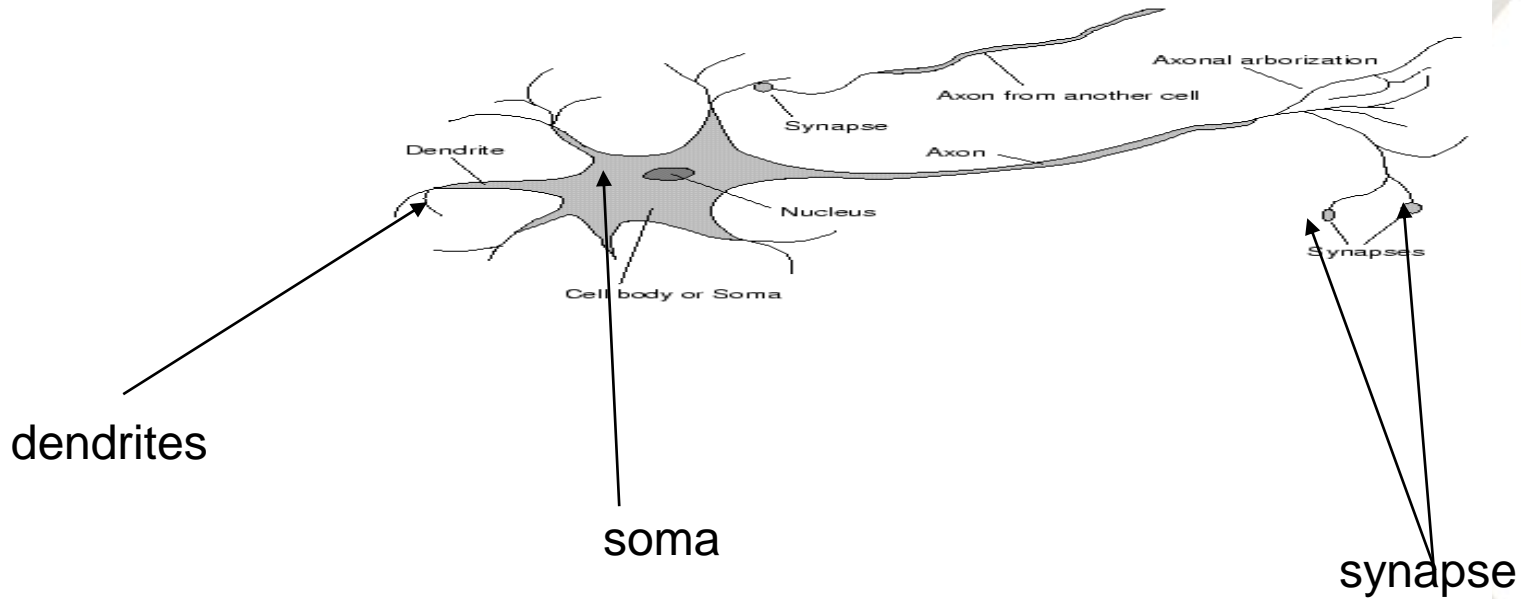
- Biological model. It has the objective of replicating biological neural systems, i.e. visual and auditive functionalities. These models are used to validate and verify hypothesis about biological systems.
- The second type is focused on the applications. The models are strongly influenced by application needs. They are called connectionist architectures.

We will focus on the second one!





Biological neuron





Biological neuron

- Human brain contain **100 million neurons**. Neuroscientific evidences show each neuron can have 10000 sinapses in input or output
- Switching time of a neuron is few **milliseconds**. It is slower than a logic gate, but it has a greater connectivity
- A neuron receives from synapses information which are summed
- If the excitatory signal is leading, the neuron is activated and it generates information through the synapse





Neural Network structure

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A neural network is composed by:

- A set of nodes (**neurons**), which is the basic unit
- A set of **weights** linked to connections.
- A set of **thresholds** or activation levels

The network **design** requires:

1. Number of basic unit.
2. Morphological structure.
3. Learning example encoding (input and output of the net)
4. Initialization and training of the weights linked to the connections, through a learning example set.





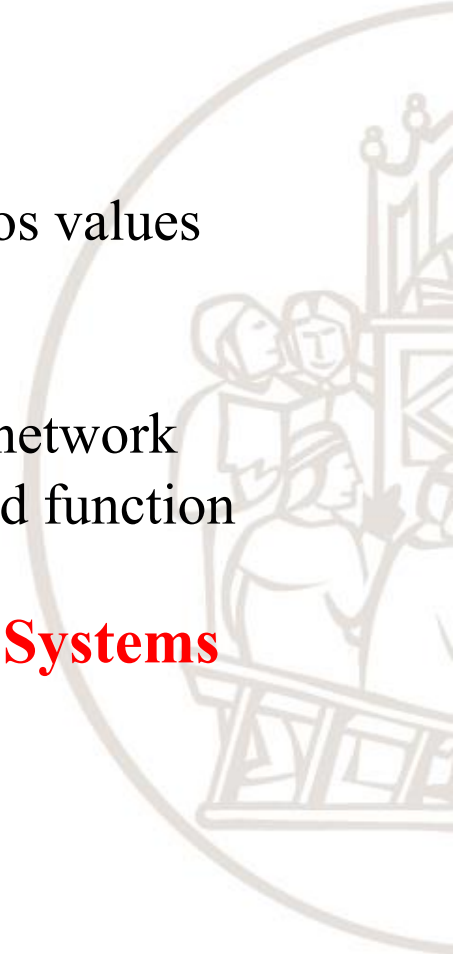
Neural network applications

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Main features:

- The objective function can have discrete/continuous values
- Learning data can be noisy
- Learning time is NOT real-time
- Fast evaluation of the learning rate of the neural network
- It is not crucial to get the **semantics** of the learned function

Robotics, Image Understanding, Biological Systems





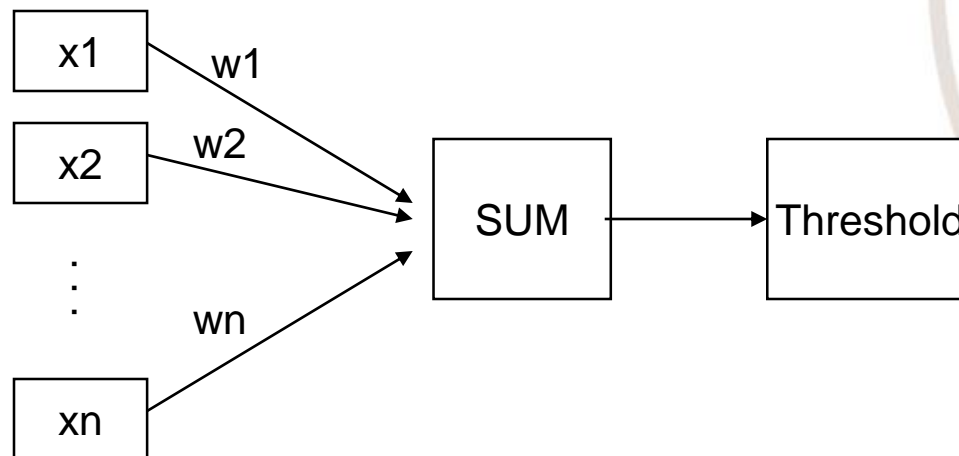
- **Supervised Learning**
 - MLP and recurrent NN
- **Unsupervised Learning**
 - Clustering
- **Competitive Learning**
 - Kohonen networks
- **Reinforcement Learning**





The perceptron

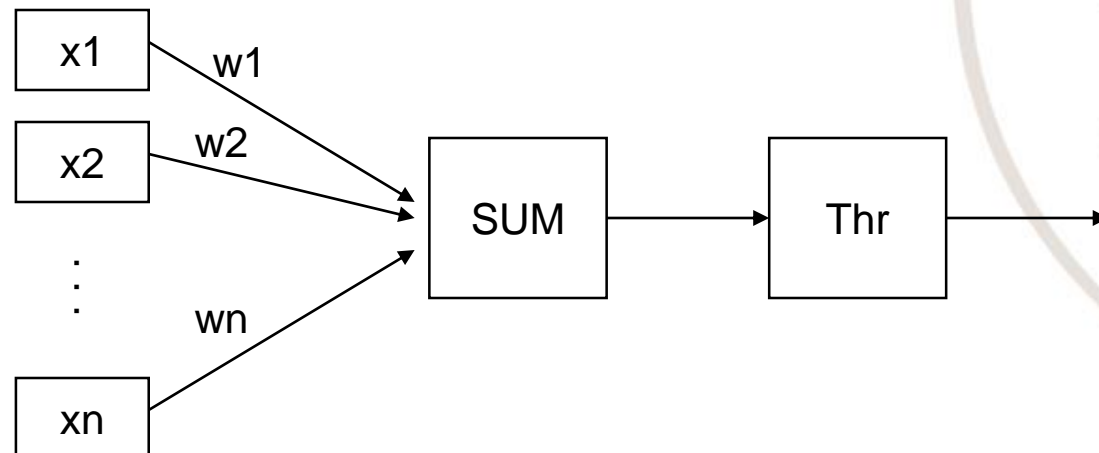
- The perceptron is the neural network basic unit
- It was defined by Rosenblatt (1962)
- Try to replicate the single neuron function





The perceptron

- Output values are boolean: 0 – 1
- Inputs x_i and weights w_i are real (positive or negative)
- Learning consists in selecting value for weights and threshold

