

Humanoid Robots and biped locomotion



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Outline

- What is a Humanoid?
- Why Develop Humanoids?
- Challenges in Humanoid robotics
- Active vs Passive Locomotion
- Active Bipedal Locomotion
 - Humanoid Robot kinematics
 - Humanoid Robot Dynamics (Stability via ZMP)

What is a Humanoid?

- Humanoid refers to any being whose body structure resembles that of a human: head, torso, legs, arms, hands.
- But it is also a robot made to resemble a human both in **appearance** and **behavior**.
- The difference between a **robot** and **android** is only skin-deep, looks exactly like humans on the outside, but with internal mechanics of humanoid robot.

Why Develop Humanoids?

- More rational reasons
 - They can work in human environment without a need to adapt themselves or to change the environment
 - Our environment and our tools are adapted for us
 - Why adapt all to robots?!
 - It is easier for a human being to interact with a human-like being

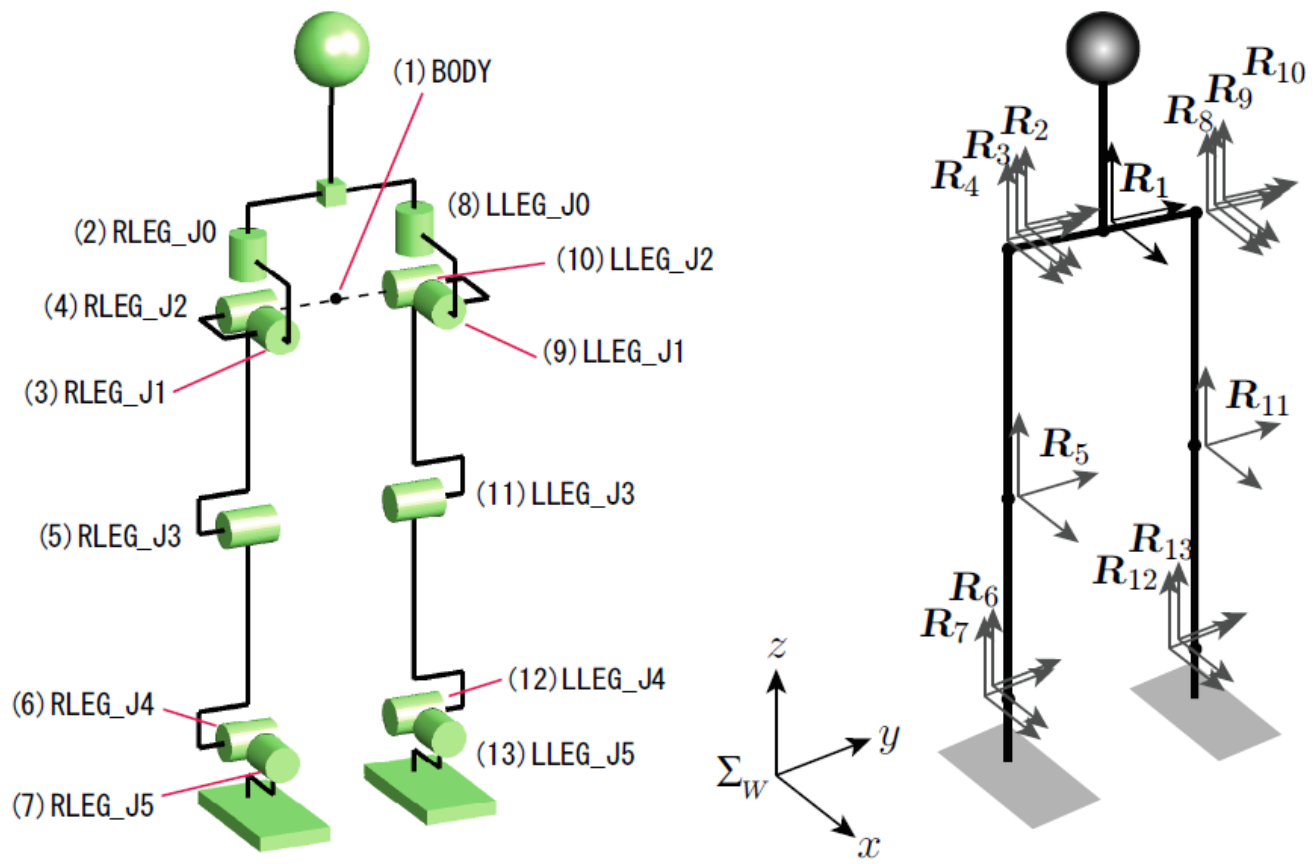
Challenges in Humanoids

- Bipedal human-like locomotion
 - Stable gait
 - Changing model during one/two feet support walking
 - Two legs, two arms, head, torso
- Hyper DOF system (>20)
 - Complex kinematics and dynamics
- Complex real-time control architecture

