Succinct Data Structures

Auto-completion as our target application

Rossano Venturini

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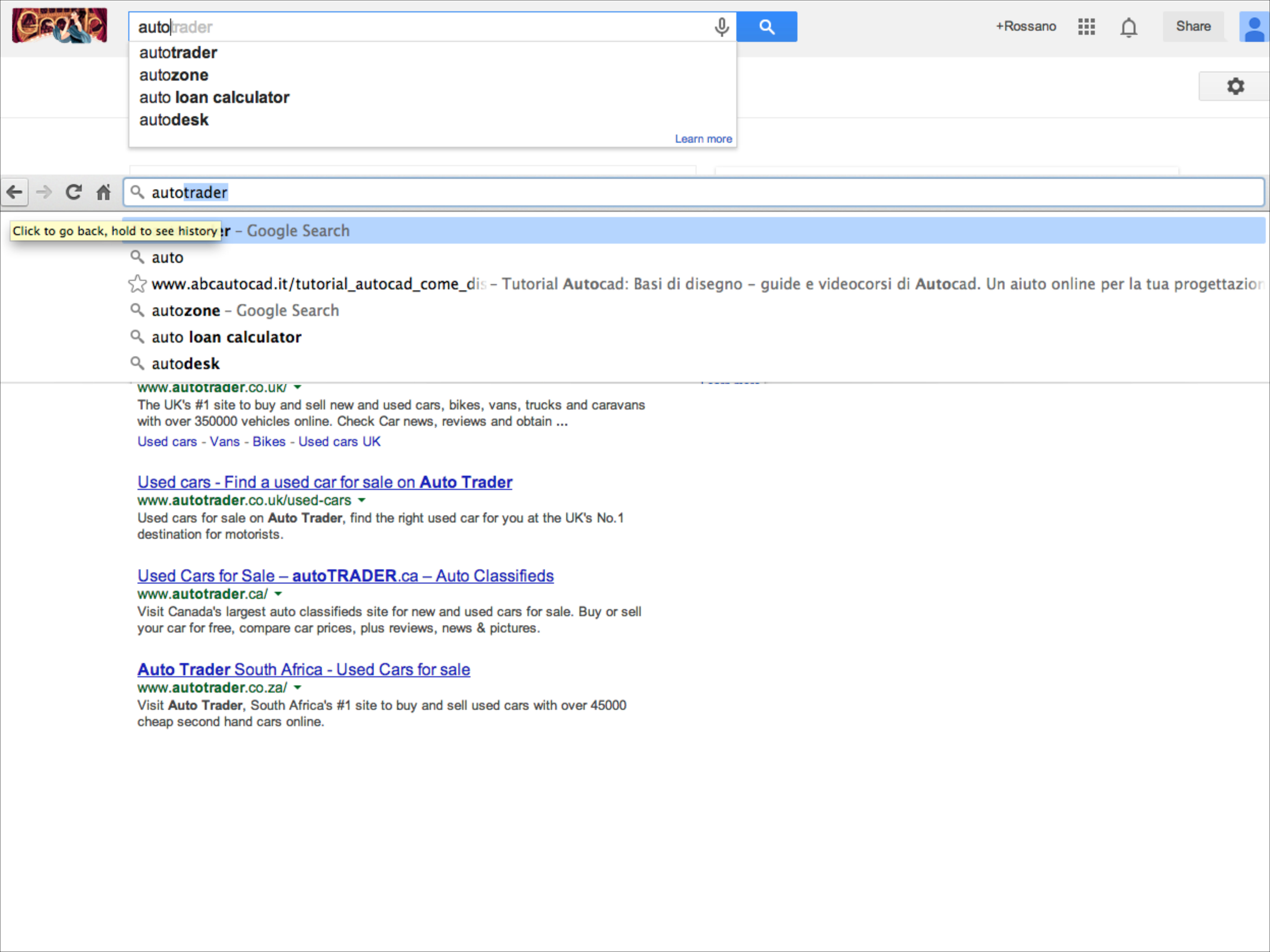
