[Ex. 1] Determine the type of the HOFL term
\[ t \overset{\text{def}}{=} \text{rec } x. ((\lambda y. \text{if } y \text{ then } 0 \text{ else } 0) x). \]
Then compute its (lazy) canonical form.

[Ex. 2] Determine the type of the HOFL term
\[ \text{map} \overset{\text{def}}{=} \lambda f. \lambda x. ((f \text{ fst}(x)), (f \text{ snd}(x))) \]
Then, compute the (lazy) canonical forms of the terms
\[ t_1 \overset{\text{def}}{=} \text{map } (\lambda z. 2 \times z) (1, 2) \quad t_2 \overset{\text{def}}{=} \text{fst } (\text{map } (\lambda z. 2 \times z) (1, 2)) \]

[Ex. 3] Let \((D, \sqsubseteq_D)\) be a CPO and \(f : D \to D\) be a continuous function. Prove that the set of fixpoints of \(f\) is itself a CPO (ordered by \(\sqsubseteq_D\)).

[Ex. 4] (Test for convergence) We would like to modify the denotational semantics of HOFL assigning to the construct
\[ \text{if } t \text{ then } t_0 \text{ else } t_1 \]
• the semantics of \(t_1\) if the semantics of \(t\) is \(\bot\), and
• the semantics of \(t_0\) otherwise.
Is it possible? If not, why?

[Ex. 5] (Strict conditional) Modify the operational semantics of HOFL by taking the following rules for conditionals:
\[
\begin{align*}
\text{if } t \text{ then } t_0 \text{ else } t_1 & \to c_0 \\
\text{if } t \to n \quad n \neq 0 & \to t_0 \to c_0 \quad t_1 \to c_1
\end{align*}
\]
Without changing the denotational semantics, prove that:
1. for any term \(t\) and canonical form \(c\), we have \(t \to c \Rightarrow \forall \rho. \llbracket t \rrbracket \rho = \llbracket c \rrbracket \rho\);
2. in general \(t \Downarrow \neq t \Downarrow\) (exhibit a counterexample).

[Ex. 6] Determine the type of the HOFL term
\[ t \overset{\text{def}}{=} \text{rec } f. (\lambda x.1, \text{fst}(f) \ 0) \]
Then, compute the (lazy) denotational semantics of \(t\).