Models of computation (MOD) 2016/17

Appello straordinario – April 6, 2017

[Ex. 1] Let us extend IMP with the command

$$x := c$$

whose denotational semantics is

$$\mathscr{C}[\![x := c]\!]\sigma \stackrel{\mathsf{def}}{=} \left(\lambda\sigma_1.\ \sigma[^{\sigma_1(x)}/_x]\right)^* (\mathscr{C}[\![c]\!]\sigma)$$

1. Define the operational semantics for the new construct.
2. Extend the proofs of correctness and completeness between the operational and the denotational semantics to account for the new construct.

[Ex. 2] Let $(S, \prec)$ be a set S with a binary relation $\prec \subseteq S \times S$ such that for all $s \in S$ the set $[s) \stackrel{\mathsf{def}}{=} \{\ x \mid x \prec s\ \}$ is finite, and let $f : (\wp(S), \subseteq) \to (\wp(S), \subseteq)$ be the function over the $\mathrm{CPO}_\bot$ $(\wp(S), \subseteq)$ such that, for any $X \in \wp(S)$

$$f(X) \stackrel{\mathsf{def}}{=} \{\ y \mid [y) \subseteq X\ \}$$

1. Is $(S, \prec)$ always well founded? (If not, exhibit a counterexample)
2. Prove that f is monotone.
3. Prove that f is continuous.

[Ex. 3] Let us consider the HOFL term

$$t \stackrel{\mathsf{def}}{=} \mathbf{rec}\ f.\ \lambda x.\ (\mathbf{fst}(x), \mathbf{snd}(f\ x))$$

1. Find the principal type of t.
2. Compute the denotational semantics of t.

[Ex. 4] Let us consider the CCS process

$$p \stackrel{\mathsf{def}}{=} \mathbf{rec}\ x.\ (\ (\alpha.x \mid \overline{\alpha}.\mathbf{nil})\backslash\alpha + \beta.x\ )$$

and let $p\backslash^0_\alpha \stackrel{\mathsf{def}}{=} p$ and $p\backslash^{n+1}_\alpha \stackrel{\mathsf{def}}{=} (p\backslash^n_\alpha|\mathbf{nil})\backslash\alpha$ for $n \geq 0$.

1. Draw, at least in part, the LTS for the process p.
2. What are the transitions leaving from $p\backslash^n_\alpha$?
3. Prove that $p\backslash^n_\alpha$ is strongly bisimilar to $p\backslash^m_\alpha$ for any $n, m \geq 0$.