Models of computation (MOD) 2015/16
Exam – Jan. 20, 2017

[**Ex. 1**] Let the transition relation $\to^n$ with $n \in \mathbb{N}$ be defined by the following inference rules

$$\frac{}{\langle c, \sigma \rangle \to^0 \sigma} \qquad \frac{\langle c, \sigma \rangle \to \sigma'' \quad \langle c, \sigma'' \rangle \to^n \sigma'}{\langle c, \sigma \rangle \to^{n+1} \sigma'}$$

1. Prove the determinacy of $\to^n$ (for any $n \in \mathbb{N}$).
2. Prove that for all c, σ, σ', n:

$$\langle c, \sigma \rangle \to^n \sigma' \;\Rightarrow\; \forall k < n.\ \exists \sigma''.\ \langle c, \sigma \rangle \to^k \sigma''.$$

[**Ex. 2**] Let S be a non-empty, finite set. A *multiset* over S is a function $M : S \to \mathbb{N}$ that assigns to each element $s \in S$ its multiplicity $M(s)$. Let $\mu(S)$ denote the set of all multisets over S, ordered by the relation

$$M \sqsubseteq M' \quad \stackrel{\text{def}}{=} \quad \forall s \in S.\ M(s) \leq M'(s)$$

1. Prove that $(\mu(S), \sqsubseteq)$ is a partial order with bottom.
2. Exhibit a set S and a chain $\{M_i\}_{i \in \mathbb{N}}$ in $(\mu(S), \sqsubseteq)$ that has no lub.
3. Let $\delta : (\mu(S), \sqsubseteq) \to (\mathbb{N}, \leq)$ be defined by $\delta(M) \stackrel{\text{def}}{=} |\{s \in S \mid M(s) > 0\}|$. Prove that δ is monotone.

[**Ex. 3**] Consider the HOFL term

$$t \stackrel{\text{def}}{=} \mathbf{rec}\ f.\ \lambda x.\ \mathbf{if}\ \mathbf{fst}(x)\ \mathbf{then}\ \mathbf{snd}(x)\ \mathbf{else}\ (f\ (\mathbf{snd}(x), \mathbf{fst}(x)))$$

1. Under which hypothesis is t typable?
2. Compute the (lazy) denotational semantics of t.

[**Ex. 4**] Consider the CCS processes

$$p \stackrel{\text{def}}{=} \mathbf{rec}\ x.\ (\alpha.\mathbf{nil} + \tau.\mathbf{rec}\ y.\ (\beta.y + \tau.x)) \qquad q \stackrel{\text{def}}{=} \mathbf{rec}\ z.\ (\alpha.\mathbf{nil} + \beta.z)$$

1. Draw the LTSs of p and q.
2. Prove that p and q are weakly bisimilar.
3. Are p and q weakly observational congruent? (Explain)