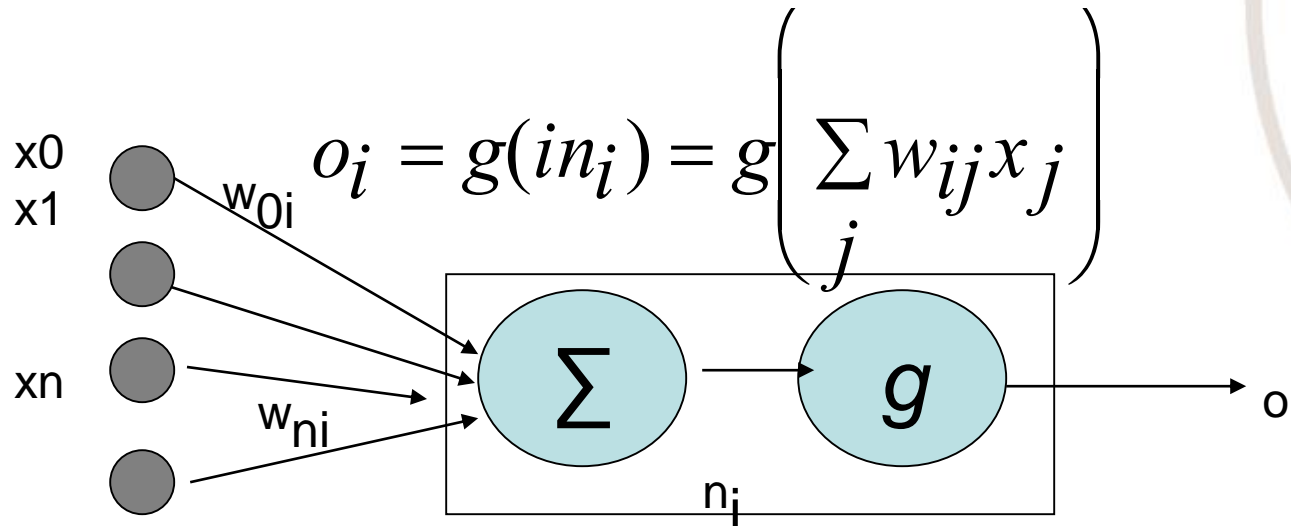




a) **Input function**, *linear (SUM)*

$$in_i = \sum_j w_{ij} x_j = w_i x_i$$

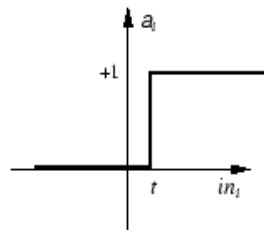
b) **Activation function**, *non linear (THRESHOLD)*



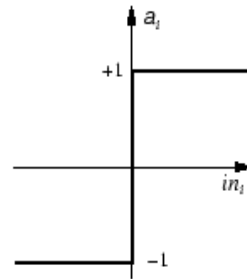


Activation functions

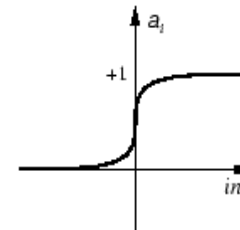
$$step_t(x) = \begin{cases} 1, & \text{if } x > t \\ 0, & \text{else} \end{cases}$$



(a) Step function



(b) Sign function



(c) Sigmoid function

$$sign(x) = \begin{cases} +1, & \text{if } x \geq 0 \\ -1, & \text{else} \end{cases}$$

$$sigmoid(x) = \frac{1}{1 + e^{-x}}$$





Objective function

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- If the threshold function is $sign()$ and $x_1..x_n$ are the input values:

$$o(x) = 1 \text{ if } w_0 + w_1x_1 + w_2x_2 + \dots + w_nx_n > 0$$

$$o(x) = -1 \text{ else}$$

- Vector notation:

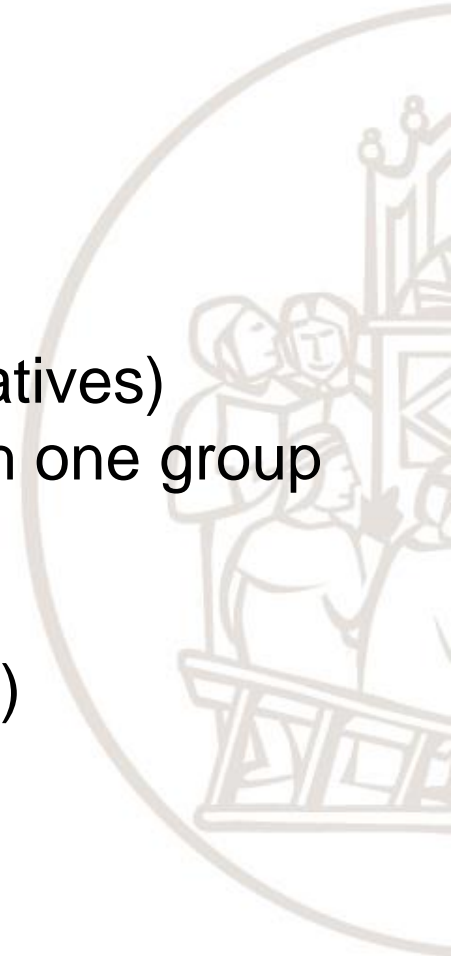
$$o(\vec{x}) = sign(\vec{w} \cdot \vec{x})$$





The Perceptron (classification generalization)

- Learning problem:
 - Set of points in a *n-dimensional* space
 - 1 classify into two groups (positives and negatives)
 - 2 Then, given a new point P , associate P with one group
 - 1 Classification problem
 - 2 generalization problem (learning concepts)





Perceptron – Training algorithm

- Initialize weights randomly
- Gives an example from the dataset $\langle x, c(x) \rangle$
- Compute $o(x)$
- IF $o(x) \neq c(x)$ then update:
- η is the *learning rate*
- x_i is the i th feature value of x
- The perceptron error (E) is equal to $(c-o)$

$$w_i \leftarrow w_i + \Delta w_i$$

$$\Delta w_i = \eta (c(x) - o(x)) x_i$$



Example test

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- Suppose that $o(x)=-1$ (if the threshold function is $\text{sign}(x)$) and $c(x)=1$
- It is needed to modify weights
- Example:

$$x_i = 0,8, \quad \eta = 0,1, \quad c = 1, \quad o = -1$$

$$\Delta w_i = \eta(c - o)x_i = 0,1(1 - (-1))0,8 = 0,16$$

- The w_i value increases in order to reduce the error
- IF $c(x)=-1$ e $o(x)=1$

$$\Delta w_i = \eta(c - o)x_i = 0,1(-1 - (+1))0,8 = -0,16$$





The perceptron

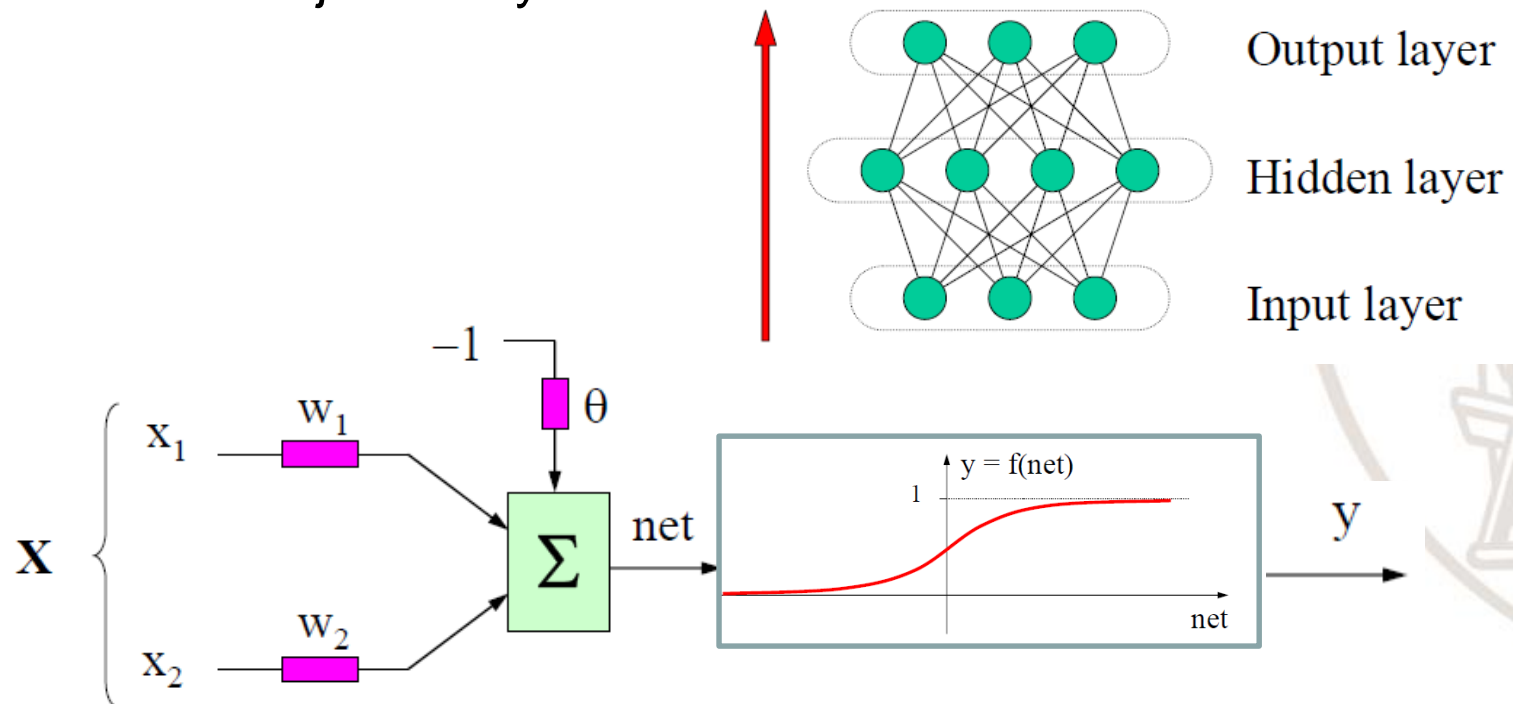
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- Convergence theorem of *perceptron* (Roseblatt, 1962)
 - The perceptron is a linear classifier, therefore it will never get to the state with all the input vectors classified correctly if the training set is not linearly separable
 - In other words.. No local minima!
- The way to solve nonlinear problems is using multiple layers
- Solution: **Feed forward neural network and recurrent neural network**



