

University of Pisa

Master of Science in Computer Science

**Course of Robotics (ROB)**

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Scuola Superiore  
Sant'Anna

## Robot Sensors

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<http://didawiki.cli.di.unipi.it/doku.php/magistraleinformatica/rob/start>





# Robot definition

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*A robot* is an autonomous system  
which exists in the physical world,  
can sense its environment,  
and can act on it to achieve some goals

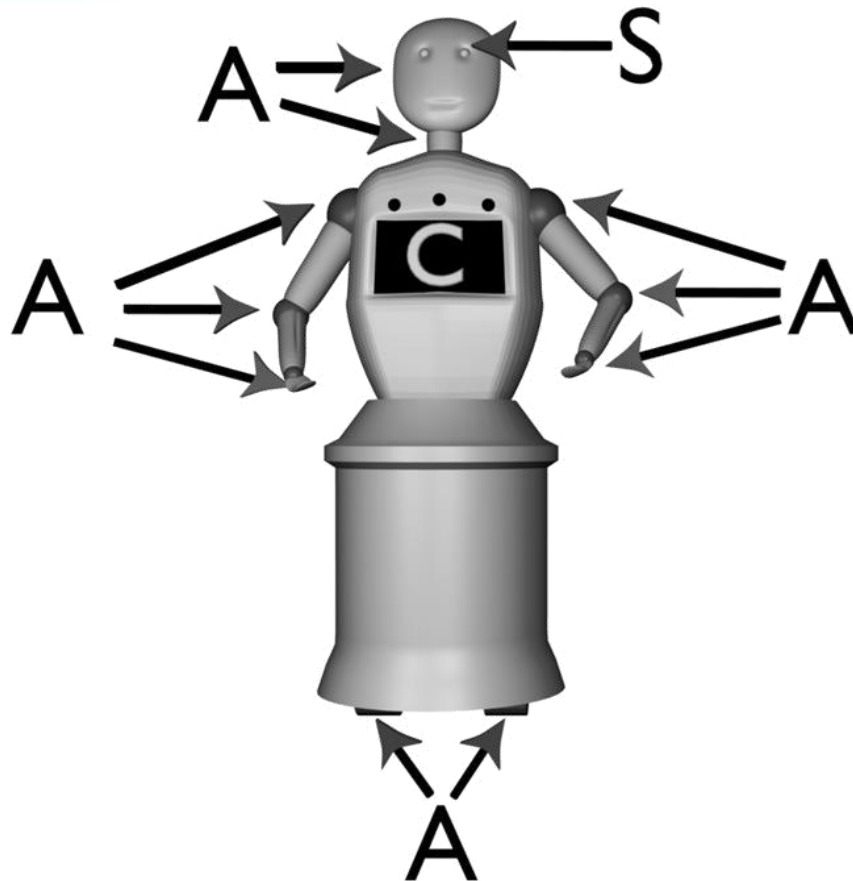




# What's in a robot?

## Robot components

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### Legend

Actuator  
Controller  
Sensor

# Outline of the lesson

- Definitions of sensor and transducer
- Classification of transducers
- Fundamental properties of sensors
- Position sensors: switches, encoders, potentiometers, Hall-effect sensors
- Range/Distance sensors: ultrasound sensors and laser range finders
- Proximity sensors: Hall-effect and infrared sensors
- Force sensors: strain gauges and force/torque sensors
- Inertial sensors



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# Definitions of sensor and transducer

- **SENSOR:**  
device sensitive to a physical quantity and able to transform it in a measurable and transferable signal
- **TRANSDUCER:**  
device receiving in input a kind of energy and producing in output energy of a different kind, according to a known relation between input and output, not necessarily for measurement purposes



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# First classification:

- Passive sensors:
  - convert directly input energy in output, without external energy sources
- Active sensors:
  - require external energy (excitation) for energy conversion





# Classification of transducers

*based on the kind of input energy, output energy, or external energy*

- Radiant – electromagnetic waves:
  - intensity, frequency, polarization and phase
- Mechanical – external parameter of materials:
  - position, velocity, dimension, compliance, force
- Thermal:
  - temperature, gradient of temperature, heat
- Electrical:
  - voltage, current, resistivity, capacity
- Magnetic:
  - field intensity, flow density, permeability
- Chemical – internal structure of materials:
  - concentrations, crystal structure, aggregation state



# Trasformations of energy in a transducer

*INPUT ENERGY* ➡ *AUSILIARY ENERGY* ➡ *OUTPUT ENERGY*

CHEMICAL

MAGNETIC

ELECTRICAL

THERMAL

MECHANICAL

RADIANT

CHEMICAL

MAGNETIC

ELECTRICAL

THERMAL

MECHANICAL

RADIANT

NONE

CHEMICAL

MAGNETIC

ELECTRICAL

THERMAL

MECHANICAL

RADIANT



# Trasformations of energy in a transducer

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# Fundamental properties of a sensor

- TRANSFER FUNCTION
- CALIBRATION
- LINEARITY
- HYSTERESIS
- ACCURACY
- REPEATABILITY
- RESOLUTION
- SENSITIVENESS
- SENSITIVENESS TO NOISE
- LIFETIME
- STABILITY



# Transfer function

The *transfer function* (or *characteristic function*) is the relation between the quantity to measure (input to the sensor) and the output of the sensor



# Calibration

The *calibration* procedure consists of measuring the output of the sensor for known quantities

*Calibration cycle* means a trial that covers the whole working range of the sensor; the trial is divided in two parts, one with increasing values and the other with decreasing values



# Linearity

If the transfer function of a sensor is represented in a linear plot, *linearity* is a measure of the deviation of the transfer function from a line.

The line can be chosen in two ways:

- 1) the line between the output of the sensor for the input values corresponding to 0% and 100% of its working range
- 2) the line that best fits the sensor transfer function, with the minimum squares method

*Linearity* is measured as the maximum difference, expressed in % of the maximum value of the transfer function, between the transfer function and the reference line





# Hysteresis

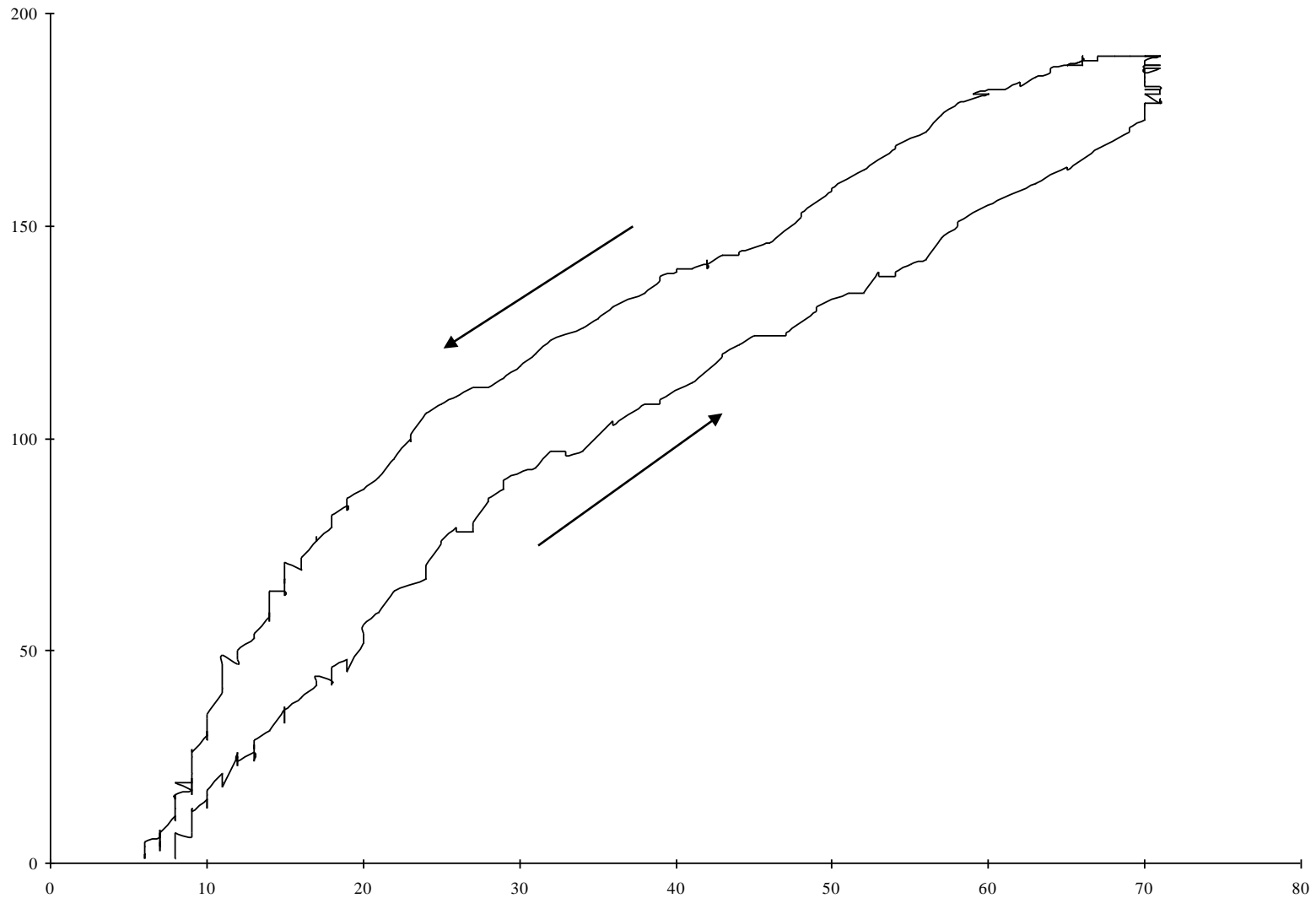
If a sensor has *hysteresis*, for a same input value, the output may vary, depending on the fact that the input values are increasing or decreasing.

*Hysteresis* is measured as the maximum difference between the two output curves of the sensor during the calibration cycle.

It is expressed as a % of the maximum value for the transfer function



# Example of hysteresis in a tactile sensor



# Accuracy

*Accuracy* represents the maximum error between the actual value and the value measured by the sensor.



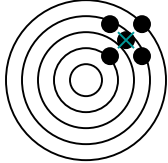
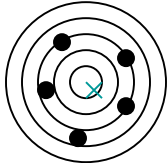
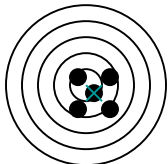
# Repeatability

When a same input value is applies to a sensor, *repeatability* is a measure of the variability of the output of the sensor.



# Accuracy and Repeatability

- accuracy
  - $100 (x_m - x_v) / x_v$
  - $x_m$  = average value
  - $x_v$  = actual value
- repeatability
  - dispersion of measures

measure	Repeatable	Accurate
	YES	NO
	NO	YES
	YES	YES



# Resolution

*Resolution* is the minimum variation of the input which gives a variation of the output of the sensor.



# Sensitiveness

A small variation of the input causes a corresponding small variation of the output values.

*Sensitiveness* is the ratio between the output variation and the input variation.



# Noise

*Noise* is the amount of signal in the sensor output which is not given by the input.





# Stability

*Stability* is the capability of the sensor to keep its working characteristics for a given time (short, medium, long).



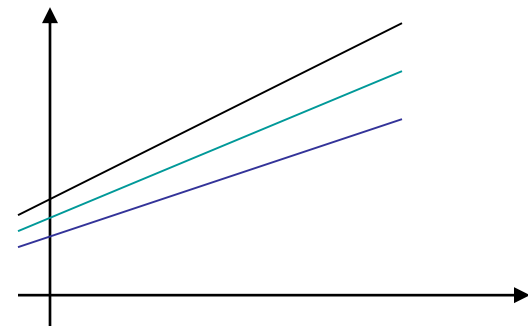
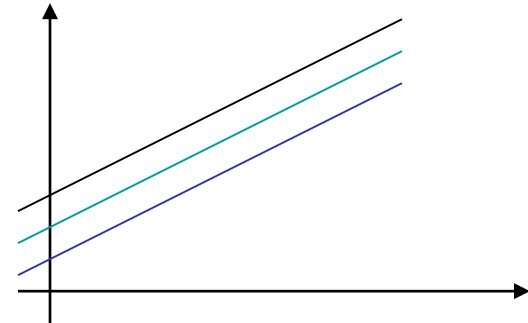
# Other static parameters

- Response time
- Input range
- Cost, size, weight
- Response in frequency
- Environmental factors
- Maximum/minimum temperature
- Warm-up time
- Presence of smoke, gas, ...
- ...



# Dynamic parameters

- zero drift
  - For instance,  
due to temperature
- sensitiveness drift



# Role of sensors in a robot

- Perception of the external state: measurement of variables characterizing the working environment. For instance, distance, proximity, force.



# Role of sensors in a robot

- Perception of the internal state: measurement of variables internal to the system that are used to control the robot. For instance, joint position.

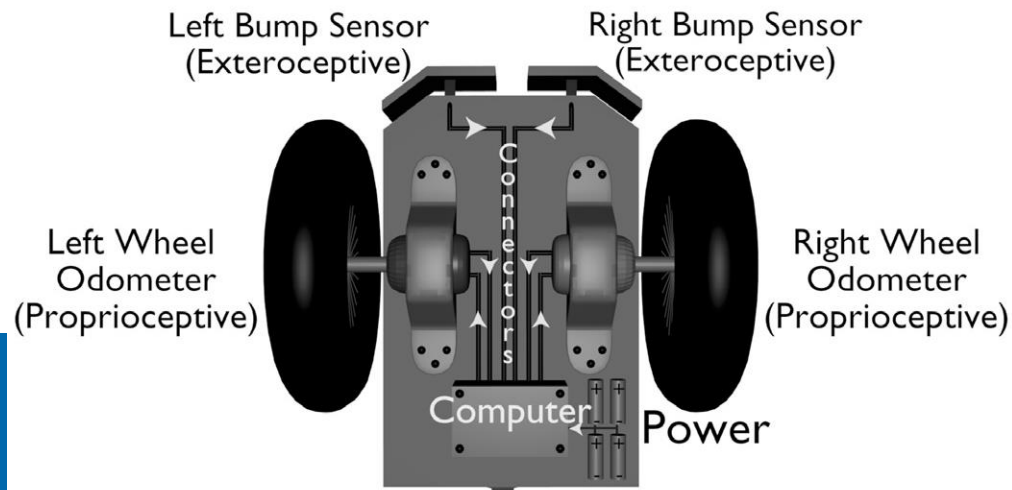


# Role of sensors in a robot

- Sensing the external state (**exteroception**): measurement of variables characterizing the working environment.
- Examples:

Physical Property	→	Sensing Technology
Contact	→	bump, switch
Distance	→	ultrasound, radar, infra red
Light level	→	photocells, cameras
Sound level	→	microphones
Strain	→	strain gauges
Rotation	→	encoders and potentiometers
Acceleration	→	accelerometers and gyroscopes
Magnetism	→	compasses
Smell	→	chemical sensors
Temperature	→	thermal, infra red
Inclination	→	inclinometers, gyroscopes
Pressure	→	pressure gauges
Altitude	→	altimeters

- Sensing the internal state (**proprioception**): measurement of variables internal to the system that are used to control the robot.
- Examples:
  - Joint position / encoders
  - Battery level



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# Position sensors

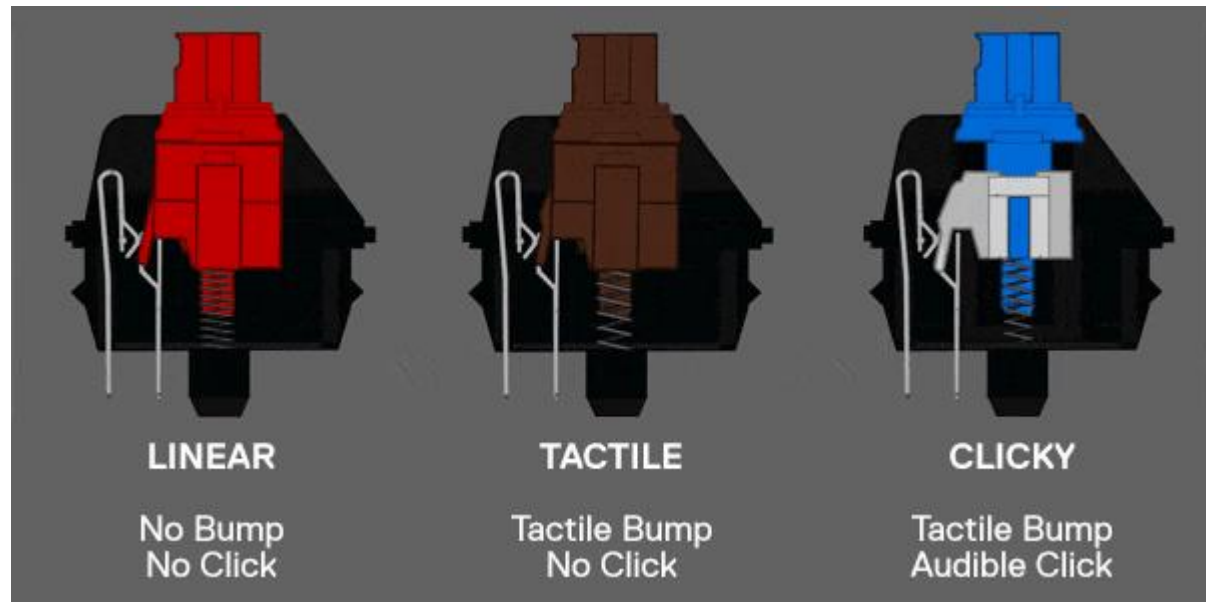
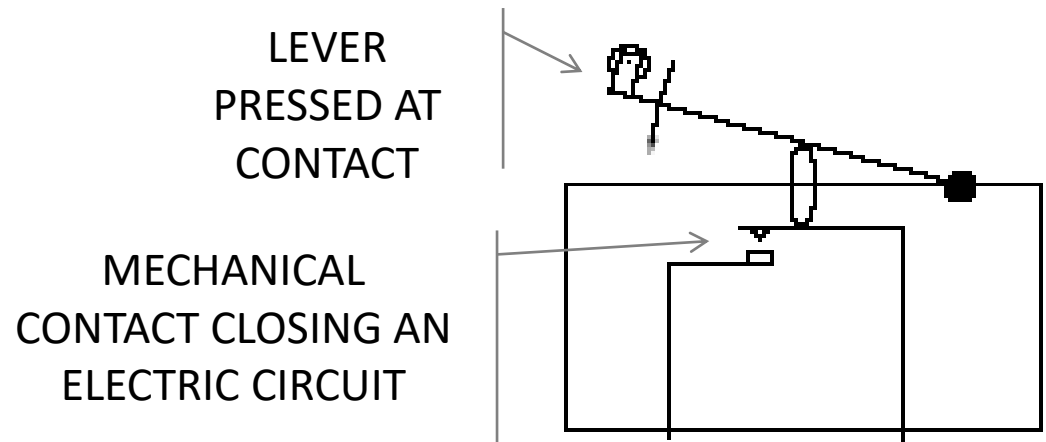
- Switches
- Optical encoders
- Potentiometers
- Hall-effect sensors