Active vs. Passive Locomotion

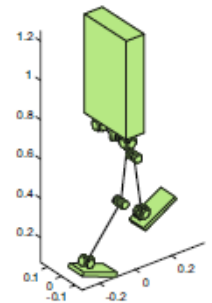
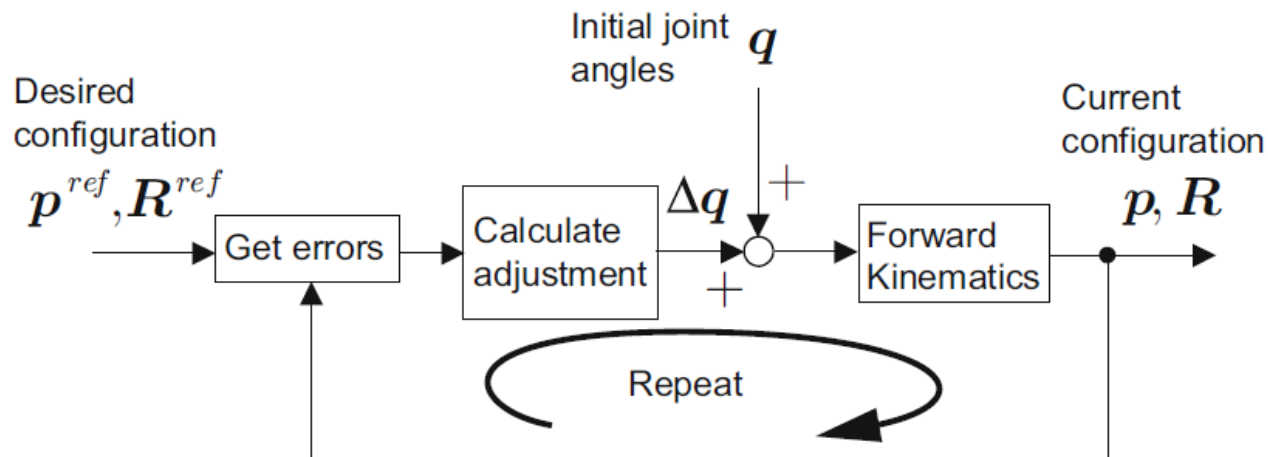
- Common humanoid uses all their DOF to perform the movement:
 - Continuous motor consumption (including arms)
 - Continuous motor control and synchronization
 - Extremely complex real-time control
- How is possible to reduce complexity?
 - Reducing number of active DOF
 - Using DOF only when it is strictly necessary
 - Using energy of previous step to generate the next
 - These actions reduce also the consumption
 - Robot not fully controllable

Kinematics of Humanoid Robots



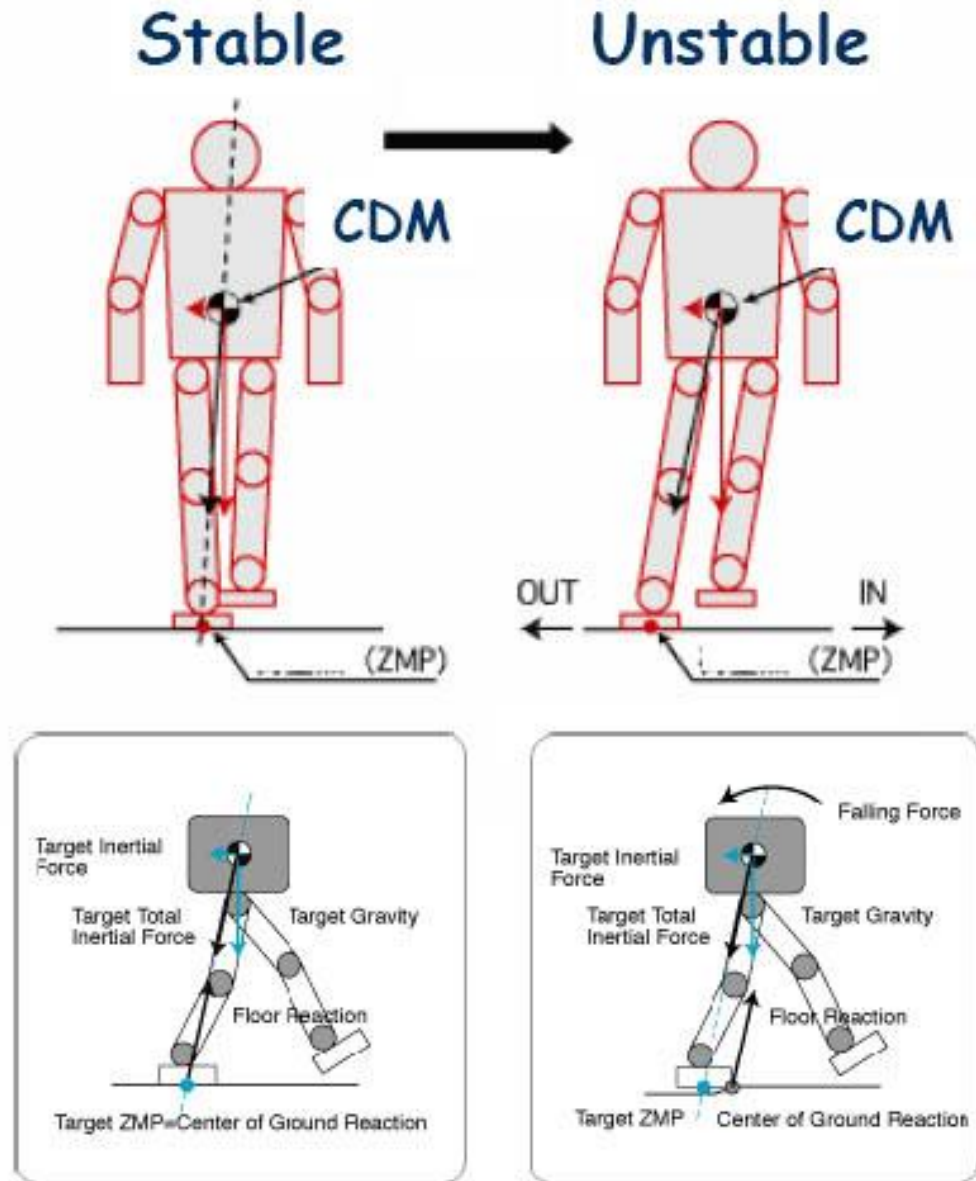
Kinematics of Humanoid Robots (2)

