Suppose we extend IMP with the arithmetic expression for integer division \(a_0/a_1\), whose operational semantics is defined by the inference rule:

\[
\frac{\langle a_0, \sigma \rangle \rightarrow n_0 \quad \langle a_1, \sigma \rangle \rightarrow n_1 \quad n_1 \neq 0}{\langle a_0/a_1, \sigma \rangle \rightarrow n_0 \mathsf{div} n_1}
\]

1. Prove that termination does not hold anymore for arithmetic and boolean expressions.

2. Prove that determinacy continues to hold for arithmetic and boolean expressions.

3. Redefine the denotational semantics of \(Aexp\), \(Bexp\) and \(Com\) to take into account non-termination of arithmetic expressions.

Let \(c \stackrel{\mathsf{def}}{=} \mathsf{while} \; x \neq y \; \mathsf{do} \; (x := x + 1; y := y - 1)\). Find the largest set of memories \(S\) such that \(\forall \sigma \in S. \langle c, \sigma \rangle \not\rightarrow\).

Let \((D, \sqsubseteq_D)\) be a CPO. Given any chain \(\{d_n\}_{n \in \mathbb{N}}\) in \(D\) prove that

\[
\bigsqcup_{n \in \mathbb{N}} d_n = \bigsqcup_{n \in \mathbb{N}} d_{2n}
\]

**Hint:** Prove that the chains \(\{d_n\}_{n \in \mathbb{N}}\) and \(\{d_{2n}\}_{n \in \mathbb{N}}\) have the same set of upper bounds and therefore the same lub.

Write a Haskell function \(\mathsf{nodup}\) that takes a list and checks that no element occurs more than once in the list. For example, \(\mathsf{nodup} [1,5,3]\) must return \(\mathsf{true}\), while \(\mathsf{nodup} [1,5,3,5]\) must return \(\mathsf{false}\). Write down also the type of \(\mathsf{nodup}\).

Which of the following HOFL pre-terms are well-formed? If they are, write their principal type. If they are not, explain why.

1. \(\lambda x. \mathsf{if} \; x \; \mathsf{then} \; \mathsf{fst}(x) \; \mathsf{else} \; \mathsf{snd}(x)\)
2. \(\mathsf{rec} \; f. \; \lambda x. \; f(\mathsf{snd}(x), \mathsf{fst}(x))\)
3. \(\mathsf{rec} \; f. \; \lambda x. \; \mathsf{if} \; \mathsf{fst}(x) \; \mathsf{then} \; \mathsf{fst}(x) \; \mathsf{else} \; f(\mathsf{snd}(x), 0)\)
4. \(\mathsf{rec} \; f. \; \lambda g. \; \lambda x. \; g(f(x))\)
5. \(\lambda f. \; (\lambda x. \; f(x)) \; (\lambda x. \; f(x))\)
6. \(\lambda n. \; \lambda f. \; \lambda x. \; f((n)(f)x)\)

(substitution preserves types) Let \(\tilde{e} : \tilde{\tau}\) stand for \(e_1 : \tau_1, \ldots, e_n : \tau_n\), and \(t[\tilde{e}/\tilde{x}]\) for \(t[e_1/x_1, \ldots, e_n/x_n]\). Prove that \(\forall t. \; P(t)\) where

\[
P(t) \overset{\mathsf{def}}{=} \forall \tau, \tilde{x}, \tilde{e}, \tilde{\tau}. \; (t : \tau \land \tilde{x} : \tilde{\tau} \land \tilde{e} : \tilde{\tau}) \rightarrow t[\tilde{e}/\tilde{x}] : \tau
\]