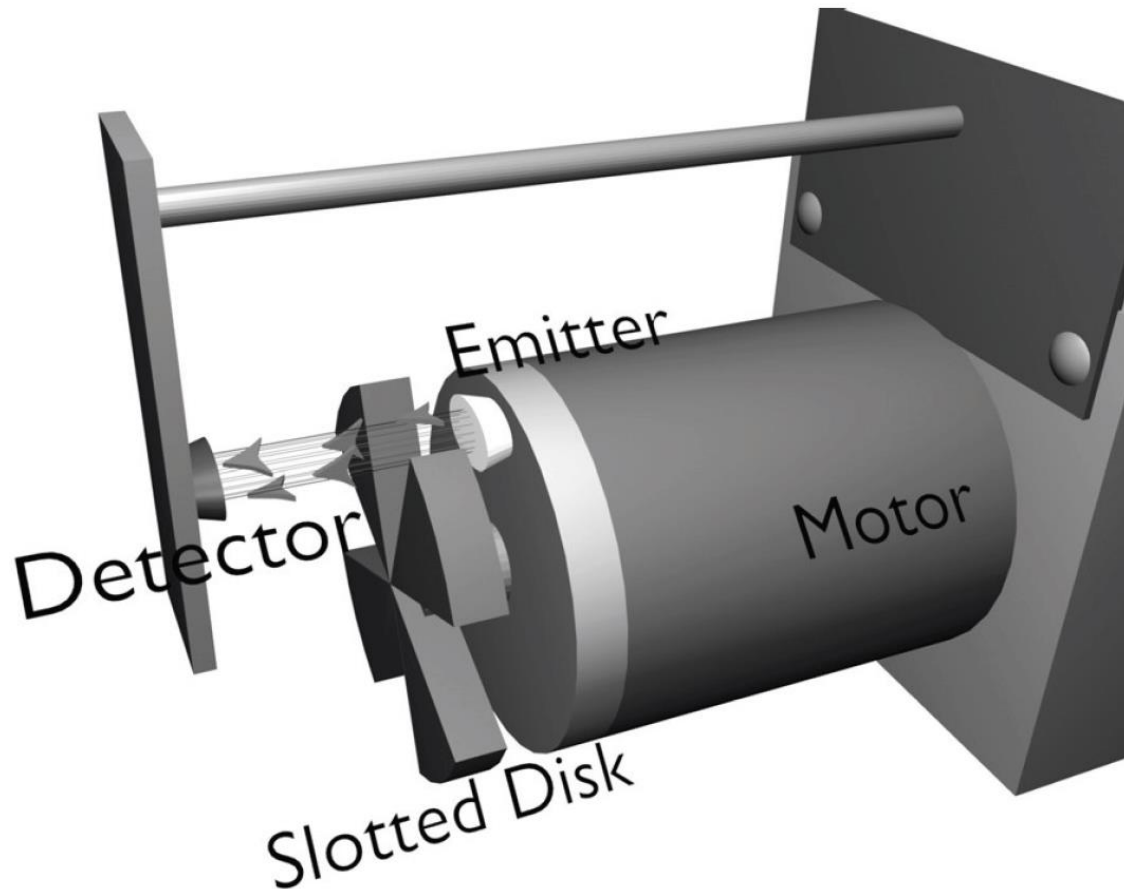
# Mechanical switches

- Simplest contact sensors
- Provide binary data:  
contact / no contact
- Applications in robotics:
  - impact sensors on mobile robots
  - whiskers
  - endstop sensors for manipulator joints

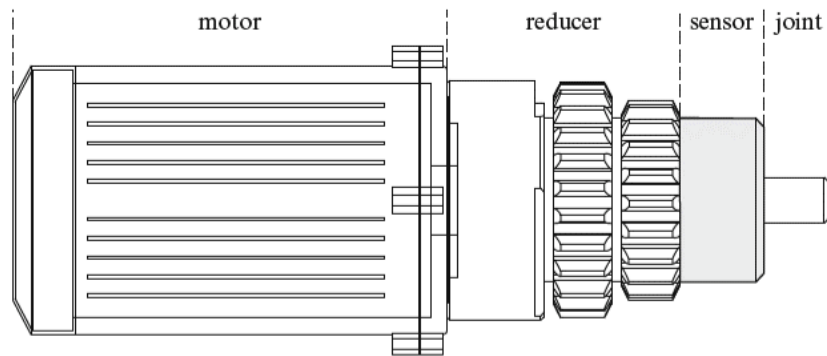


# Optical encoders

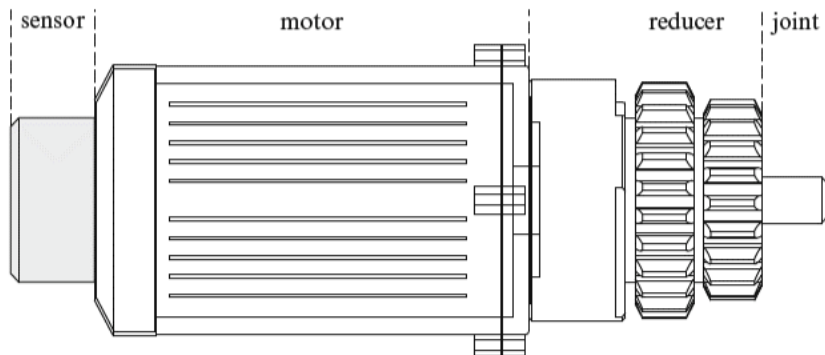
- Measurement of angular rotation of a shaft or an axle



# Placement of position sensors



After reducer



Before reducer

$\theta$ : joint angular position

$\theta_m$ : motor angular position

$k$ : motor reduction ratio

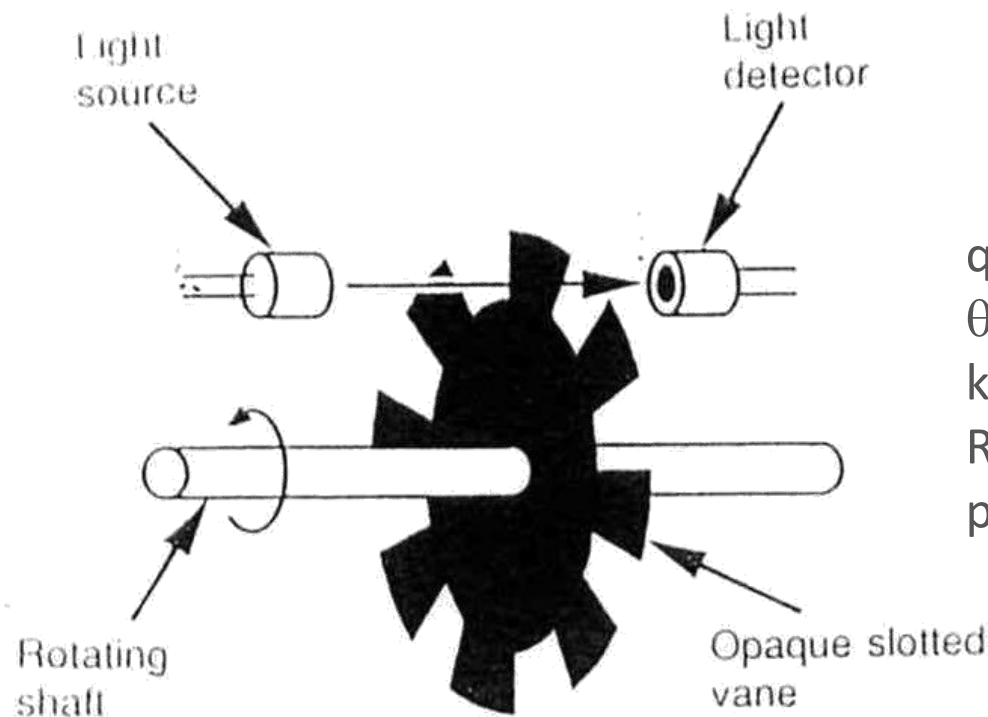
$$\theta = \frac{\theta_m}{k}$$

$$\frac{d\theta}{d\theta_m} = \frac{1}{k} \Rightarrow d\theta = \frac{1}{k} d\theta_m \quad \Rightarrow \text{The sensor error is reduced of a factor } k$$



# Optical encoders

**Rotation** is measured by counting the **pulses** and by knowing the number of the disk **steps**



$$q = \frac{\theta \times 360^\circ}{R \times k}$$

q: joint angular position (in degrees)

$\theta$ : joint position in encoder steps

k: motor reduction ratio

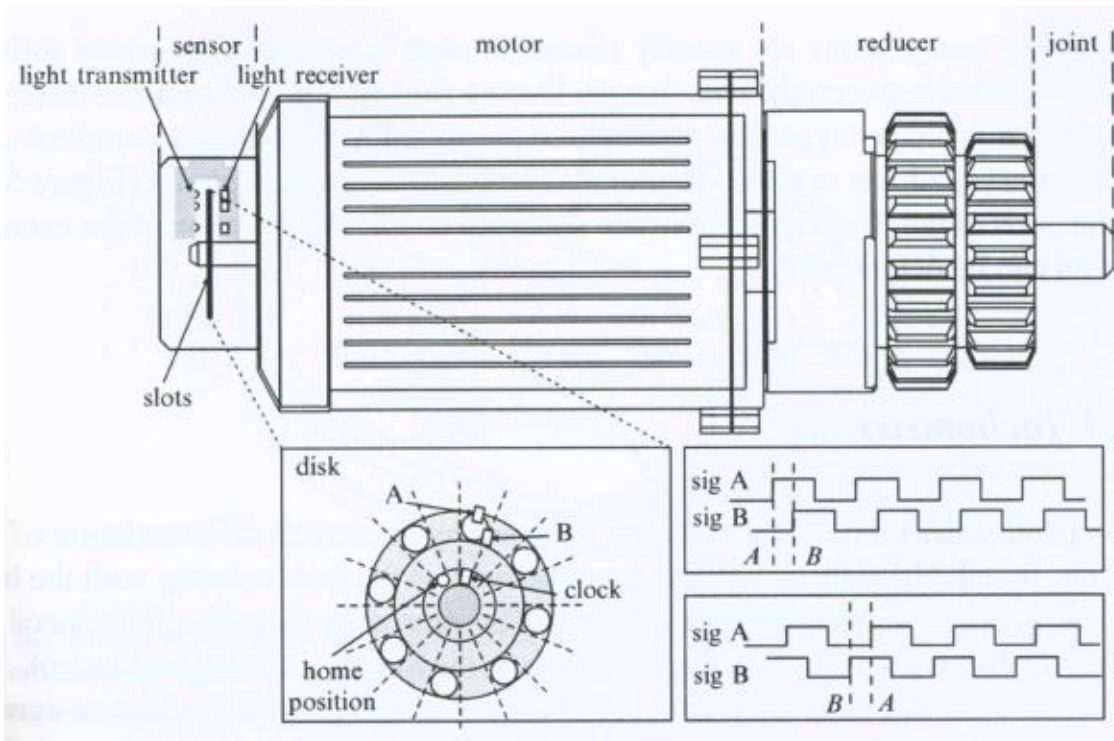
R: encoder resolution (number of steps per turn)

The **frequency** of the pulse train is proportional to **angular velocity**



# Incremental encoders

By using 2 photo-switches it is possible to detect the rotation direction, by means of the relation between the phases of their pulse trains



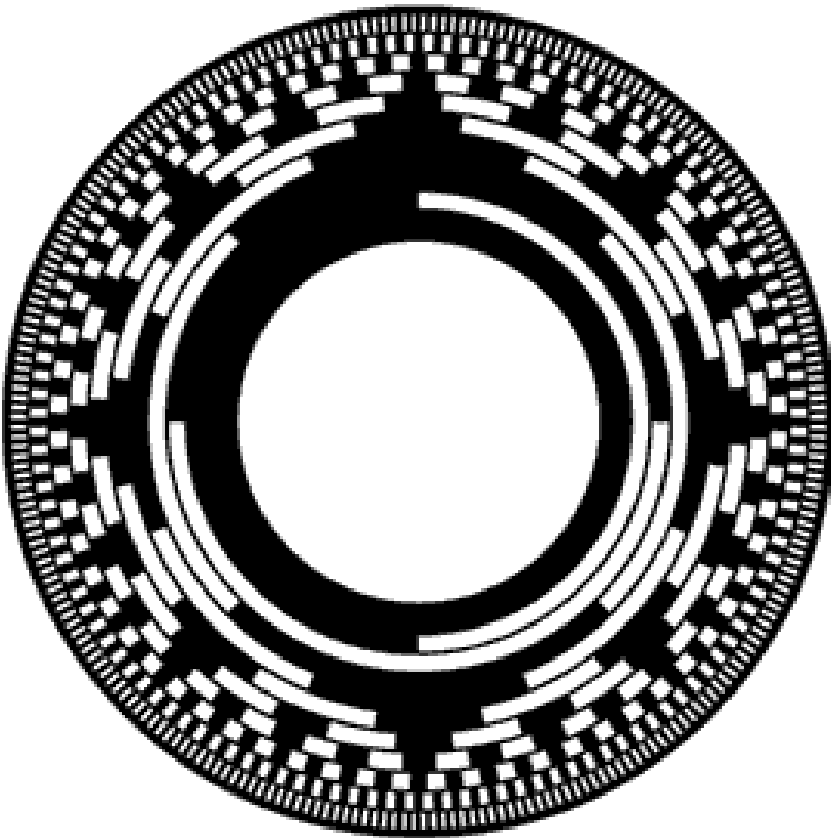
A and B are out of phase of  $\frac{1}{4}$  of cycle

An increase of A with B=0 corresponds to a clockwise rotation

An increase of A with B=1 corresponds to a counterclockwise rotation



# Absolute encoder



k photo-switches

k code tracks

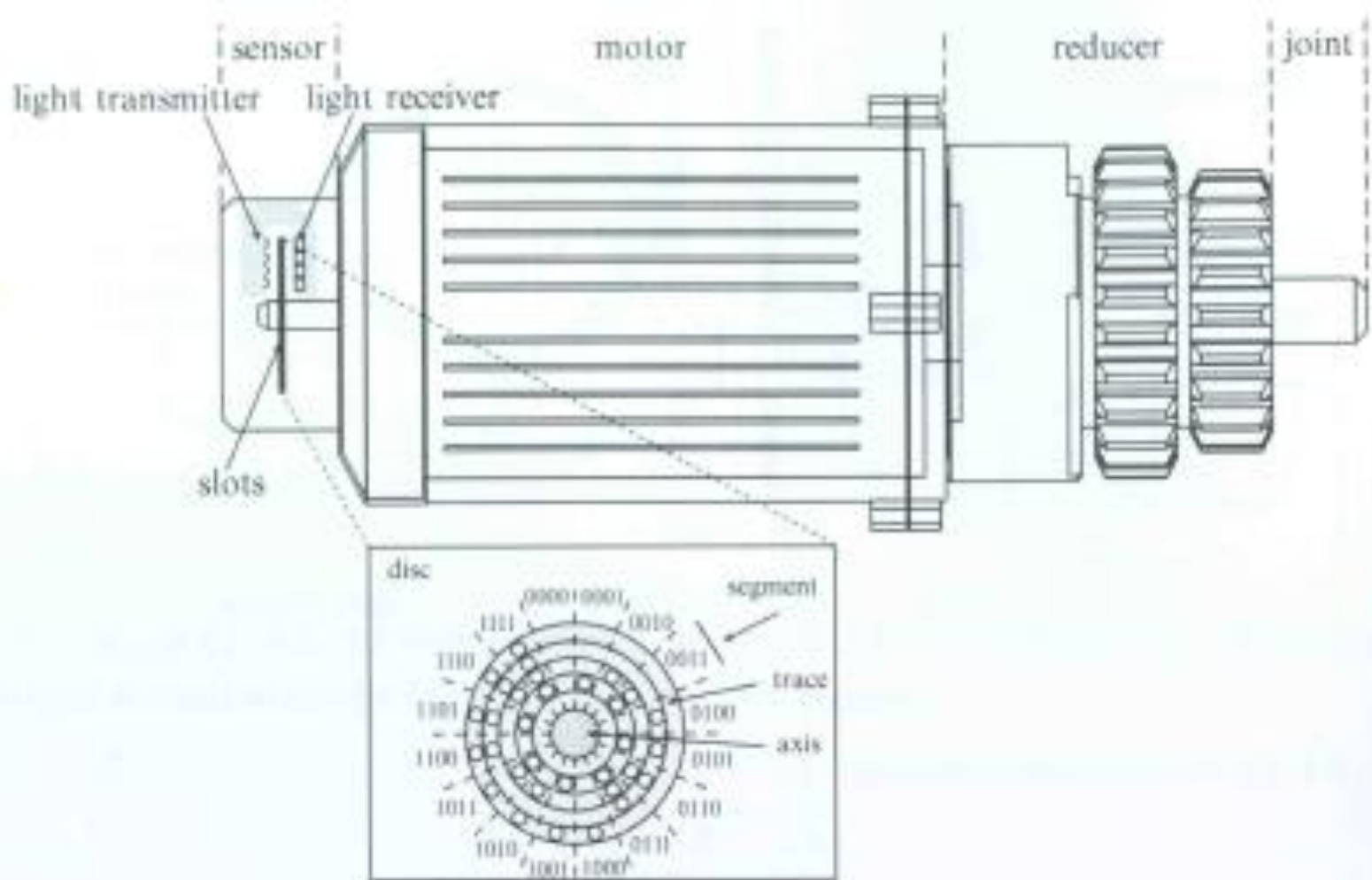
Binary word of k bits,  
representing  $2^k$  different disk  
orientations