Supervised learning MLP networks

- All the neurons of a layer are connected to all the neurons of the next layer
- There are no connections between neurons in the same layer and between non adjacent layers





Feed forward neural network: back propagation algorithm

- Objectives:
 - Perceptron weights initialized randomly
 - Fast learning
 - Generalization capability

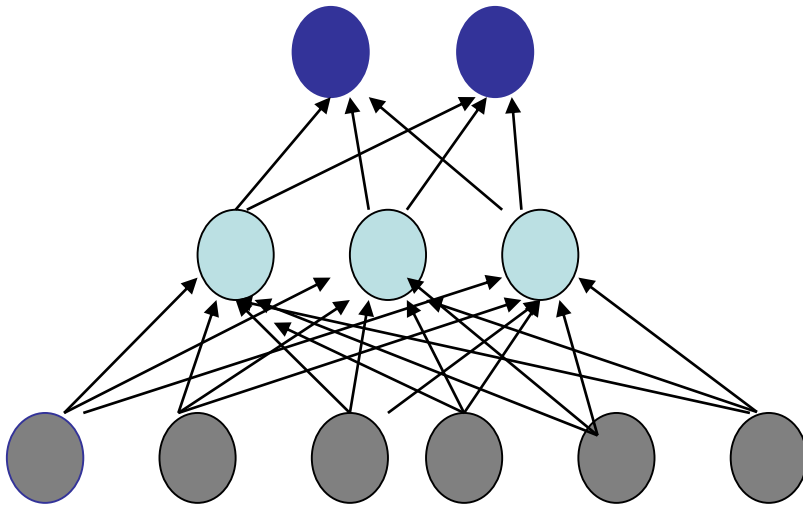





Backpropagation (2)

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-  I_i Input units
-  H_j Hidden units
-  O_k Output units

Threshold function:
sigmoid

$$o(\vec{x}) = \sigma(\vec{w} \cdot \vec{x}) = \frac{1}{1 + e^{\vec{w} \cdot \vec{x}}}$$

Error function is as follow:

$$E(\vec{w}) \equiv \frac{1}{2} \sum_{x \in D} (t(x) - o(x))^2 =$$

$$\frac{1}{2} \sum_{x \in D} \sum_{k \in N_{out}} (t_k(x) - o_k(x))^2$$



Backpropagation (3)

Gola: **minimize error**
between expected and real
output

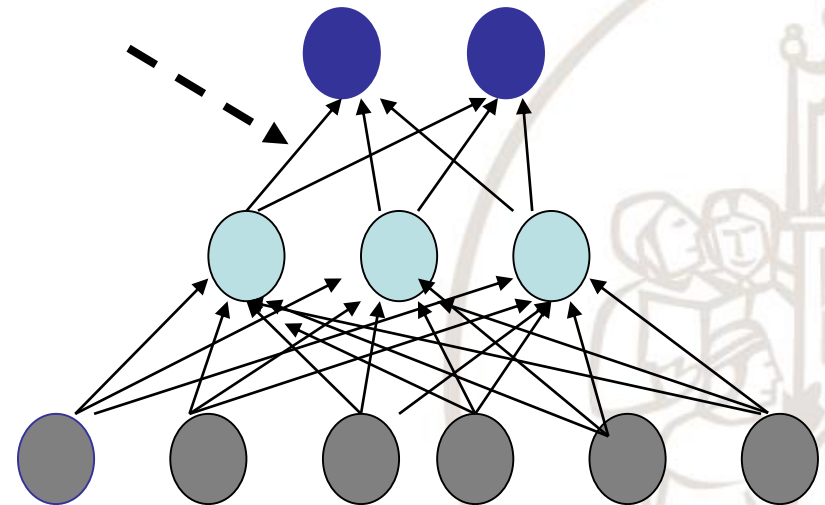
Update weights rule:

$$w_{ji} \leftarrow w_{ji} + \Delta w_{ji}$$

$$\Delta w_{ji} = \eta \delta_j x_{ji}$$

$$\delta_j = o_j(1 - o_j)(t_j - o_j)$$

Weights w_{ij} (from n_j to n_i)



NNs produce **m** output values



Backpropagation (4)

- **While** unreached termination condition, **execute**:
- For each value $v \in D: (x, t(x))$ ($x=(x_1, x_2, \dots, x_n)$, $t(x)=(t_1, t_2, \dots, t_m)$):
 - I set of the input nodes $(1, 2, \dots, n)$, O set of output nodes $(1, 2, \dots, m)$, N set of the net nodes
 - Compute the output of the net generated by the input v and the output of each node of the net $n_u \in N$ (x_i input of the input node $i_i \in I$, o_j output yielded by the node $n_j \in N$)
 - Compute error of the output node $o_k \in O$ as follows:

$$\delta_k = o_k(1 - o_k)(t_k - o_k)$$
 - Compute the error for the hidden units $h_n \in H = (N - O \cup I)$ connected the the output nodes, as follows:

$$\delta_h = o_h(1 - o_h) \sum_{n_k \in O} w_{kh} \delta_k$$
 - Compute the error for the other nodes

$$w_{ji} \leftarrow w_{ji} + \Delta w_{ji}$$
 - Updates the net weights as follow:

$$\Delta w_{ji} = \eta \delta_j x_{ji}$$

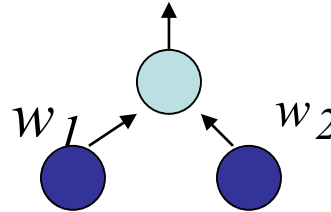


Gradient computation

$$\Delta w_1 = -\eta \frac{\partial E(w_1 x_1 + w_2 x_2)}{\partial w_1} = -\eta \frac{\partial E}{\partial net_1} \frac{\partial net_1}{\partial w_1} = -\eta \frac{\partial E}{\partial net_1} x_1$$

$$net_1 = w_1 x_1 + w_2 x_2$$

$$E = \frac{1}{2} (t - o)^2$$



$$\frac{\partial E}{\partial net_1} = \frac{\partial E}{\partial o} \frac{\partial o}{\partial net_1} \quad \frac{\partial E}{\partial o} = \frac{1}{2} 2(t - o) \frac{\partial (t - o)}{\partial o} = -(t - o)$$

$$\frac{\partial o}{\partial net_1} = \frac{\partial \sigma(net_1)}{\partial net_1} = o(1 - o)$$

$$\partial(\sigma(x)) = \sigma(x)(1 - \sigma(x))$$

$$\Delta w_1 = \eta o(1 - o)(t - o)x_1$$

What we used in the BP algorithm!!





Termination condition

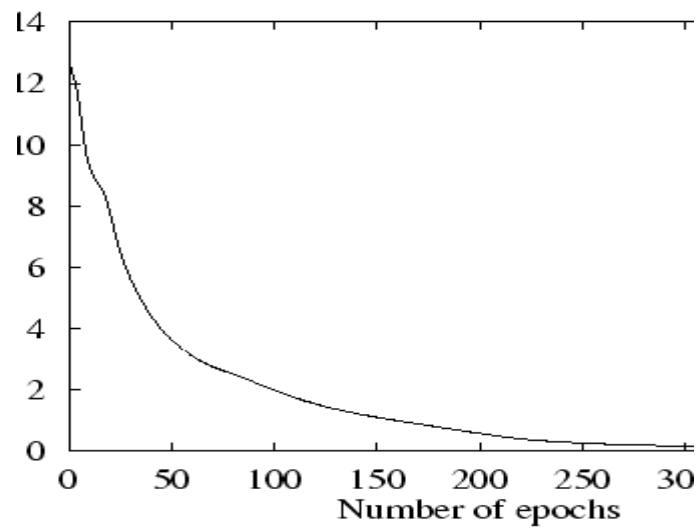
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- The process continues until all the examples ($\langle x, t(x) \rangle$) have been processed
- When does the process stop? Minimize errors on the train set is not the best solution (*overfitting*)
- Minimize errors on a test set (T), this means to split D in $D' \cup T$, training using D' and using T to verify the termination condition



Error in the training set

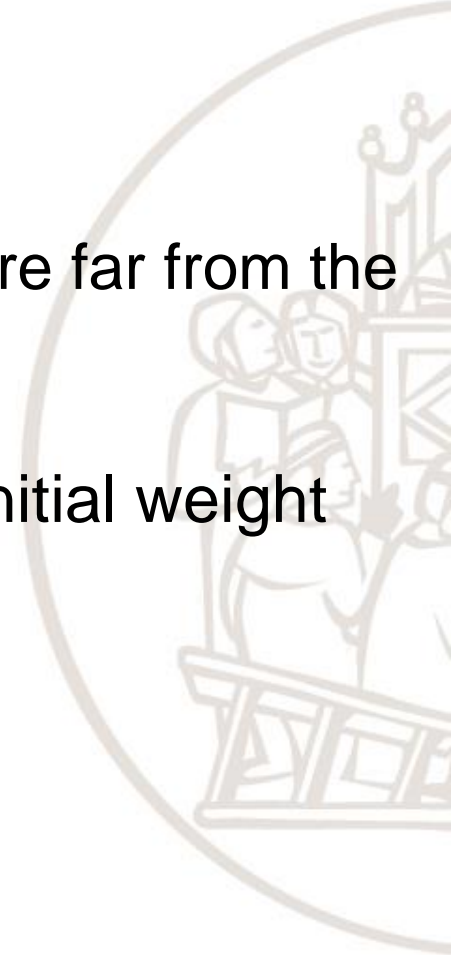
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Does the algorithm converge?

