# Models of computation (MOD) 2015/16

Second Mid-Term Exam – May 25, 2016

[**Ex. 1**] Add to IMP the construct **try** $c$ **guarding** $x$ whose operational semantics is defined by the inference rule:

$$\frac{\langle c, \sigma\rangle \rightarrow \sigma' \quad \sigma(x) = \sigma'(x)}{\langle \textbf{try } c \textbf{ guarding } x, \sigma\rangle \rightarrow \sigma'}$$

1. Define a function $\mathit{Guard} : \Sigma_\perp \times \textbf{Loc} \times \mathbb{Z} \rightarrow \Sigma_\perp$ such that the denotational semantics of the new construct is defined by letting:

   $$\mathcal{C}[\![\textbf{try } c \textbf{ guarding } x]\!]\sigma \stackrel{\text{def}}{=} \mathit{Guard}(\mathcal{C}[\![c]\!]\sigma, x, \sigma(x))$$

   Make sure that the function $\mathit{Guard}$ is monotone (and therefore continuous) on its first argument.

2. Extend the proof of completeness between operational and denotational semantics to take into account the new construct.

3. Extend the proof of correctness between operational and denotational semantics to take into account the new construct.

[**Ex. 2**] Consider the HOFL term

$$t \stackrel{\text{def}}{=} \textbf{rec } f.\ \lambda x.\ \lambda y.\ \textbf{if } (x - y) \textbf{ then } 0 \textbf{ else } ((f\ y)\ x)$$

1. Prove that the term is typable and give its principal type.

2. Compute the denotational semantics of $t$.

[**Ex. 3**] Let us consider the CCS processes

$$p \stackrel{\text{def}}{=} \textbf{rec } X.\ (\alpha.\textbf{nil} + (\textbf{rec } Y.\ (\alpha.\textbf{nil} + \beta.Y + \gamma.X))) \quad q \stackrel{\text{def}}{=} \textbf{rec } Z.\ (\alpha.\textbf{nil} + \beta.Z + \gamma.Z)$$

1. Prove that the processes $p$ and $q$ are guarded.

2. Draw the LTSs of $p$ and $q$.

3. Prove that $p$ and $q$ are bisimilar.

[**Ex. 4**] Two automatically driven shuttles $A_1$ and $A_2$ are serving three stations $S_0, S_1, S_2$ on a railway ring, travelling in opposite directions. Let $R_i$ denote the railway segment that connects $S_i$ with $S_{(i+1)\textbf{mod } 3}$. Given the propositions $in_{i,j}$ that asserts that the shuttle $A_i$ is at station $S_j$ and $mv_{i,j}$ that asserts that the shuttle $A_i$ is moving along the railway $R_j$:

1. Use LTL temporal operator $F$ and ordinary logical connectives to specify that the two shuttles will never be moving along the railway segment $R_1$ at the same time.

2. Use CTL to specify that whenever the shuttle $A_1$ is at station $S_2$ it will remain at that station until it starts moving along the segment $R_2$.

3. Use $\mu$-calculus to specify that, at any time, the shuttle $A_2$ has the possibility to reach station $S_0$.

Keep the formulas as simple as possible.