Angular resolution of  $360^\circ / 2^k$

- It gives the absolute rotation angle
- Each position is uniquely determined



# Absolute encoder



# Absolute encoder

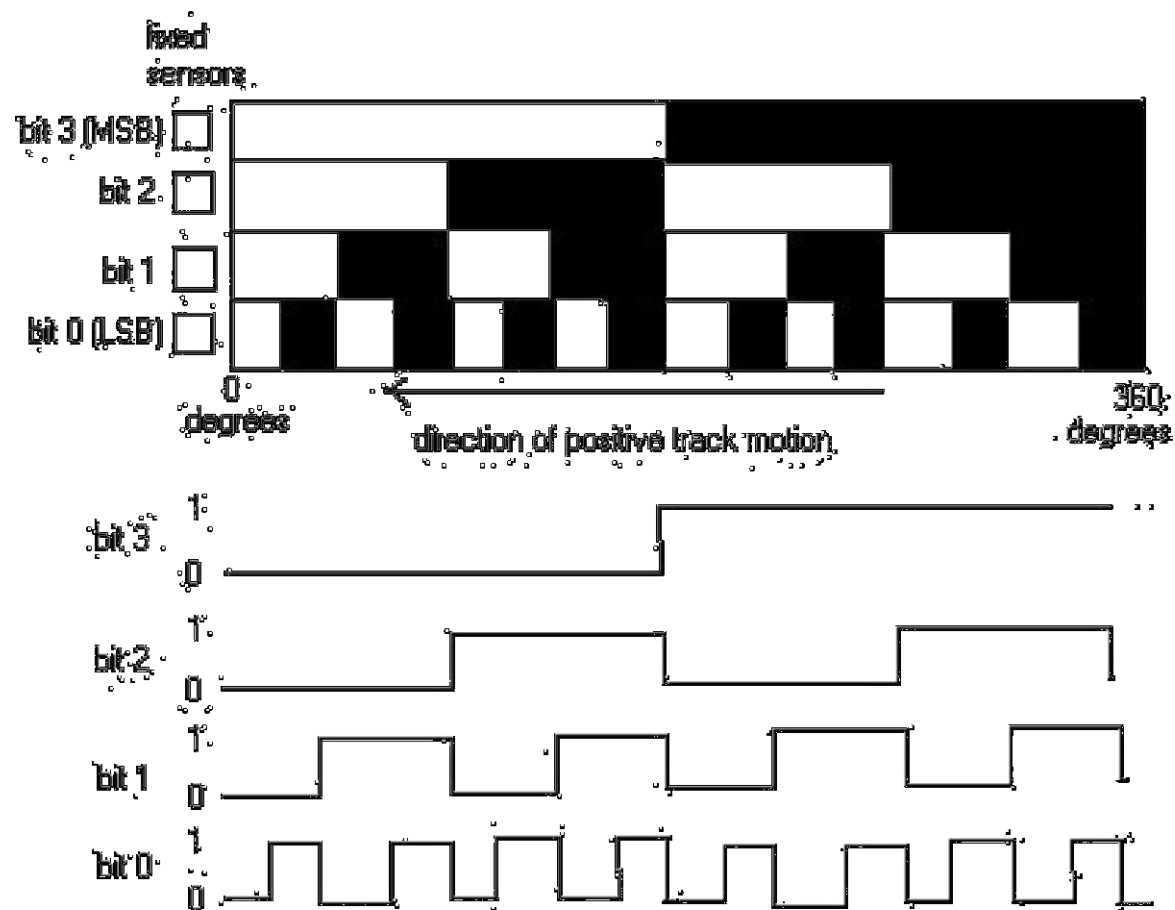


Fig 3 4-Bit binary code absolute encoder disk track patterns





# Absolute encoder - Gray Code

## Single transition

Decimal	Binary	Gray Code
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101

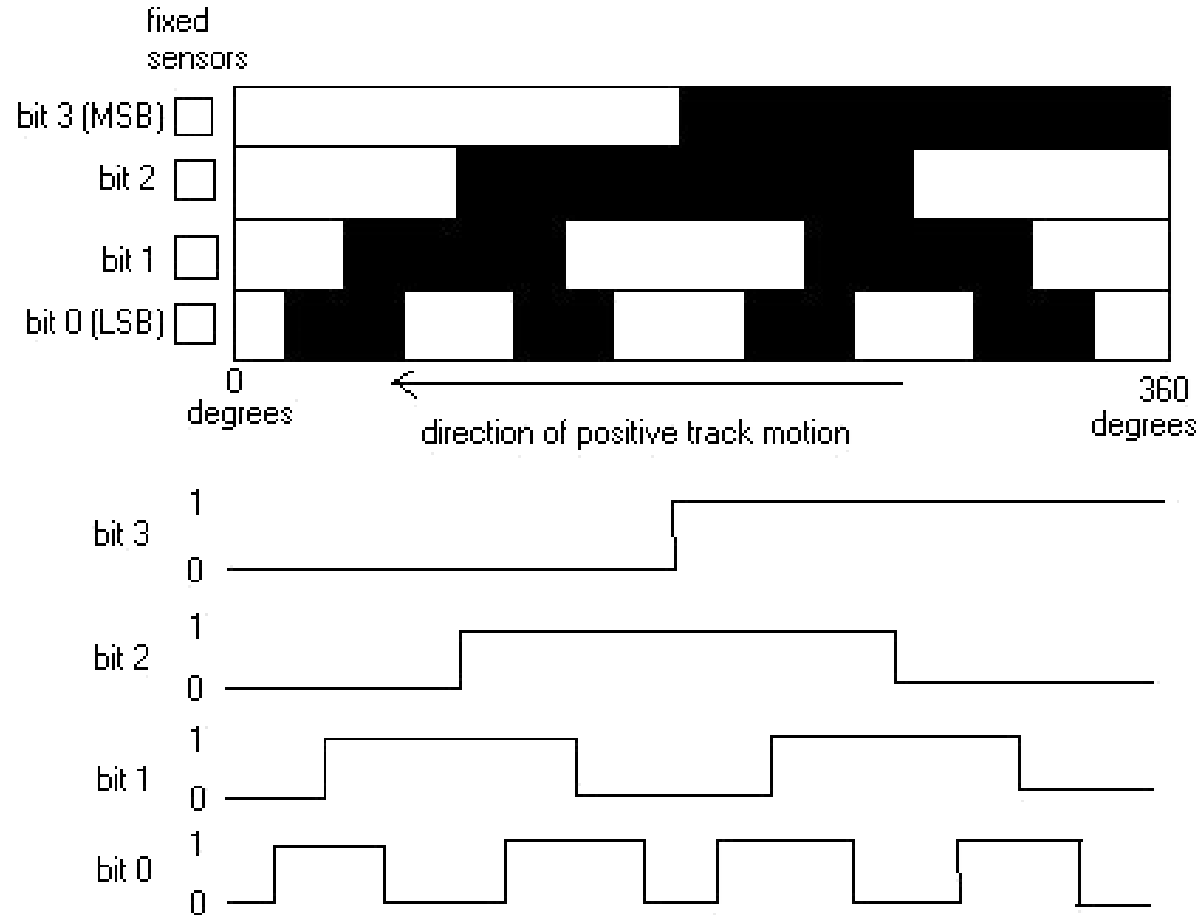
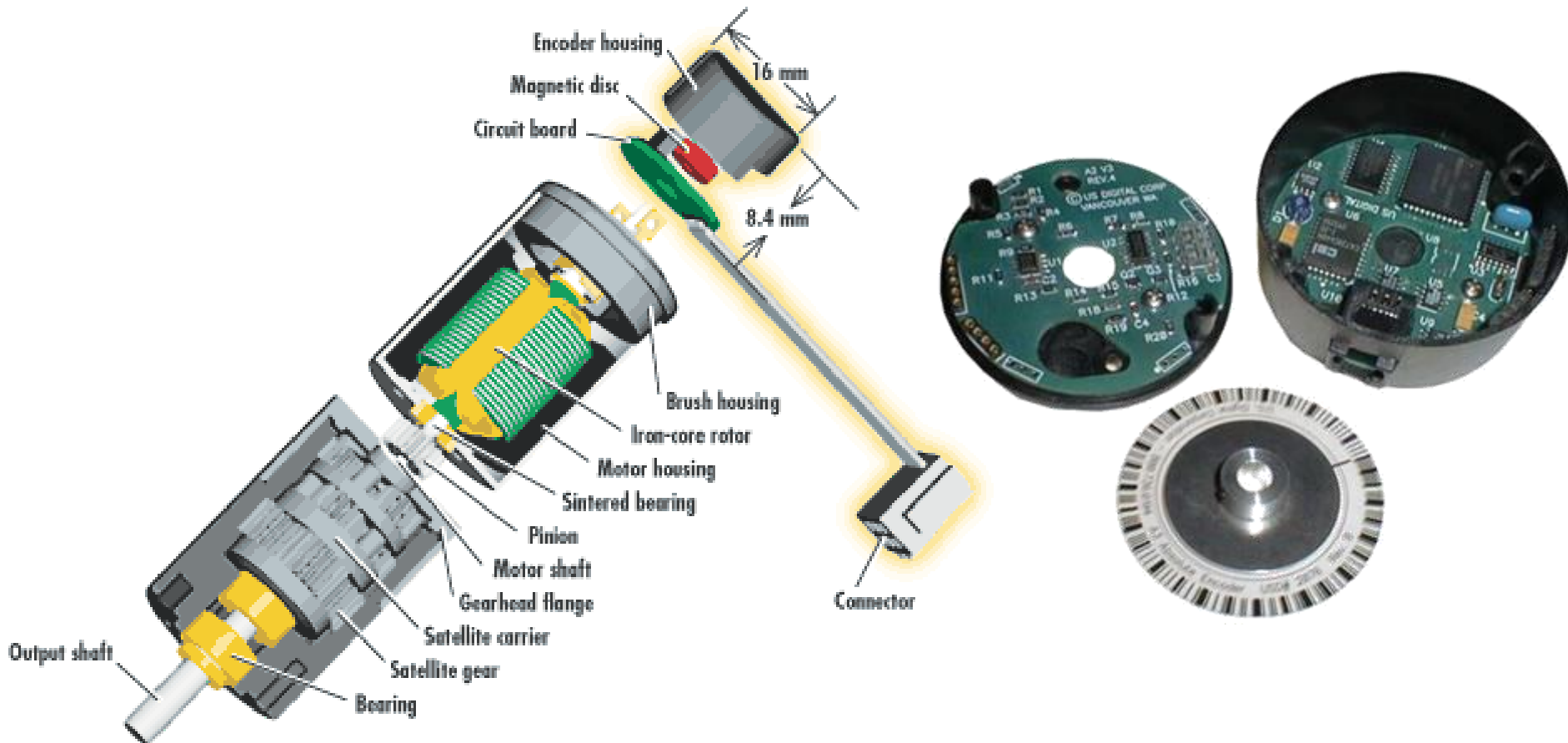


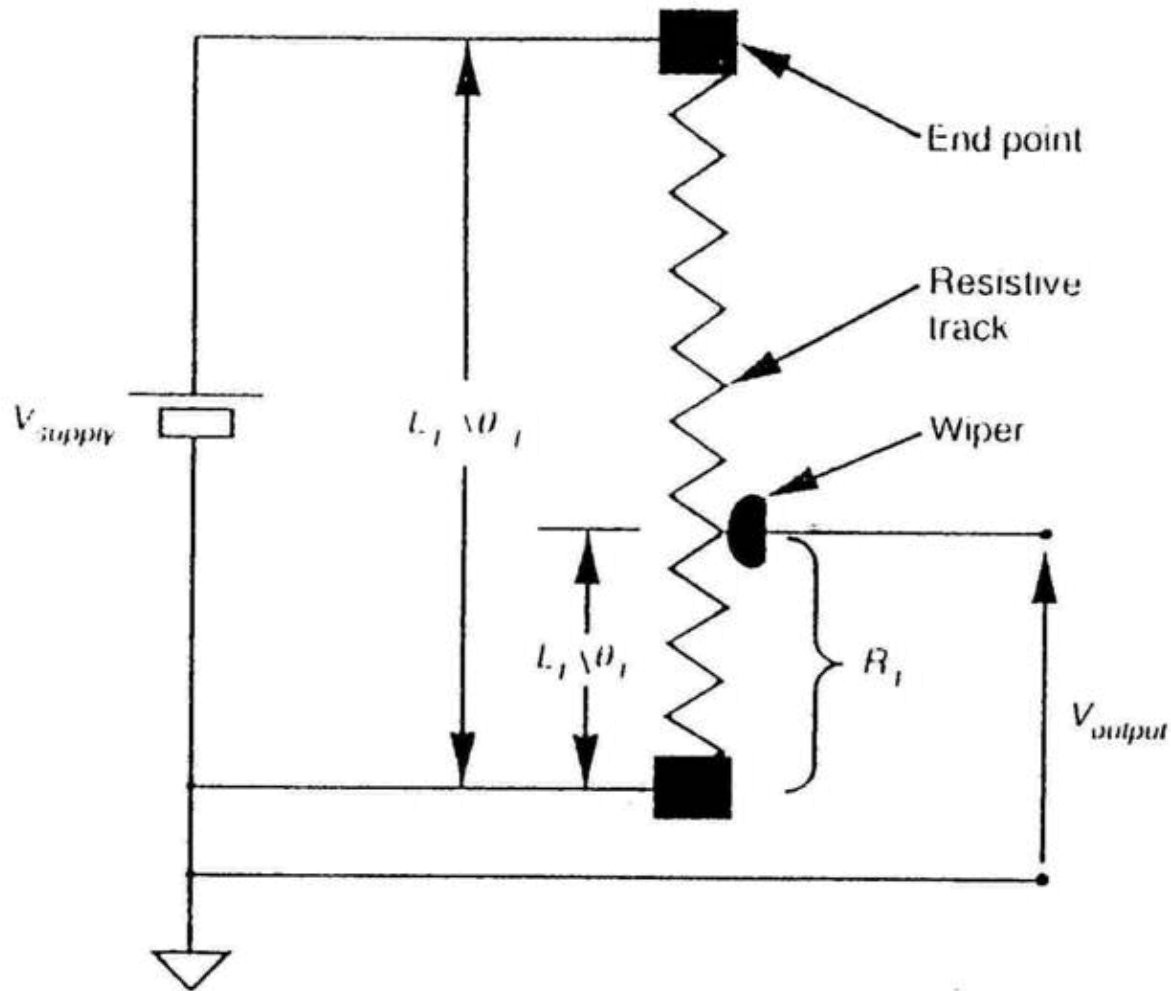
Fig 2. 4-Bit gray code absolute encoder disk track patterns



# Optical encoder in an electric motor



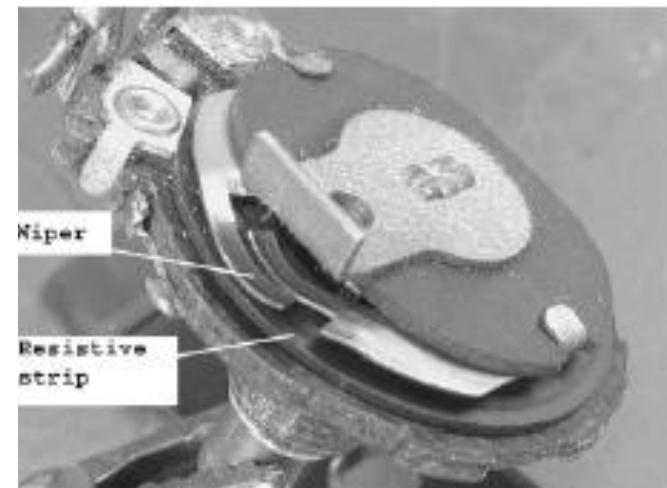
# Potentiometers



Variable resistor

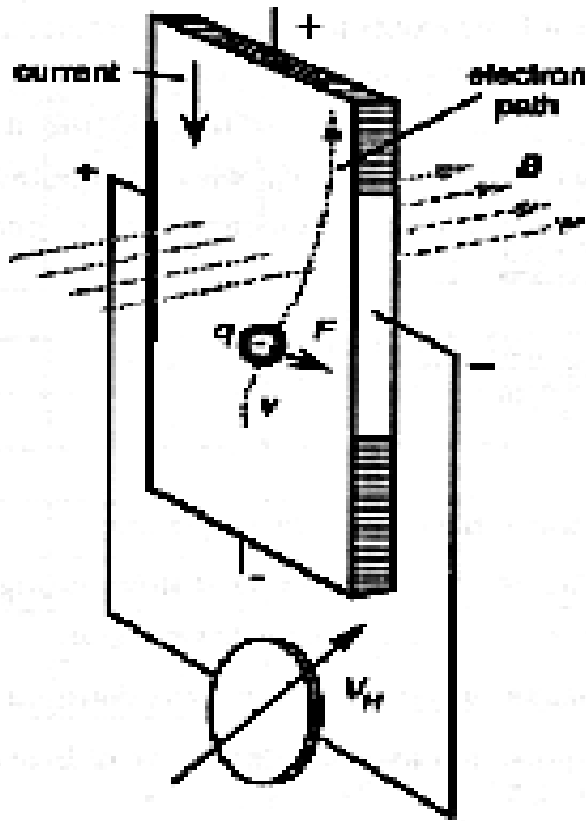
$$L_1 = R_1 L_T / R_T =$$

$$= V_{output} L_T / V_{supply}$$



# Hall-Effect sensors

In a conductor where a current  $i$  flows, immersed in a magnetic field of intensity  $B$ , a voltage  $V$  originates in the direction normal both to the current and to the magnetic field.



Voltage is proportional to:

- intensity of the current  $i$
  - intensity of the magnetic field  $B$ ,
- while it is inversely proportional to:
- material thickness  $d$ :

