[Ex. 1] Let us extend IMP with the command

\[ x := c \]

whose denotational semantics is

\[ \mathcal{C}[x := c] \sigma \overset{\text{def}}{=} (\lambda \sigma_1. \sigma[\sigma_1(x)/x])^* (\mathcal{C}[c] \sigma) \]

1. Define the operational semantics for the new construct.

2. Extend the proofs of correctness and completeness between the operational and the denotational semantics to account for the new construct.

[Ex. 2] Let \( c \), \( w \) and \( w' \) be the IMP commands defined below:

\[
\begin{align*}
  c & \overset{\text{def}}{=} (z := x; (x := y; y := z)) \\
  w & \overset{\text{def}}{=} \text{while } x \neq y \text{ do } c \\
  w' & \overset{\text{def}}{=} \text{while } x \neq y \text{ do skip}
\end{align*}
\]

Compute \( \mathcal{C}[c] \sigma \). Then prove that \( \mathcal{C}[w] = \mathcal{C}[w'] \).

[Ex. 3] Let \((S, \prec)\) be a set \( S \neq \emptyset \) with a binary relation \( \prec \subseteq S \times S \) such that for all \( s \in S \) the set \( [s] \overset{\text{def}}{=} \{ x \mid x \prec s \} \) is finite. Let \( f : (\wp(S), \subseteq) \rightarrow (\wp(S), \subseteq) \) be the function over the CPO \( \wp(S) \) such that, for any \( X \in \wp(S) \)

\[ f(X) \overset{\text{def}}{=} \{ y \mid [y] \subseteq X \} \]

1. Is \((S, \prec)\) always well founded? (If not, exhibit a counterexample)

2. Prove that \( f \) is monotone.

3. Prove that \( f \) is continuous.

[Ex. 4] Let us consider expressions of the form \( e := n \mid e^* \) with \( n \in \mathbb{N} \), whose operational semantics is defined by the rules

\[
\begin{align*}
  n & \rightarrow n \quad \text{(num)} \\
  e & \rightarrow n \quad \text{(once)} \\
  e^* & \rightarrow n_1 \quad e^* \rightarrow n_2 \quad \text{(more)}
\end{align*}
\]

It is evident that \( e^* \rightarrow n \) implies \( (e^*)^* \rightarrow n \) (by the second rule). Prove the converse by rule induction, i.e., that \( \forall e, n. \ ( (e^*)^* \rightarrow n \Rightarrow e^* \rightarrow n ) \).

[Ex. 5] Compute the most general type of the HOFL term

\[ t \overset{\text{def}}{=} \text{rec } f. \lambda x. (\text{fst}(x), \text{snd}(f x)) \]