- Numerical solution to Inverse Kinematics

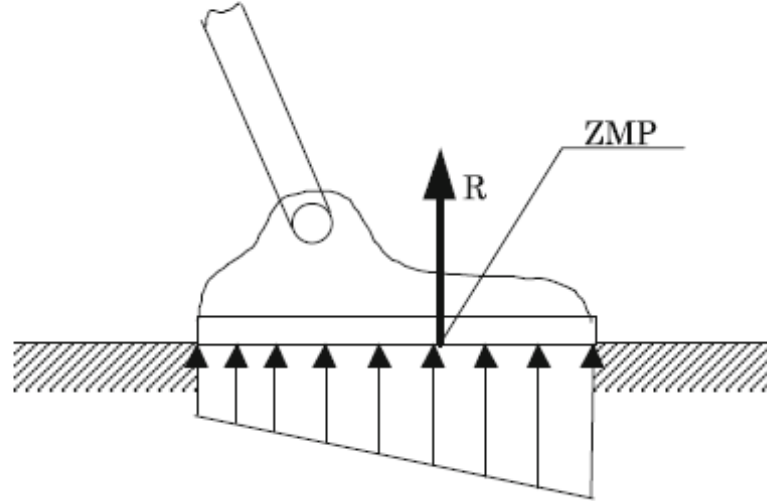


Bipedal Locomotion and ZMP

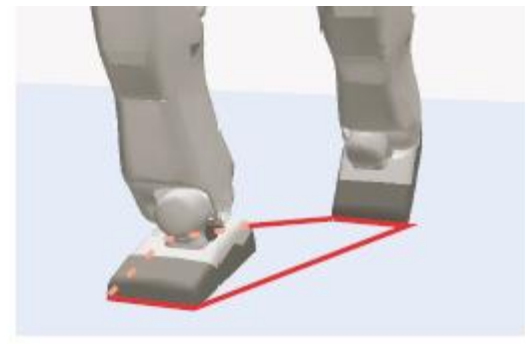
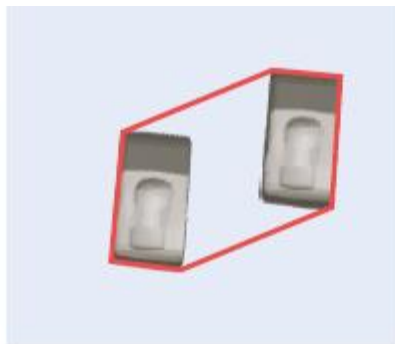
- **ZMP (Zero Moment Point)** specifies the point with respect to which dynamic reaction force at the contact of the foot with the ground does not produce any moment, i.e. the point where total inertia force equals 0 (zero).
- **ZMP is the indicator** of the stability of the robot:
 - if it is in the foot shadow (**Support Polygon**) – stable,
 - if not – unstable.
- The shadow depends on single or double support phase.



Support Polygon

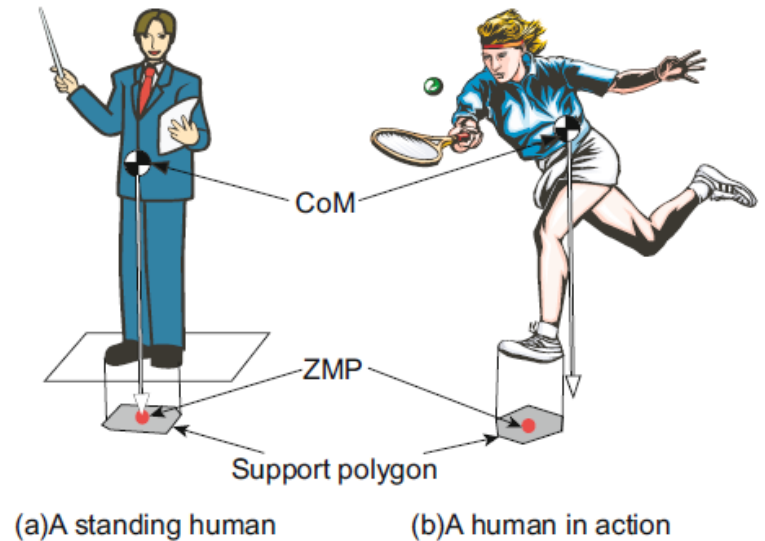


- Consider the region formed by enclosing all the contact points between the robot and the ground by using an elastic cord braid. We call this region as the **support polygon**.
- Mathematically the support polygon is defined as a convex hull, which is the smallest convex set including all contact points.



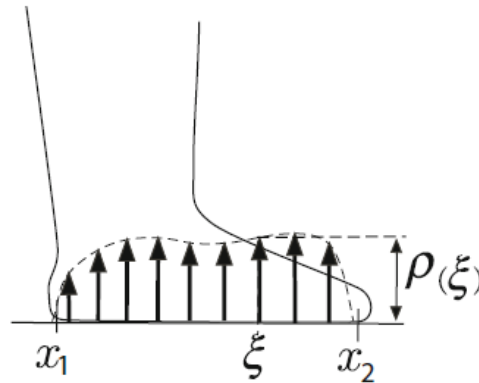
ZMP

- When a human stands on the ground, the ZMP coincides with the ground projection of CoM.
- A human can keep balance if the ground projection of CoM is included strictly inside of the support polygon.
- when a human moves dynamically the ground projection of CoM may exist outside the support polygon.
- However, the ZMP never exists outside the support polygon.

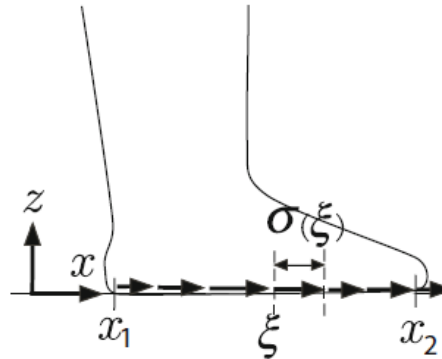


The ZMP always exists inside of the support polygon.

