[Ex. 1] Complete the proof of termination of boolean expressions by structural induction.

[Ex. 2] Extend the syntax of arithmetic expressions with the operator $a_0 \sqcap a_1$ whose big-step operational semantics is given by the rules:

$$
\frac{\langle a_0, \sigma \rangle \rightarrow n \quad \langle a_1, \sigma \rangle \rightarrow n}{\langle a_0 \sqcap a_1, \sigma \rangle \rightarrow n}
$$

1. Prove termination or exhibit a counterexample.
2. Prove determinacy or exhibit a counterexample.

[Ex. 3] Extend the syntax of arithmetic expressions with the operator $a_0 \sqcup a_1$ whose big-step operational semantics is given by the rule:

$$
\begin{align*}
\frac{\langle a_0, \sigma \rangle \rightarrow n_0 \quad \langle a_1, \sigma \rangle \rightarrow n_1}{\langle a_0 \sqcup a_1, \sigma \rangle \rightarrow n_0} & \quad \frac{\langle a_0 \sqcup a_1, \sigma \rangle \rightarrow n_1}{\langle a_1, \sigma \rangle \rightarrow n_1}
\end{align*}
$$

1. Prove termination or exhibit a counterexample.
2. Prove determinacy or exhibit a counterexample.

[Ex. 4] Consider the command

$$
w \overset{\text{def}}{=} \textbf{while } x > y \textbf{ do } (x := x + 1 ; \ y := y - 1)
$$

Find out the set $S$ of memories $\sigma$ such that $\langle w, \sigma \rangle \not\rightarrow$ and prove that this is the case by using the inference rule for divergence.

[Ex. 5] Prove determinacy of boolean expressions by rule induction.

[Ex. 6] Let $b$ be a boolean expression and $c$ a command. Consider the command

$$
w \overset{\text{def}}{=} \textbf{while } b \textbf{ do } c
$$

Prove by rule induction that:

$$
\forall \sigma, \sigma'. \langle w, \sigma \rangle \rightarrow \sigma' \Rightarrow \langle b, \sigma' \rangle \rightarrow \text{false}
$$