- Gradient algorithm issues:
 - Can stop on local minima
 - A local minimum can give solutions which are far from the global minimum
 - Sometimes there are a lot of local minima...
- A possible solution: training with changing initial weight values





Weights Updating Rule

- Considering the n -th value of d_i in D , updating rule becomes:

$$\Delta w_{ij}(n) \leftarrow \eta \delta_j x_{ij} + \alpha \Delta w_{ij}(n-1) \quad \text{Momentum}$$

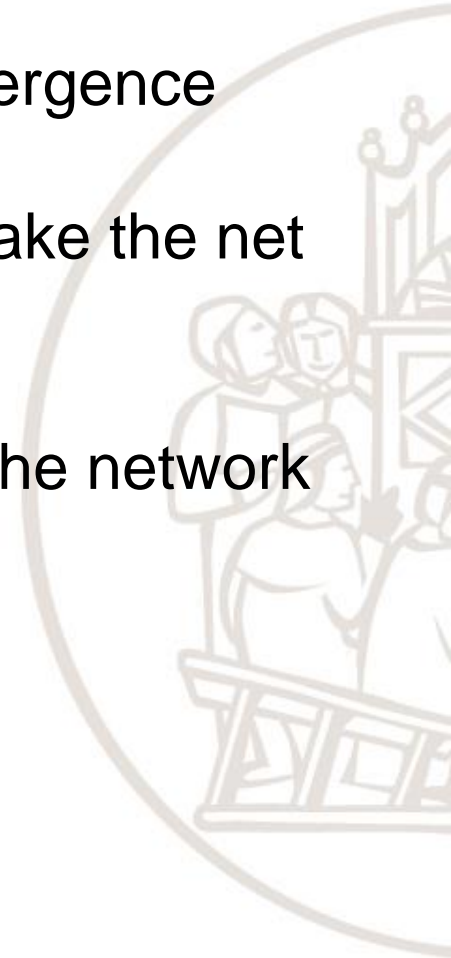
- Pros:
 - Overcoming local minima
 - Keeping stable value for the weights in the «flat zones»
 - Increase velocity when gradient does not change
- Cons:
 - If momentum value is too high can stop on local MAXIMA
 - One more tuning value





A few considerations...

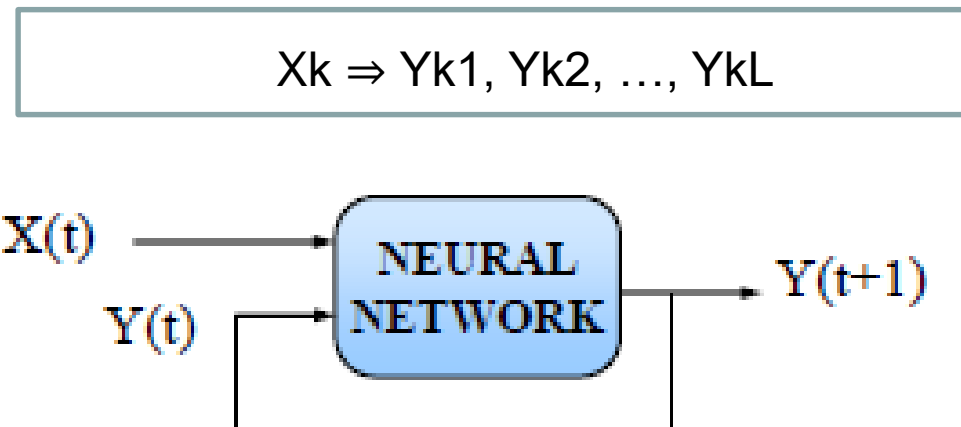
- Initializing weight values is basic to reach convergence
- BP depends on the learning rate η . This can make the net diverging.
- It can be useful to use different values of η for the network layers





Recurrent neural networks

They are networks that learn to associate an input pattern with a sequence of output patterns



A recurrent neural network (RNN) is a class of neural networks where connections between units form a directed cycle. This creates an internal state of the network which allows it to exhibit dynamic temporal behavior.



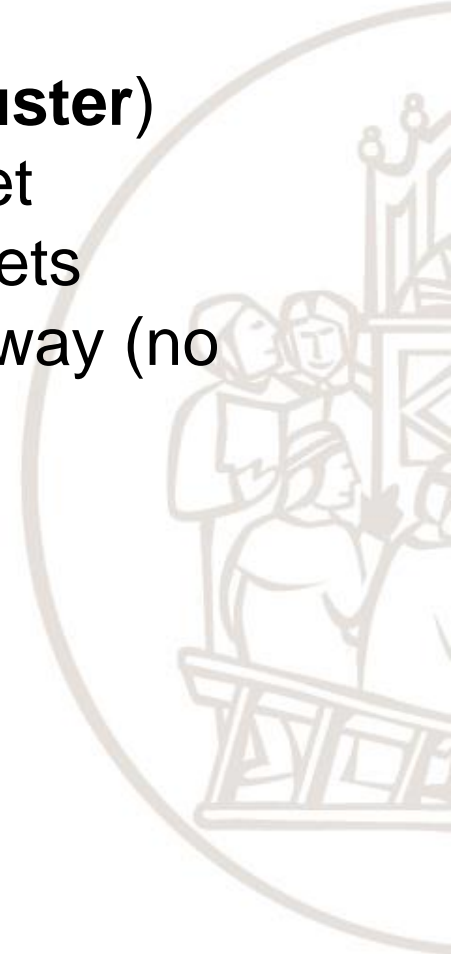
- Supervised Learning
 - MLP and Recurrent NN
- Unsupervised Learning
 - Clustering
- Competitive Learning
 - Kohonen networks
- Reinforcement Learning





Unsupervised learning

- Split non labeled input values in subsets(**cluster**)
 - Similar input values are in the same subset
 - Different input values are in different subsets
- Find new in an subsets in an unsupervised way (no labels provided)





- Supervised Learning
 - MLP e reti neurali ricorrenti
 - RBF
- Unsupervised Learning
 - Clustering
- Competitive Learning
 - Reti di Kohonen
- Reinforcement Learning





Competitive Learning

- In some cases, the network output can be ambiguous
- Thanks to the **lateral inhibition**, neurons start competing to respond to a stimulus.
- The neuron having the greatest output wins the competition and specializes itself to recognize that stimulus.
- Thanks to the **excitatory connections**, neurons near the winner are also sensitive to similar inputs

An isomorphism is created between input and output space

Competitive Learning - Implementation

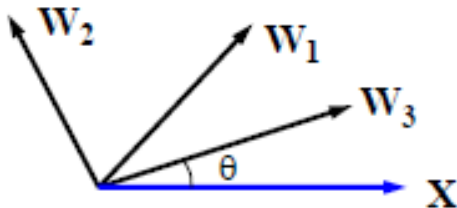
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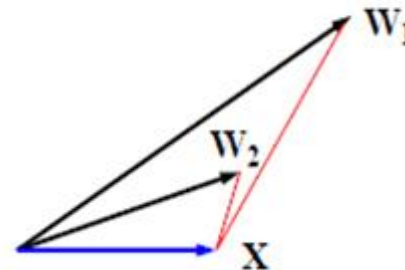
- The winning neuron is selected using a global strategy just by comparing the outputs of the other neurons.
- Two techniques can be used:
 1. Select the neuron with the maximum output;
 2. Select the neuron whose weight vector is more similar

$$y_j = \sum_{i=1}^n w_{ji} x_i = W_j \bullet X = |W_j| |X| \cos \theta$$



METHOD 1 - The winner on an input X is the one with the greatest output

$$j_0 : \text{DIS}(W_{j_0}, X) < \text{DIS}(W_j, X) \quad \forall j \neq j_0$$



METHOD 2 - The winner neuron on input X is that having its weight vector more similar to X

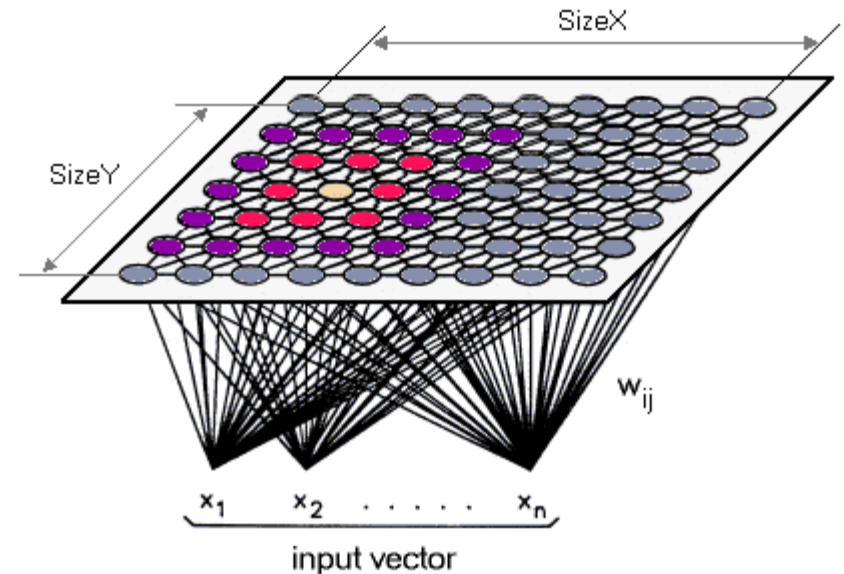


Kohonen networks

The Kohonen network (or "self-organizing map", or SOM, for short) has been developed by Teuvo Kohonen.

