



PSC 2021/22 (375AA, 9CFU)

Principles for Software Composition

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27 - PEPA

PEPA

Performance Evaluation Process Algebra

Building models

Conceptualise your system as a Markov chain

Construct your Markov chain (infinitesimal generator matrix)

Solve your equations to derive quantitative information

Building models

Conceptualise your system as a Markov chain

Construct your Markov chain (infinitesimal generator matrix)

Solve your equations to derive quantitative information

Monolithic approach: not suitable for complex systems

PEPA project



the PEPA project started in Edinburgh in 1991

motivated by the performance analysis
of large computer and communication systems

exploit interplay between Process Algebras and CTMC

Process Algebras (PA):

compositional description of complex systems,
formal reasoning (for correctness)

CTMC:

numerical analysis

compositional construction of CTMC

PEPA meets CTMC

PA

mutual influence

CTMC

interaction designed around CTMC

ease of construction

actions have durations

design of independent components

add rates to labels

cooperation between components

probabilistic branching

explicit interaction

quantitative measures

reusable sub-models

probabilistic model checking

easy to understand models

quantitative logics

space reduction techniques

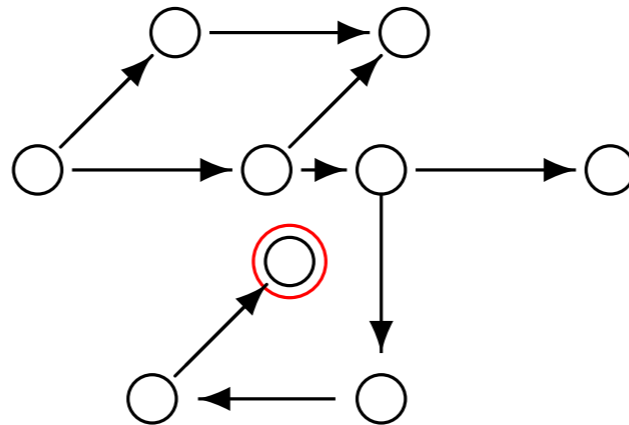
functional verification

Formal models

qualitative

vs

quantitative



reachability:
will the system arrive to a
particular state?

how long will it take
the system to arrive to a
particular state?

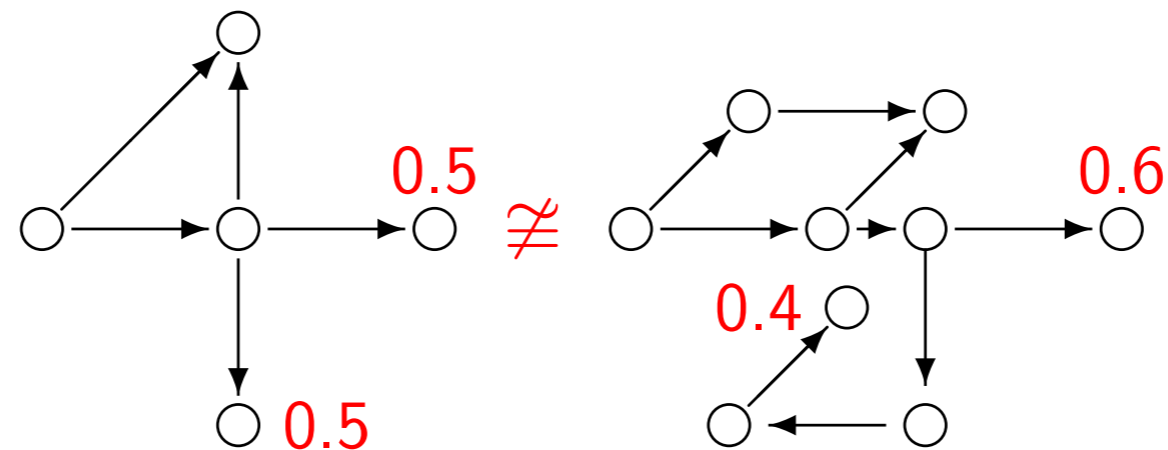
(taken from Jane Hillston's slides)

Formal models

qualitative

vs

quantitative



conformance:
does system behaviour
match its specification?

how likely is that
system behaviour will
match its specification?

does the frequency profile
of the system match
that of its specification?

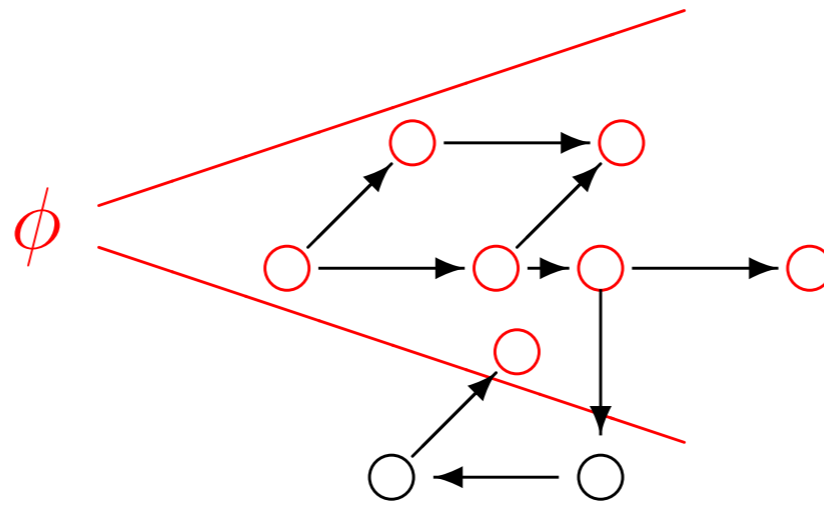
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Formal models

qualitative

vs

quantitative



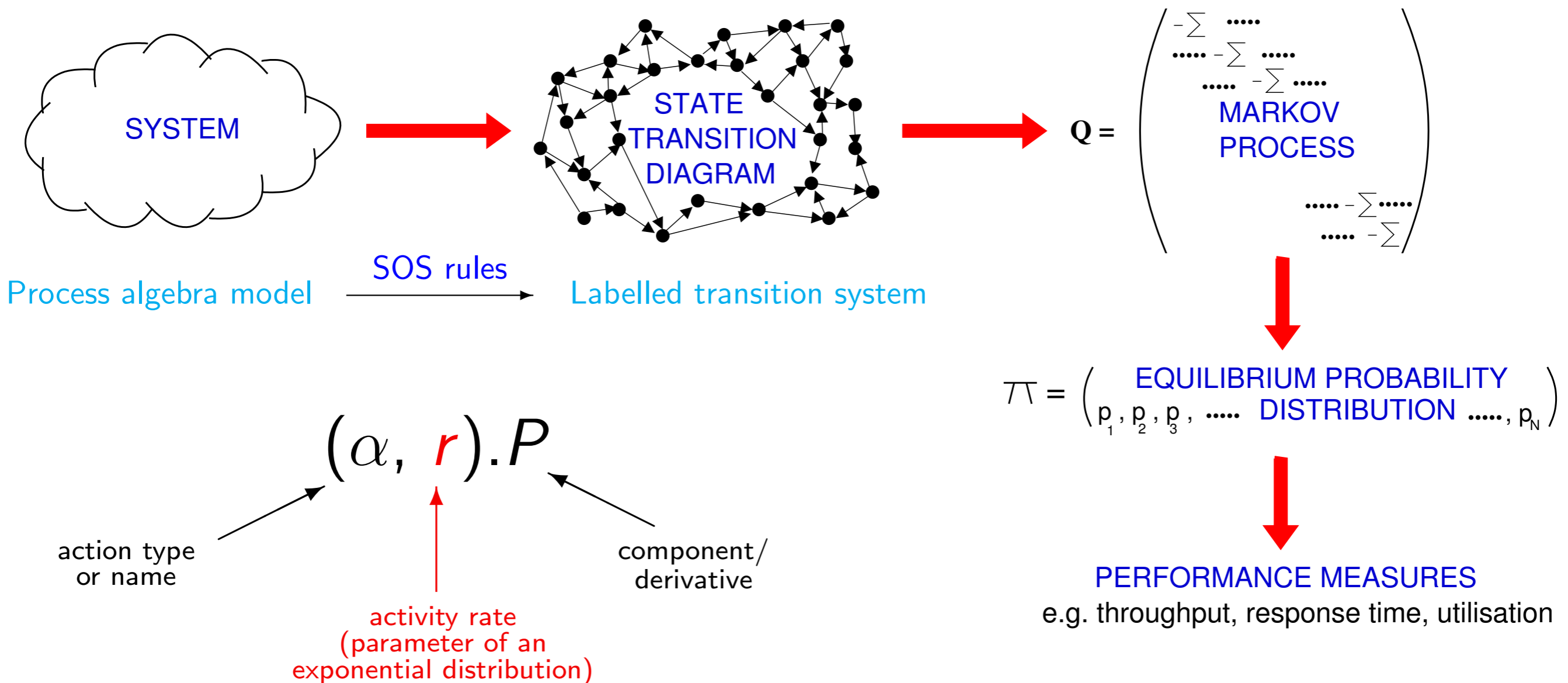
verification:
does a given property
hold within the system?