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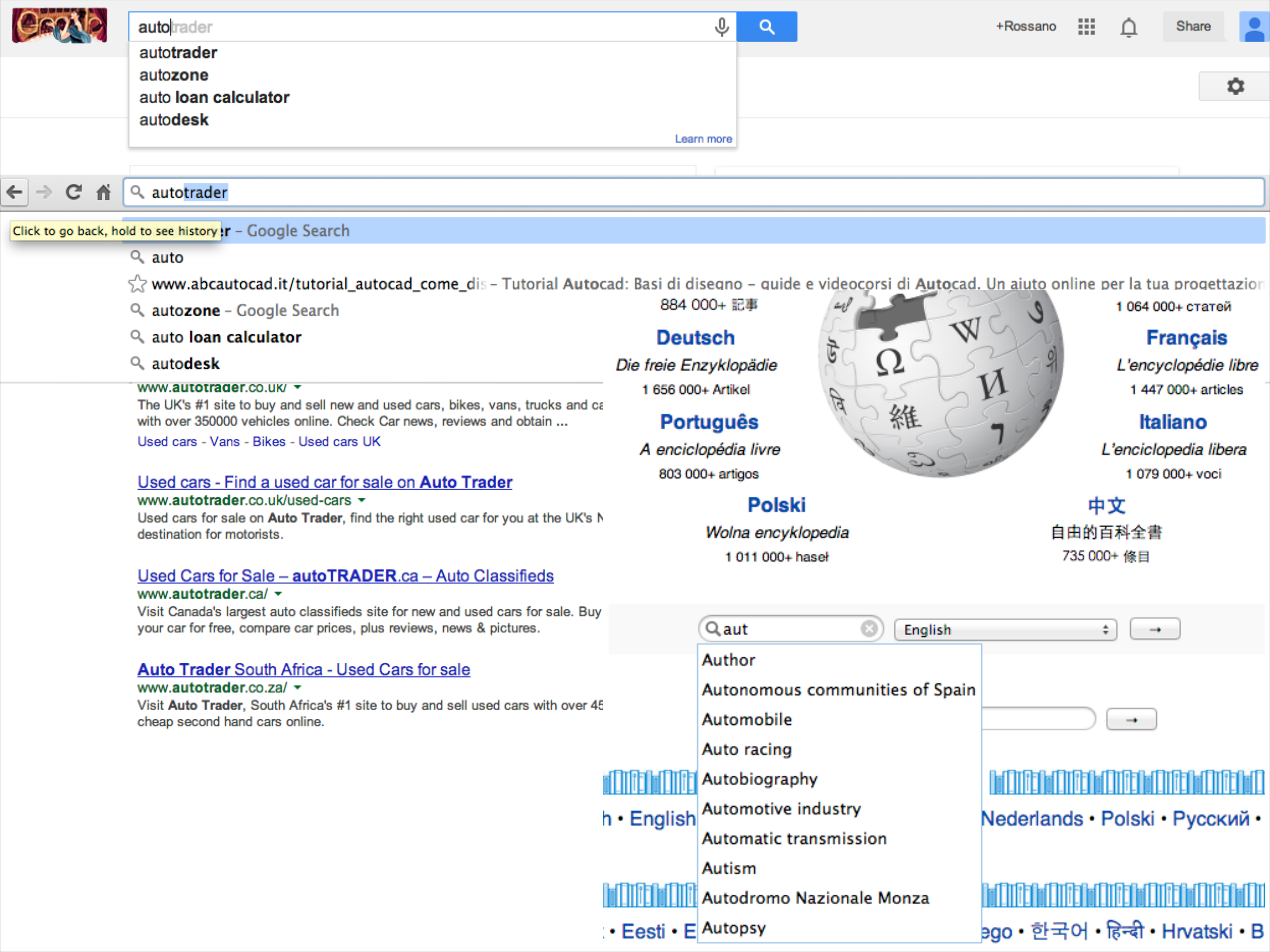
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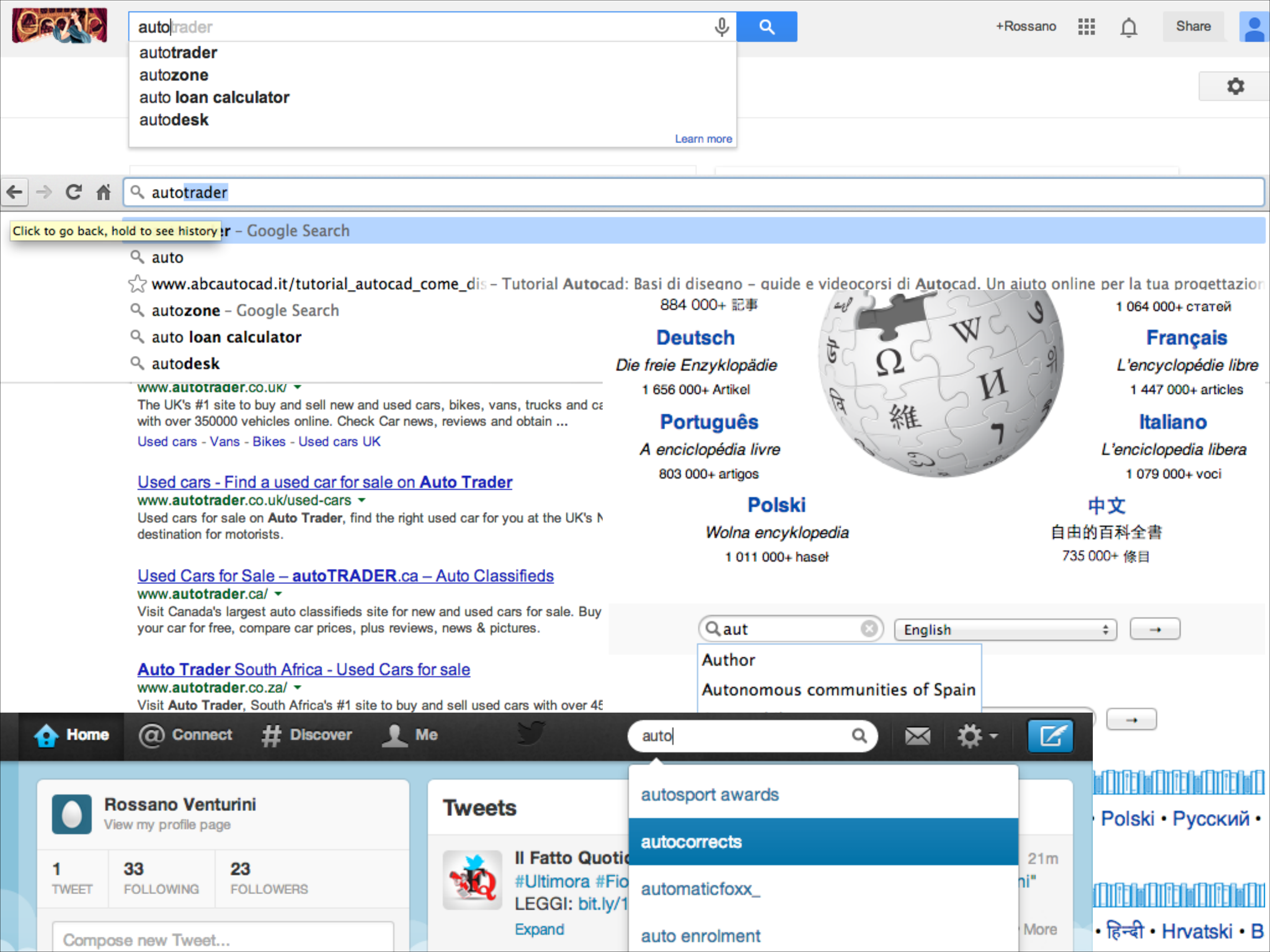