The basic idea behind the Kohonen network is to setup a structure of interconnected processing units (neurons) which *compete* for the signal.

The SOM defines a **mapping** from the input data space spanned by $x_1 \dots x_n$ onto a one- or two-dimensional array of nodes. The mapping is performed in a way that the topological relationships in the n -dimensional input space are maintained when mapped to the SOM. In addition, the local density of data is also reflected by the map: areas of the input data space which are represented by more data are mapped to a larger area of the SOM.



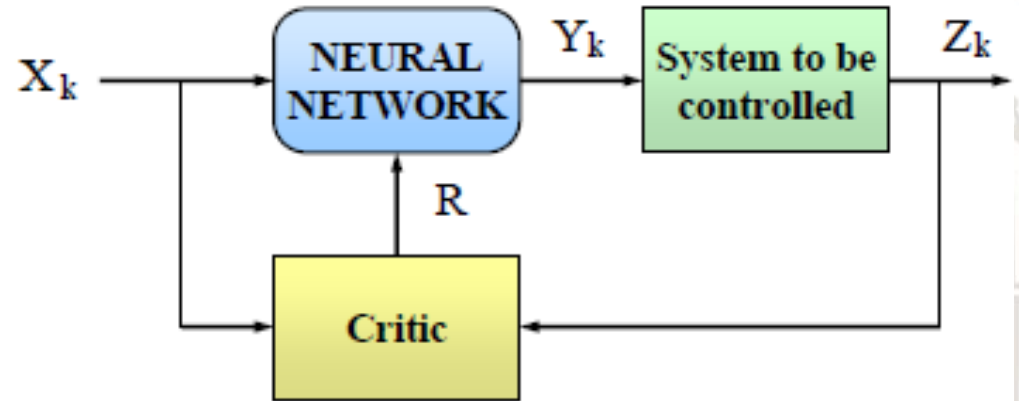


- **Supervised Learning**
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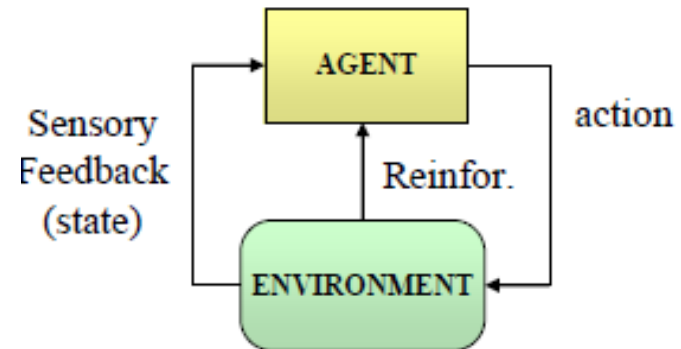
Reinforcement learning

Several actions are executed.
Successful actions are stored
(by weight variations).



Punishments and rewards

An agent operates in the environment and modify actions based on the produced consequences.



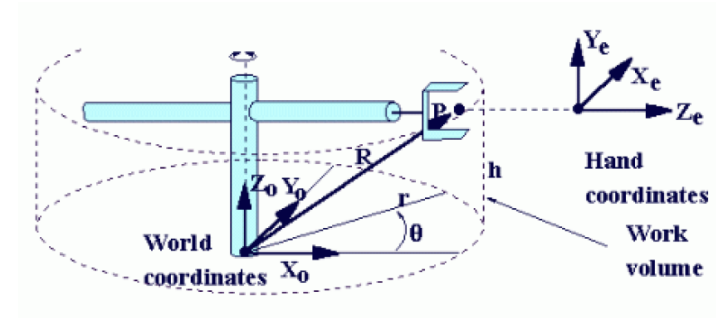
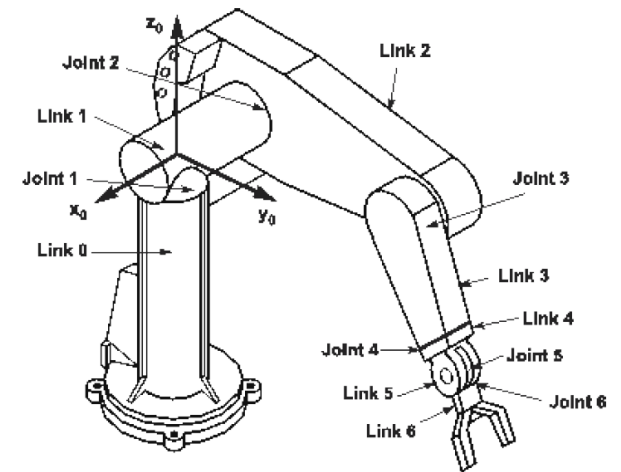
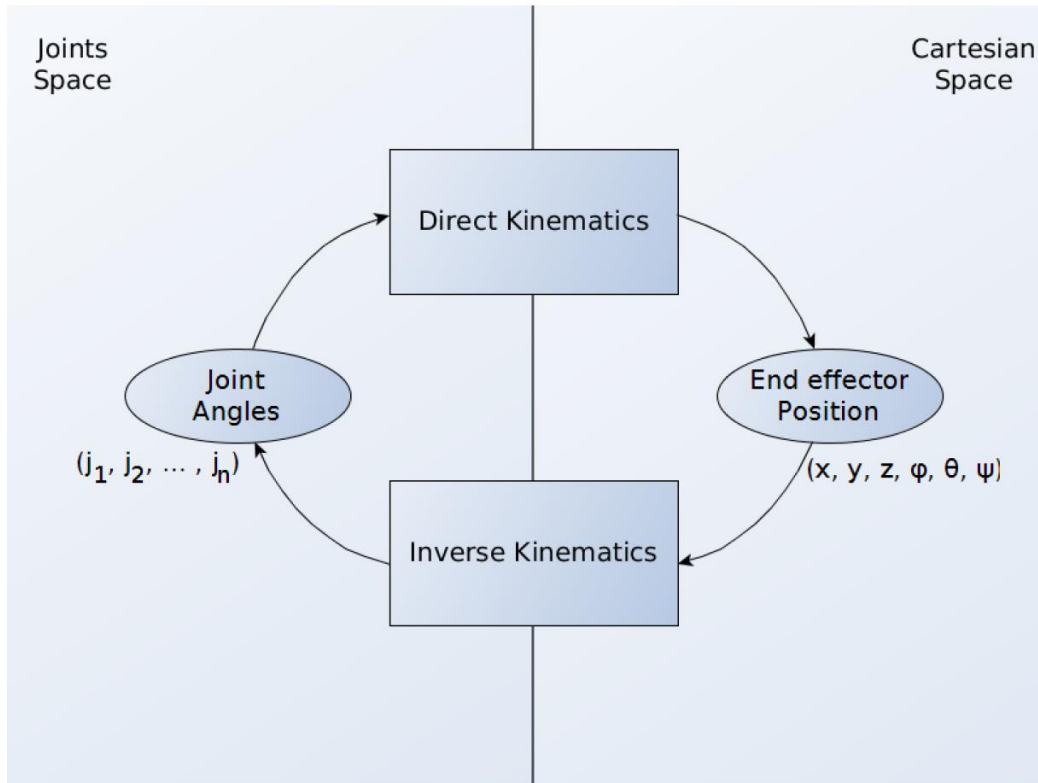
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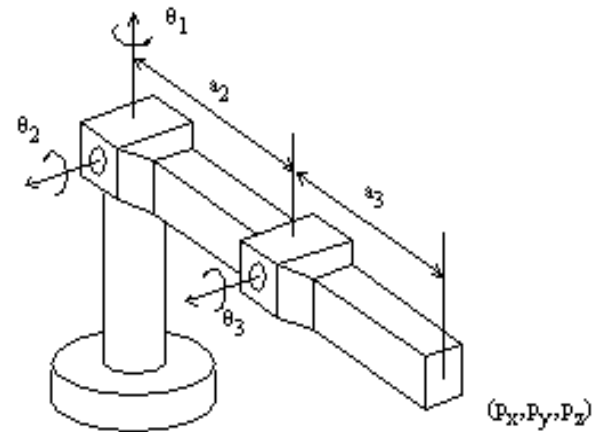
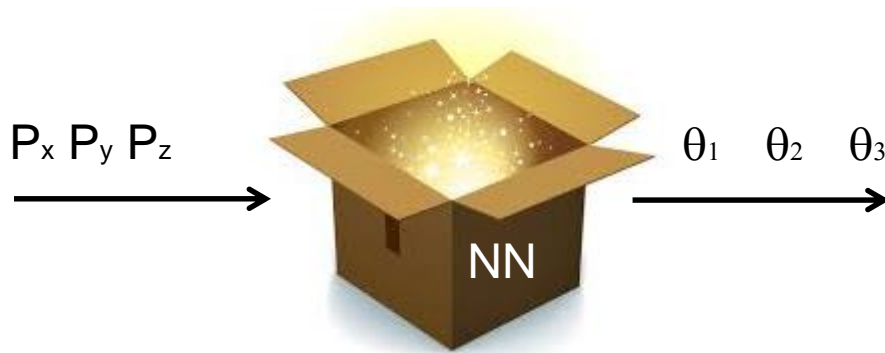
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Robot control and neurocontrollers

