Models of computation (MOD) 2016/17

Appello straordinario – October 31, 2017

[**Ex. 1**] Let the set $a[^{a_0}/_y]$ denote the syntactic replacement of all occurrences of y by the expression a_0 in the expression a, defined by structural recursion over IMP-expressions as follows:

$$n[^{a_0}/_y] \stackrel{\text{def}}{=} n \qquad x[^{a_0}/_y] \stackrel{\text{def}}{=} \begin{cases} a_0 & \text{if } x = y \\ x & \text{if } x \neq y \end{cases} \qquad (a_1 \text{ op } a_2)[^{a_0}/_y] \stackrel{\text{def}}{=} (a_1[^{a_0}/_y]) \text{ op } (a_2[^{a_0}/_y])$$

1. Prove by rule induction that $\forall a \in \mathbf{Aexp}.\, \forall \sigma \in \Sigma.\, \forall k, m \in \mathbb{Z}.\, \forall y \in \mathbf{Loc}$:

   $$\langle a, \sigma[^k/_y]\rangle \to m \;\Rightarrow\; \forall a_0 \in \mathbf{Aexp}.\; (\; \langle a_0, \sigma\rangle \to k \;\Rightarrow\; \langle a[^{a_0}/_y], \sigma\rangle \to m \;)$$

2. Prove that $\forall a, a_0 \in \mathbf{Aexp}.\, \forall \sigma \in \Sigma.\, \forall y \in \mathbf{Loc}$:

   $$\mathcal{A}\,[\![a[^{a_0}/_y]]\!]\,\sigma = \mathcal{A}\,[\![a]\!]\,(\sigma[^{\mathcal{A}[\![a_0]\!]\sigma}/_y])$$

3. Is it possible to find three expressions a, a_0, a_1 such that $a[^{a_0}/_y][^{a_1}/_y]$ is not denotationally equivalent to $a[^{a_1}/_y]$?

[**Ex. 2**] Consider the HOFL term

$$t \stackrel{\text{def}}{=} \mathbf{rec}\ f.\ (\ \lambda x.1\ ,\ (\mathbf{fst}\ f)\ 0\ )$$

1. Find the principal type of t.
2. Compute the (lazy) denotational semantics of t.

[**Ex. 3**] Let us consider the CCS processes

$$p \stackrel{\text{def}}{=} \mathbf{rec}\ x.(\alpha.x + \beta.x) \quad q \stackrel{\text{def}}{=} \mathbf{rec}\ y.(\overline{\alpha}.\mathbf{nil} + \gamma.y) \quad r \stackrel{\text{def}}{=} \mathbf{rec}\ z.(\overline{\beta}.\mathbf{nil} + \overline{\gamma}.z)$$

1. Draw the LTS of the process $s \stackrel{\text{def}}{=} (p|q|r)\backslash\alpha\backslash\beta\backslash\gamma$.
2. Show that s is strong bisimilar to the process $\mathbf{rec}\ w.(\tau.w + \tau.\tau.\mathbf{nil})$.

[**Ex. 4**] Alice, Bob and Carol play frisbee. Alice always throws to Bob, Bob always throws to Carol, and Carol throws to Alice 1/3 of the time and to Bob 2/3 of the time.

1. Model the system as a DTMC.
2. Who has the least chance to be found with the frisbee on the long run?
3. If Carol has the frisbee, what is the chance that Carol has the frisbee back after three throws?