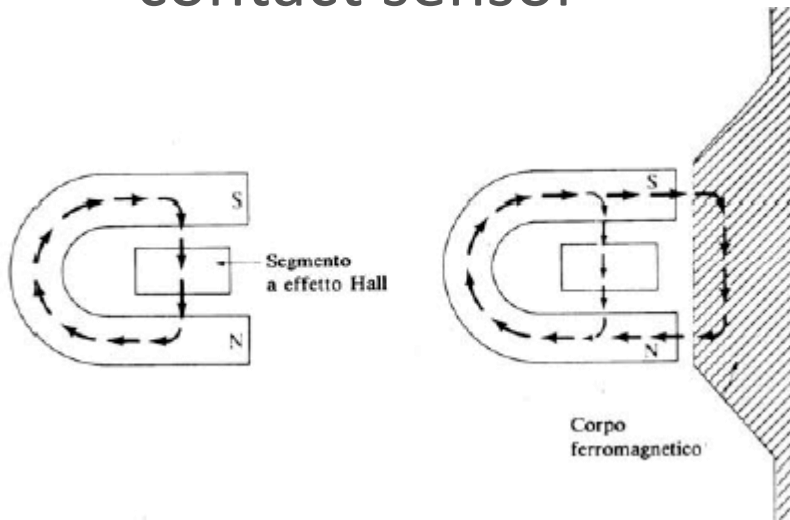
$$V = R i B / d$$

where  $R$  = Hall constant or coefficient

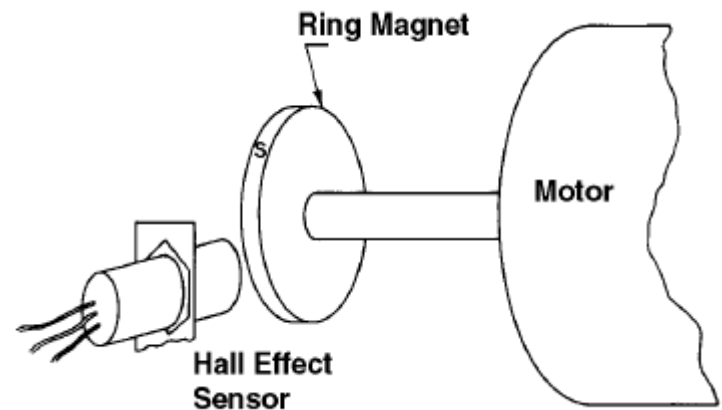


# Hall-effect sensors

## Hall-effect proximity and contact sensor



## Hall-effect position sensor



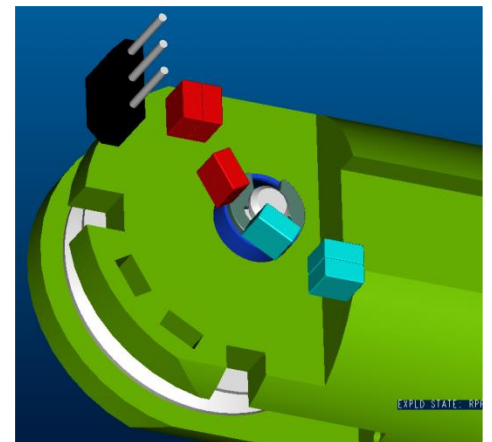
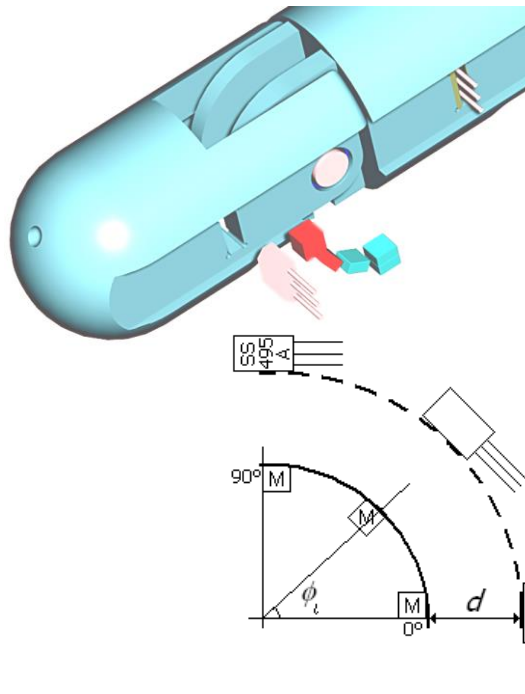
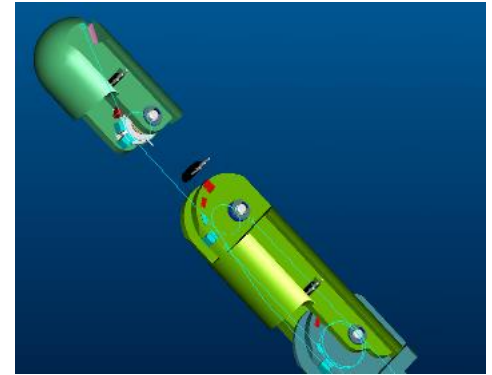
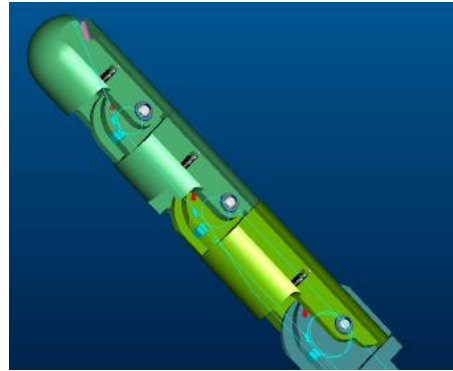
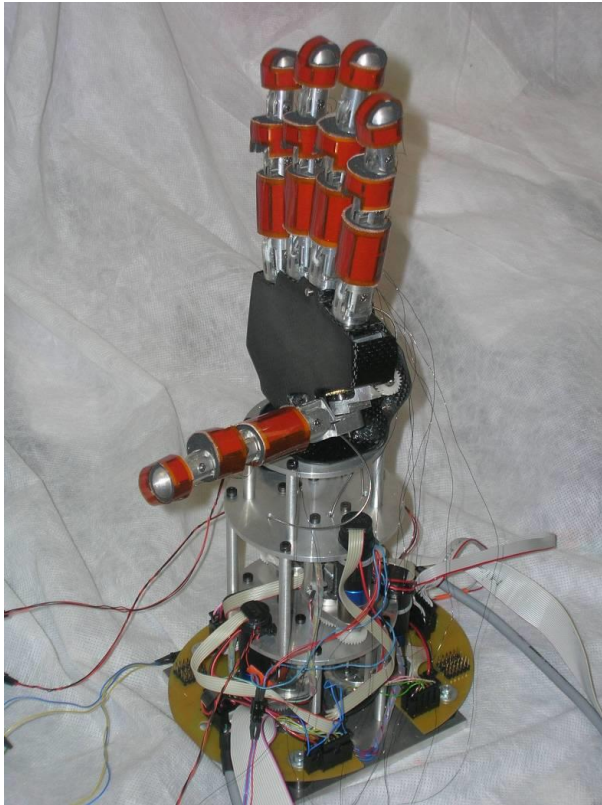
A permanent magnet generates a magnetic field.  
The contact with a ferromagnetic object modifies the magnetic field.  
The Hall effect measures this variation as a voltage



# Hall-effect sensors as position sensors in robotics

## 15 Embedded Joint Angle Sensors (Hall effect)

(Operational range: 0 – 90 degrees,  
Resolution: <5 degrees).





# HUMANGLOVE

## Studia la postura della mano

MOTION  
LINE

Patent IT/PI1997A000026

**Humanglove** è un guanto sensorizzato a 22 gradi di libertà in grado di rilevare in tempo reale i movimenti della mano durante qualsiasi attività. Può essere utilizzato per applicazioni in Medicina, Neuro-Riabilitazione, Telerobotica e Realtà Virtuale.



**HumanGlove** è compatibile con lo standard di trasmissione dati Bluetooth. In questo modo, dopo averlo indossato è possibile muoversi liberamente, anche in ambienti esterni.

Il guanto è realizzato in materiale elastico e può essere indossato da utenti con mani di taglia diversa. Grazie ad una rapida operazione di calibrazione è possibile adattare le letture dei sensori per un nuovo utente ed i parametri di calibrazione possono essere salvati e riutilizzati successivamente.

Il software mostra i dati in formato numerico, analogico e grafico.



Modulo sensore (brevettato)



### INDOSSABILITÀ

- Il dispositivo offre un elevato comfort grazie all'impiego di tessuti sintetici leggeri ed elastici e all'ingombro molto ridotto dei componenti.
- Il peso complessivo è ca. 290g
- Il sistema può anche lavorare in un ambiente non dedicato (ad es. all'aperto) perchè non necessita di collegamento via cavo.

**HumanGlove** fa uso di ventidue sensori:

- tre sensori di flessione-estensione ed un sensore di abduzione-adduzione per ciascun dito (pollice compreso)
- un sensore di flessione-estensione ed un sensore di abduzione-adduzione per il polso

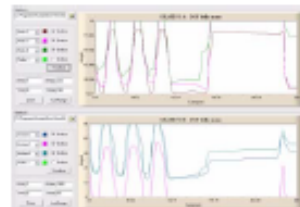
L'utilizzo di sensori ad effetto Hall garantisce una risposta lineare ed un elevato grado di robustezza e affidabilità.



### CARATTERISTICHE DEL SISTEMA

- Accuratezza dei sensori: 0.1V / 2.5V
- Linearità dei sensori: < 2.0%
- Range dei sensori: > 110°
- Converter: 12 bit A/D
- Alimentazione: 4 batterie AAA
- Trasmissione dati: Bluetooth
- Freq. campionamento: max 100 Hz

La connessione Bluetooth concede all'utente ampia libertà di movimento. La connessione alla periferica avviene attraverso una porta seriale virtuale RS-232 su USB; in questo modo essa può essere collegata a qualsiasi tipo di workstation.



**Humanware** è una società costituita da specialisti in varie discipline, dall'ingegneria meccanica all'informatica ed è una spin off della Scuola Superiore Sant'Anna di Pisa.

# Example of application of Hall-effect sensors

Sensorized glove for detecting finger movements



Modulo sensore (brevettato)



**Humanware**  
S.p.A.

Via Garibaldi, 1 - 56126 Pisa (P)  
Tel: +39 050 576023 - Fax: +39 050 573270  
web: www.humanware.it - mail: info@humanware.it



RINA  
Certified

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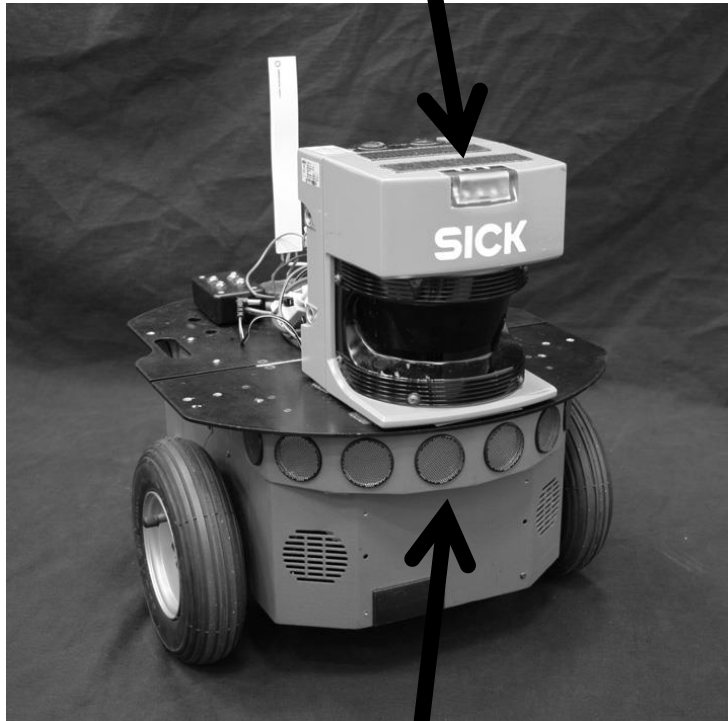
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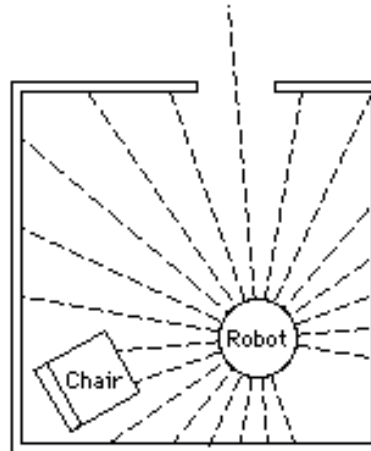
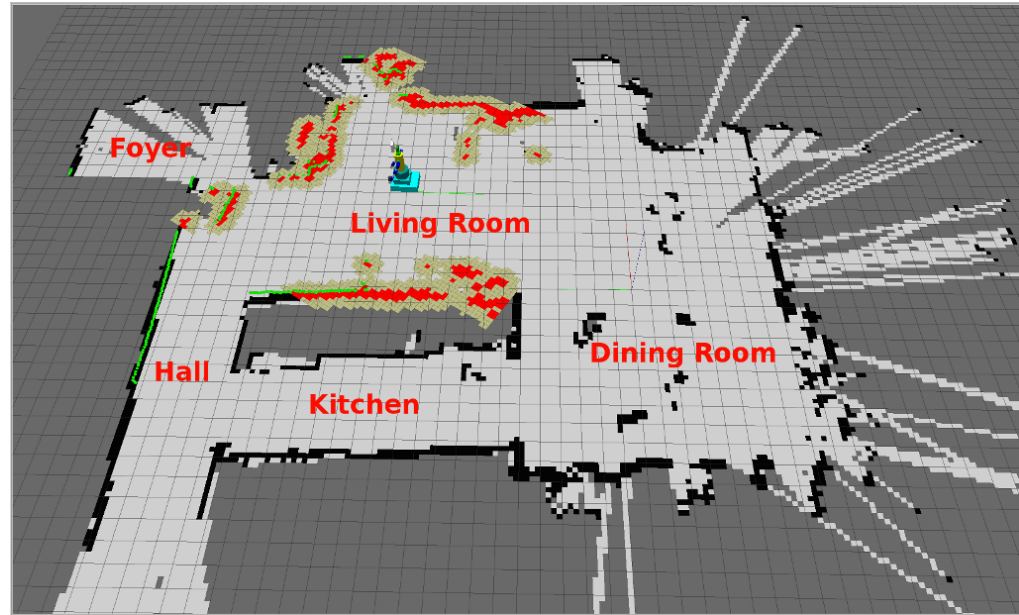


# Range\*/distance sensors

Laser scanner



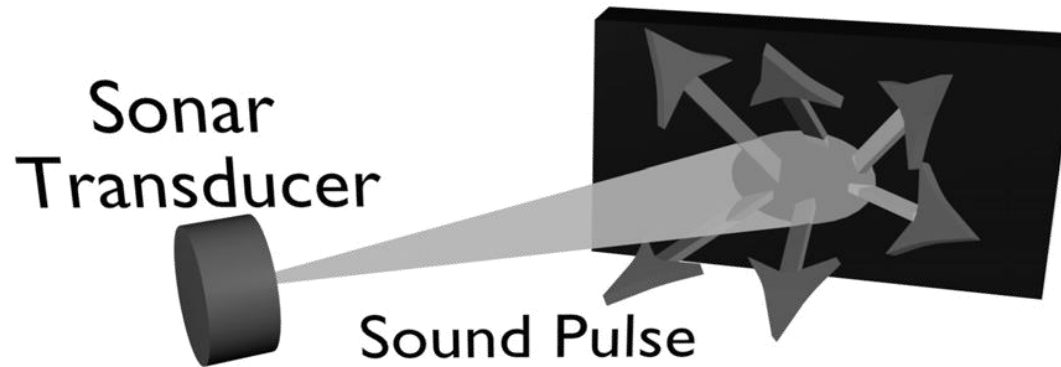
US (ultrasound) sensors



\*Range is the distance between the sensor and the object detected



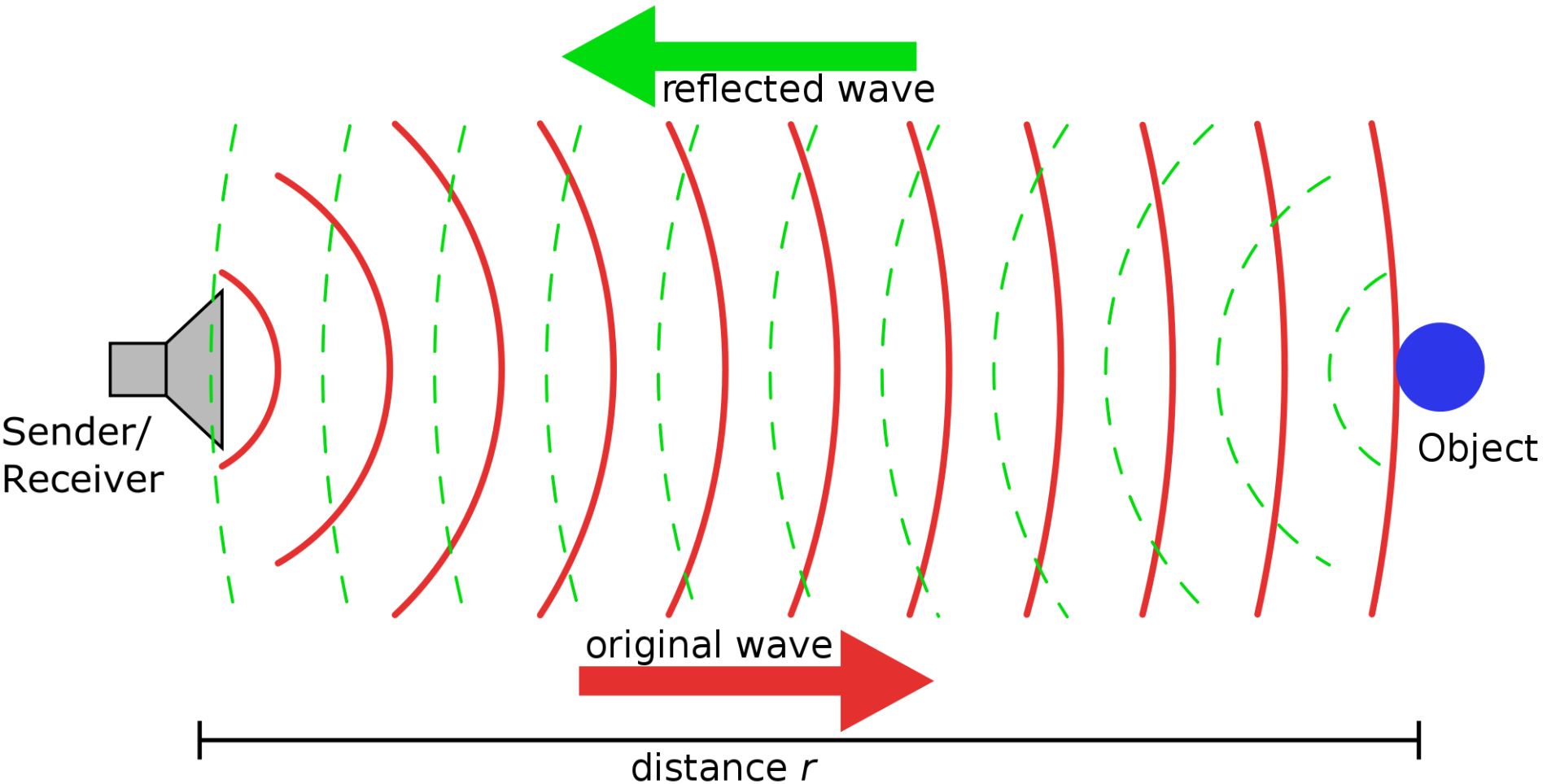
# Ultrasound sensors



Measurement of **range**  
based on **time of flight**



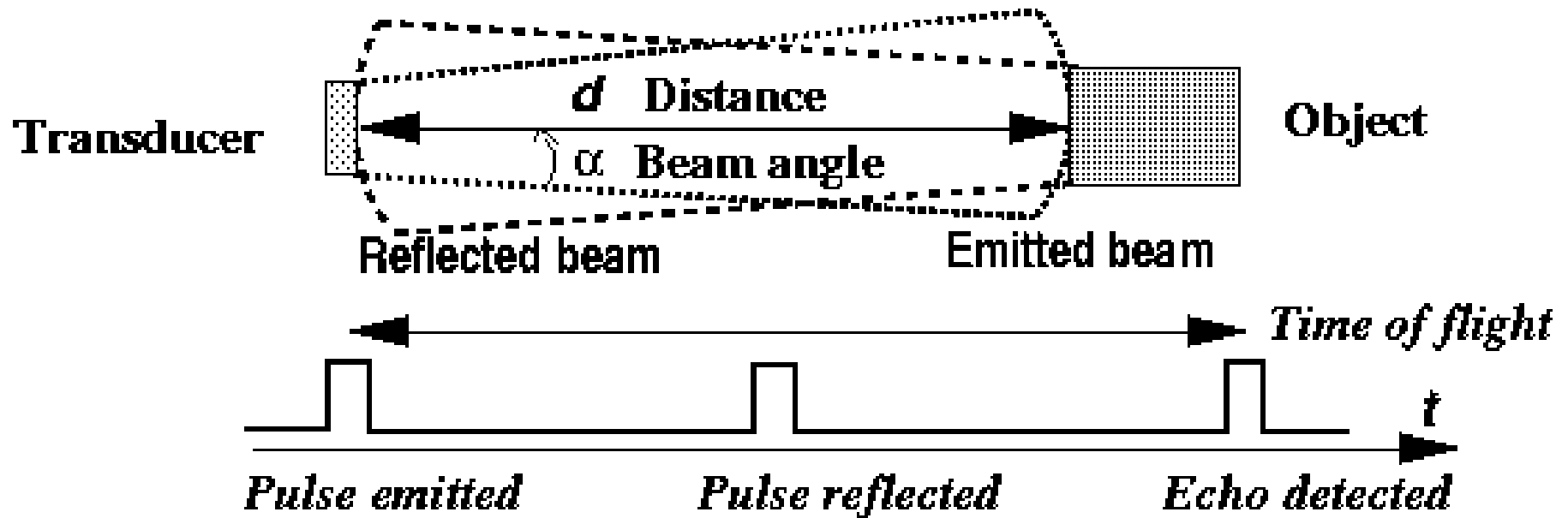
# Time-of-flight distance measurement



# Time-of-flight distance measurement

$$d = 0.5 t_e v$$

where  $v$  is the average speed of the signals emitted (air or water) and  $t_e$  is the time between the signal emitted and the signal echo received.



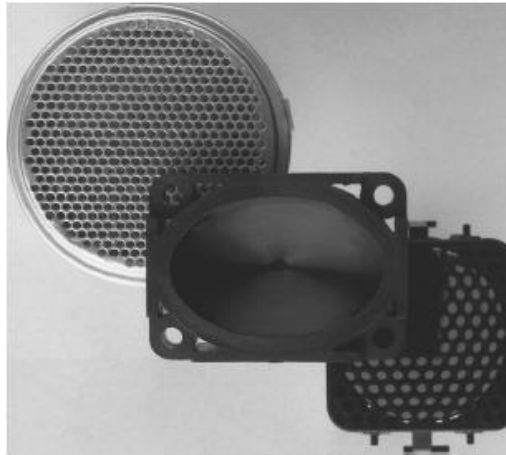
# Ultrasound sensors

2 main components:

- ultrasound transducer (working both as emitter and as receiver)
- electronics for computing the distance

Typical working cycle:

- the electronics controls the transducer to send ultrasounds
- the receiver is disabled for a given time, in order to avoid false responses due to residual signal in the transducer
- the received signal is amplified with an increasing gain, to compensate the reduction of intensity with distance
- echos above a given threshold are considered and associated to the distances measured from the time passed from transmission



Range: 0.3m to 10.5m  
Beam amplitude: 30°  
Accuracy: ca. 25mm



# Examples of application of ultrasound sensors on mobile robots



**Pioneer I – Real  
Word Interface,  
USA**

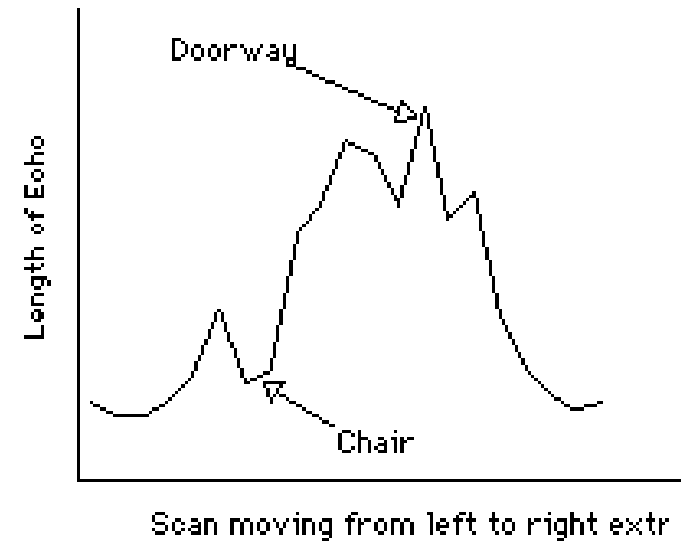
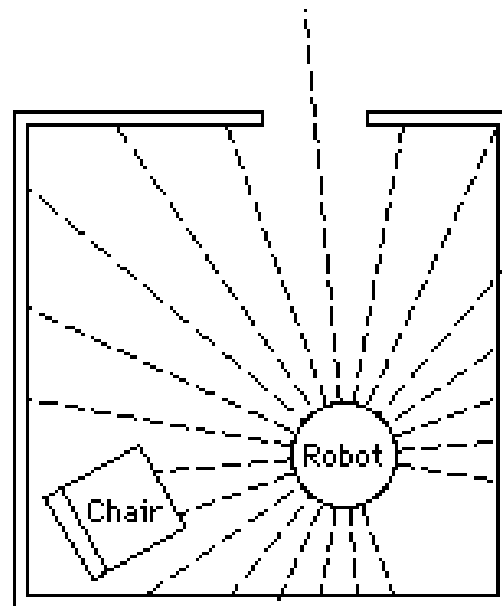
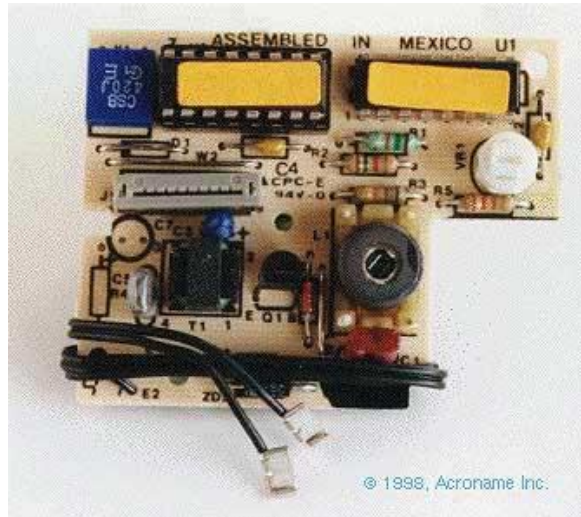


**B21r – Real Word  
Interface, USA**



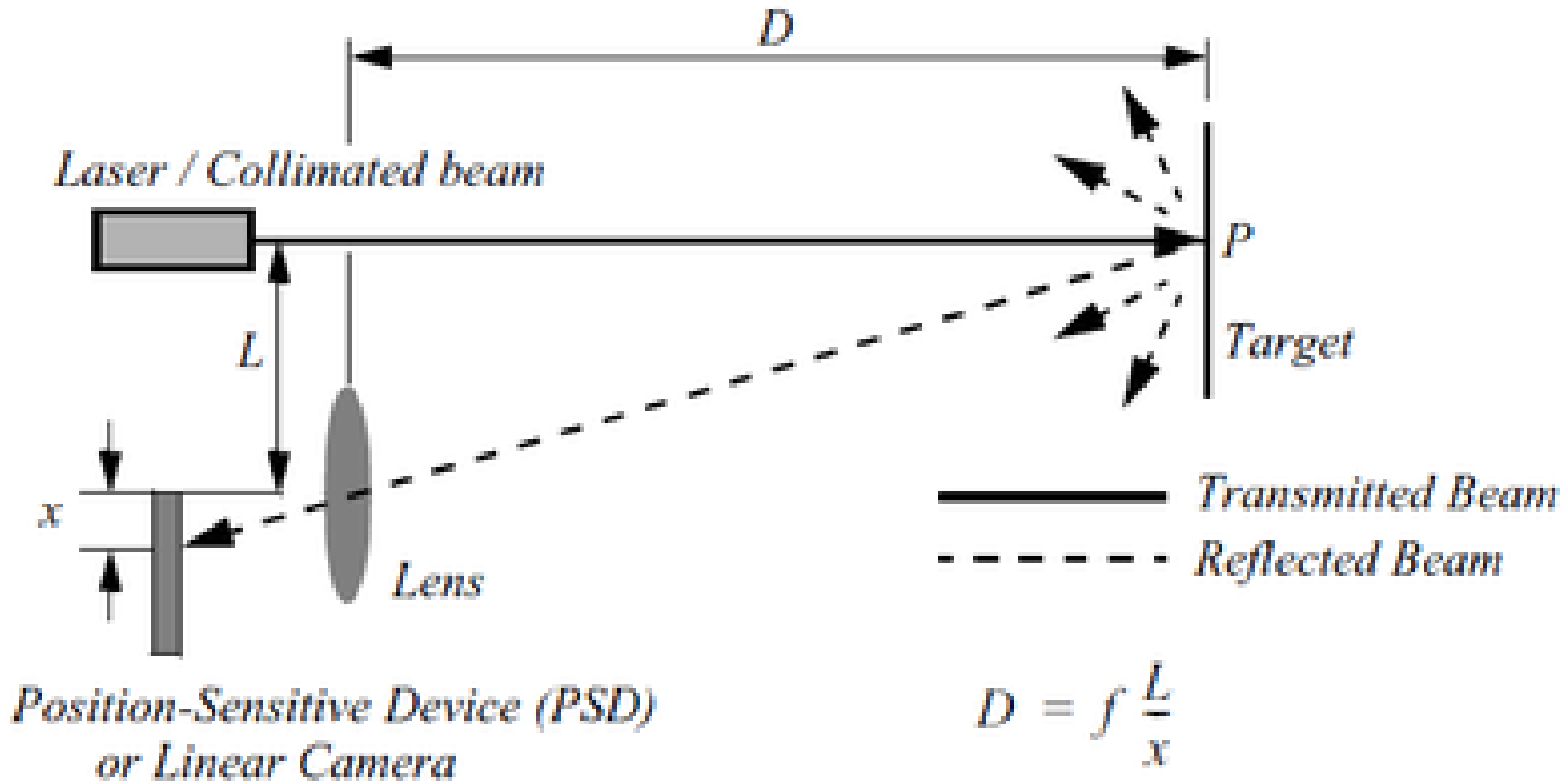


# Examples of application of ultrasound sensors on mobile robots



# Laser range finders

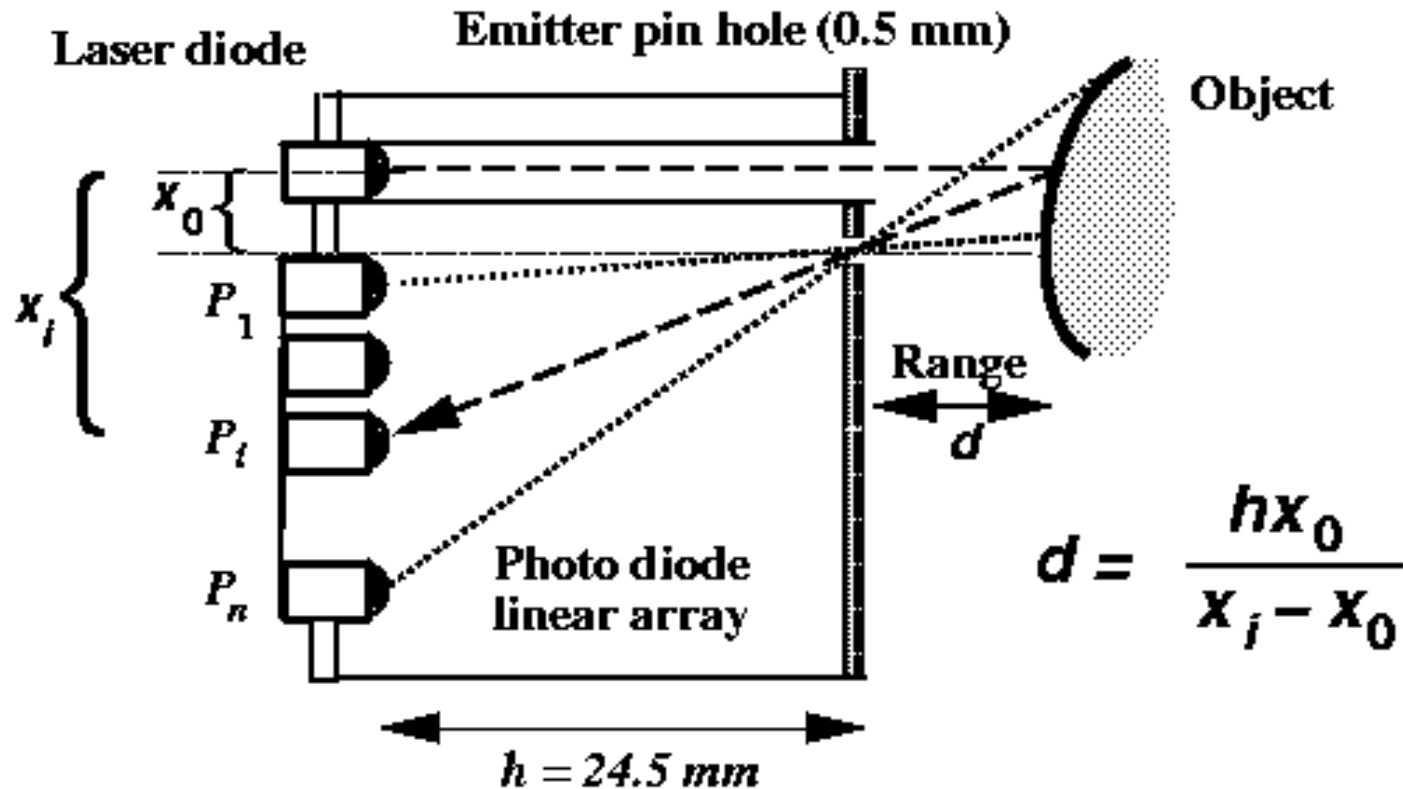
Measurement of range  
based on **phase-shift**





# Laser range finders

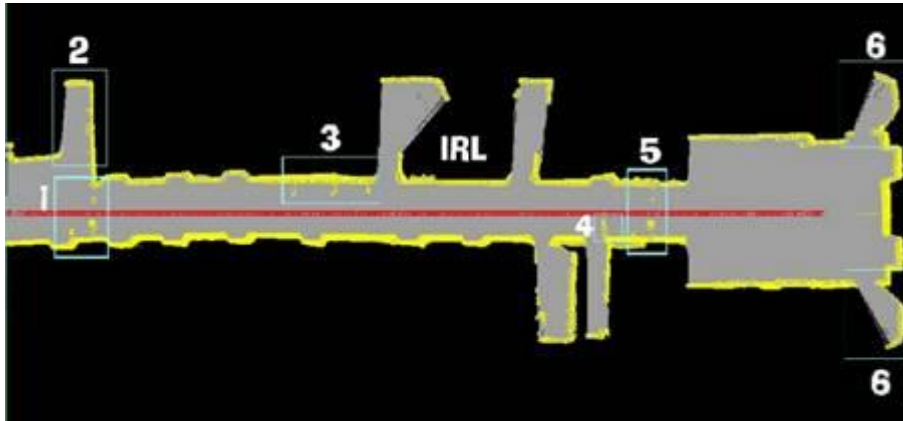
Measurement of range  
based on **phase-shift**



A simple **pin-hole short-range-finding sensor** uses a laser diode as a light source, and a linear photo-diode array as a detector. The range from a sensor to the object is a function of the position of the maximum detected light along the array.



# Example of application of laser finder on mobile robots



Map building using the  
LMS 200 laser scanner



## Technical specification

	Angular Resolution		1° / 0,5° / 0,25°
	Response Time (ms)		13 / 26 / 53
	Resolution (mm)		10
	Systematic Error (mm mode)		+/- 15 mm
	Statistic Error (1 Sigma)		5 mm
	Laser Class		1
	Max. Distance (m)		80
	Data Interface		RS422 / RS232



# Outline of the lesson

- Definitions of sensor and transducer
- Classification of transducers
- Fundamental properties of sensors
- Position sensors: switches, encoders, potentiometers, Hall-effect sensors
- Range/Distance sensors: ultrasound sensors and laser range finders
- **Proximity sensors: Hall-effect and infrared sensors**
- Force sensors: strain gauges and force/torque sensors
- Inertial sensors



# Proximity sensors

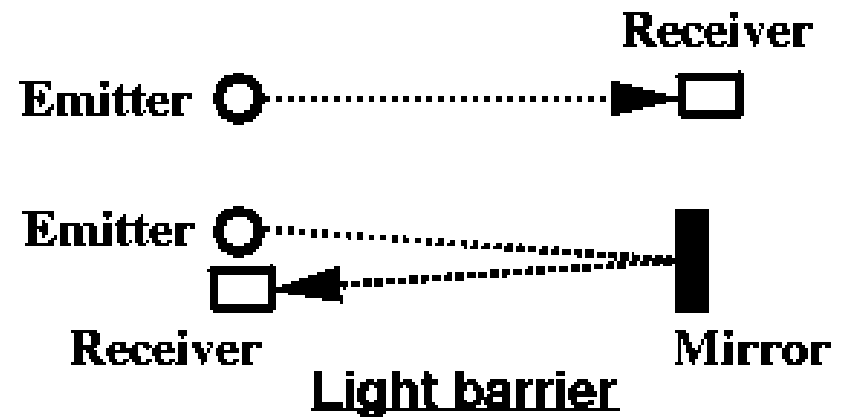
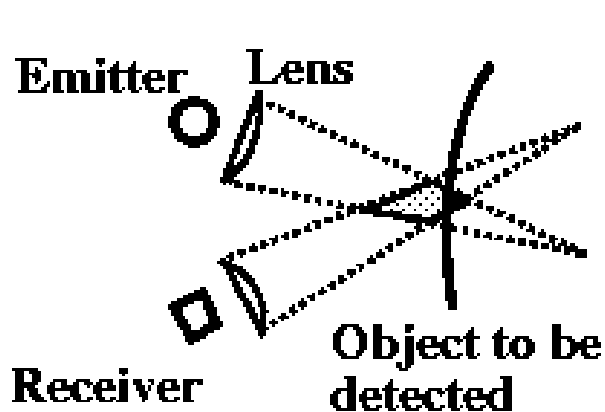
Sensing the presence of an object in a **spatial neighborhood**

**Passive proximity sensors** detect perturbations of the environment, like for instance modifications of the magnetic or the electric field

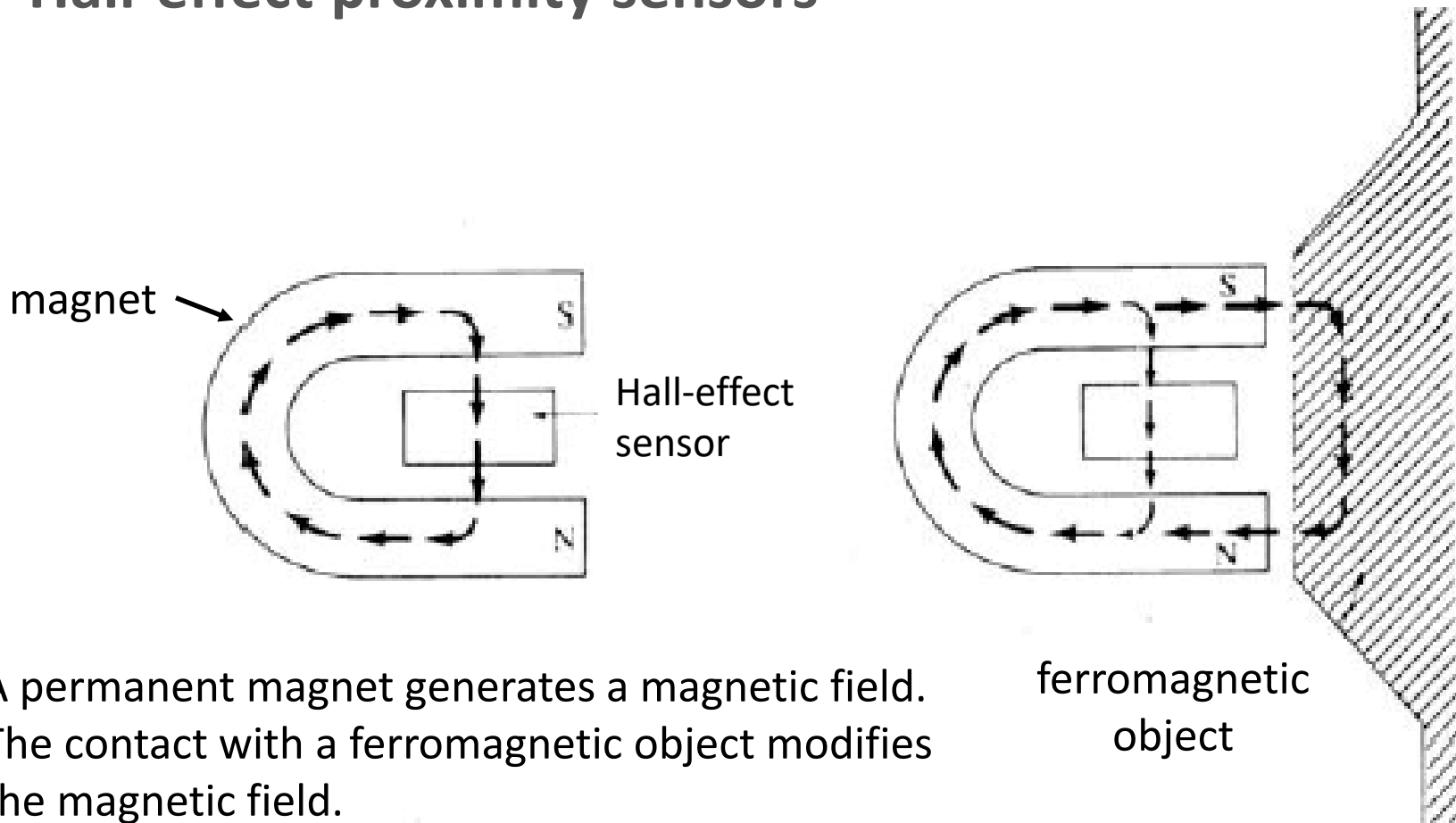
**Active proximity sensors** emit a signal and detect it back, detecting variations or interruptions of the signal received

Ex: magnetic passive sensors: Hall-effect sensors

Ex: active optical sensors: emitter and receiver of light signals



# Hall-effect proximity sensors

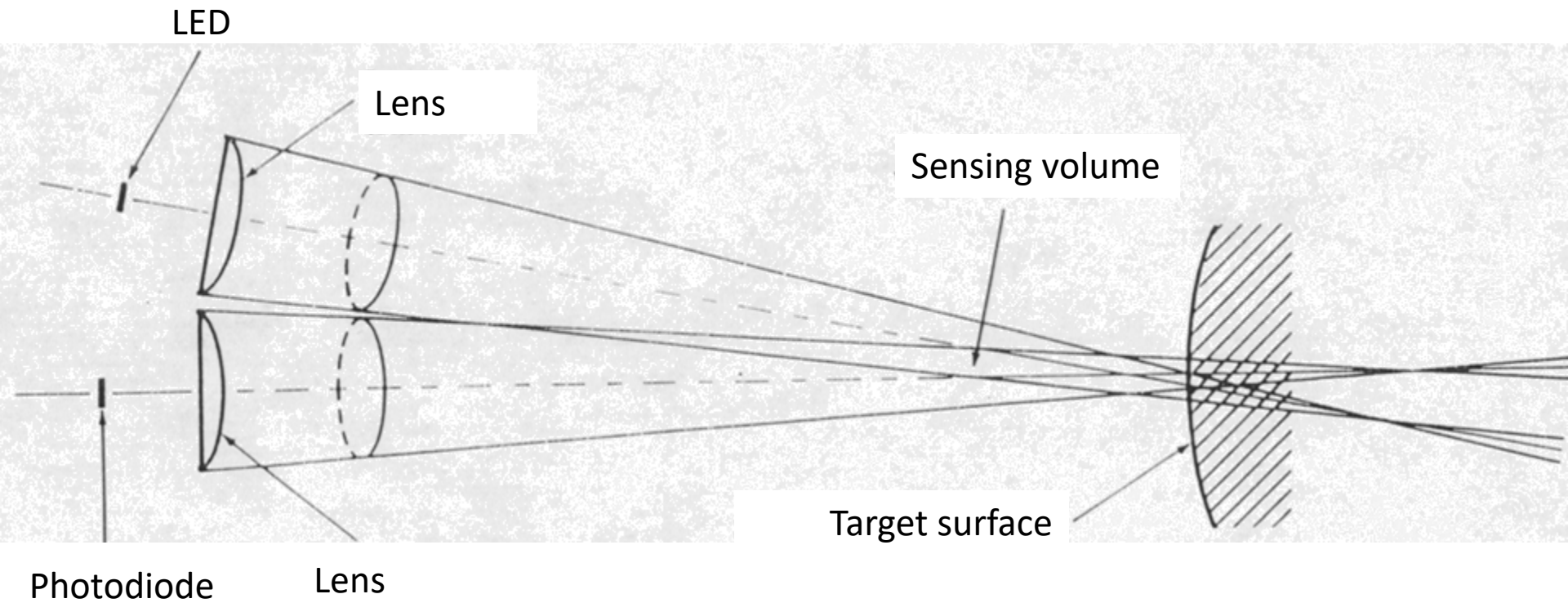


A permanent magnet generates a magnetic field.  
The contact with a ferromagnetic object modifies  
the magnetic field.

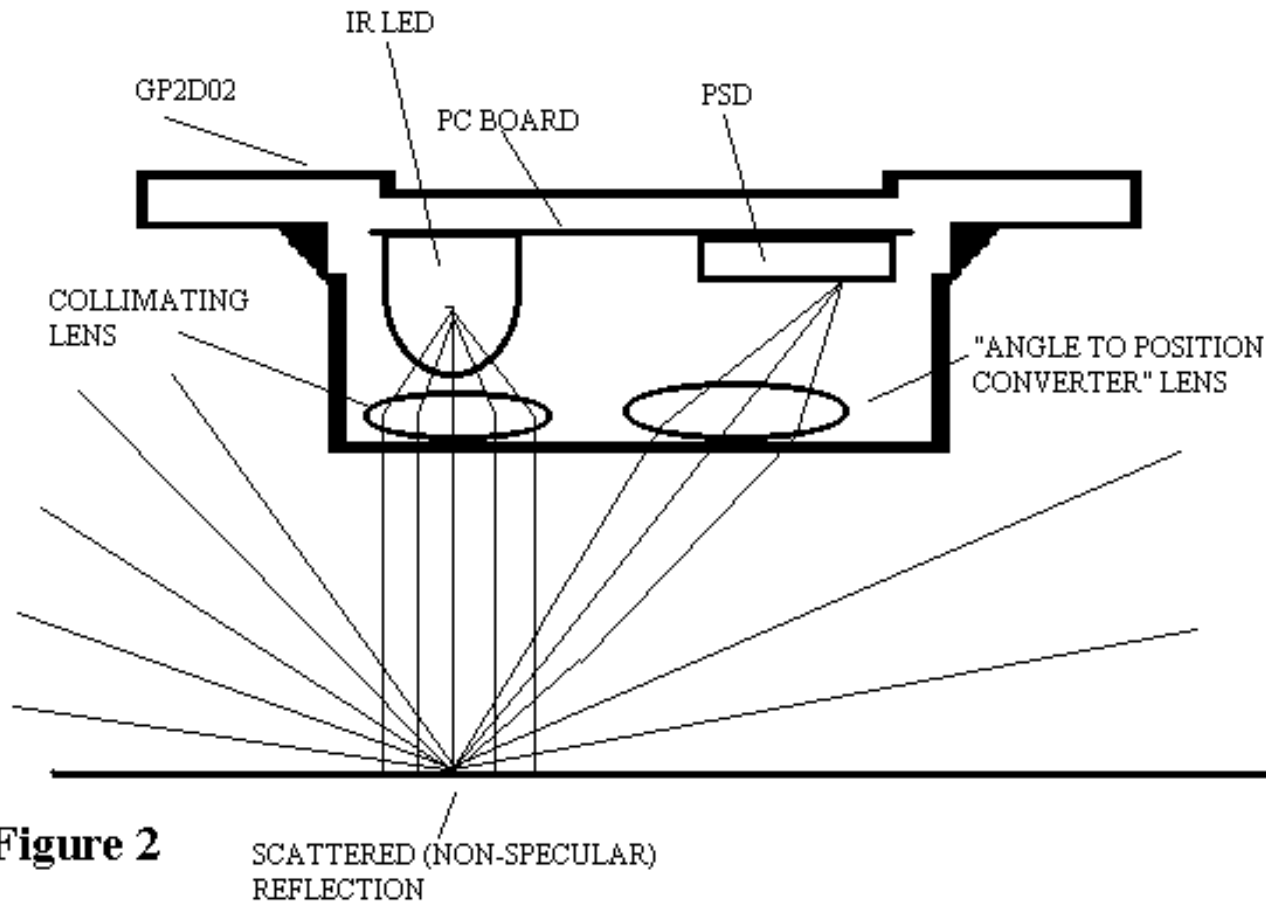
The Hall effect allows to measure this variation as  
a voltage



# Optical sensors



# Example of application of infrared optical sensor on mobile robots



**Figure 2**



# Outline of the lesson

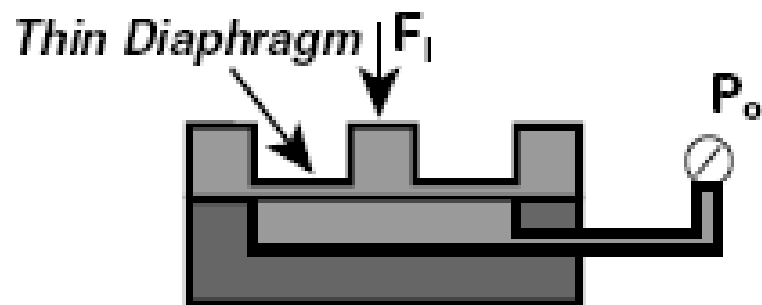
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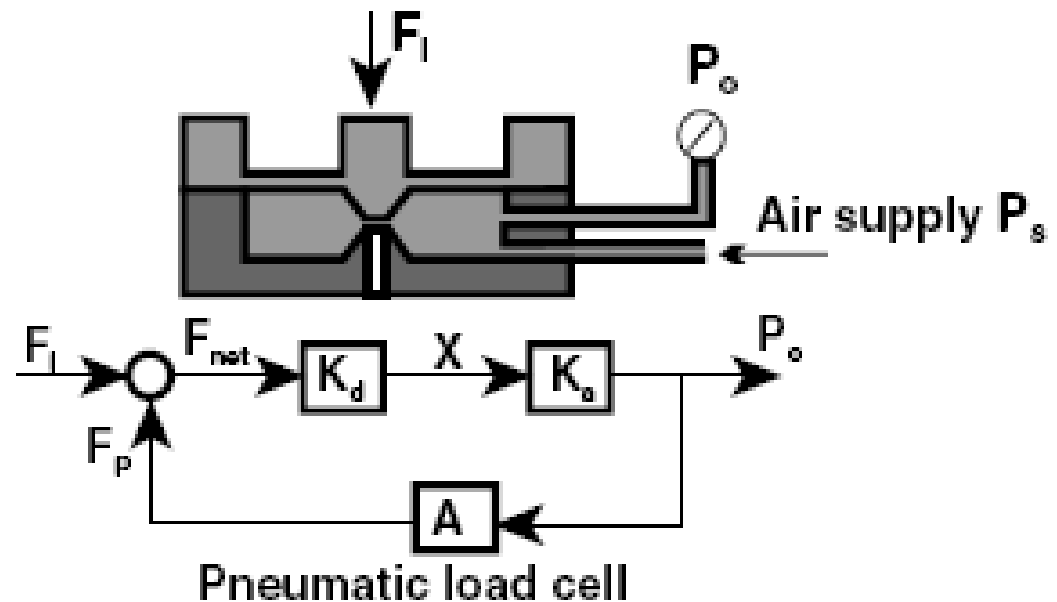


# Load cell structures

- Rigid external structure
- Indirect measure of the applied force
- Measuring element



Hydraulic load cell



Pneumatic load cell



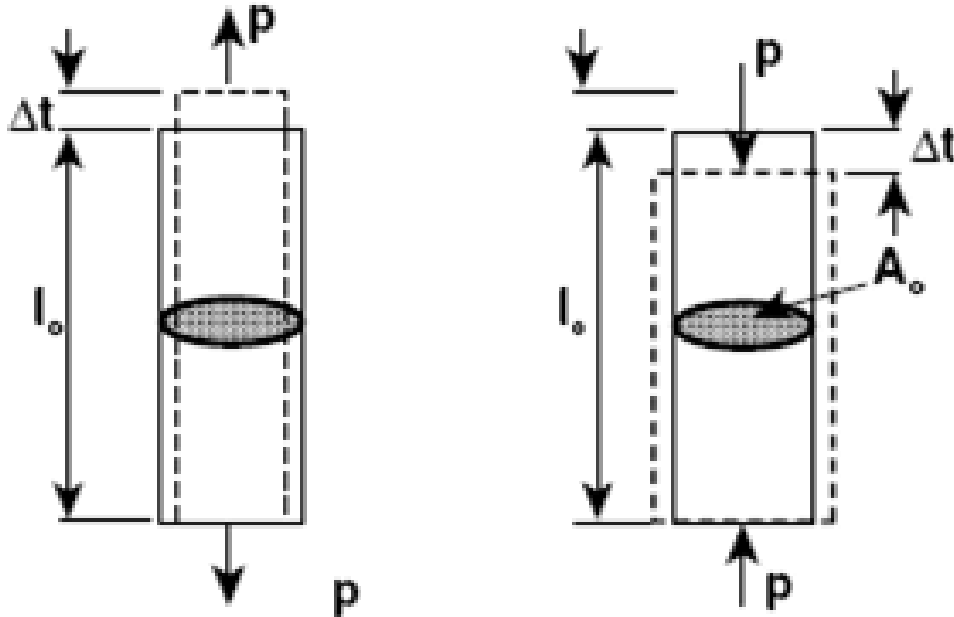
# Piezoresistive effect

Every material changes its electrical resistance with **strain**



# Basics of mechanical behavior of materials

Stress applied to a material causes strain. The material has an elastic behavior until a stress threshold (elastic limit), beyond which the material deformation is plastic



*stress*

$$\sigma = P / A_0$$

*strain*

$$\varepsilon = \Delta l / l_0$$

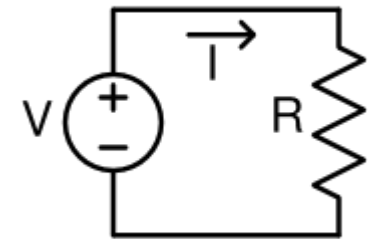
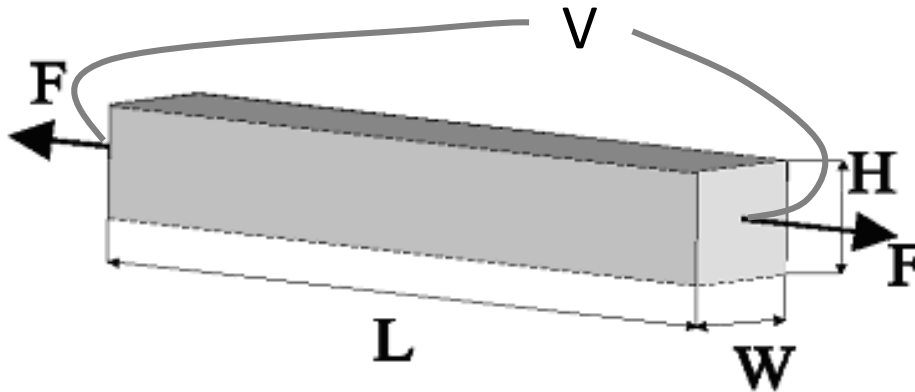
Poisson's ratio: 
$$\nu = - \frac{\delta A / A_0}{\varepsilon}$$

Elasticity module: 
$$E = \frac{\sigma}{\varepsilon}$$



# Piezoresistive effect

Every material changes its electrical resistance with **strain**



$$V=RI$$

In a metal block: 
$$R = \rho \frac{L}{WH}$$

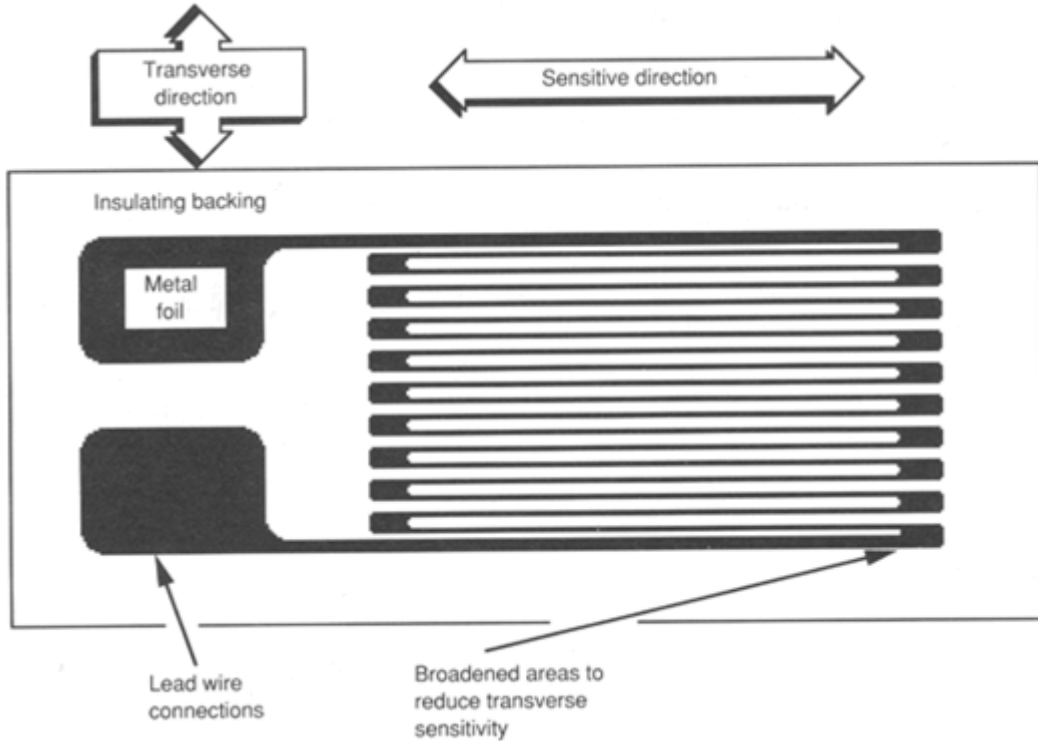
with  $\rho$  = resistivity of the material,  
 $L, W, H$  = dimensions of the block

$$\frac{\Delta R}{R} = \varepsilon + 2\nu\varepsilon + \frac{\Delta\rho}{\rho}$$

$\nu$  = Poisson's ratio of the material



# Strain gauge



The sensor shape increases sensitivity in one direction

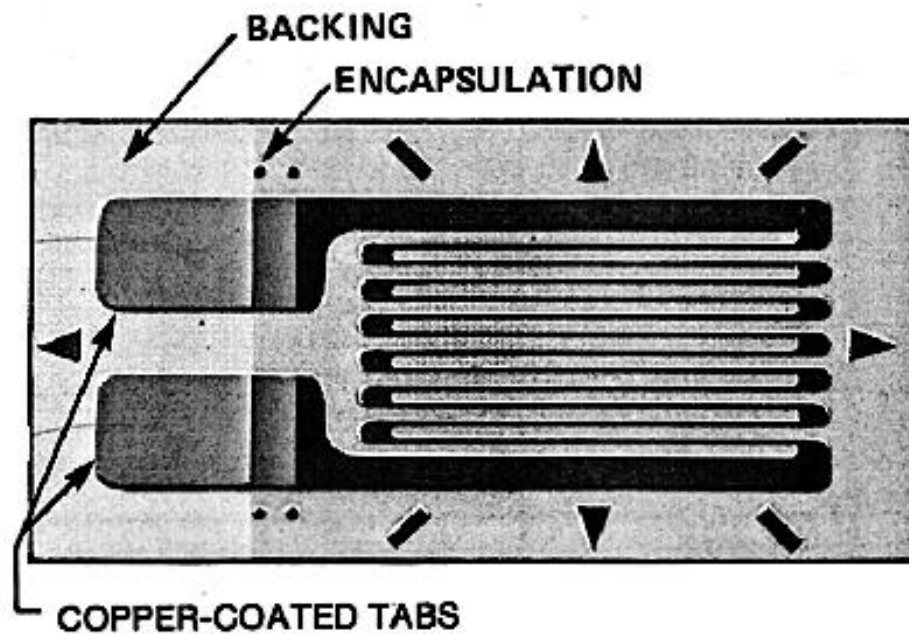
Gauge factor:

$$G = \frac{\Delta R/R}{\varepsilon} = 1 + 2\nu + \frac{\Delta\rho/\rho}{\varepsilon}$$

$\nu$  = Poisson's ratio of the material



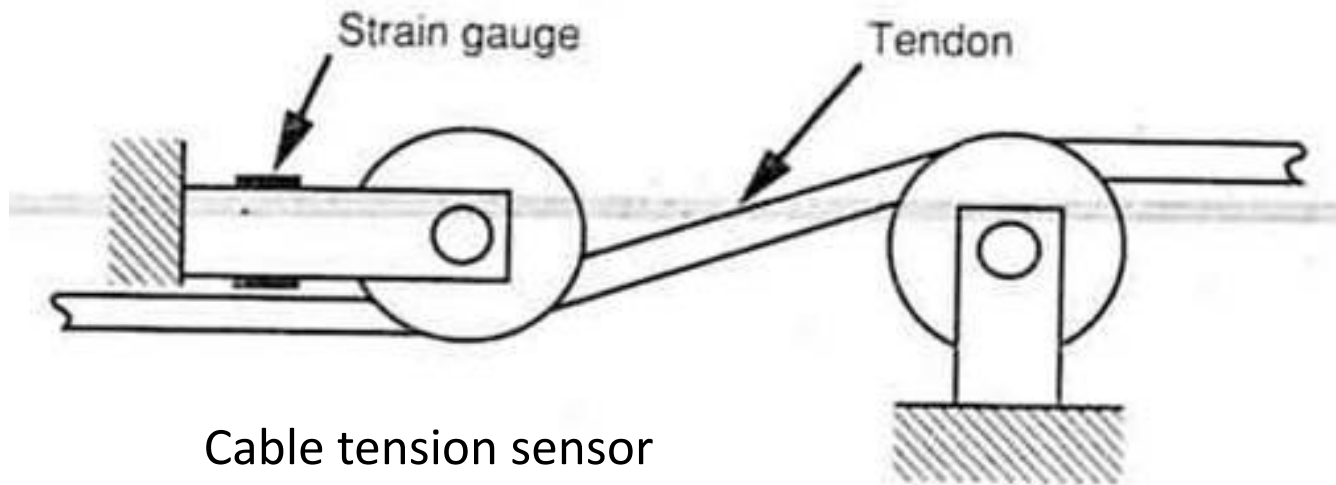
# Strain gauges



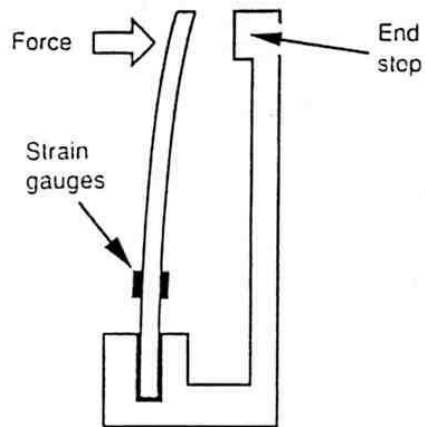
CODES FOR BASIC PATTERNS

N	Q
R	Y
T	C
U	X
Z	P

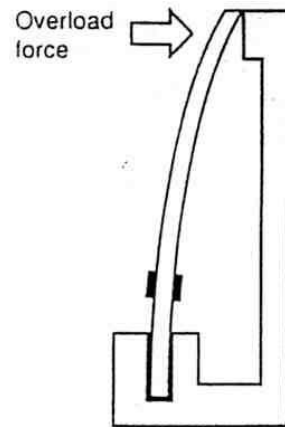
# Sensors with strain gauges



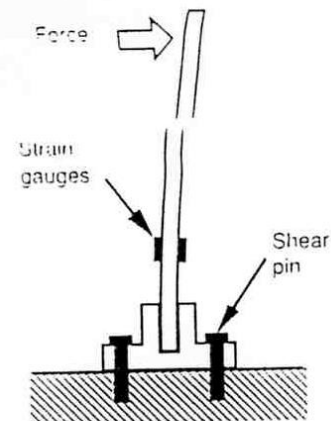
Cable tension sensor



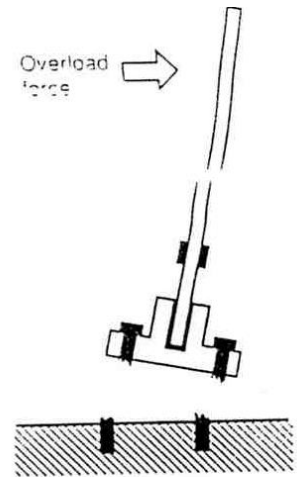
(a) Small applied force



(b) Overload force applied



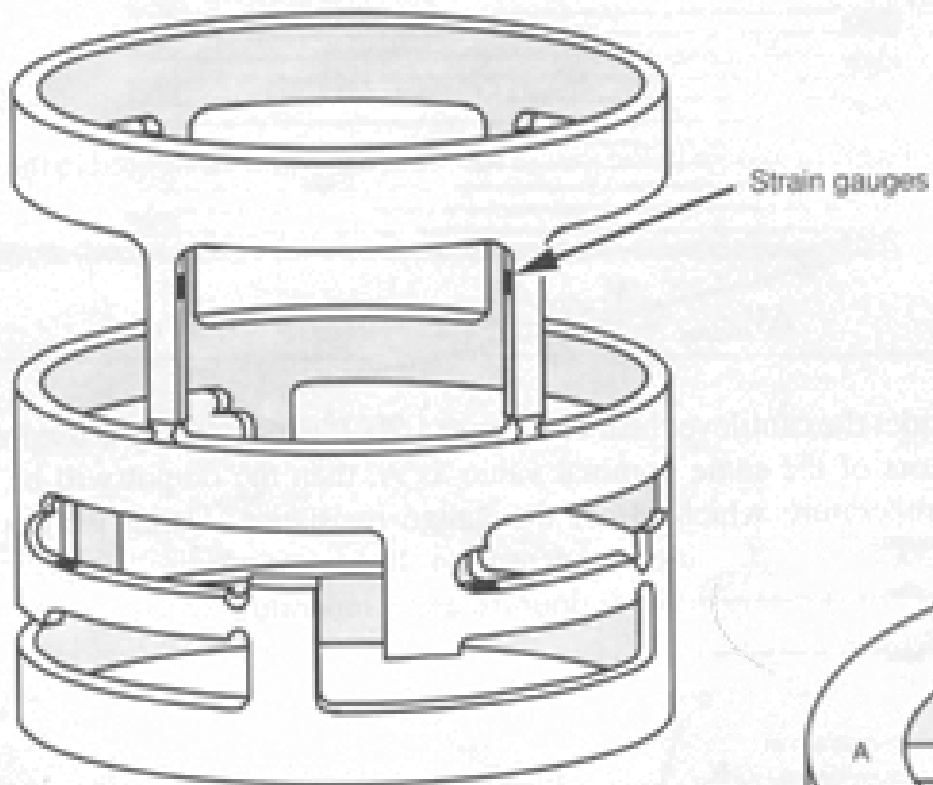
(a) Small applied force



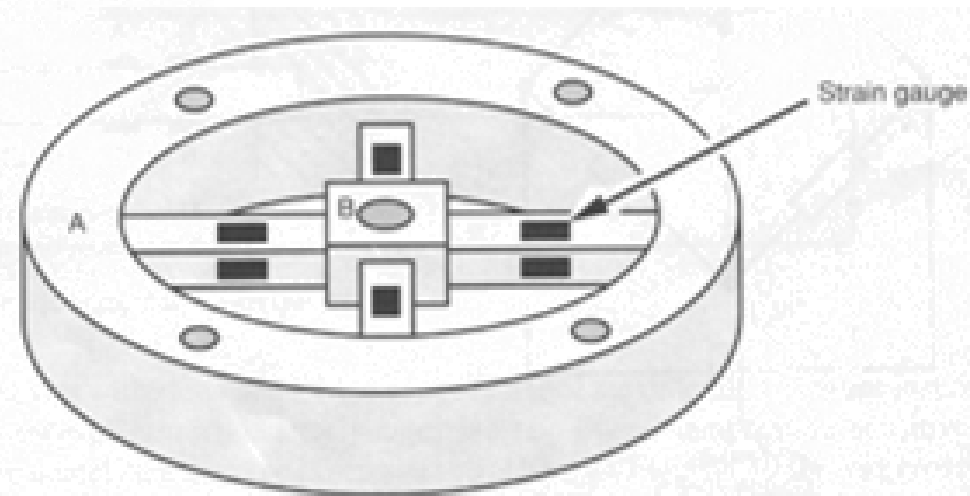
(b) Overload force applied



# Three-axial force/torque sensors



- Mechanical structure with preferred strain directions, along 3 axes
- Strain gauges arranged accordingly

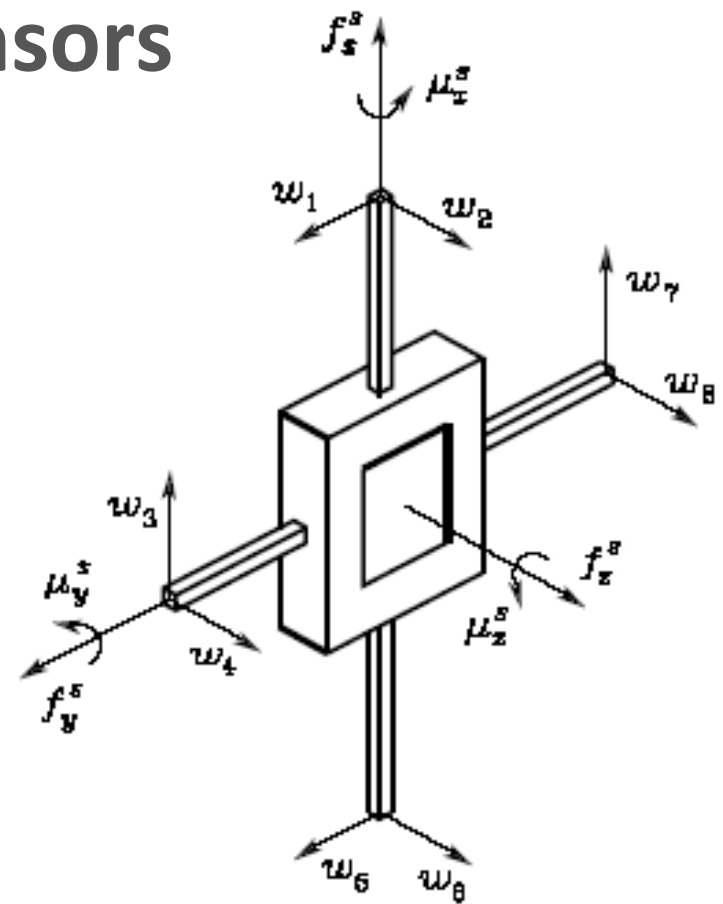




# Three-axial force/torque sensors

- Forces and torques are measured from measures of the resistance variations of the strain gauges, multiplied by a coefficient array, typical for each sensor
- The coefficient array is built by a calibration procedure in which known forces are applied

$$\begin{bmatrix} f_x^s \\ f_y^s \\ f_z^s \\ \mu_x^s \\ \mu_y^s \\ \mu_z^s \end{bmatrix} = \begin{bmatrix} 0 & 0 & c_{13} & 0 & 0 & 0 & c_{17} & 0 \\ c_{21} & 0 & 0 & 0 & c_{25} & 0 & 0 & 0 \\ 0 & c_{32} & 0 & c_{34} & 0 & c_{36} & 0 & c_{38} \\ 0 & 0 & 0 & c_{44} & 0 & 0 & 0 & c_{48} \\ 0 & c_{52} & 0 & 0 & 0 & c_{56} & 0 & 0 \\ c_{61} & 0 & c_{63} & 0 & c_{65} & 0 & c_{67} & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \\ w_8 \end{bmatrix}$$



# Example of sensors of a mobile robotic system

Hall-effect sensors on  
finger joints

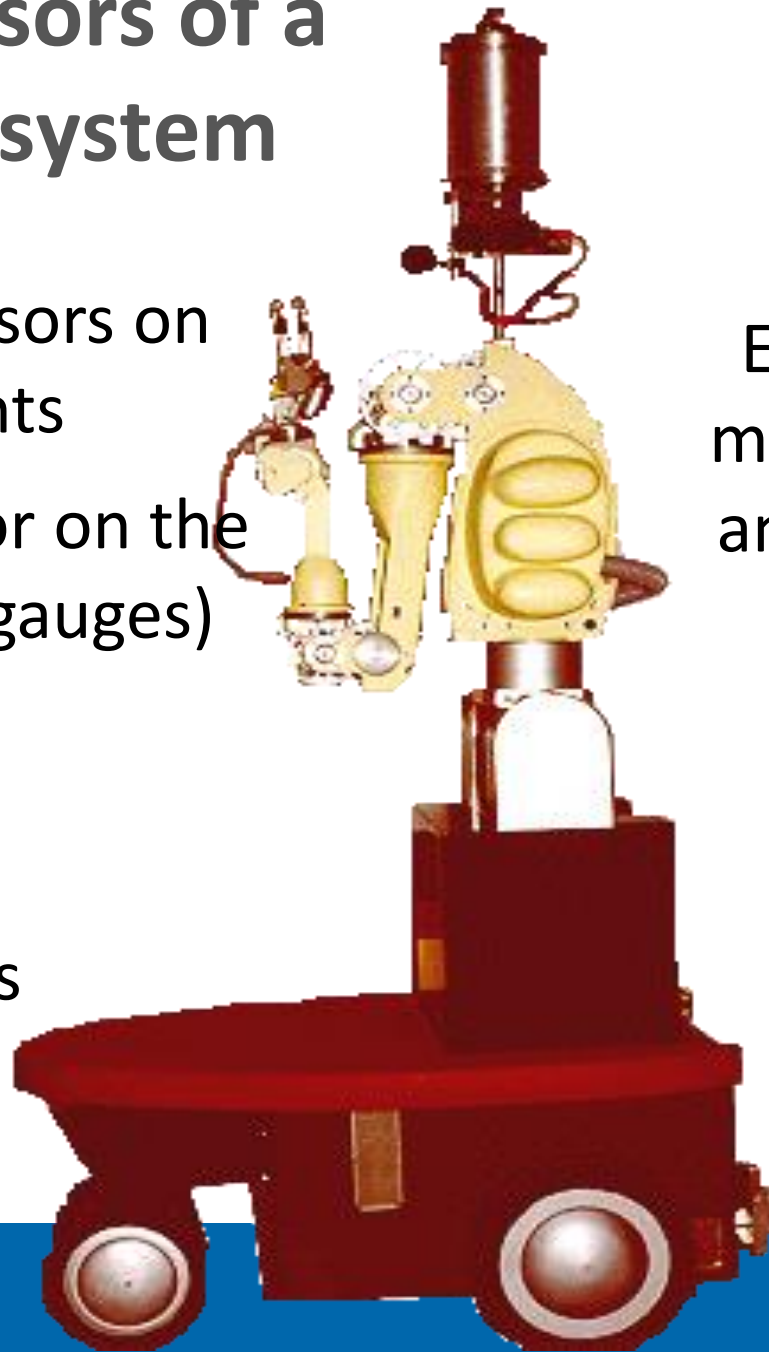
Force/torque sensor on the  
wrist (with strain gauges)

Ultrasound sensors

Switches on the  
bumper

Encoders on the  
motors of the arm  
and of the mobile  
base

Potentiometers  
in the docking  
system



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# Kinematic quantities

## ■ Position

□  $x(t); q(t)$

## ■ Velocity

□  $v(t); w(t)$

## ■ Acceleration

□  $a(t); a(t)$

## ■ Jerk

□ ...

$$\frac{d}{dt}$$

$$\int dt$$

$$\frac{d}{dt}$$

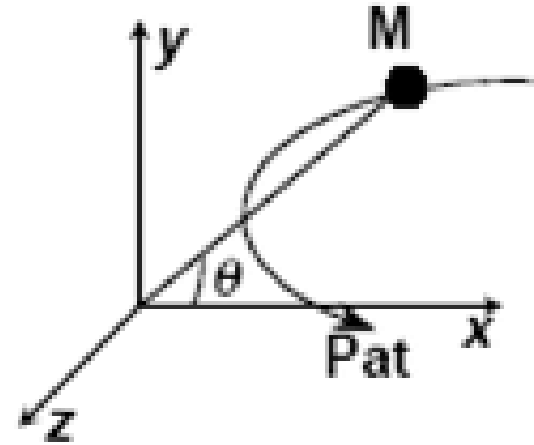
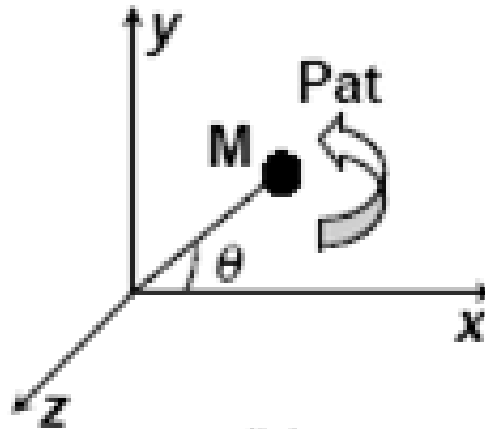
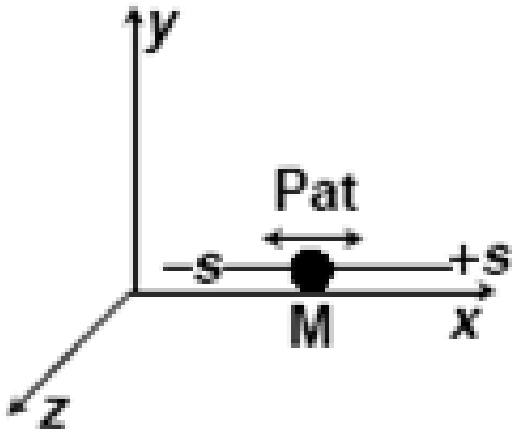
$$\int dt$$

$$\frac{d}{dt}$$

$$\int dt$$



# Types of motion



■ Linear:

$$a = \frac{dv}{dt} = \frac{d(ds/dt)}{dt} = \frac{d^2s}{dt^2}$$

■ Angular:

$$\alpha = \frac{d\omega}{dt} = \frac{d(d\theta/dt)}{dt} = \frac{d^2\theta}{dt^2}$$