ZMP in 2D



Vertical Force



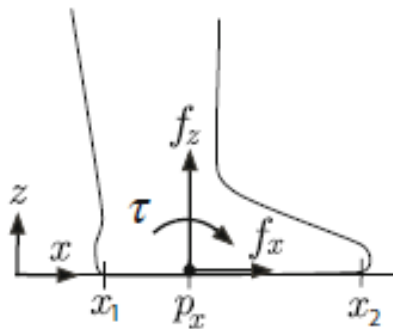
Horizontal force

The force f_x and f_z and the moment $\tau(p_x)$ at the point p_x are expressed as follows:

$$f_x = \int_{x_1}^{x_2} \sigma(\xi) d\xi$$

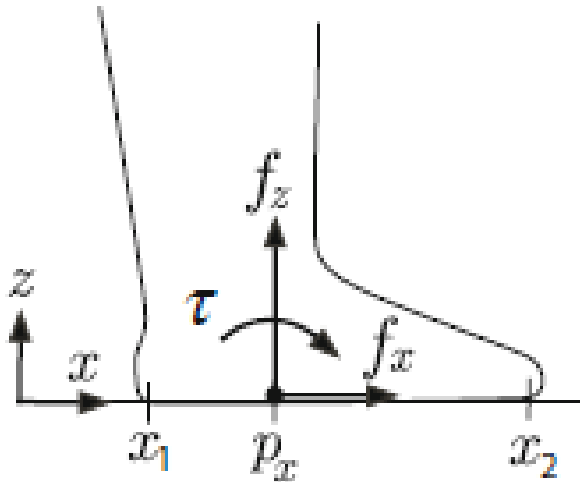
$$f_z = \int_{x_1}^{x_2} \rho(\xi) d\xi$$

$$\tau(p_x) = - \int_{x_1}^{x_2} (\xi - p_x) \rho(\xi) d\xi.$$



ZMP in 2D (2)

Considering the point where $p(x)$ becomes zero



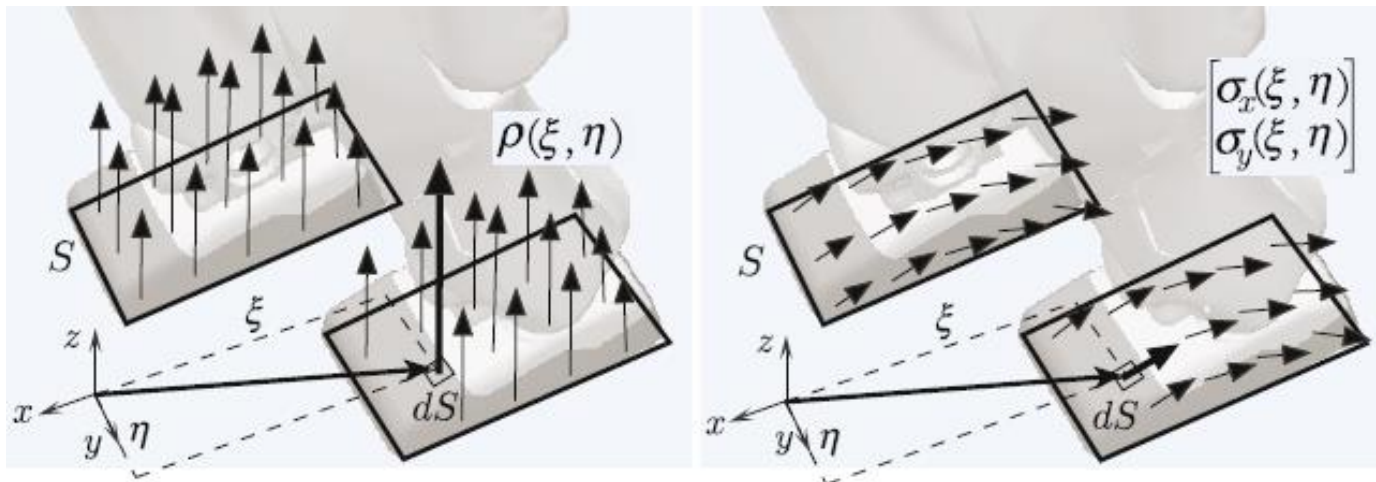
$$p_x = \frac{\int_{x_1}^{x_2} \xi \rho(\xi) d\xi}{\int_{x_1}^{x_2} \rho(\xi) d\xi}.$$

Thus $p(x)$ is the center of pressure and is the **ZMP** previously defined

ZMP in 3D

- Let $\mathbf{r}=[\xi \ \eta \ 0]^T$ the position vector defined on the ground with a vertical component $\rho=(\xi,\eta)$. The sum of the vertical component is defined as follows:

$$\mathbf{f}_z = \int_S \rho=(\xi,\eta) dS,$$



ZMP in 3D (vertical component)

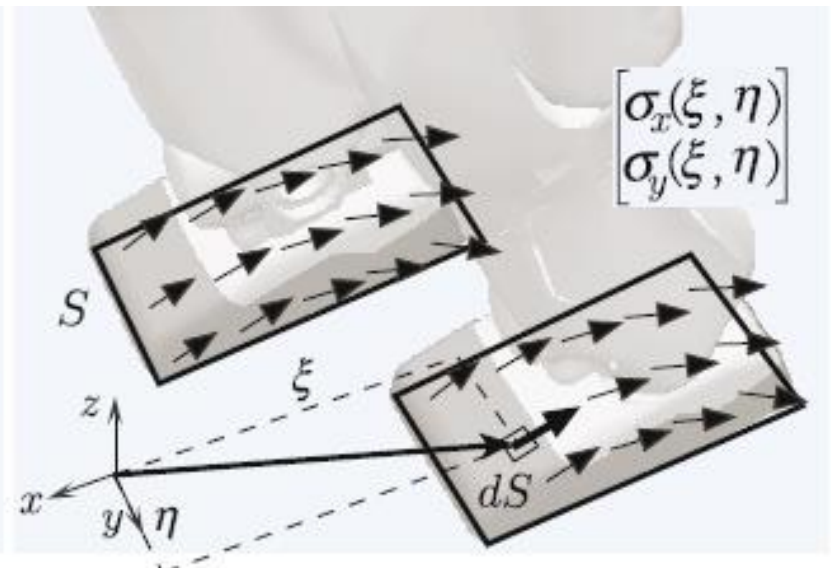
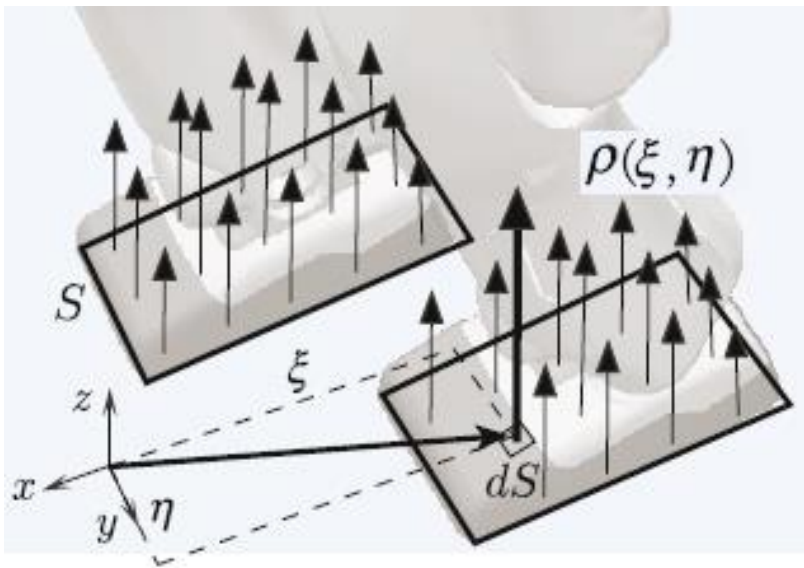
\int_S denotes the integration area at the contact point. The moment $\tau_n(p)$ is calculated as:

$$\tau_n(p) \equiv [\tau_{nx} \ \tau_{ny} \ \tau_{nz}]^T$$

$$\tau_{nx} = \int_S (\eta - p_y) \rho(\xi, \eta) dS$$

$$\tau_{ny} = - \int_S (\xi - p_x) \rho(\xi, \eta) dS$$

$$\tau_{nz} = 0.$$



ZMP in 3D (vertical component)

Assuming as for the 2D case:

$$\tau_{nx} = 0$$

$$\tau_{ny} = 0$$

The point where the moment of the vertical component of the ground reaction force becomes zero can be expressed as:

$$p_x = \frac{\int_S \xi \rho(\xi, \eta) dS}{\int_S \rho(\xi, \eta) dS}$$
$$p_y = \frac{\int_S \eta \rho(\xi, \eta) dS}{\int_S \rho(\xi, \eta) dS}.$$

p is the center of pressure or ZMP

ZMP in 3D (horizontal component)

$$\boldsymbol{\tau}_t(\mathbf{p}) \equiv [\tau_{tx} \ \tau_{ty} \ \tau_{tz}]^T$$

$$\tau_{tx} = 0$$

$$\tau_{ty} = 0$$

$$\tau_{tz} = \int_S \{(\xi - p_x)\sigma_y(\xi, \eta) - (\eta - p_y)\sigma_x(\xi, \eta)\} dS.$$

$$f_x = \int_S \sigma_x(\xi, \eta) dS$$

$$f_y = \int_S \sigma_y(\xi, \eta) dS.$$

These equations mean that the **horizontal ground reaction forces** generate the **vertical component** of the moment.

ZMP in 3D (horizontal component)

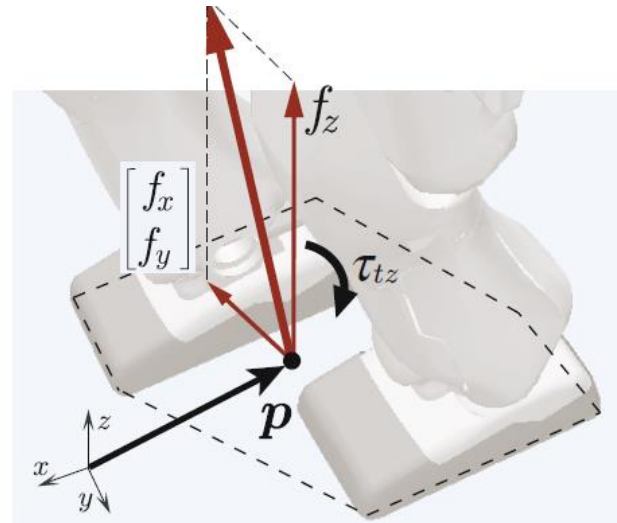
The ground reaction forces distributed over the surface of the sole can be replaced by the force

$$\mathbf{f} = [f_x \ f_y \ f_z]^T$$

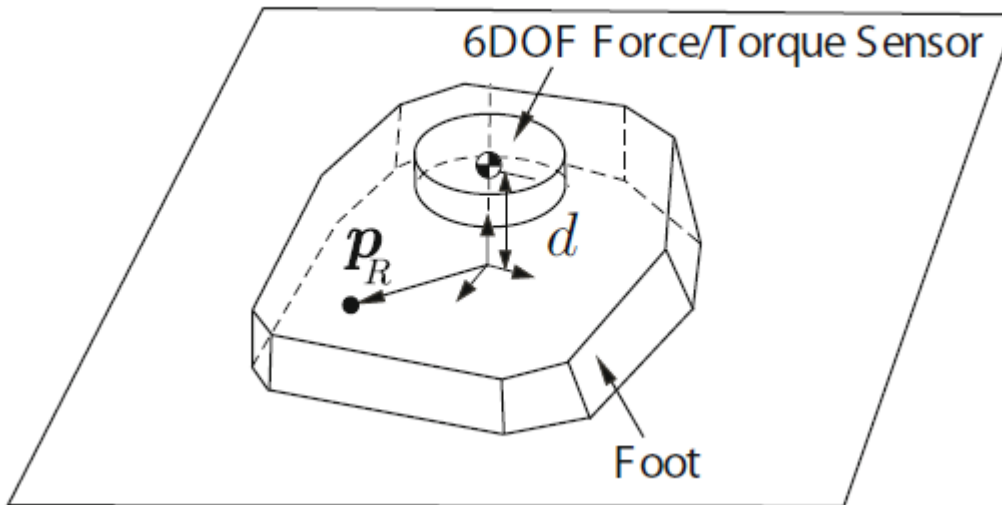
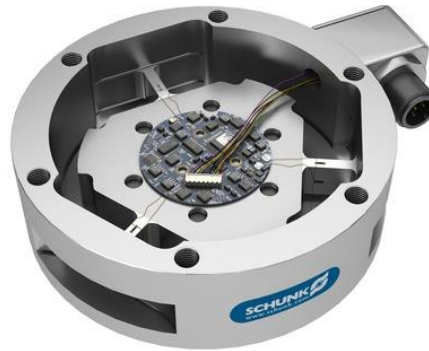
and the moment :