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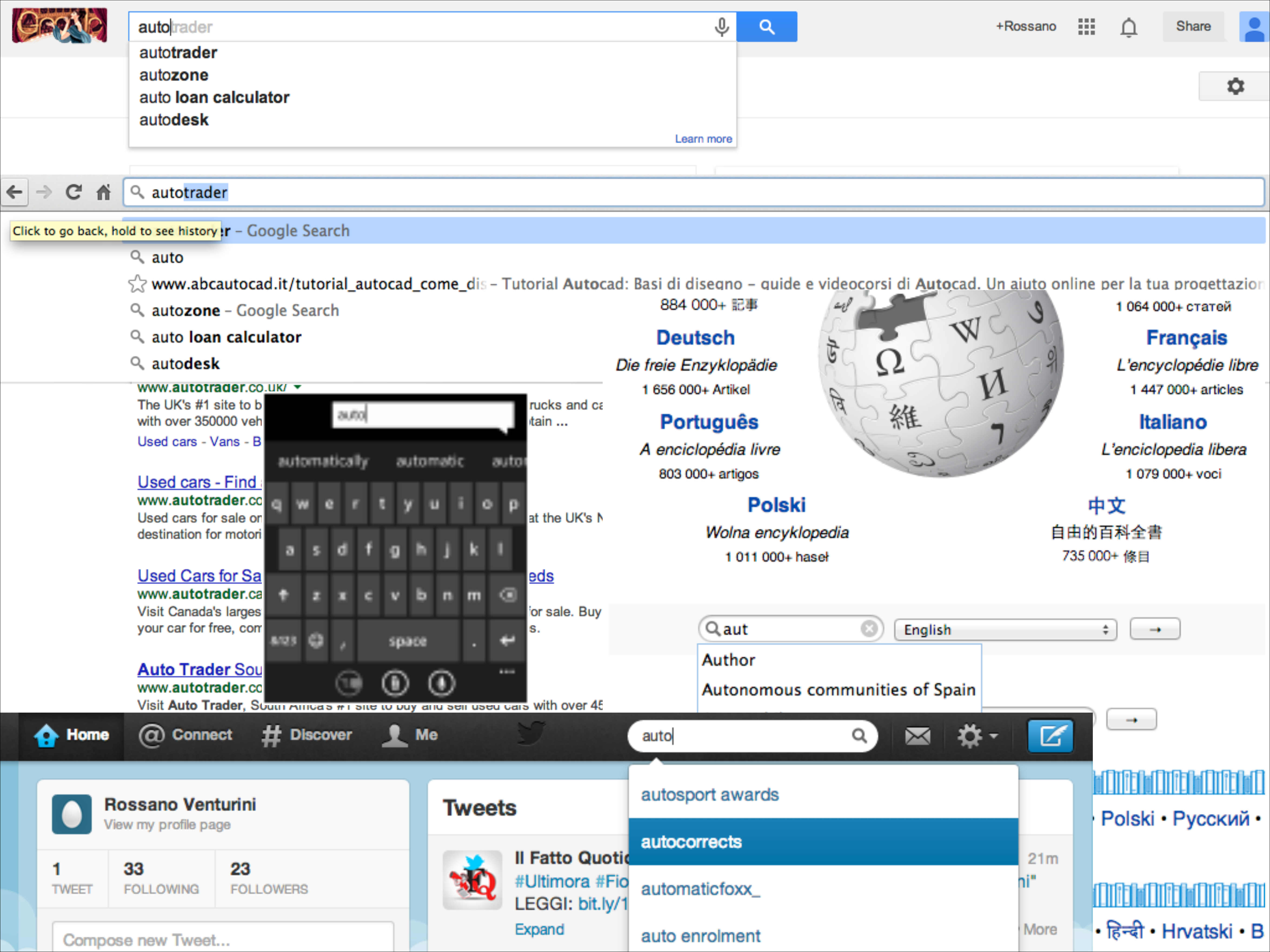
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Dataset?

