Models of computation (MOD) 2015/16
Exam – June 30, 2016

[Ex. 1] Suppose we extend IMP with the new command construct $c_0 \Rightarrow c_1$, whose operational semantics is defined by the rules:

$$\frac{\langle c_0, \sigma\rangle \to \sigma'' \quad \langle c_1, \sigma''\rangle \to \sigma'}{\langle c_0 \Rightarrow c_1, \sigma\rangle \to \sigma'} \qquad \frac{\langle c_1, \sigma\rangle \to \sigma'}{\langle c_0 \Rightarrow c_1, \sigma\rangle \to \sigma'}$$

1. Prove that $\exists c_0, c_1, \sigma, \sigma'$ such that $\langle c_1, \sigma\rangle \not\to$ and $\langle c_0 \Rightarrow c_1, \sigma\rangle \to \sigma'$.
2. Prove that $\forall c_0, c_1, c_2.\ (c_0 \Rightarrow (c_1; c_2)) \sim ((c_0 \Rightarrow c_1); c_2)$.

(Recall that $\sim$ denotes operational equivalence).

[Ex. 2] Consider the HOFL term

$$t \stackrel{\text{def}}{=} \lambda x.\ \mathbf{rec}\ f.\ \lambda y.\ \mathbf{if}\ x\ \mathbf{then}\ y\ \mathbf{else}\ (f\ y).$$

1. Under which hypothesis is t typable?
2. Let $t_0, t_1 : int$ be two closed terms. Under which hypotheses $((t\ t_0)\ t_1) \downarrow$?
3. Compute the (lazy) denotational semantics of t.

[Ex. 3] Consider the CCS processes

$$\begin{array}{lllllll} p & \stackrel{\text{def}}{=} & \mathbf{rec}\ x.\ (\alpha.x + \gamma.\mathbf{nil}) & \quad & r & \stackrel{\text{def}}{=} & p|q \\ q & \stackrel{\text{def}}{=} & \mathbf{rec}\ y.\ (\beta.y + \gamma.\mathbf{nil}) & \quad & s & \stackrel{\text{def}}{=} & \mathbf{rec}\ z.\ (\alpha.z + \beta.z + \gamma.(z\backslash\alpha) + \gamma.(z\backslash\beta)) \end{array}$$

1. Assuming the structural laws $s\backslash\alpha\backslash\beta \equiv s\backslash\beta\backslash\alpha$ and $s\backslash\alpha\backslash\alpha \equiv s\backslash\alpha$ (for any process s and actions α, β), draw the LTSs of processes r and s.
2. Prove that $r \not\simeq s$ (where $\simeq$ denotes strong bisimilarity).

[Ex. 4] You have 3 umbrellas, some at home, some in the office. You keep moving between home and office. You take an umbrella with you from one place to another only if it rains. If it does not rain you do not carry any umbrella. It may happen that you must leave one place and it starts raining, but you do not have any umbrella with you, so you get wet. Suppose you have been doing this for several years and that the probability of rain is (and has always been) p. You are about to leave one place, what is the probability that you get wet? Use DTMCs to answer the question.
Hint: In the modelling of states, the fact that you are at home or in the office is irrelevant.