$$\begin{aligned}\boldsymbol{\tau}_p &= \boldsymbol{\tau}_n(\mathbf{p}) + \boldsymbol{\tau}_t(\mathbf{p}) \\ &= [0 \ 0 \ \tau_{tz}]^T,\end{aligned}$$

Thus the ZMP is defined as **the point where the horizontal component of the moment of the ground reaction forces becomes zero** for 3D cases.



Force sensors



Measurement of ZMP

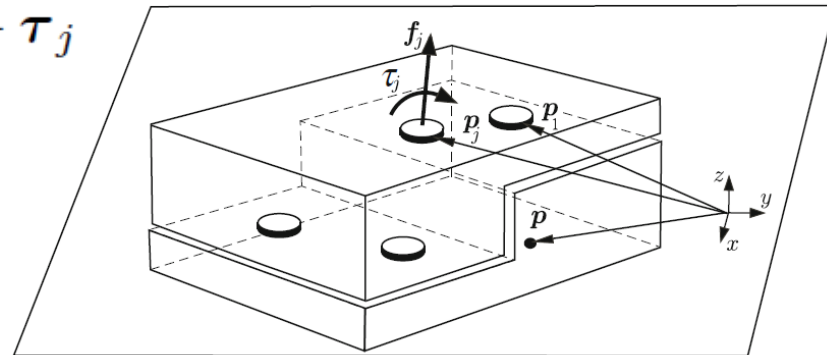
- **The ZMP of each foot** considering the reaction force between either one of the feet and the ground, and the ZMP considering the reaction force between both feet and the ground.
- During the double support phase, these two ZMPs becomes different.

Measurement of ZMP (II)

- Let us assume that, at the points \mathbf{p}_j ($j = 1, \dots, N$) with respect to the reference coordinate system, the forces \mathbf{f}_j and moments $\boldsymbol{\tau}_j$ are measured. Here, the moment about the point $\mathbf{p} = [p_x \ p_y \ p_z]$ is

$$\boldsymbol{\tau}(\mathbf{p}) = \sum_{j=1}^N (\mathbf{p}_j - \mathbf{p}) \times \mathbf{f}_j + \boldsymbol{\tau}_j$$

Setting the x and y component to zero and solving by $p_x \ p_y$



Basic equations for position of ZMP

$$p_x = \frac{\sum_{j=1}^N \{-\tau_{jy} - (p_{jz} - p_z) f_{jx} + p_{jx} f_{jz}\}}{\sum_{j=1}^N f_{jz}}$$

$$p_y = \frac{\sum_{j=1}^N \{\tau_{jx} - (p_{jz} - p_z) f_{jy} + p_{jy} f_{jz}\}}{\sum_{j=1}^N f_{jz}}$$

$$\mathbf{f}_j = [f_{jx} \ f_{jy} \ f_{jz}]^T$$

$$\boldsymbol{\tau}_j = [\tau_{jx} \ \tau_{jy} \ \tau_{jz}]^T$$

$$\mathbf{p}_j = [p_{jx} \ p_{jy} \ p_{jz}]^T$$

Measurement of ZMP

(contact between one foot and the ground)

- The ground reaction force applied to the sole is transmitted to the sensor mount through rubber bushes and dampers.
- A 6 axis force/torque sensor is attached at the sensor mount, and the force is transmitted to the ankle of the robot through this sensor
- A 6 axis force/torque sensor is coordinated to simultaneously measure the force $\mathbf{f} = [f_x, f_y, f_z]$ and the moment $\boldsymbol{\tau} = [\tau_x \tau_y \tau_z]$ applied from outside the robot.

Measurement of ZMP

(contact between one foot and the ground)

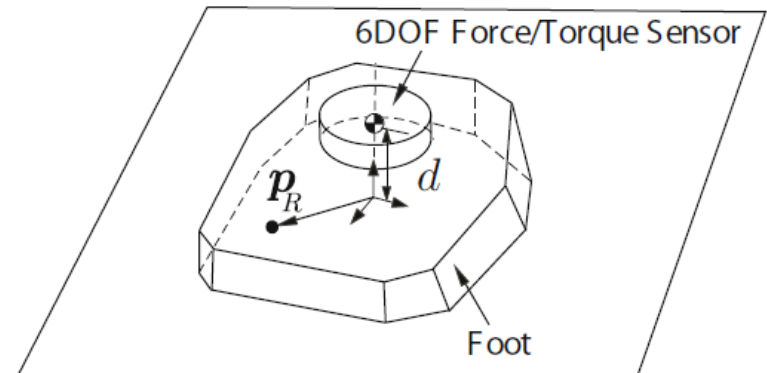
- Let the position of the ZMP in the right and the left foot be \mathbf{p}_R and \mathbf{p}_L
- The position of the ZMP (for the right foot) can be obtained as:

$$p_{Rx} = (-\tau_{1y} - f_{1x}d)/f_{1z}$$

$$p_{Ry} = (\tau_{1x} - f_{1y}d)/f_{1z}$$

$$\mathbf{p}_R = [p_{Rx} \ p_{Ry} \ p_{Rz}]^T$$

$$\mathbf{p}_1 = [0 \ 0 \ d]^T .$$



Measurement of ZMP

(both feet contact)

The position of the ZMP of each foot can be obtained as \mathbf{p}_R and \mathbf{p}_L . The ground reaction forces \mathbf{f}_R and \mathbf{f}_L are also obtained from the sensor information.