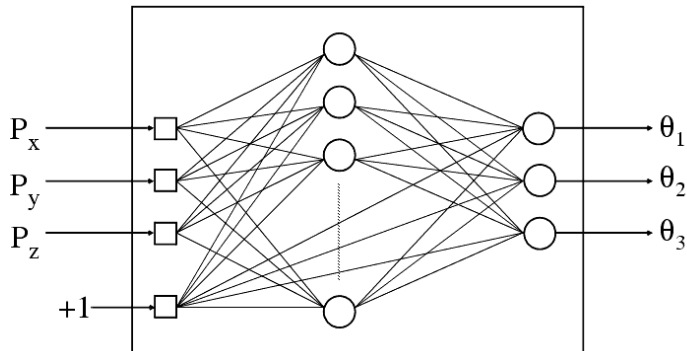




Computing the Inverse Kinematics solution using a NN



How do you make the network learning?



- ✓ Creating a dataset of $\langle \text{joint_positions}, \text{end effector_positions} \rangle$ using the direct kinematics



Learning the Inverse Static Solution

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- It is not always possible to compute the inverse kinematic solution using the joint positions
- For soft continuum robots actuated by cables it is possible to exploit the relation between the cable tendion and the end effector position, in order to control the tip

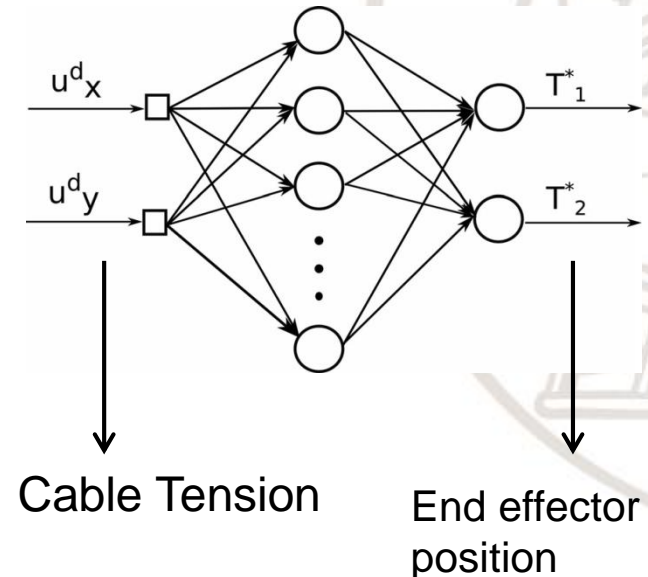




Learning the Inverse Static Solution (II)

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- **Control of the soft arm through the learning of the inverse model that allows to control the end effector position through the cable tension**
- **The inverse problem can be learned collecting points and exploiting the approximation capability of the NN as for the rigid robots**





Learning the Inverse Static Solution: an adaptive approach (I)

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A Neural Network can be used to solve the inverse solution generating an adaptive approach :

The direct model relation is :

$$\mathbf{x} = f(\mathbf{q}) \quad (1)$$

This particular representation is not invertible when $m < n$ (redundant)

where, $\mathbf{x} \in \mathcal{R}^m$ **is the position and orientation vector of the end effector**; $\mathbf{q} \in \mathcal{R}^n$ is the joint vector f is a surjective function



Learning the Inverse Static Solution: an adaptive approach (II)

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We can develop local representations by linearizing the function at a point (\mathbf{q}^0) thereby obtaining :

$$\delta \mathbf{x} = J(\mathbf{q}^0) \delta \mathbf{q} \quad (I)$$

Here $J(\mathbf{q}^0)$ is the Jacobian matrix at the point \mathbf{q}^0 ; $\delta \mathbf{x}$ and $\delta \mathbf{q}$ are infinitesimally small changes in \mathbf{x} e \mathbf{q} . The differential IK method involves generating of $(\delta \mathbf{x}, \delta \mathbf{q}, \mathbf{q})$ and learning the mapping $(\delta \mathbf{x}, \mathbf{q}^0) \rightarrow \delta \mathbf{q}$

The learning is feasible since the differential IK solutions form a convex set and therefore averaging multiple solutions still results in a valid solution



Learning the Inverse Static Solution: an adaptive approach (III)

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The method we have proposed involves expanding Eq. I and expressing it in terms of absolute positions, as shown below:

$$J(\mathbf{q}_i)\mathbf{q}_{i+1} = \mathbf{x}_{i+1} - f(\mathbf{q}_i) + J(\mathbf{q}_i)\mathbf{q}_i \quad (\text{II})$$

\mathbf{q}_{i+1} is the next actuator configuration for reaching a point \mathbf{x}_{i+1} from the present configuration \mathbf{q}_i . Note that Eq. II is only valid when the configurations are infinitesimally close. However, for practical purposes this can be a good approximation for larger regions.



Learning the Inverse Static Solution: an adaptive approach (IV)

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The analytical solution for the equation II can be written as:

$$\mathbf{q}_{i+1} = G(\mathbf{x}_{i+1} - f(\mathbf{q}_i) + J\mathbf{q}_i) + (I_n - GJ)\mathbf{z}$$

where G is a generalized inverse of $J(\mathbf{q}_i)$ and I_n is the identity matrix, and \mathbf{z} is an arbitrary n -dimensional vector. The first component represents the particular solution to the non-homogenous problem prescribed in Eq. II and the second component represents the infinite homogenous solutions. It can be proved that the solution space still forms a convex set. Therefore, a **universal function approximator (i.e. NN)** can be used for learning the mapping

$$(\mathbf{q}_i, \mathbf{x}_{i+1}) \rightarrow (\mathbf{q}_{i+1})$$

The samples $(\mathbf{q}_i, \mathbf{q}_{i+1}, \mathbf{x}_{i+1})$ generated are such that

$$\|\mathbf{q}_{i+1} - \mathbf{q}_i\| < \epsilon$$

An appropriate value of ϵ is between 10%-5% of the maximum actuator range



Learning the Inverse Static Solution: an adaptive approach (V)

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- We use a feed-forward NN to learn the relation:

- $$(\mathbf{q}_i, \mathbf{x}_i, \mathbf{x}_{i+1}) \rightarrow (\mathbf{q}_{i+1})$$

- The values of $\mathbf{x}_i, \mathbf{x}_{i+1}$** are generated using the direct model as showed for the learning of the IK of a rigid manipulator

TRAINING PHASE

TEST PHASE

$(\mathbf{q}_{i-1}, \mathbf{x}_i, \mathbf{x}_{i-1})$ \longrightarrow INPUT

$(\mathbf{q}_i, \mathbf{x}_i, \mathbf{x}_{i+1})$ \longrightarrow INPUT

(\mathbf{q}_i) \longrightarrow Desired
OUTPUT

(\mathbf{q}_{i+1}) \longrightarrow Network
OUTPUT



Learning the Inverse Static Solution: an adaptive approach (V): real robot implementation

Six DoF Hybrid System (Pneumatic and Tendon)

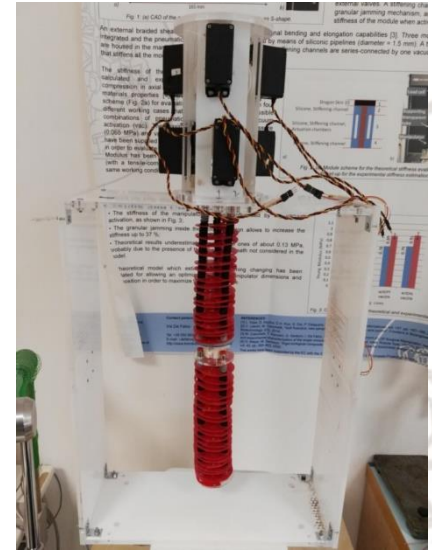
LEARNING

- 2000 sample points divided in the ratio 70:30 for training and testing respectively
- 2 hours for data collection, training and setting-up