Does a given property
hold within the system
with a given probability?

How long is it until
a given probability hold?

(taken from Jane Hillston's slides)

PEPA workflow



(taken from Jane Hillston's slides)

Communication style

PEPA parallel composition is based on Hoare's CSP

CCS-style

actions and co-actions

binary synchronisation

conjugate sync

result in a silent action

restriction

parallel composition

one operator

CSP-style

no i/o distinction

multiple cooperation

shared name sync

result in the same name

hiding

cooperation combinator

parametric operator

CSP cooperation combinator

$$P \bowtie_L Q$$

cooperation set

interleaving

$$\frac{P_1 \xrightarrow{\alpha} Q_1 \quad \boxed{\alpha \notin L}}{P_1 \bowtie_L P_2 \xrightarrow{\alpha} Q_1 \bowtie_L P_2} \quad \frac{P_2 \xrightarrow{\alpha} Q_2 \quad \boxed{\alpha \notin L}}{P_1 \bowtie_L P_2 \xrightarrow{\alpha} P_1 \bowtie_L Q_2}$$

cooperation

$$\frac{P_1 \xrightarrow{\alpha} Q_1 \quad P_2 \xrightarrow{\alpha} Q_2 \quad \boxed{\alpha \in L}}{P_1 \bowtie_L P_2 \xrightarrow{\alpha} Q_1 \bowtie_L Q_2}$$

pure interleaving

$$P \parallel Q \triangleq P \bowtie_{\emptyset} Q$$

PEPA

syntax and semantics

PEPA syntax

P, Q	$::=$	nil	inactive process
		$(\alpha, r).P$	action prefix
		$P + Q$	choice
		$P \boxtimes_L Q$	cooperation combinator
		P/L	hiding
		C	process constant

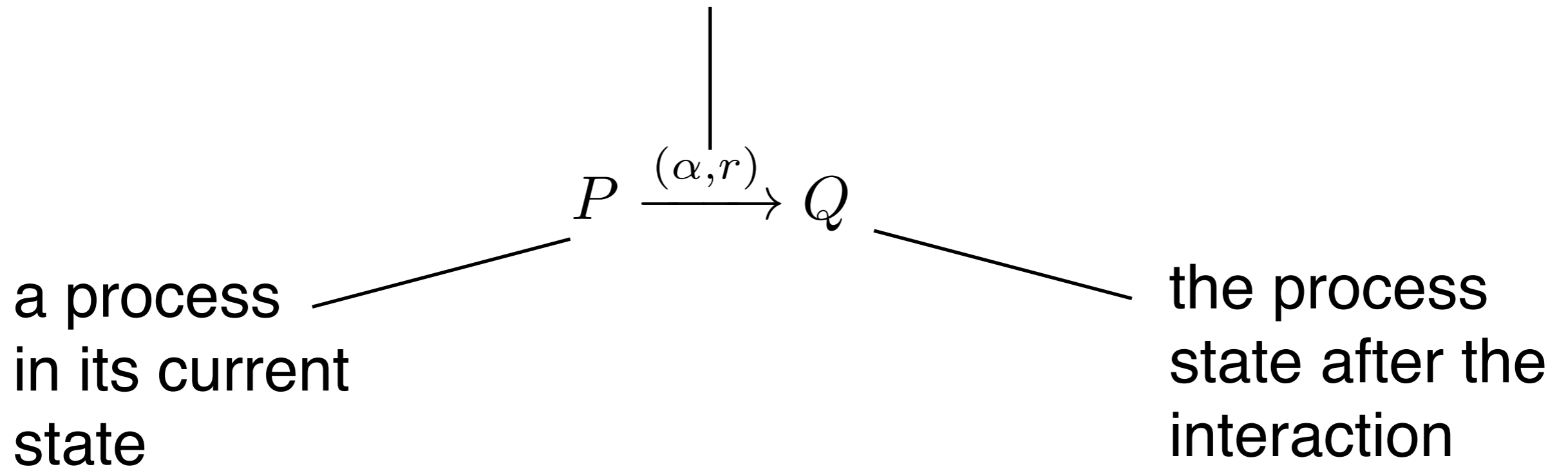
$\alpha \in \Lambda$ action

$L \subseteq \Lambda$ set of actions

$\Delta = \{C_i \triangleq P_i\}_{i \in I}$ set of process declarations

PEPA LTS

ongoing interaction
with the environment
(with other processes)
and its rate



small-step semantics

PEPA semantics (basics)

$$\frac{}{(\alpha, r).P \xrightarrow{(\alpha, r)} P}$$

$$\frac{P_1 \xrightarrow{(\alpha, r)} Q}{P_1 + P_2 \xrightarrow{(\alpha, r)} Q}$$

$$\frac{P_2 \xrightarrow{(\alpha, r)} Q}{P_1 + P_2 \xrightarrow{(\alpha, r)} Q}$$

$$\frac{C \triangleq P \in \Delta \quad P \xrightarrow{(\alpha, r)} Q}{C \xrightarrow{(\alpha, r)} Q}$$

Example

Server \triangleq $(get, \top).(download, \mu).(rel, \top).Server$

extremely high rate
cannot influence the overall rate
of interacting components

Browser \triangleq $(display, \lambda_1).(cache, m).Browser$
+ $(display, \lambda_2).(get, g).(download, \top).(rel, r).Browser$

a local choice
taken with probability $\frac{\lambda_i}{\lambda_1 + \lambda_2}$

Hiding and interleaving

$$\frac{P \xrightarrow{(\alpha, r)} Q \quad \boxed{\alpha \notin L}}{P/L \xrightarrow{(\alpha, r)} Q/L}$$

$$\frac{P \xrightarrow{(\alpha, r)} Q \quad \boxed{\alpha \in L}}{P/L \xrightarrow{(\tau, r)} Q/L}$$

$$\frac{P_1 \xrightarrow{(\alpha, r)} Q_1 \quad \boxed{\alpha \notin L}}{P_1 \boxtimes_L P_2 \xrightarrow{(\alpha, r)} Q_1 \boxtimes_L P_2}$$

$$\frac{P_2 \xrightarrow{(\alpha, r)} Q_2 \quad \boxed{\alpha \notin L}}{P_1 \boxtimes_L P_2 \xrightarrow{(\alpha, r)} P_1 \boxtimes_L Q_2}$$