■ Curve:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2x}{dt^2}\mathbf{i} + \frac{d^2y}{dt^2}\mathbf{j} + \frac{d^2z}{dt^2}\mathbf{k}$$

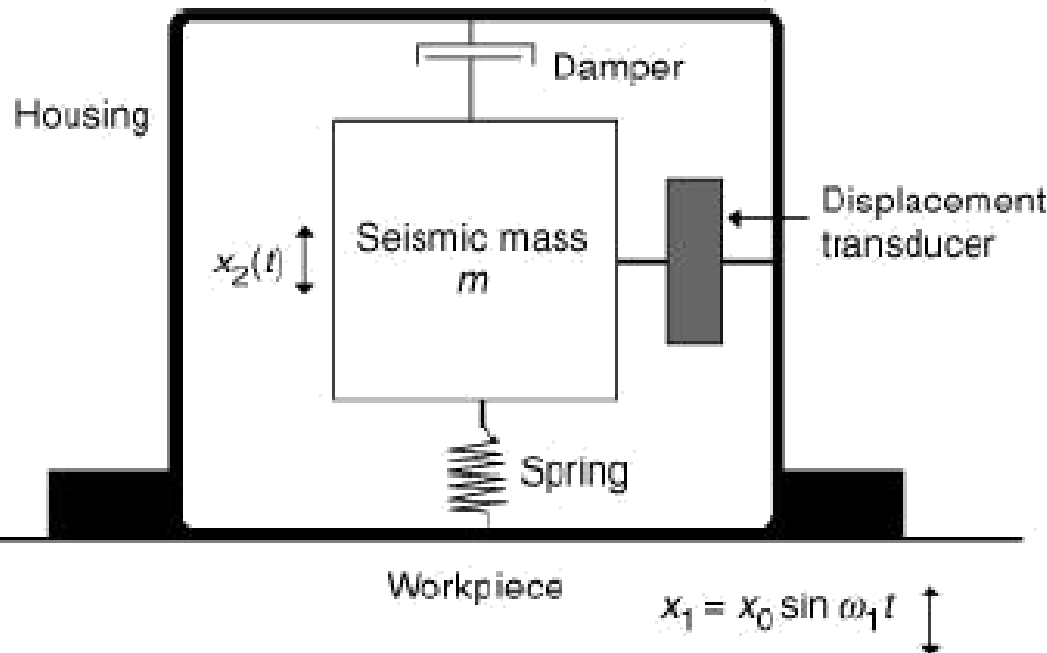


# Acceleration measure

- DIRECT: through accelerometers
- INDIRECT: by deriving velocity
- In linear or angular motion direct measurement is preferable
- In curve motion acceleration is measured with indirect methods



# Working principle of accelerometers



$$f(t) = m \frac{d^2 x}{dt^2} + c \frac{dx}{dt} + kx$$

$$m \frac{d^2 z}{dt^2} + c \frac{dz}{dt} + kz = mg \cos(\theta) - m \frac{d^2 x_1}{dt^2}$$

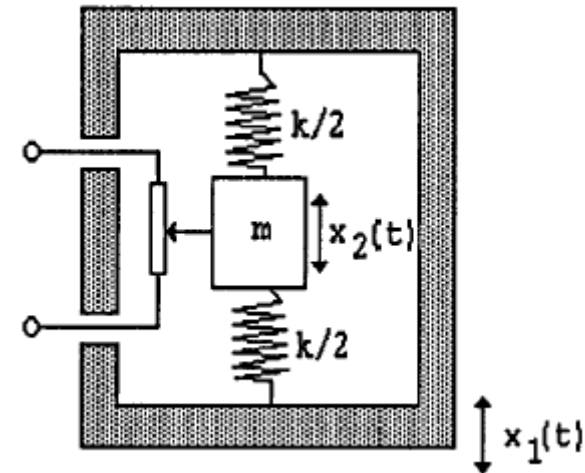
$$z = x_2 - x_1$$

$\theta = \text{angle with respect to gravity}$



# Potentiometer accelerometers

- A potentiometer is used to measure the relative displacement between the seismic mass and the base
  - A viscous fluid continuously interact with the base and the mass to provide damping
  - Low frequency of operation (lower than 100 Hz)
- Dynamic range:  $\pm 1g$  to  $\pm 50g$  fs.
- Natural frequencies: 12 - 89 Hz,
- Damping ratio  $\zeta$ : 0.5 - 0.8
- Potentiometer resistance: 1000–10000 $\Omega$ 
  - Corresponding resolution: 0.45–0.25% fs.
- Cross-axial sensibility:  $< \pm 1\%$ .
- Accuracy:  $\pm 1\%$  fs at environmental temperature.
- Dimension: 50mm<sup>3</sup> (<0.1 gr.)

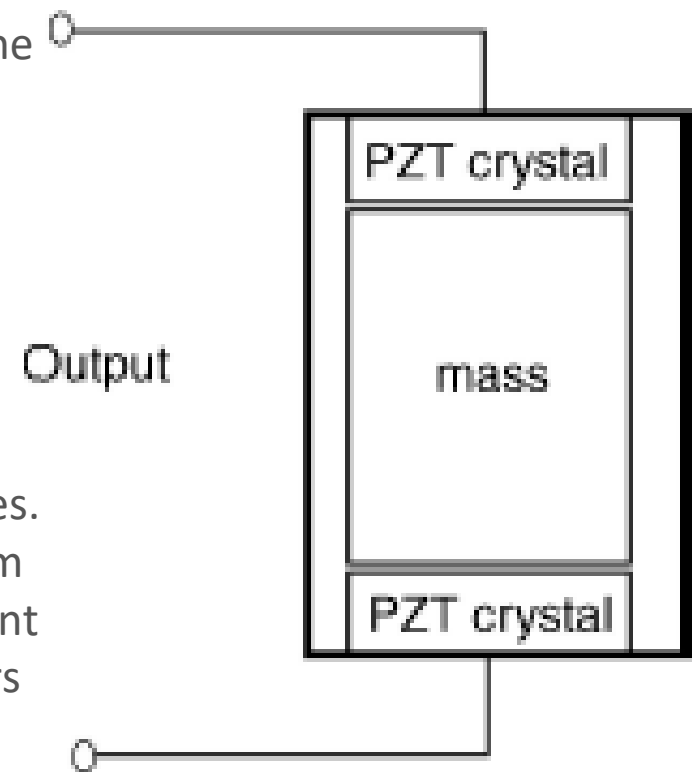




# Piezoelectric accelerometers

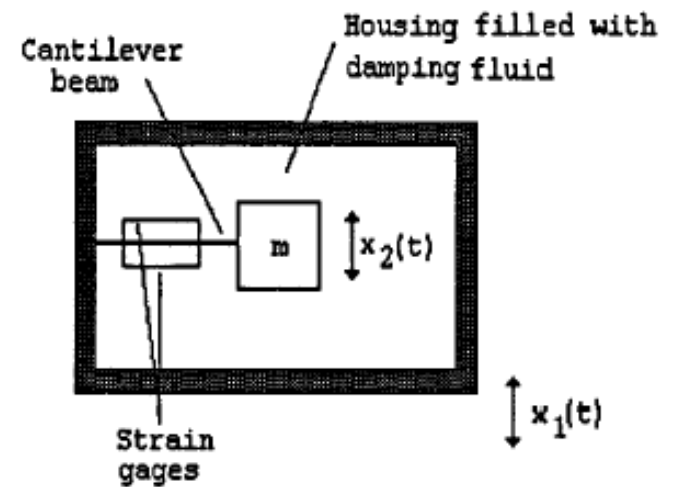
- **Piezoelectric accelerometers** are widely used for general-purpose acceleration, shock, and vibration measurements. They are basically motion transducers with large output signals and comparatively small sizes.
- When a varying motion is applied to the accelerometer, the crystal experiences a varying force excitation ( $F = ma$ ), causing a proportional electric charge  $q$  to be developed across it.
- These accelerometers are useful for high-frequency applications.
- Piezoelectric accelerometers are available in a wide range of specifications. They are manufactured as small as 3 x 3 mm in dimension with about 0.5 g in mass, including cables. They have excellent temperature ranges and some of them are designed to survive the intensive radiation environment of nuclear reactors. However, piezoelectric accelerometers tend to have larger cross-axis sensitivity than other types, about 2–4%.

A mass in direct contact with the piezoelectric component or crystal



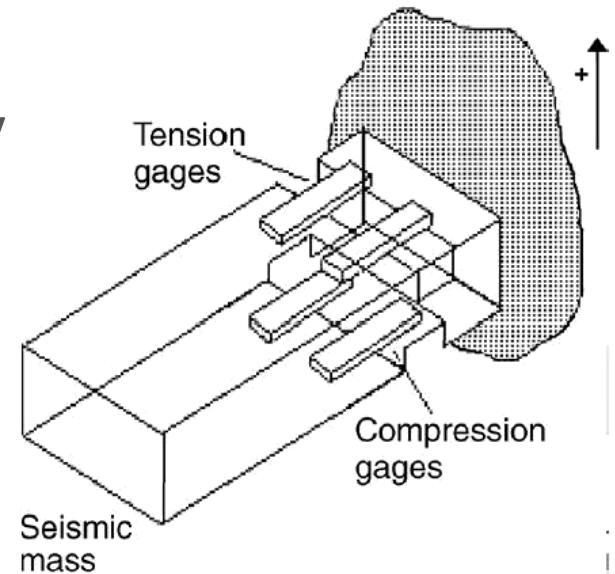
# Strain gauge accelerometers

- Electric resistance strain gauges are also used for displacement sensing of the seismic mass
  - the seismic mass is mounted on a cantilever beam rather than on springs.
- Resistance strain gauges are bonded on each side of the beam to sense the strain in the beam resulting from the vibrational displacement of the mass.
- Damping for the system is provided by a viscous liquid filling the housing.
- The output of the strain gauges is connected to an appropriate bridge circuit.
- The natural frequency of such a system is about 300 Hz.
  - The low natural frequency is due to the need for a sufficiently large cantilever beam to accommodate the mounting of the strain gauges.



# Piezoresistive accelerometers

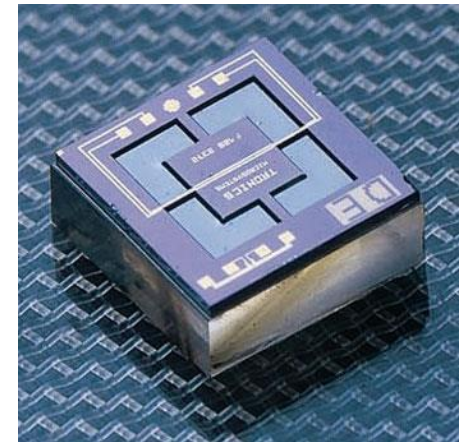
- Piezoresistive accelerometers are essentially semiconductor strain gauges with large gauge factors. The sensitivity of a piezoresistive sensor comes from the elastic response of its structure and resistivity of the material.
- Piezoresistive accelerometers are useful for acquiring vibration information at low frequencies. They are suitable to measure shocks well above 100,000g.



## Characteristics

- Frequency: Less than 1Hz-20kHz
- Limited temperature range: Calibration
- Light weight: Less than 1 to 10g
- AC/DC response
- Less than .01g to 200,000g

pressure changes the resistance by mechanically deforming the sensor



# Velocity measurement

- Methods based on a reference
  - Measurements done on the object in motion and on a reference
  - Average speed

$$v_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

- Inertial methods
  - Do not require contact with a reference
  - Provide the velocity relative to the initial velocity of the sensor

$$v(t) = v_i + \int_{t_i}^t a(\tau) d\tau$$



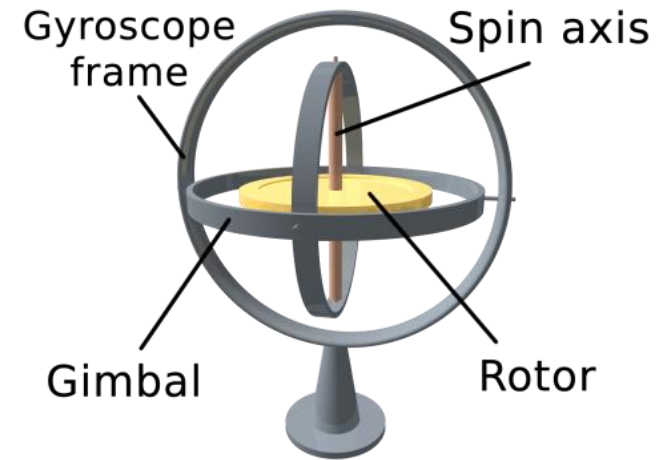
# Gyroscopes for measuring angular velocities

Physical rotating device, which tends to keep its rotational axis constant, due to the effect of the angular momentum conservation law

A gyroscope is a device composed of:

- Rotor, with a toroidal shape, rotating around its axis (Spin axis)
- Gimbal, which set the rotor free to orient in the 3 3D space directions

if the rotor is rotating, its axis tends to keep its orientation, even if the support changes its orientation

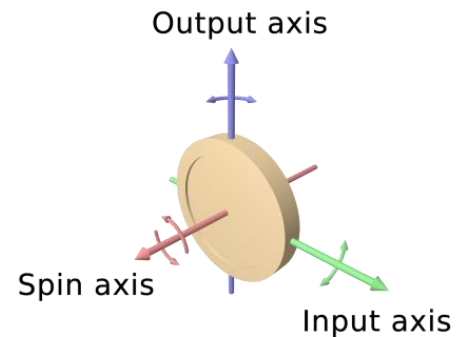
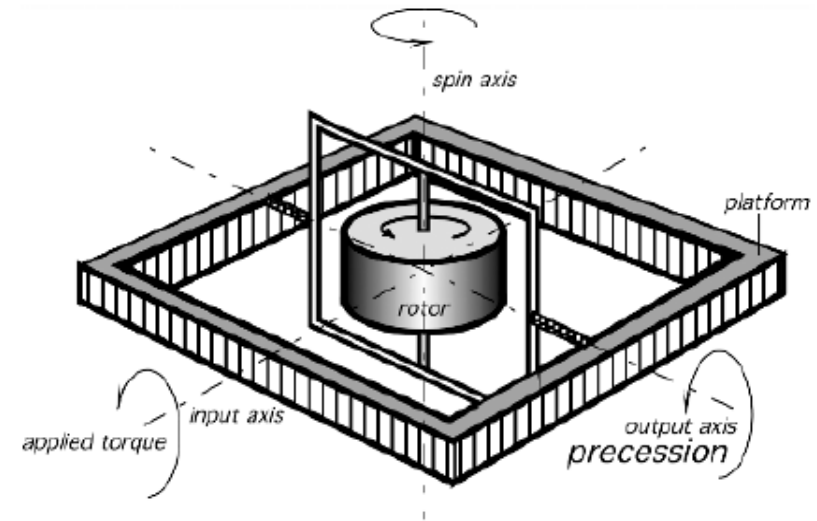


(Mechanism invented in 1852 by the physicist Jean Bernard Léon Foucault in the framework of his studies on earth rotation)



# Mechanical rotating gyroscope

- A disk (rotor) is free to rotate with respect to one/two spin axes (1/2-DOF gyroscope)
- If a rotation is applied to the gyroscope support around the *input* axis, then the gyroscope tends to rotate around a perpendicular axis (*output* axis)
- The gyroscope generated an output signal which is proportional to the angular velocity on an axis perpendicular to the *spin* axis



$$T = I\omega\Omega$$

$T$  : applied torsion

$I$ : inertia

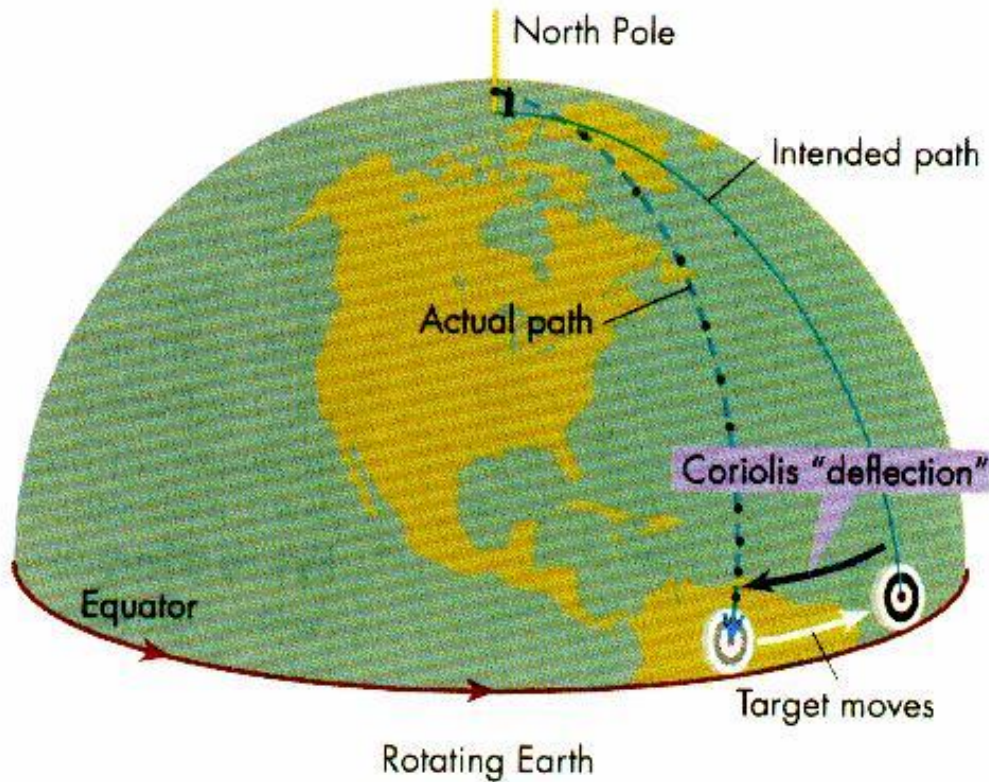
$\omega$ : constant rotor velocity

$\Omega$ : angular velocity around the output axis





# Coriolis effect



The mathematical relation expressing the **Coriolis force** is:

$$\vec{F}_C = 2m(\vec{v} \times \vec{\omega})$$

$\vec{F}_C$  is the Coriolis force,  
 $m$  is the mass,  
 $\vec{v}$  is the linear velocity,  
 $\vec{\omega}$  is the angular velocity of the rotation system.



# Coriolis-based accelerometers

## Vibrating mass gyroscopes

A vibrating element (vibrating resonator) creates an oscillatory linear velocity

If the sensor is rotated about an axis orthogonal to this velocity, a Coriolis acceleration is induced

The vibrating element is subjected to the Coriolis effect that causes secondary vibration orthogonal to the original vibrating direction.

By sensing the secondary vibration, the *rate of turn* can be detected.

The Coriolis force is given by:

$$\mathbf{F}_C = -2m(\boldsymbol{\omega} \times \mathbf{v})$$

