Models of computation (MOD) 2016/17

Exam – July 6, 2017

[**Ex. 1**] Extend IMP syntax with conditional expressions $b\ ?\ a_0 : a_1$ whose operational semantics if defined by the rules

$$\frac{\langle b,\sigma\rangle \to \mathbf{true} \quad \langle a_0,\sigma\rangle \to n}{\langle b\ ?\ a_0 : a_1,\sigma\rangle \to n} \qquad \frac{\langle b,\sigma\rangle \to \mathbf{false} \quad \langle a_1,\sigma\rangle \to n}{\langle b\ ?\ a_0 : a_1,\sigma\rangle \to n}$$

1. Extend the proof of determinacy for (boolean and) arithmetic expressions to take the new construct into account.
2. Define the denotational semantics of conditional expressions.
3. Extend the proof of correspondence between the operational and denotational semantics of (boolean and) arithmetic expressions to take the new construct into account.

[**Ex. 2**] Given a partial order $(D, \sqsubseteq_D)$, let $C : (D, \sqsubseteq_D) \to (\wp(D), \subseteq)$ be the function that assigns to each element $d \in D$ the set of elements in D that are *comparable* with d, i.e.

$$C(d) \stackrel{\text{def}}{=} \{\ x \in D \ \mid\ x \sqsubseteq_D d \ \vee\ d \sqsubseteq_D x\ \}$$

1. Show that in general C is not monotone.
   Hint: it is enough to consider a three elements set D.
2. Prove that $\forall d_0, d_1.\ C(d_0) \subseteq C(d_1) \Rightarrow d_1 \in C(d_0)$.

[**Ex. 3**] Consider the HOFL term

$$t \stackrel{\text{def}}{=} \lambda x.\ \mathbf{rec}\ y.\ \mathbf{if}\ x\ \mathbf{then}\ 0\ \mathbf{else}\ (\ (\lambda z.\ y)\ x\ )$$

1. Find the principal type of t.
2. Compute the (lazy) denotational semantics of t.

[**Ex. 4**] Let $\simeq$ denote strong bisimilarity.

1. Prove that $\forall \alpha, p, q.\ p \simeq q \ \Rightarrow\ p\backslash\alpha \simeq q\backslash\alpha\backslash\alpha$.
2. Let
   $$p \stackrel{\text{def}}{=} \mathbf{rec}\ x.((\beta.x)\backslash\alpha + \alpha.x) \qquad q \stackrel{\text{def}}{=} p\backslash\alpha + p\backslash\beta$$
   Prove that $p \not\simeq q$ by exhibiting a suitable HM-formula.