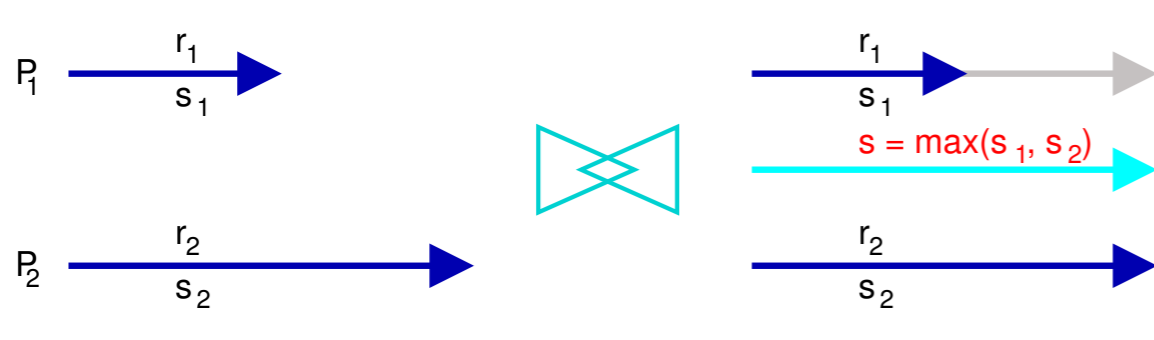
Cooperation

$$\frac{P_1 \xrightarrow{(\alpha, r_1)} Q_1 \quad P_2 \xrightarrow{(\alpha, r_2)} Q_2 \quad \boxed{\alpha \in L}}{P_1 \bowtie_L P_2 \xrightarrow{(\alpha, r)} Q_1 \bowtie_L Q_2}$$

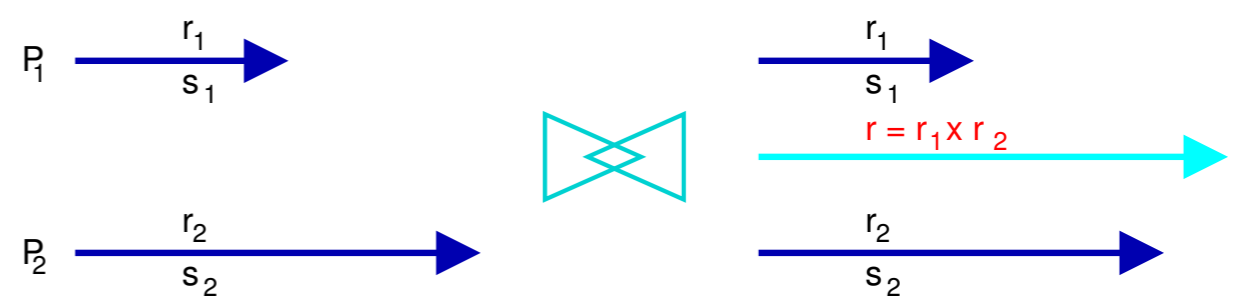
which rate should we put here?

Which rate for sync?

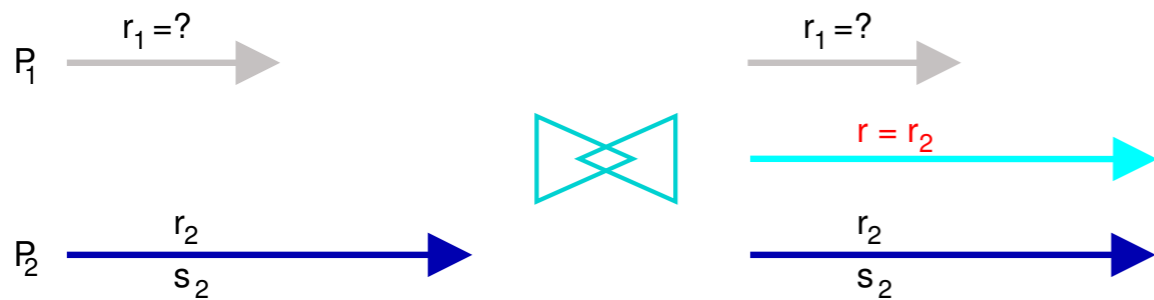
stochastic PA differ for the treatment of rates of synchronised actions



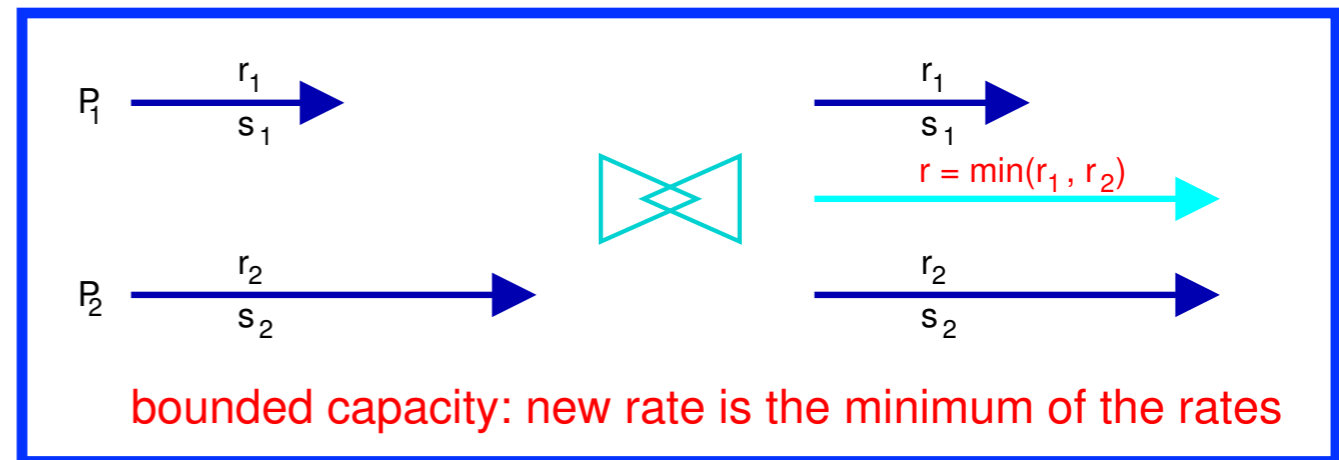
s is no longer exponentially distributed



TIPP: new rate is product of individual rates



EMPA: one participant is passive



bounded capacity: new rate is the minimum of the rates

PEPA's approach

(taken from Jane Hillston's slides)

PEPA: bounded capacity

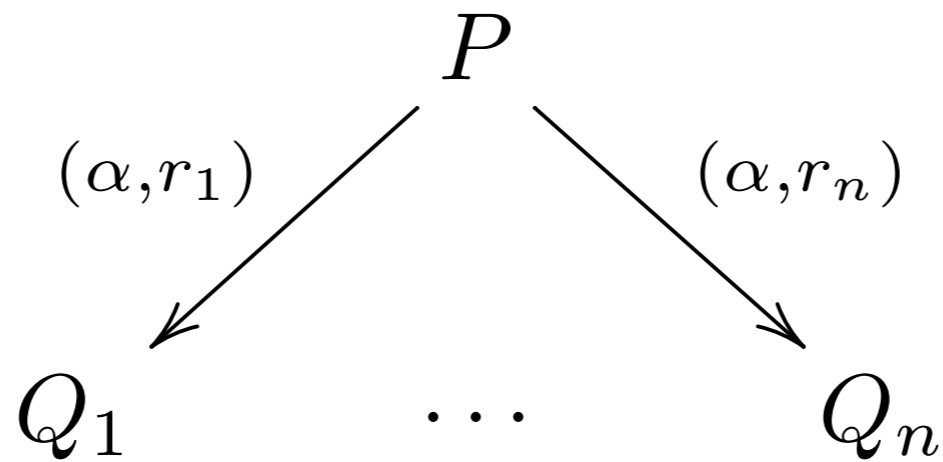
No component can be made to carry out an action in cooperation faster than its own defined rate for the actions

thus shared actions proceed at the minimum of the rates in the participating components

the apparent rates of independent actions is instead the sum of their rates within independent concurrent components