Dataset?

All the past queries

Dataset?

Searches?

All the past queries

Dataset?

Searches?

All the past queries

Prefix search

Dataset?

Searches?

Data structure?

All the past queries

Prefix search

Dataset? All the past queries

Searches? Prefix search

Data structure? Trie

Dataset?	All the past queries
Searches?	Prefix search
Data structure?	Trie

How to find top-k efficiently?

Trie

Trie

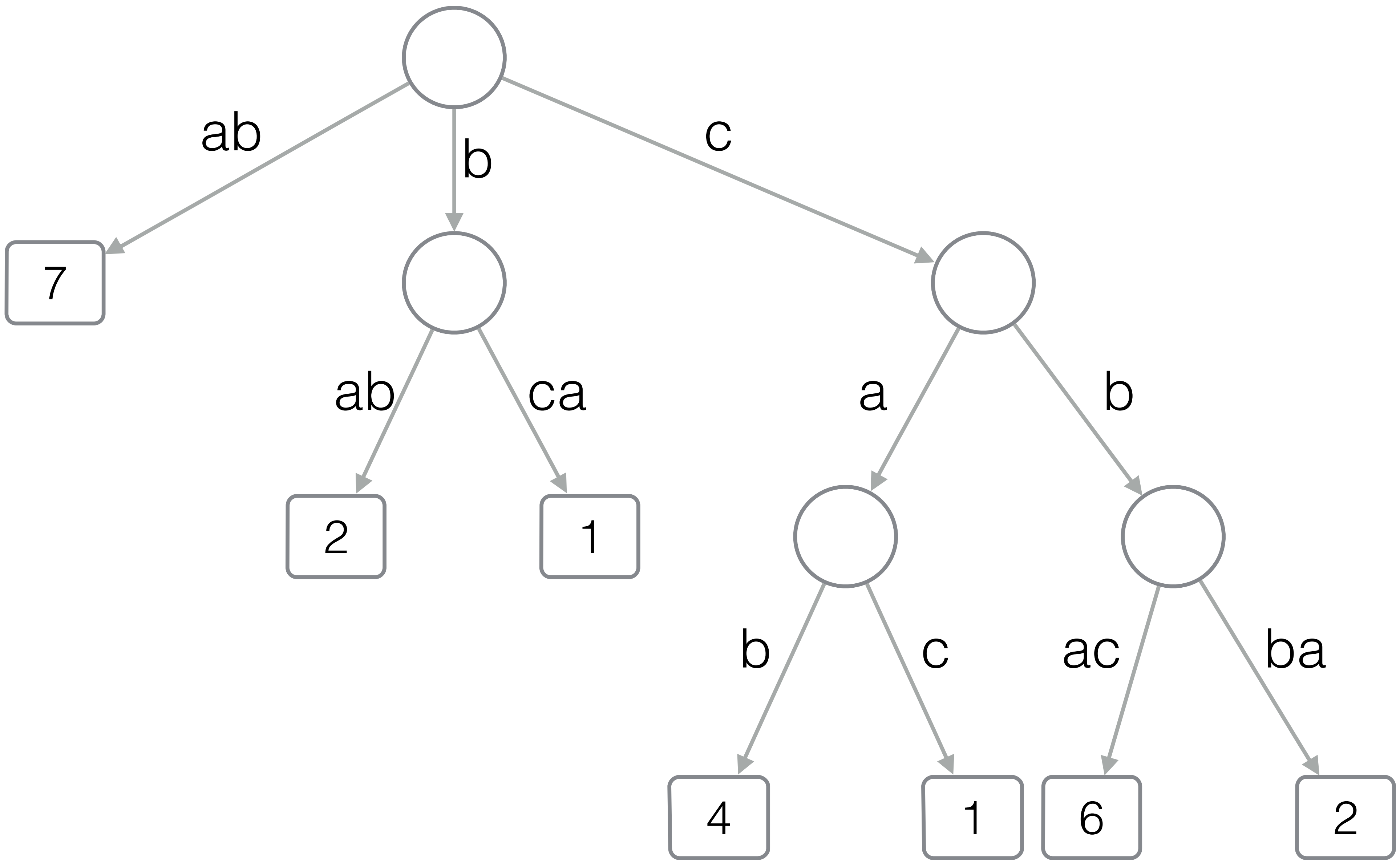
D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

Trie

D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Trie



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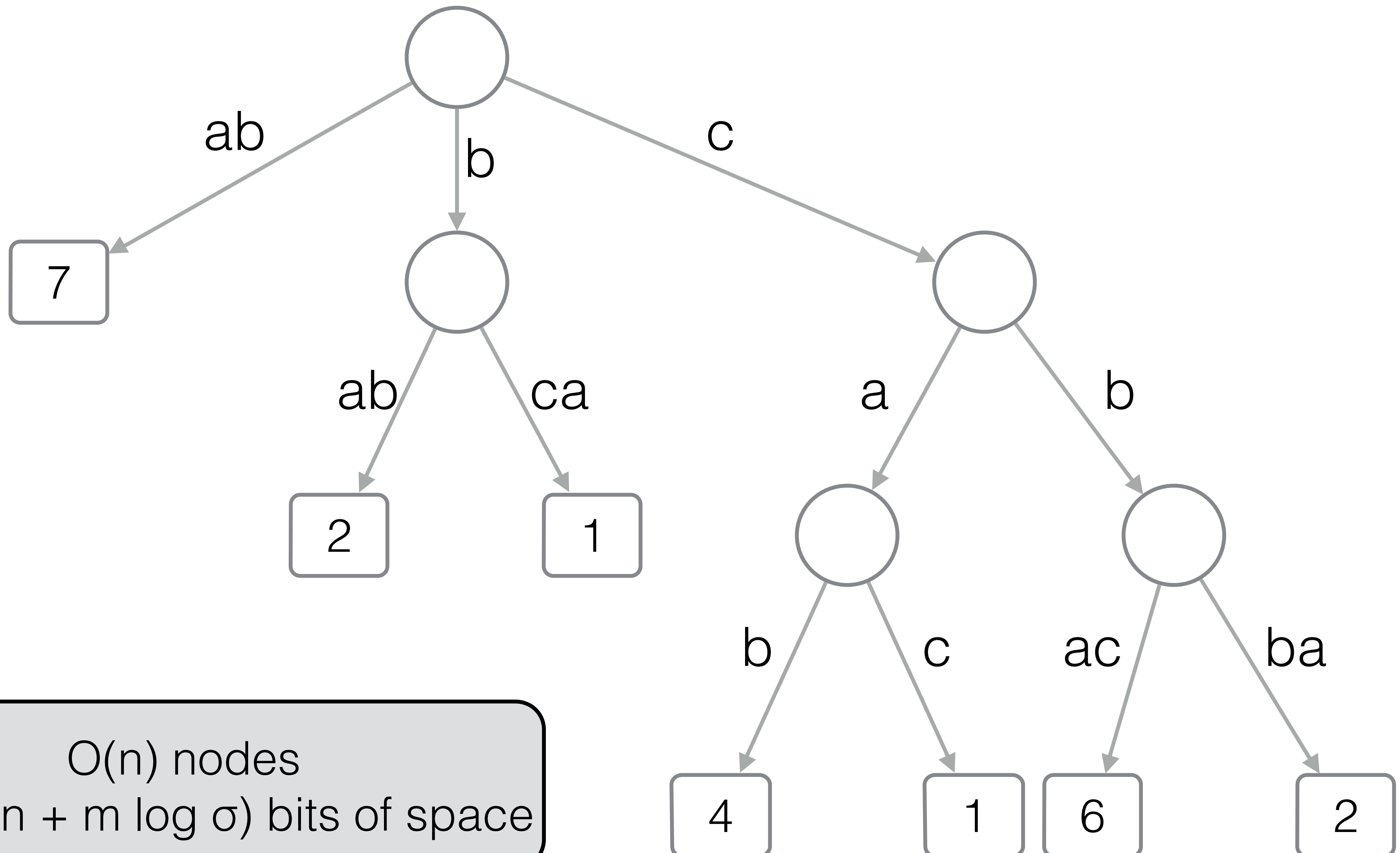
n = |D|, m total length of strings in D

