[Ex. 1] Determine the type of the HOFL term
\[ t \overset{\text{def}}{=} \text{rec } x. ( \lambda y. \text{if } y \text{ then } 0 \text{ else } 0 ) x. \]
Then compute its (lazy) canonical form.

[Ex. 2] Determine the type of the HOFL term
\[ \text{map } \overset{\text{def}}{=} \lambda f. \lambda x. ( ( f \ \text{fst}(x) ), ( f \ \text{snd}(x) ) ) \]
Then, compute the (lazy) canonical forms of the terms
\[ t_1 \overset{\text{def}}{=} \text{map } ( \lambda z. 2 \times z ) (1, 2) \quad t_2 \overset{\text{def}}{=} \text{fst } ( \text{map } ( \lambda z. 2 \times z ) (1, 2) ) \]

[Ex. 3] Let \((D, \sqsubseteq_D)\) be a CPO and \(f : D \rightarrow D\) be a continuous function. Prove that the set of fixpoints of \(f\) is itself a CPO (ordered by \(\sqsubseteq_D\)).

[Ex. 4] (Test for convergence) We would like to modify the denotational semantics of HOFL assigning to the construct
\[ \text{if } t \text{ then } t_0 \text{ else } t_1 \]
- the semantics of \(t_1\) if the semantics of \(t\) is \(\bot_Z\), and
- the semantics of \(t_0\) otherwise.
Is it possible? If not, why?

[Ex. 5] (Strict conditional) Modify the operational semantics of HOFL by taking the following rules for conditionals:

\[
\begin{align*}
& t \rightarrow 0 \quad t_0 \rightarrow c_0 \quad t_1 \rightarrow c_1 \quad t \rightarrow n \quad n \neq 0 \quad t_0 \rightarrow c_0 \quad t_1 \rightarrow c_1 \\
& \text{if } t \text{ then } t_0 \text{ else } t_1 \rightarrow c_0 \\
& \text{if } t \text{ then } t_0 \text{ else } t_1 \rightarrow c_1.
\end{align*}
\]
Without changing the denotational semantics, prove that:
1. for any term \(t\) and canonical form \(c\), we have \(t \rightarrow c \Rightarrow \forall \rho. [t]\rho = [c]\rho; \]
2. in general \(t \downarrow \neq t \downarrow\) (exhibit a counterexample).

[Ex. 6] Determine the type of the HOFL term
\[ t \overset{\text{def}}{=} \text{rec } f. ( \lambda x. 1 , \text{fst}(f) 0 ) \]
Then, compute the (lazy) denotational semantics of \(t\).