PEPA: apparent rate

$r_\alpha(P)$ is the observed rate of action α in P



$$r_\alpha(P) = r_1 + \dots + r_n$$

PEPA: apparent rate

$r_\alpha(P)$ is the observed rate of action α in P

$$r_\alpha(\mathbf{nil}) \triangleq 0$$

$$r_\alpha((\beta, r).P) \triangleq \begin{cases} r & \text{if } \alpha = \beta \\ 0 & \text{if } \alpha \neq \beta \end{cases}$$

$$r_\alpha(P + Q) \triangleq r_\alpha(P) + r_\alpha(Q) \quad (+ \text{ is not idempotent!})$$

$$r_\alpha(P/L) \triangleq \begin{cases} r_\alpha(P) & \text{if } \alpha \notin L \\ 0 & \text{if } \alpha \in L \end{cases}$$

actions are
interleaved

$$r_\alpha(P \bowtie_L Q) \triangleq \begin{cases} r_\alpha(P) + r_\alpha(Q) & \text{if } \alpha \notin L \\ \min \{r_\alpha(P), r_\alpha(Q)\} & \text{if } \alpha \in L \end{cases}$$

the slowest must
be waited for

$$r_\alpha(C) \triangleq r_\alpha(P) \quad \text{if } C \triangleq P \in \Delta$$

Cooperation

$$\frac{P_1 \xrightarrow{(\alpha, r_1)} Q_1 \quad P_2 \xrightarrow{(\alpha, r_2)} Q_2 \quad \boxed{\alpha \in L}}{P_1 \bowtie_L P_2 \xrightarrow{(\alpha, r)} Q_1 \bowtie_L Q_2}$$

$$r = r_\alpha(P_1 \bowtie_L P_2) \cdot \frac{r_1}{r_\alpha(P_1)} \cdot \frac{r_2}{r_\alpha(P_2)}$$

apparent rate

probability of specific action (α, r_i)
among the α -transitions of P_i

the sum of the rates of all the
 α -transitions that $P_1 \bowtie_L P_2$ can do

Example

Server \triangleq (*get*, \top).(*download*, μ).(*rel*, \top).Server

S \triangleq (*get*, \top).S1

S1 \triangleq (*dnd*, μ).S2

S2 \triangleq (*rel*, \top).S

Browser \triangleq (*display*, λ_1).(*cache*, m).Browser

+ (*display*, λ_2).(*get*, g).(*download*, \top).(*rel*, r).Browser

B \triangleq (*dis*, λ_1).B1 + (*dis*, λ_2).B2

B1 \triangleq (*cac*, m).B

B2 \triangleq (*get*, g).B3

B3 \triangleq (*dnd*, \top).B4

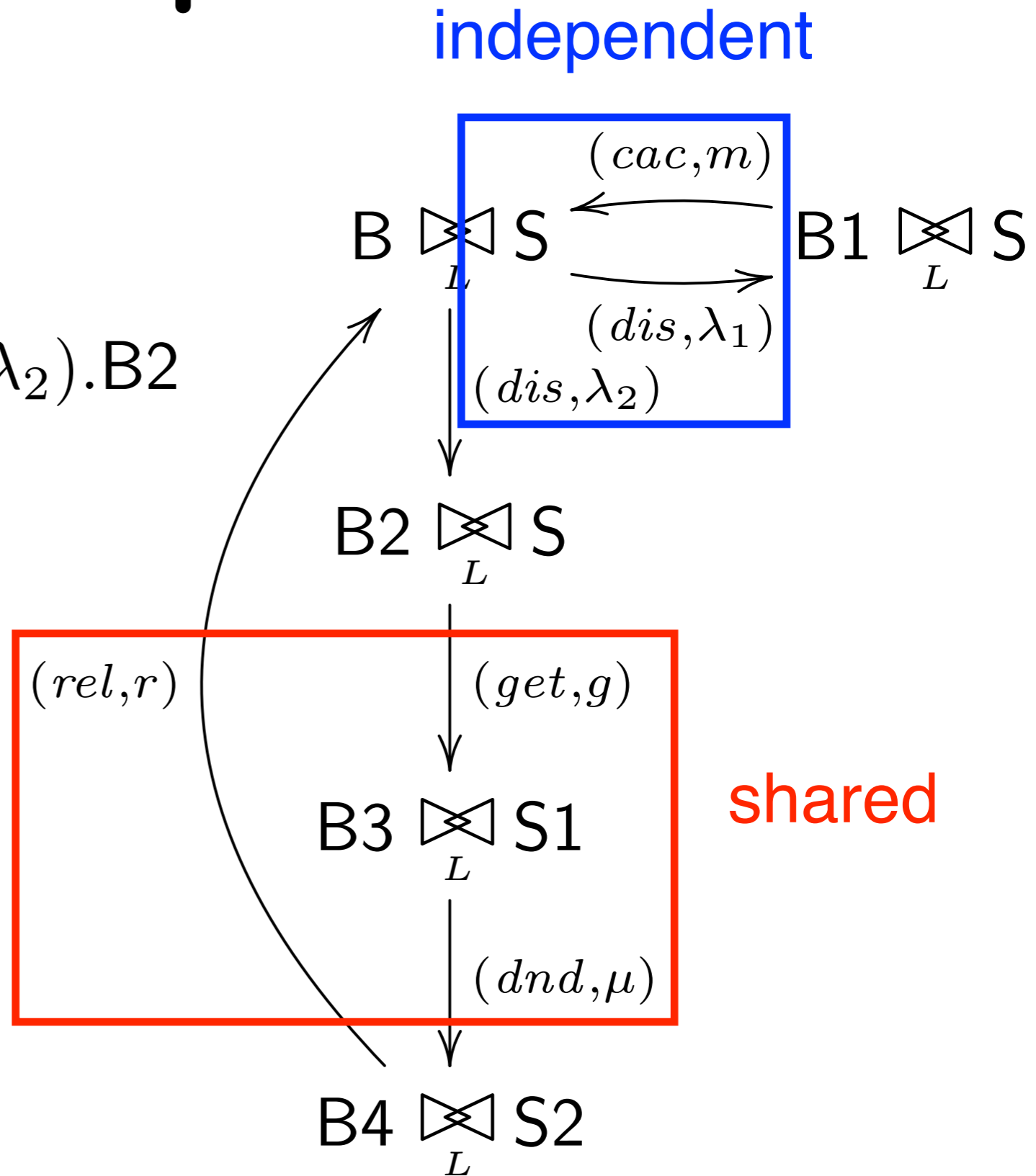
B4 \triangleq (*rel*, r).B

Example

$S \triangleq (get, \top).S1$
 $S1 \triangleq (dnd, \mu).S2$
 $S2 \triangleq (rel, \top).S$
 $B \triangleq (dis, \lambda_1).B1 + (dis, \lambda_2).B2$
 $B1 \triangleq (cac, m).B$
 $B2 \triangleq (get, g).B3$
 $B3 \triangleq (dnd, \top).B4$
 $B4 \triangleq (rel, r).B$