Trie

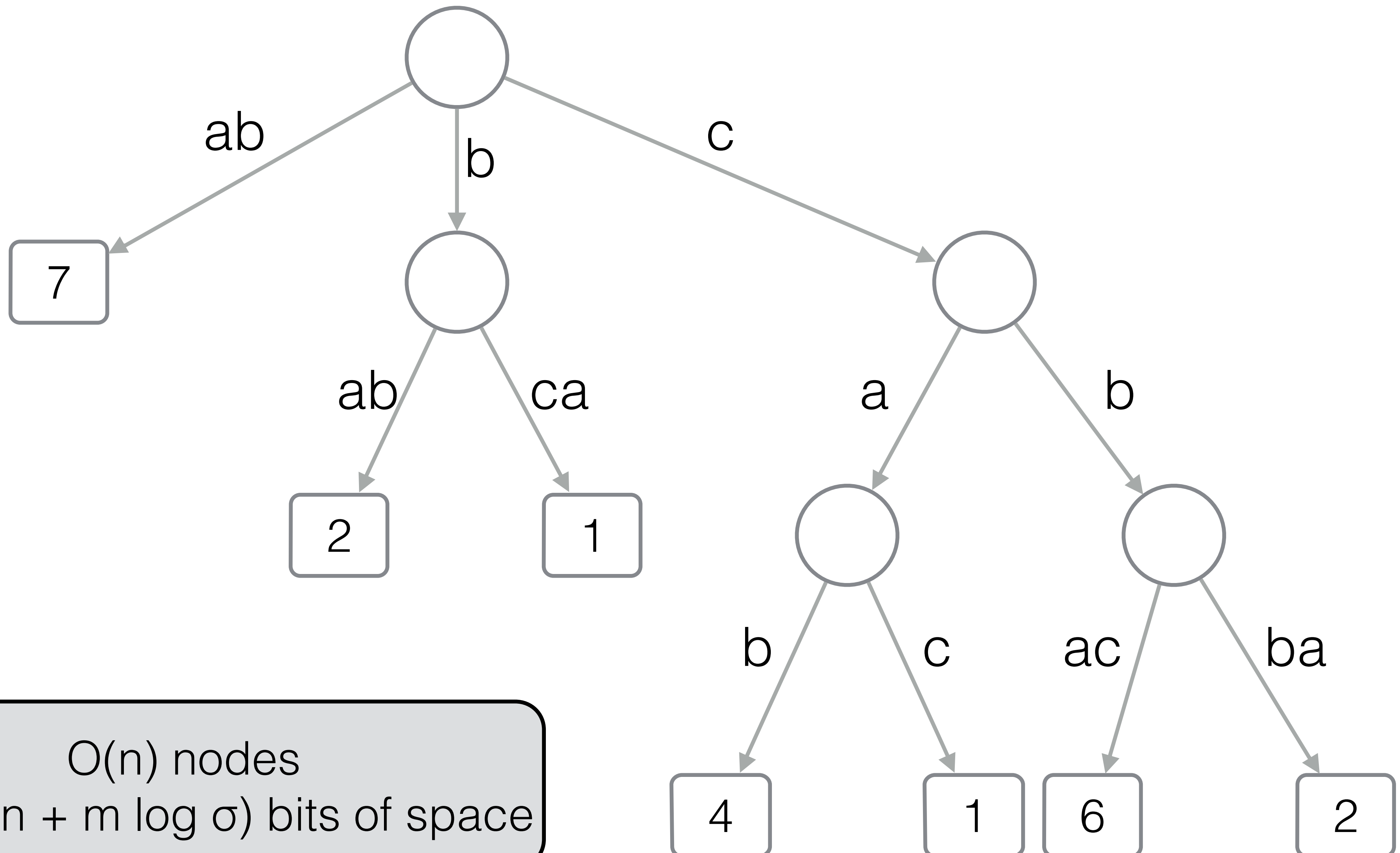


O(n) nodes
O(n log n + m log σ) bits of space

D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Trie



