Models of computation (MOD) 2013/14
Exam – January 16, 2015

[**Ex. 1**] Let us consider the IMP command

$$w = \textbf{while } x \neq 0 \textbf{ do } x := x - y.$$

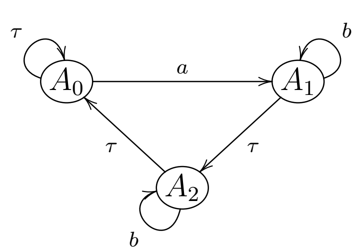
1. Prove that, for any σ, σ', if $\langle w, \sigma \rangle \to \sigma'$ then there exists an integer k such that $\sigma(x) = k \times \sigma(y)$.
2. Give a store σ such that $\sigma(x) = k \times \sigma(y)$ for an integer k but such that $\langle w, \sigma \rangle \not\to$. Explain why the program diverges for the given σ.
3. Define a command c such that, for any σ, σ', $\langle c, \sigma \rangle \to \sigma'$ iff $\sigma(x) = k \times \sigma(y)$ for some integer k. Sketch the proof of the double implication.

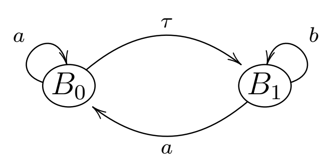
[**Ex. 2**] Let $\{\top\}$ be a one-element set and $\{\top\}_\perp$ the corresponding flat domain. Let Ω be the domain of *vertical natural numbers*

$$0 \leq 1 \leq 2 \leq 3 \leq ... \leq \infty.$$

Show that the functional domain $[\Omega \to \{\top\}_\perp]$ is in bijection with Ω.
Hint: Define what the possible continuous functions from Ω to $\{\top\}_\perp$ are.

[**Ex. 3**] Let us consider the CCS agents below:





1. Write the recursive expressions that corresponds to A_0 and B_0.

   Hint: Introduce a **rec** construct for each node in the diagram and name the process variables as the nodes for simplicity, e.g., for the expression that corresponds to A_0 write **rec** A_0. $(\tau.A_0 + ...)$.
2. Prove that $A_0 \not\approx B_0$ and $B_0 \approx B_1$, where $\approx$ is the weak bisimilarity.

[**Ex. 4**] Write a μ-calculus formula ϕ representing the statement:

'p is always true along any path leaving the current state.'

Write the denotational semantics of ϕ and evaluate it over the LTS below: