Principles for software composition 2018/19
Exam – September 10, 2019

[Ex. 1]
Let us add to IMP the command **repeat** c, whose operational semantics is defined by the rules:

$$\frac{\langle c, \sigma\rangle \to \sigma}{\langle \textbf{repeat } c, \sigma\rangle \to \sigma} \qquad \frac{\langle c, \sigma\rangle \to \sigma'' \quad \langle \textbf{repeat } c, \sigma''\rangle \to \sigma'}{\langle \textbf{repeat } c, \sigma\rangle \to \sigma'}$$

1. Prove by rule induction that

$$\langle \textbf{repeat } c, \sigma\rangle \to \sigma' \implies \langle c, \sigma'\rangle \to \sigma'$$

2. Exploit the previous property to prove that the evaluation of the command **repeat** $x := x + 1$ always diverges.

[Ex. 2]
Let $(\mathbb{N}^\infty, \leq)$ be the usual $\mathrm{CPO}_\perp$ consisting of all the natural numbers ordered by the less-or-equal-to relation and extended with a top element ∞.

1. How many monotone but not continuous functions $f : \mathbb{N}^\infty \to \mathbb{N}^\infty$ do exist such that $f(\infty) = 1$? How many fixpoints do they have?

2. How many monotone but not continuous functions $g : \mathbb{N}^\infty \to \mathbb{N}^\infty$ do exist such that $g(\infty) = 2$ and g has exactly two fixpoints?

3. How many monotone but not continuous functions $h : \mathbb{N}^\infty \to \mathbb{N}^\infty$ do exist such that $h(\infty) = 2$ and h has exactly one fixpoint?

[Ex. 3]
Consider the HOFL term

$$t \stackrel{\text{def}}{=} \textbf{rec } f.\ \lambda x.\ (x, \textbf{fst}(f\ 0))$$

1. Find the principal type of t.

2. Find the denotational semantics of t.

[Ex. 4]
Let us consider the CCS processes

$$\begin{array}{ll} p \stackrel{\text{def}}{=} \textbf{rec } x.\ \alpha.x + \beta.x + \gamma.\delta.\textbf{nil} & \quad r \stackrel{\text{def}}{=} (p|q)\backslash\alpha\backslash\beta\backslash\gamma \\ q \stackrel{\text{def}}{=} \textbf{rec } y.\ \overline{\alpha}.\overline{\alpha}.y + \overline{\beta}.y + \overline{\gamma}.y) & \quad s \stackrel{\text{def}}{=} \textbf{rec } z.\ \tau.z + \tau.\delta.\textbf{nil} \end{array}$$

1. Draw the LTSs of the processes r and s.

2. Show that r and s are not strongly bisimilar.

3. Show that r and s are weakly bisimilar.