O(n) nodes
O(n log n + m log σ) bits of space

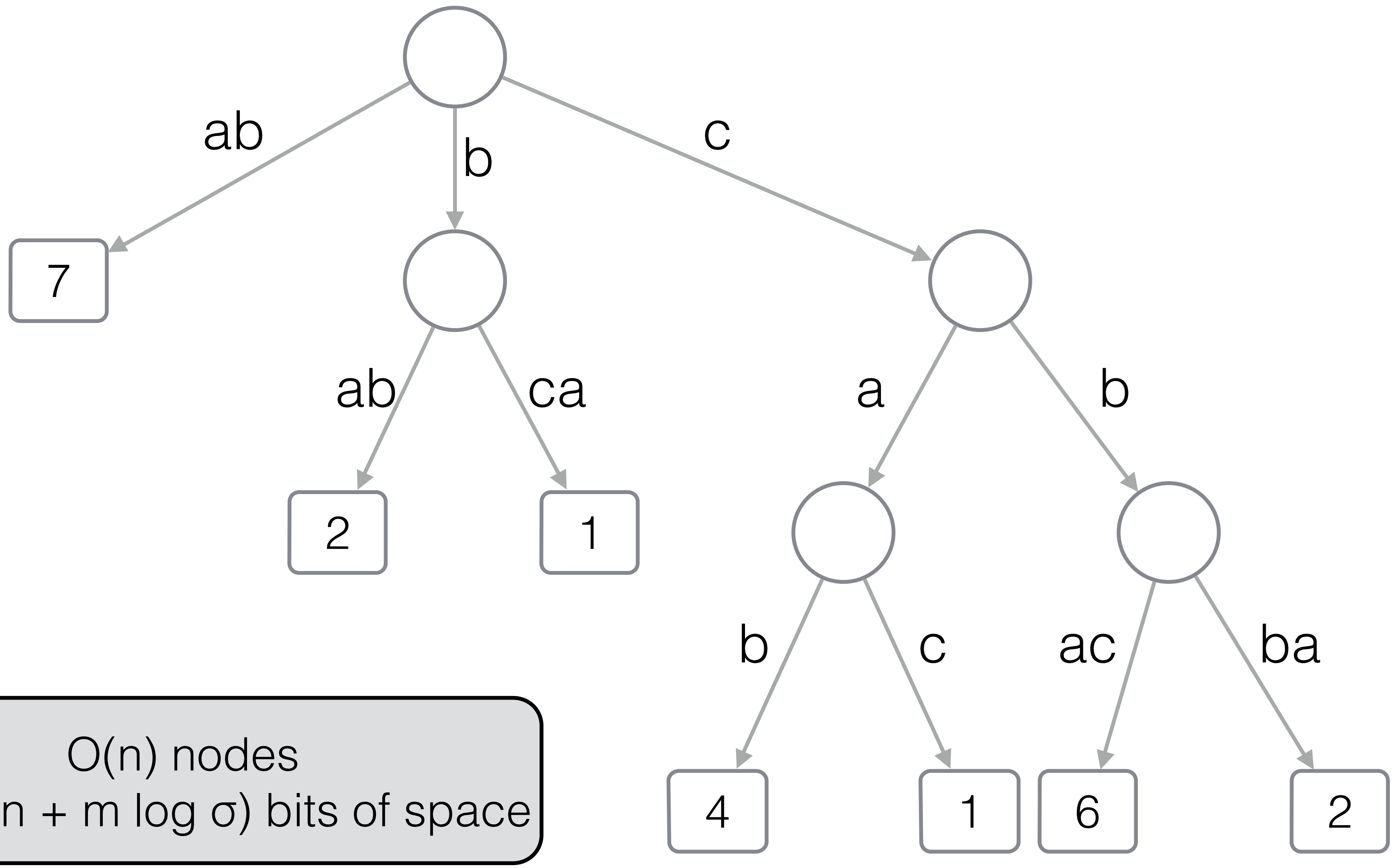
Find all the strings prefixed by any pattern P in O(|P|) time