TESTS

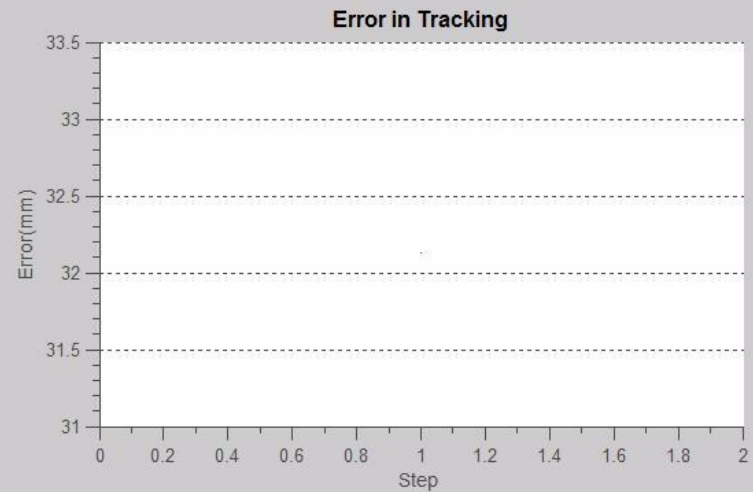
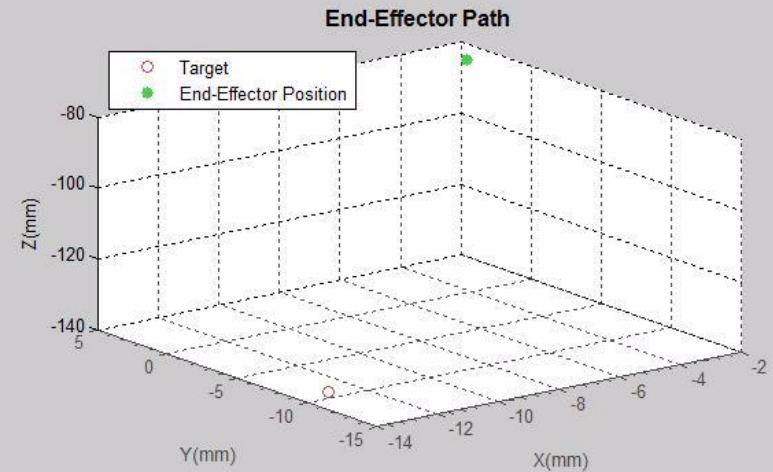
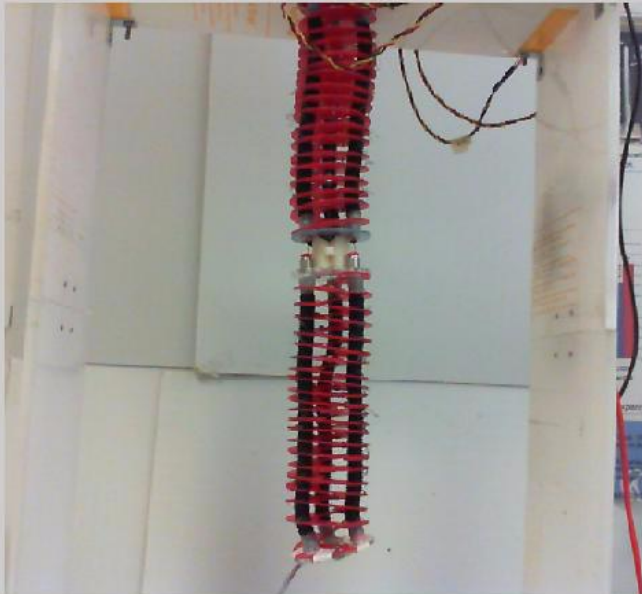
Twenty Five random points selected from workspace

	Mean Error	Standard Deviation
Position (mm)	5.58	3.08
X- axis rotation (degrees)	2.76	5.42
Y- axis rotation (degrees)	1.84	1.83
Z- axis rotation (degrees)	3.85	7.02



I-Support Prototype

External Disturbance (Only Position)



Unlike the case of rigid robots external disturbances modify the kinematics of the soft manipulator

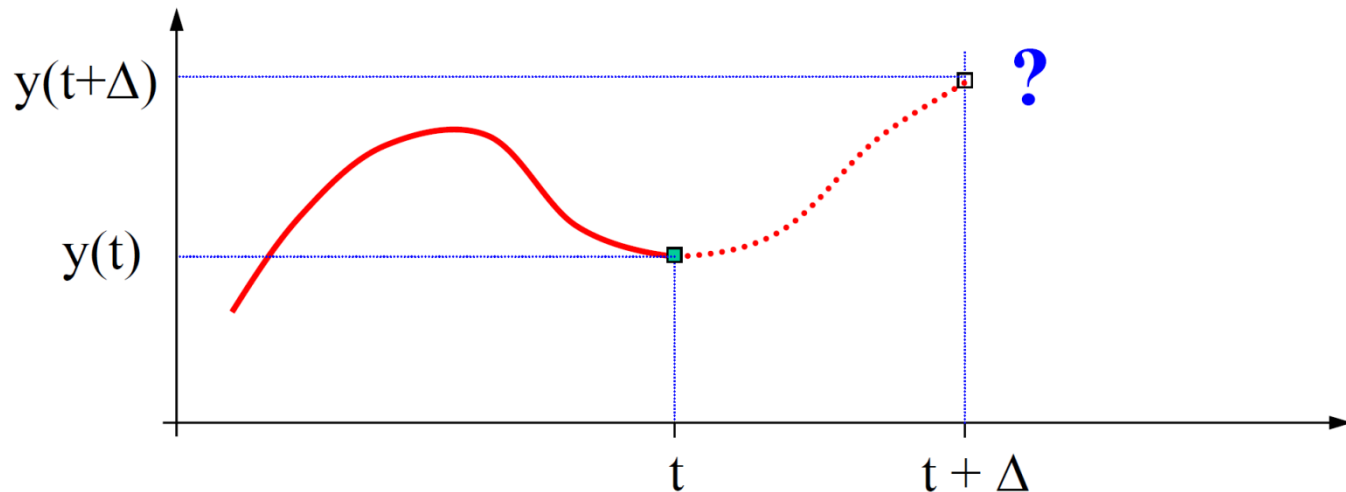
This is the first experimental implementation of soft robots tracking under external disturbances



Offline signal prediction

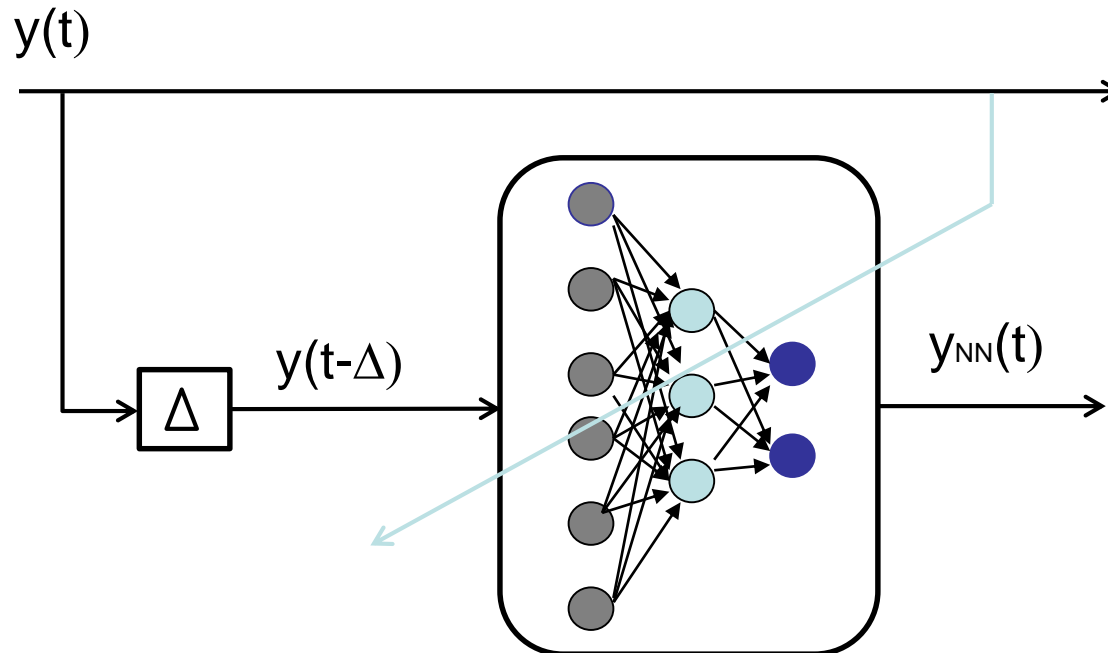
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- Using a NN to foresee the signal in the future
- Training the NN using past values