The **ZMP** in the case where both feet are **in contact with the ground** (using the basic equation for the ZMP computation) can be obtained as:

$$p_x = \frac{p_{Rx} f_{Rz} + p_{Lx} f_{Lz}}{f_{Rz} + f_{Lz}}$$

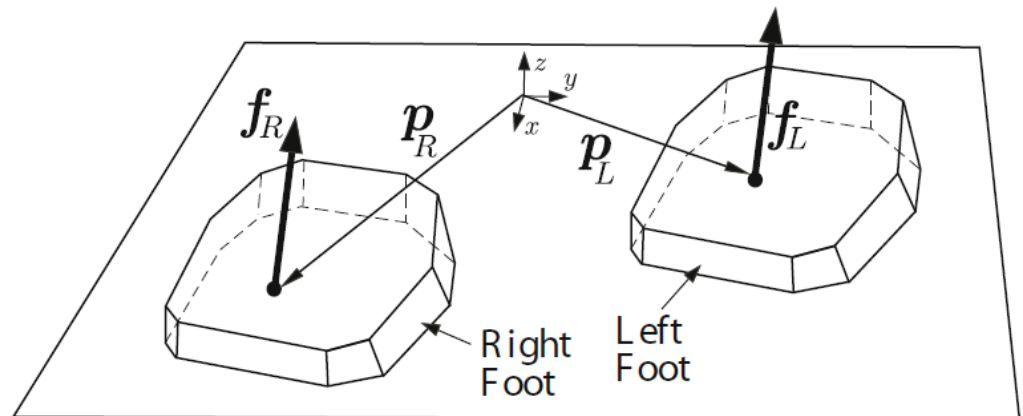
$$p_y = \frac{p_{Ry} f_{Rz} + p_{Ly} f_{Lz}}{f_{Rz} + f_{Lz}}$$

$$\mathbf{f}_R = [f_{Rx} \ f_{Ry} \ f_{Rz}]^T$$

$$\mathbf{f}_L = [f_{Lx} \ f_{Ly} \ f_{Lz}]^T$$

$$\mathbf{p}_R = [p_{Rx} \ p_{Ry} \ p_{Rz}]^T$$

$$\mathbf{p}_L = [p_{Lx} \ p_{Ly} \ p_{Lz}]^T$$




Measurement of ZMP

(both feet contact)

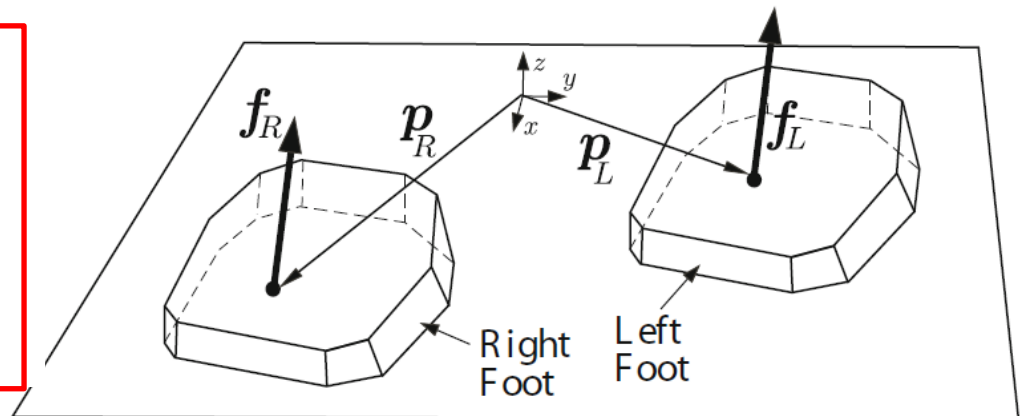
During the single support phase, since the vertical component of the ground reaction force becomes zero, the ZMP coincides with the ZMP of the supporting foot.

$$[p_x \ p_y \ p_z]^T = \begin{cases} [p_{Rx} \ p_{Ry} \ p_{Rz}]^T & \text{for support of right foot} \\ [p_{Lx} \ p_{Ly} \ p_{Lz}]^T & \text{for support of left foot.} \end{cases}$$

Considering the balance of a humanoid robot, we can use the following equations taking both feet into account regardless of the supporting foot.

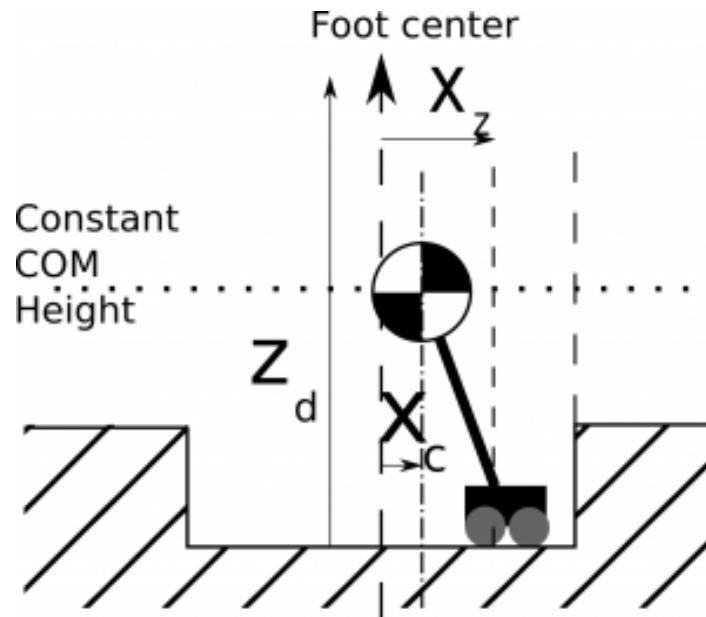

$$p_x = \frac{p_{Rx} f_{Rz} + p_{Lx} f_{Lz}}{f_{Rz} + f_{Lz}}$$

$$p_y = \frac{p_{Ry} f_{Rz} + p_{Ly} f_{Lz}}{f_{Rz} + f_{Lz}}$$



ZMP and Center of Mass (COM)

Conceptually the ZMP is equivalent to the center of pressure. Since the center of pressure (COP) is simply the average of the pressure distribution the relationship between the ZMP, COM and force at the COM can be explained analogously to a **inverted pendulum cart** ZMP point



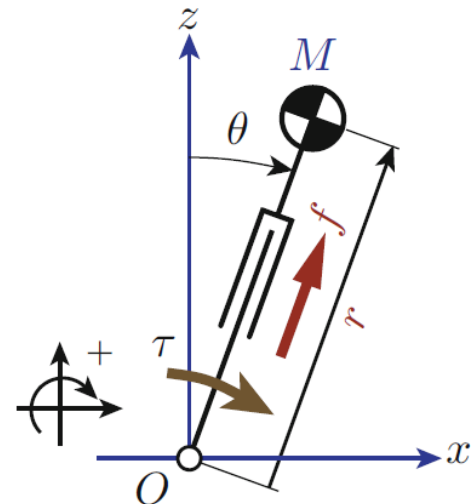
The ZMP point is moved such that it only moves inside the support area (otherwise the foot will tip), this constraint is represented by the walls in the figure.

ZMP and COM

- Based on knowledge of the joint trajectories or the center of mass trajectory we would like to calculate the ZMP position.
- For a desired ZMP position we could find the necessary joint trajectories or the required center of mass trajectory. The center of mass – ZMP relationship is shown below

$$\ddot{x} = \frac{g}{z} x.$$

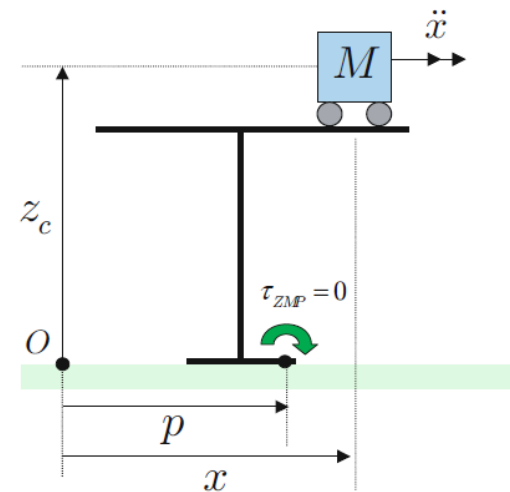
where, x , z gives the CoM of the inverted pendulum



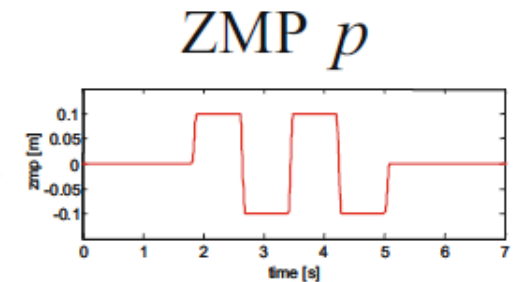
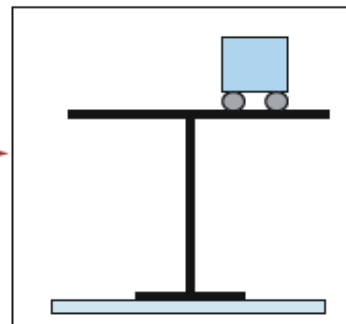
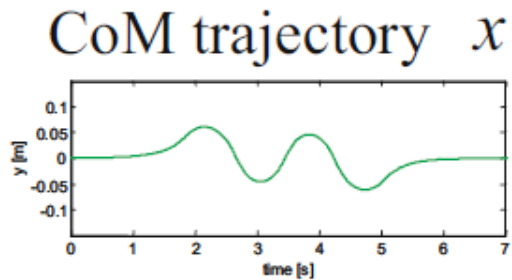
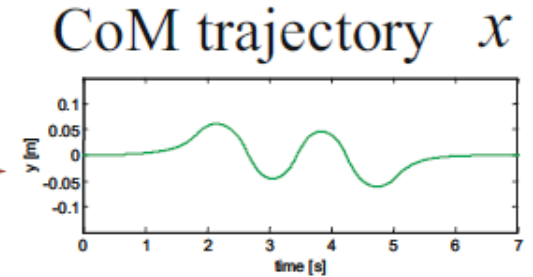
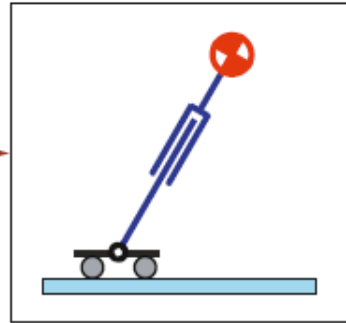
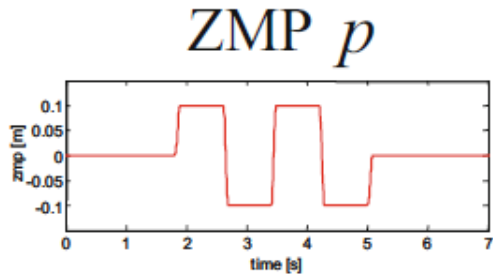
Cart-Table model

- A cart with mass M runs on a table whose mass is negligibly small. Although the table foot is too small to keep balance having a cart on the edge of the table, it can still keep an instantaneous balance if the cart runs with certain acceleration.
- We call this a ***cart-table model***. The ZMP is given as

$$p = x - \frac{z_c}{g} \ddot{x}.$$



From ZMP to COM



An example of pattern generation and computation