$L = \{get, dnd, rel\}$

$B \bowtie_L S$



Example

$$\begin{array}{ll}
 S & \triangleq (get, \top).S1 \\
 S1 & \triangleq (dnd, \mu).S2 \\
 S2 & \triangleq (rel, \top).S \\
 L & = \{get, dnd, rel\}
 \end{array}
 \qquad
 \begin{array}{ll}
 B & \triangleq (dis, \lambda_1).B1 + (dis, \lambda_2).B2 \\
 B1 & \triangleq (cac, m).B \\
 B2 & \triangleq (get, g).B3 \\
 B3 & \triangleq (dnd, \top).B4 \\
 B4 & \triangleq (rel, r).B
 \end{array}$$

$$(B \parallel B) \bowtie_L S$$

$$(B \parallel B) \bowtie_L S \xrightarrow{(dis, \lambda_2)} (B2 \parallel B) \bowtie_L S \xrightarrow{(dis, \lambda_2)} (B2 \parallel B2) \bowtie_L S$$

$$(B2 \parallel B2) \bowtie_L S \xrightarrow{(get, g)} (B3 \parallel B2) \bowtie_L S1$$

$$r_{get}(B2) = g$$

$$r_{get}(B2 \parallel B2) = 2g$$

$$r_{get}(S) = \top$$

$$\begin{array}{c} \downarrow \\ (get, g) \end{array}$$

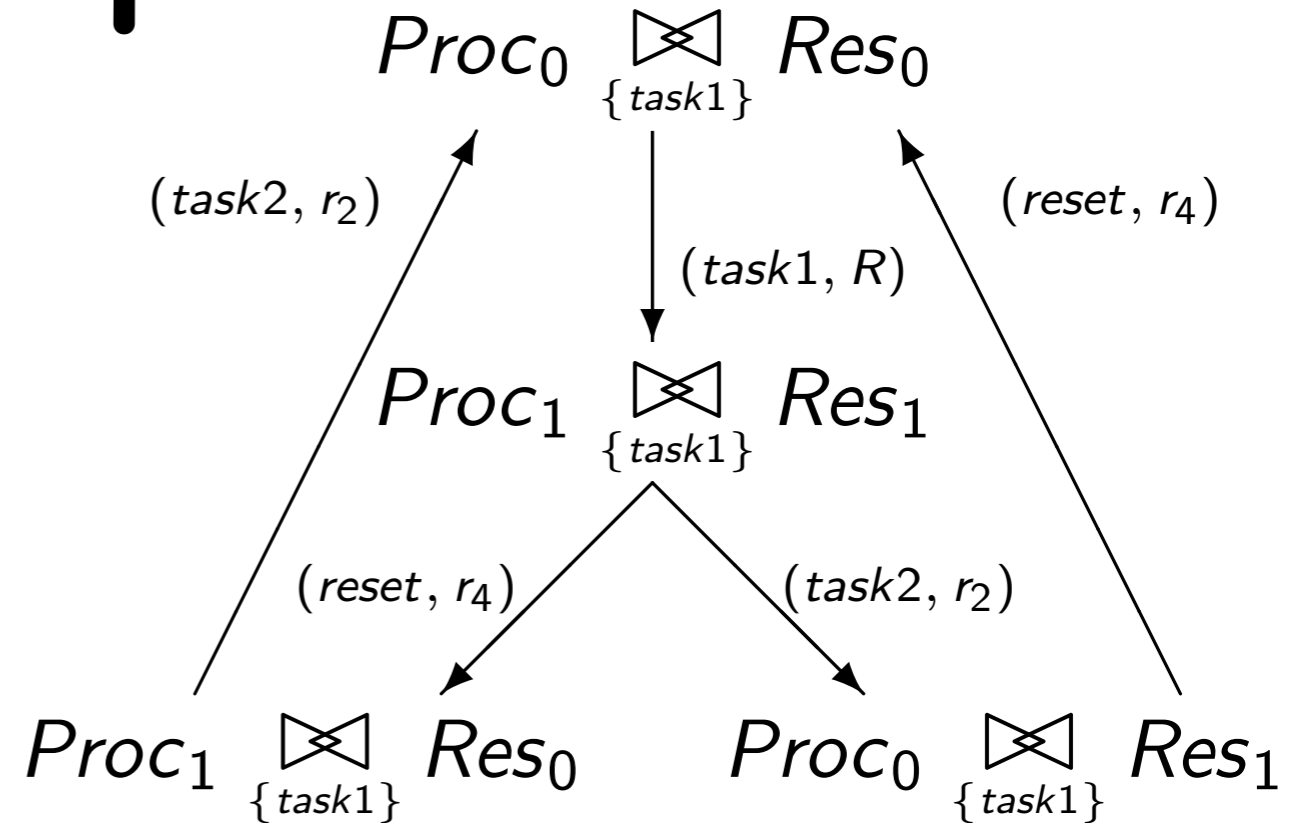
$$(B2 \parallel B3) \bowtie_L S1$$

$$r_{get}((B2 \parallel B2) \bowtie_L S) = 2g$$

Example

$Proc_0 \stackrel{def}{=} (task1, r_1).Proc_1$
 $Proc_1 \stackrel{def}{=} (task2, r_2).Proc_0$
 $Res_0 \stackrel{def}{=} (task1, r_3).Res_1$
 $Res_1 \stackrel{def}{=} (reset, r_4).Res_0$

$Proc_0 \begin{array}{c} \boxtimes \\ \{task1\} \end{array} Res_0$



$$R = \min(r_1, r_3)$$

$$\mathbf{Q} = \begin{pmatrix} -R & R & 0 & 0 \\ 0 & -(r_2 + r_4) & r_4 & r_2 \\ r_2 & 0 & -r_2 & 0 \\ r_4 & 0 & 0 & -r_4 \end{pmatrix} \quad \begin{cases} p \cdot \mathbf{Q} = 0 \\ \sum_{i=1}^N p_i = 1 \end{cases}$$

(taken from Jane Hillston's slides)

Example

$$Q = \begin{pmatrix} -R & R & 0 & 0 \\ 0 & -(r_2 + r_4) & r_4 & r_2 \\ r_2 & 0 & -r_2 & 0 \\ r_4 & 0 & 0 & -r_4 \end{pmatrix} \quad \begin{cases} p \cdot Q = 0 \\ \sum_{i=1}^N p_i = 1 \end{cases}$$

$$r_1 = 2 \quad r_2 = 2 \quad r_3 = 6 \quad r_4 = 8 \quad R = \min\{r_1, r_3\} = 2$$

$$p_1 = \frac{20}{41} \quad p_2 = \frac{4}{41} \quad p_3 = \frac{1}{41} \quad p_4 = \frac{16}{41}$$

