Principles for software composition 2020/21
02 - Structural induction and rule induction

[**Ex. 1**] Complete the proof of termination of boolean expressions by structural induction.

[**Ex. 2**] Extend the syntax of arithmetic expressions with the operator $a_0 \sqcap a_1$ whose big-step operational semantics is given by the rules:

$$\frac{\langle a_0, \sigma\rangle \longrightarrow n \quad \langle a_1, \sigma\rangle \longrightarrow n}{\langle a_0 \sqcap a_1, \sigma\rangle \longrightarrow n}$$

1. Prove termination or exhibit a counterexample.
2. Prove determinacy or exhibit a counterexample.

[**Ex. 3**] Extend the syntax of arithmetic expressions with the operator $a_0 \sqcup a_1$ whose big-step operational semantics is given by the rule:

$$\frac{\langle a_0, \sigma\rangle \longrightarrow n_0}{\langle a_0 \sqcup a_1, \sigma\rangle \longrightarrow n_0} \qquad \frac{\langle a_1, \sigma\rangle \longrightarrow n_1}{\langle a_0 \sqcup a_1, \sigma\rangle \longrightarrow n_1}$$

1. Prove termination or exhibit a counterexample.
2. Prove determinacy or exhibit a counterexample.

[**Ex. 4**] Consider the command

$$w \stackrel{\text{def}}{=} \textbf{while } x > y \textbf{ do } (x := x + 1 \ ; \ y := y - 1)$$

Find out the set S of memories σ such that $\langle w, \sigma\rangle \not\longrightarrow$ and prove that this is the case by using the inference rule for divergence.

[**Ex. 5**] Prove determinacy of boolean expressions by rule induction.

[**Ex. 6**] Let b be a boolean expression and c a command. Consider the command

$$w \stackrel{\text{def}}{=} \textbf{while } b \textbf{ do } c$$

Prove by rule induction that:

$$\forall \sigma, \sigma'. \ \langle w, \sigma\rangle \longrightarrow \sigma' \ \Rightarrow \langle b, \sigma'\rangle \longrightarrow \mathit{false}$$