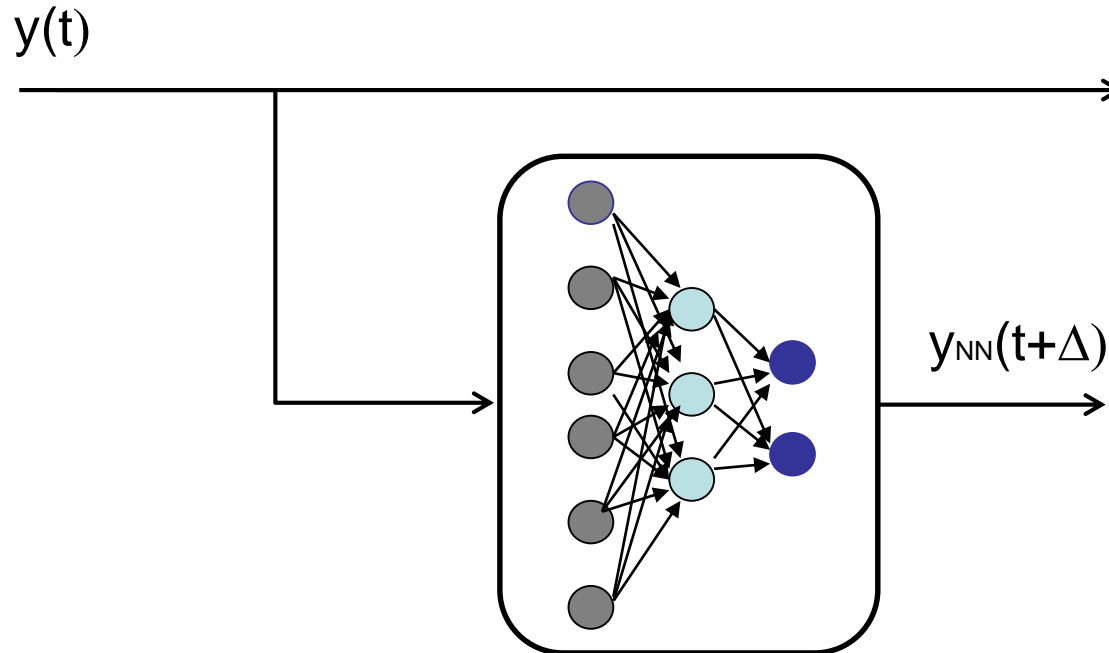
- Learning phase



- The NN learns the relation between $y(t)$ and $y(t-\Delta)$



- Test phase



- The NN yields an estimation of the $y(t+\Delta)$

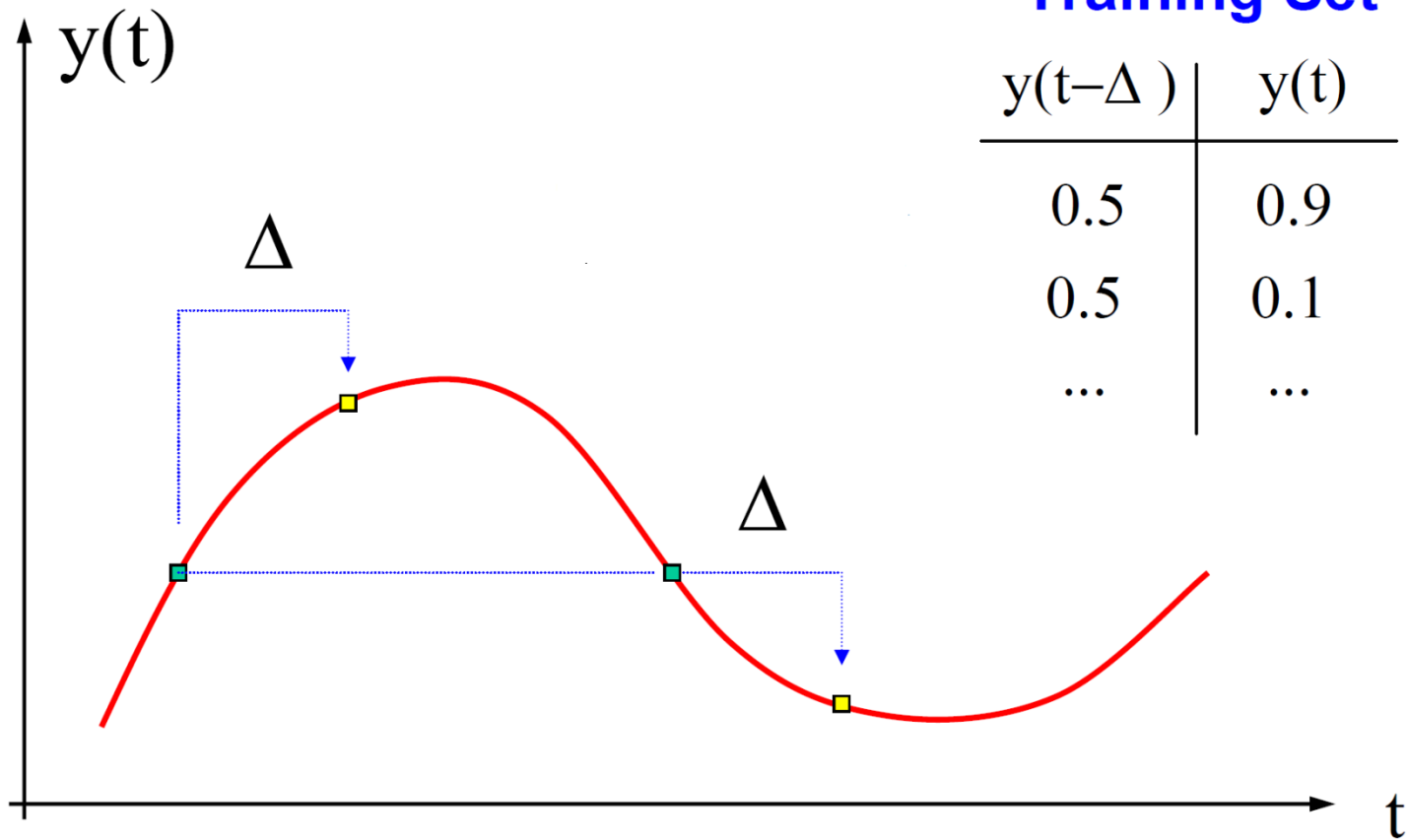




Learning and prediction

Inconsistent training set

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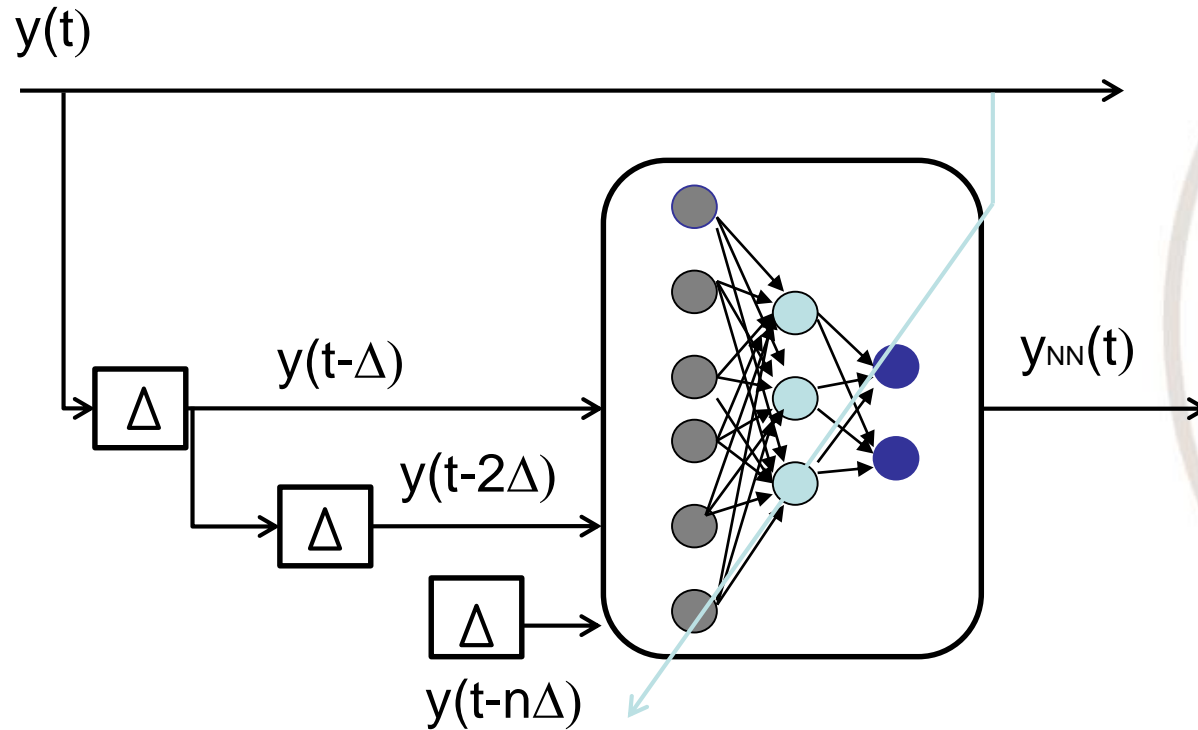
Training Set

$y(t-\Delta)$	$y(t)$
0.5	0.9
0.5	0.1
...	...



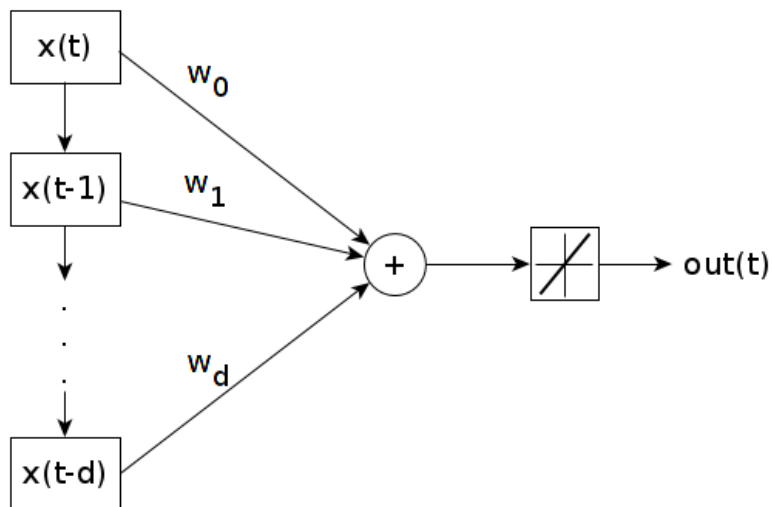
Prediction with delays

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- Generate a prediction with any a-priori knowledge of the signal
- Fast convergence with less input seen
- Simple model based on a single neuron (PERCEPTRON) receiving as input current and past values ($x(t)-x(t-d)$)



Es. Ten steps ahead prediction
 $d=10$

training set (<input, desired output>):
 $\langle X_1-X_{10}, X_{20} \rangle, \langle X_2-X_{11}, X_{21} \rangle,$
 $\dots \langle X_n-X_{n+10}, X_{n+20} \rangle$

Network output: $y_{10}, y_{11} \dots y_{n+10}$