Reward structure

\mathcal{C} a set of PEPA components

$\rho : \mathcal{C} \rightarrow \mathbb{R}$ a reward structure

p a steady state distribution

$$R_\rho \triangleq \sum_i p_i \cdot \rho(C_i)$$

sometimes rewards are defined in terms of activities

$$\rho : L \rightarrow \mathbb{R}$$

$$\rho(C) = \sum_{C \xrightarrow{(\alpha, r)} Q} \rho(\alpha)$$

Example: throughput

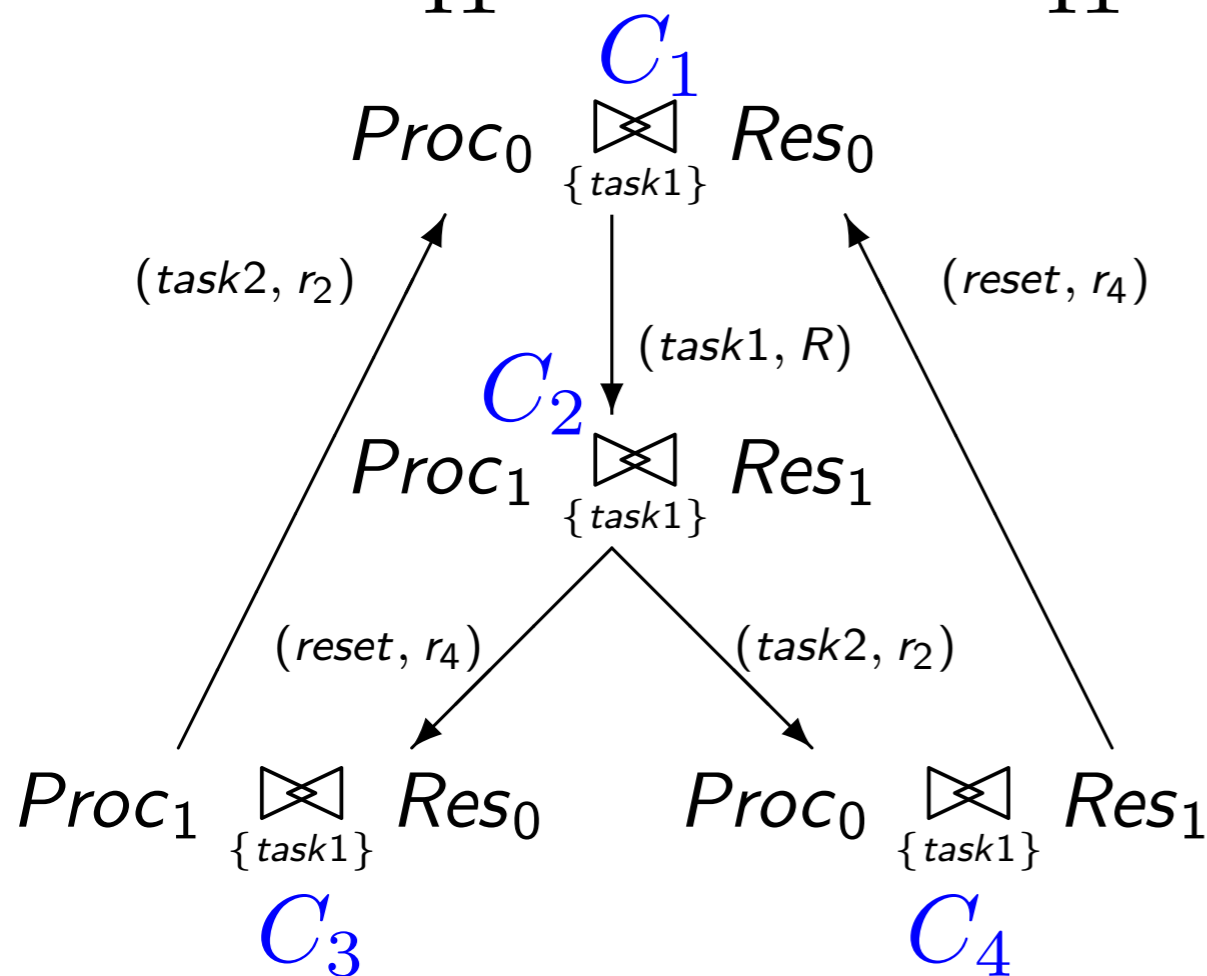
$$Q = \begin{pmatrix} -R & R & 0 & 0 \\ 0 & -(r_2 + r_4) & r_4 & r_2 \\ r_2 & 0 & -r_2 & 0 \\ r_4 & 0 & 0 & -r_4 \end{pmatrix} \begin{cases} p \cdot Q = 0 \\ \sum_{i=1}^N p_i = 1 \end{cases}$$

$$p_1 = \frac{20}{41}$$

$$p_2 = \frac{4}{41}$$

$$p_3 = \frac{1}{41}$$

$$p_4 = \frac{16}{41}$$



$$\rho(task_i) = 1 \quad \rho(reset) = 0$$

$$\rho(C_1) = \rho(C_2) = \rho(C_3) = 1$$

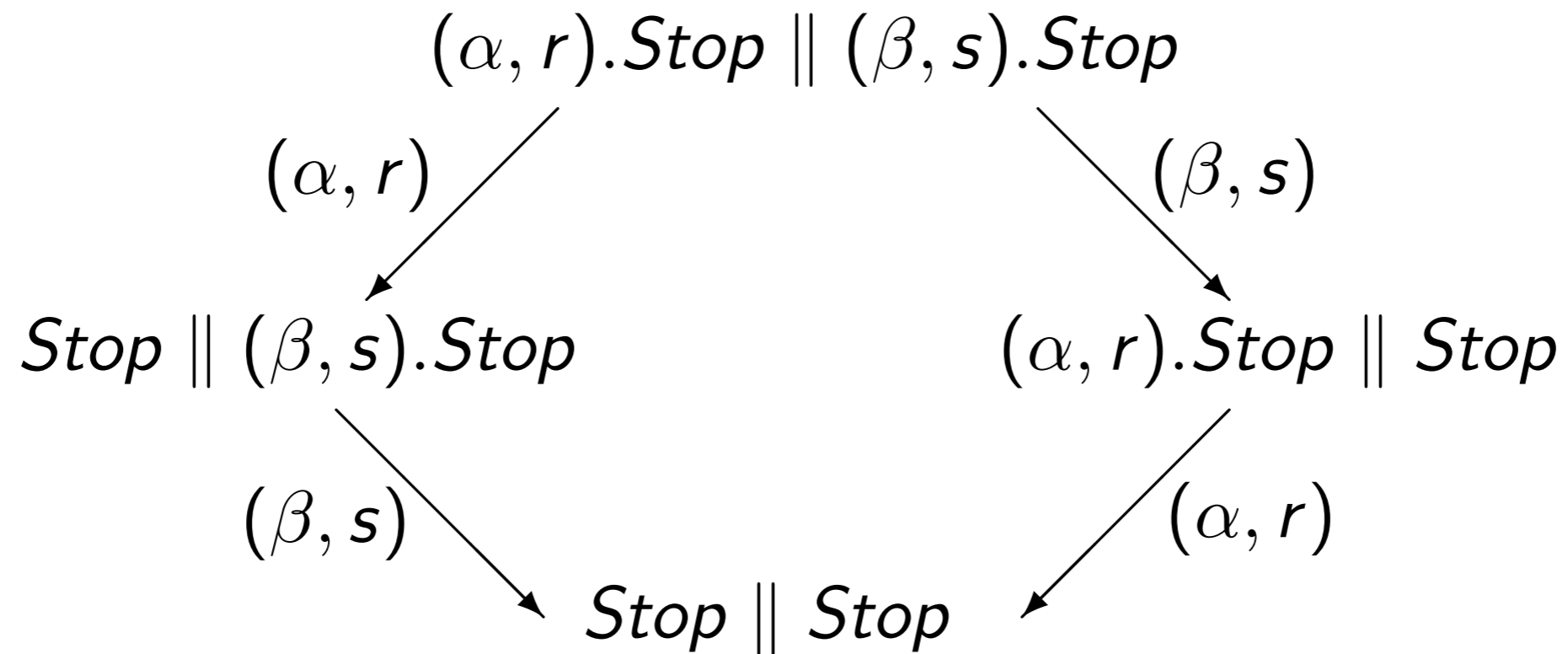
$$\rho(C_4) = 0$$

$$R = \frac{20 + 4 + 1}{41} = \frac{25}{41} = 61\%$$

PEPA

further considerations

The importance of being Exp



We retain the **expansion law** of classical process algebra:

$$\begin{aligned}
 (\alpha, r).Stop \parallel (\beta, s).Stop = \\
 (\alpha, r).(\beta, s).(Stop \parallel Stop) + (\beta, s).(\alpha, r).(Stop \parallel Stop)
 \end{aligned}$$

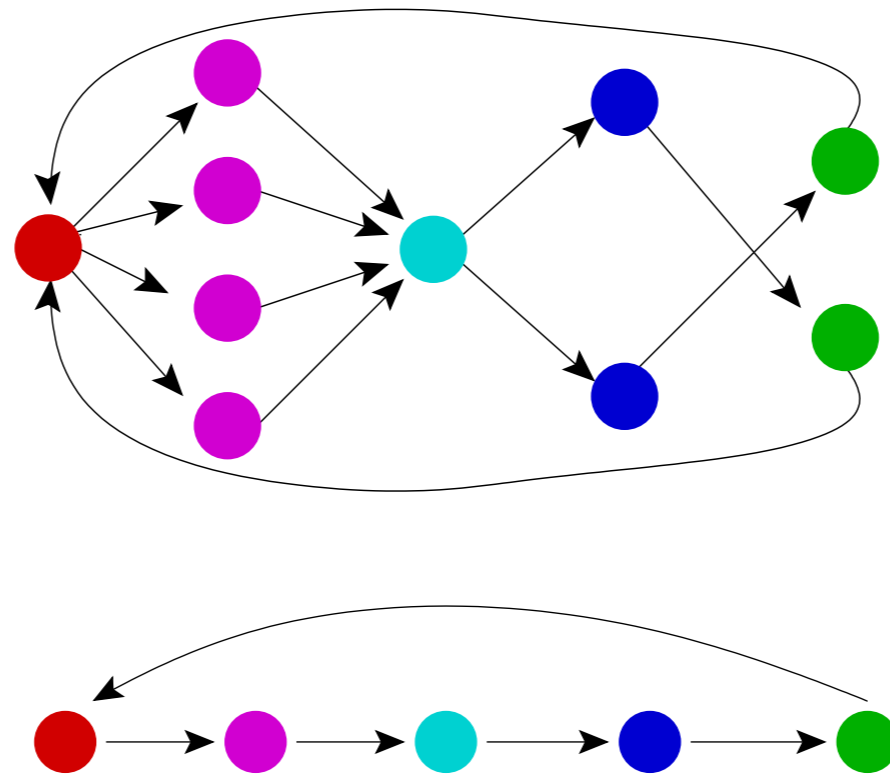
only if the **negative exponential distribution** is assumed.

(taken from Jane Hillston's slides)

Model aggregation

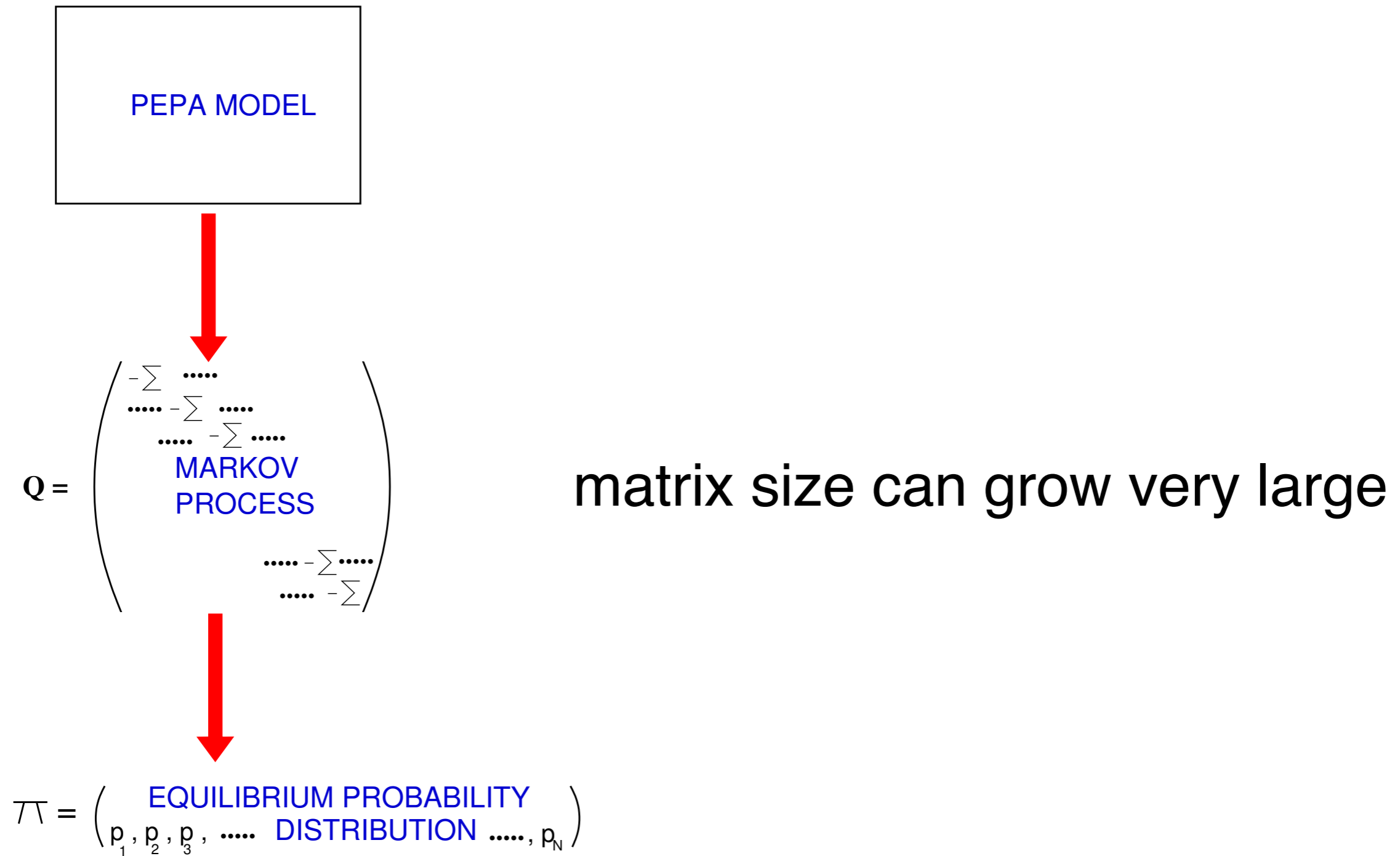
we can exploit CTMC bisimulation to reduce the state space
(notion of lumpable partition)

it is the only equivalence that preserves the Markov property



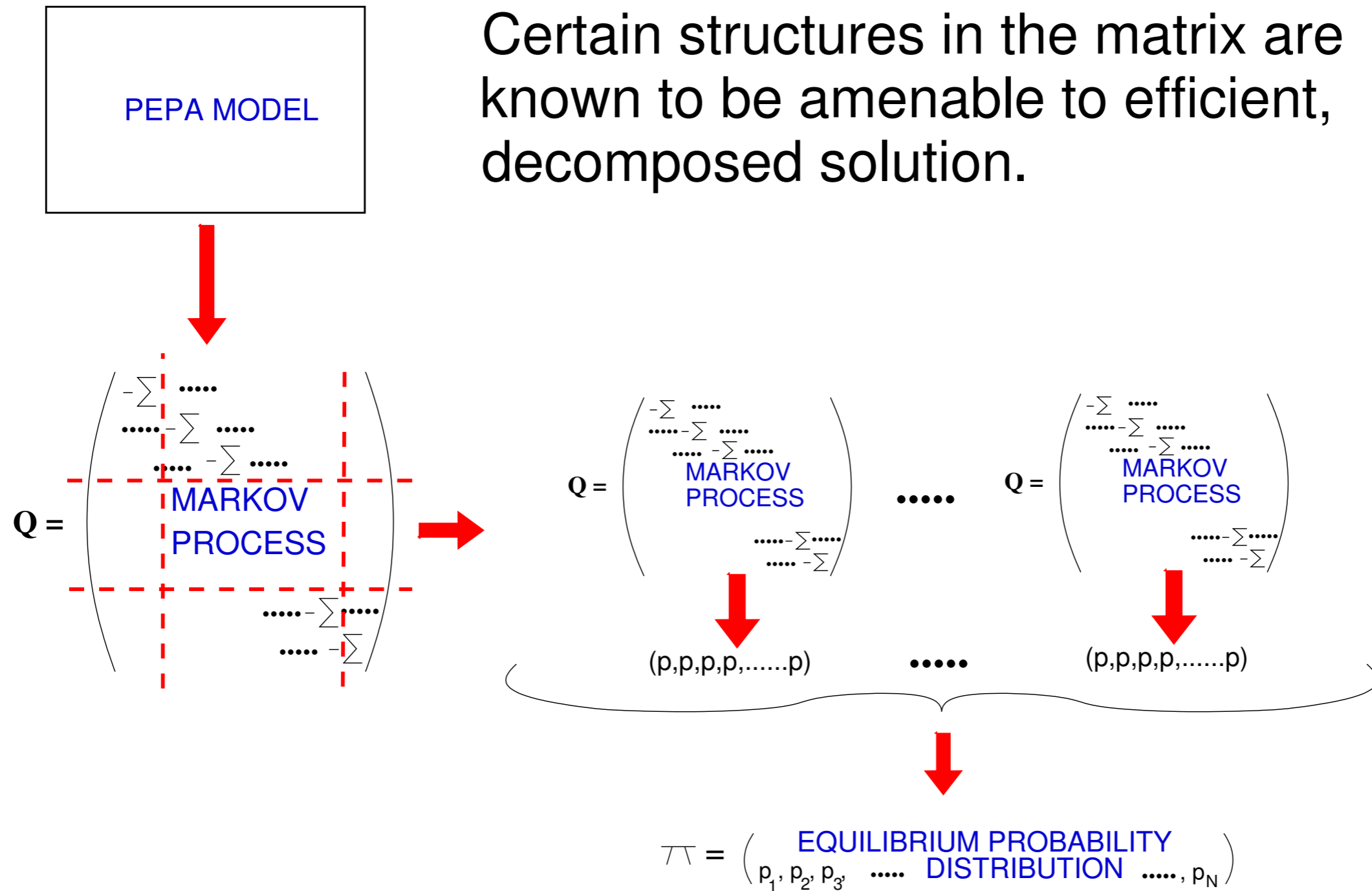
(taken from Jane Hillston's slides)

Compositionality



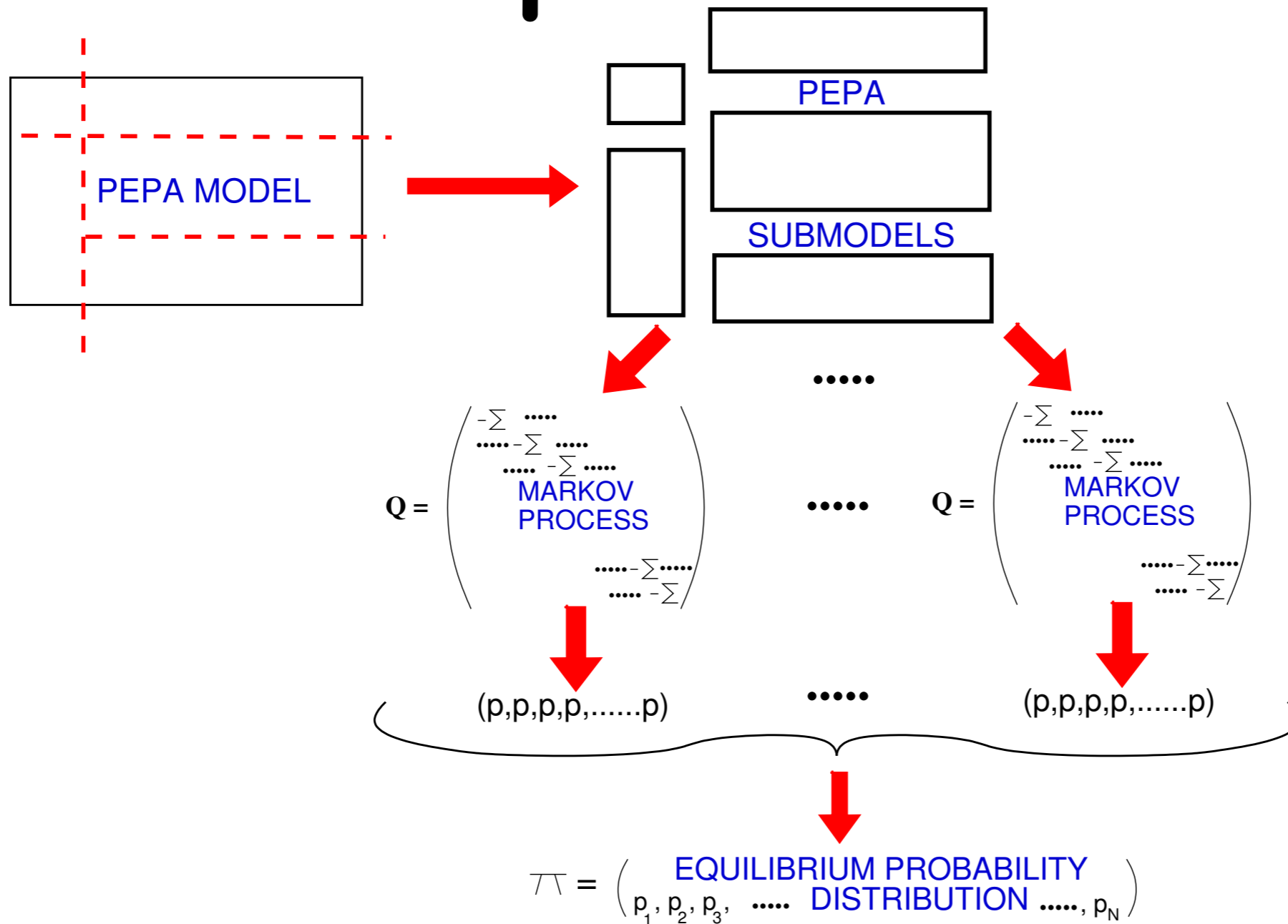
(taken from Jane Hillston's slides)

Compositionality



(taken from Jane Hillston's slides)

Compositionality



lift independent structures to the PEPA model!

(taken from Jane Hillston's slides)

Last badge



The final exam of a course consists of a list of 30 questions and a list of 30 answers. Each student has to draw a bijective correspondence between the two lists, linking each question to its answer.

The teacher will assign 1 point to each correct link and 0 to each wrong link.

Unfortunately, many students had no time to prepare for the exam, because they had a tight deadline to deliver a project and they will answer completely random.

1. What is the average score for such students?
2. Would the average score be improved by adding more questions and answers?

$$\langle c, \sigma \rangle \rightarrow \sigma' \quad p \cdot Q = 0$$

$$\bigsqcup_n f^n(\perp) \quad p \approx q$$

$\Phi(\mathbf{R})$



$$S \models \psi$$

$$s \models \varphi$$

$$p \models F$$

$$s \simeq_D u$$

$$s \simeq_R u$$

$$t \rightarrow c$$

$$\text{fix} \quad (D, \sqsubseteq)$$

$$t : \tau$$



$$s \simeq_C u$$

END.nil



$$\llbracket t \rrbracket \rho$$

$$\pi \cdot P = \pi$$

$$p \sim_L q$$

$$p \sim_E q$$

$$p \xrightarrow{\mu} q$$



$$i \xrightarrow{\lambda_{i,j}} j$$

R

$$S, \pi \models \psi$$

$$p \xrightarrow{\alpha} q$$

$$P \xrightarrow{(\alpha,r)} Q$$

$$i \xrightarrow{a_{i,j}} j$$

$$C[c]\sigma$$

$$p \simeq q$$