Models of computation (MOD) 2013/14

Exam – February 6, 2015

[**Ex. 1**] Extend IMP with the command

$$\textbf{try } c_1 \textbf{ then } c_2$$

that returns the store obtained by computing c_2 if c_1 converges and that diverges otherwise.

1. Define the operational semantics of the new command.
2. Define the denotational semantics of the new command.
3. Extend the proof of correspondence between the operational and the denotational semantics.

[**Ex. 2**] Let us consider the HOFL term

$$map \stackrel{\text{def}}{=} \lambda f.\lambda x.((f\ \mathbf{fst}(x)), (f\ \mathbf{snd}(x)))$$

1. Show that *map* is a typeable term and give its principal type.
2. Write the denotational semantics of *map* and of $(map\ \lambda z.z)$.
3. Give two terms $t_1 : int$ and $t_2 : int$ such that the terms

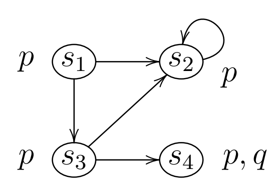
   $$(map\ \lambda z.z)(t_1, t_2) \qquad (map\ \lambda z.z)(t_2, t_1)$$

   have different canonical forms but the same denotational semantics.

[**Ex. 3**] Write a μ-calculus formula ϕ representing the statement:

'there is some path where p holds until eventually q holds.'

Write the denotational semantics of ϕ and evaluate it over the LTS below:



[**Ex. 4**] A certain calculating machine uses only the digits 0 and 1. It is supposed to transmit one of these digits through several stages. However, at every stage, there is a probability p that the digit that enters this stage will be changed when it leaves and a probability $q = 1 - p$ that it won't.

1. Form a Markov chain to represent the process of transmission. What are the states? What is the matrix of transition probabilities?
2. Assume that the digit 0 enters the machine: what is the probability that the machine, after two stages, produces the digit 0? For which value of p is this probability minimal?