Models of computation (MOD) 2016/17

Mid-term exam – April 6, 2017

[**Ex. 1**] Let us extend IMP with the command

$$x := c$$

whose denotational semantics is

$$\mathscr{C}[\![x := c]\!]\sigma \stackrel{\text{def}}{=} \left(\lambda\sigma_1.\ \sigma[^{\sigma_1(x)}/_x]\right)^* (\mathscr{C}[\![c]\!]\sigma)$$

1. Define the operational semantics for the new construct.
2. Extend the proofs of correctness and completeness between the operational and the denotational semantics to account for the new construct.

[**Ex. 2**] Let c, w and w' be the IMP commands defined below:

$$\begin{array}{rcl} c & \stackrel{\text{def}}{=} & (z := x; (x := y;\ y := z)) \\ w & \stackrel{\text{def}}{=} & \textbf{while}\ x \neq y\ \textbf{do}\ c \\ w' & \stackrel{\text{def}}{=} & \textbf{while}\ x \neq y\ \textbf{do skip} \end{array}$$

Compute$\mathscr{C}[\![c]\!]\sigma$. Then prove that $\mathscr{C}[\![w]\!] = \mathscr{C}[\![w']\!]$.

[**Ex. 3**] Let $(S, \prec)$ be a set $S \neq \emptyset$ with a binary relation $\prec \subseteq S \times S$ such that for all $s \in S$ the set $[s) \stackrel{\text{def}}{=} \{\ x \mid x \prec s\ \}$ is finite. Let $f : (\wp(S), \subseteq) \to (\wp(S), \subseteq)$ be the function over the CPO$_\perp$ $(\wp(S), \subseteq)$ such that, for any $X \in \wp(S)$

$$f(X) \stackrel{\text{def}}{=} \{\ y \mid [y) \subseteq X\ \}$$

1. Is $(S, \prec)$ always well founded? (If not, exhibit a counterexample)
2. Prove that f is monotone.
3. Prove that f is continuous.

[**Ex. 4**] Let us consider expressions of the form $e\ ::=\ n \mid e^*$ with $n \in \mathbb{N}$, whose operational semantics is defined by the rules

$$\frac{}{n \to n}(num) \qquad \frac{e \to n}{e^* \to n}(once) \qquad \frac{e^* \to n_1 \quad e^* \to n_2}{e^* \to n_1 \times n_2}(more)$$

It is evident that $e^* \to n$ implies $(e^*)^* \to n$ (by the second rule). Prove the converse by rule induction, i.e., that $\forall e, n.\ (\ (e^*)^* \to n\ \Rightarrow\ e^* \to n\ )$.

[**Ex. 5**] Compute the most general type of the HOFL term

$$t \stackrel{\text{def}}{=} \textbf{rec}\ f.\ \lambda x.\ (\textbf{fst}(x), \textbf{snd}(f\ x))$$