D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Trie

P = c



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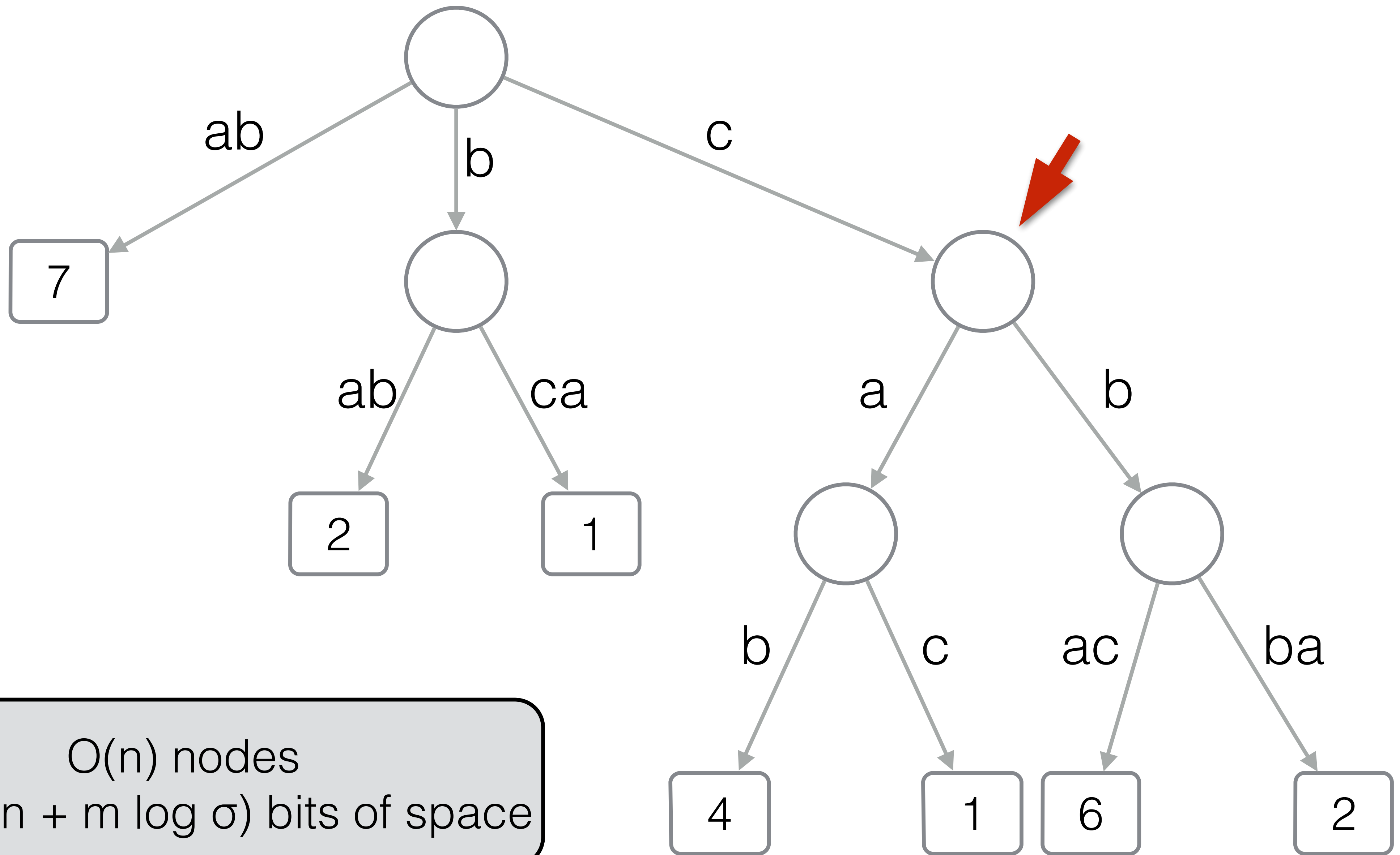

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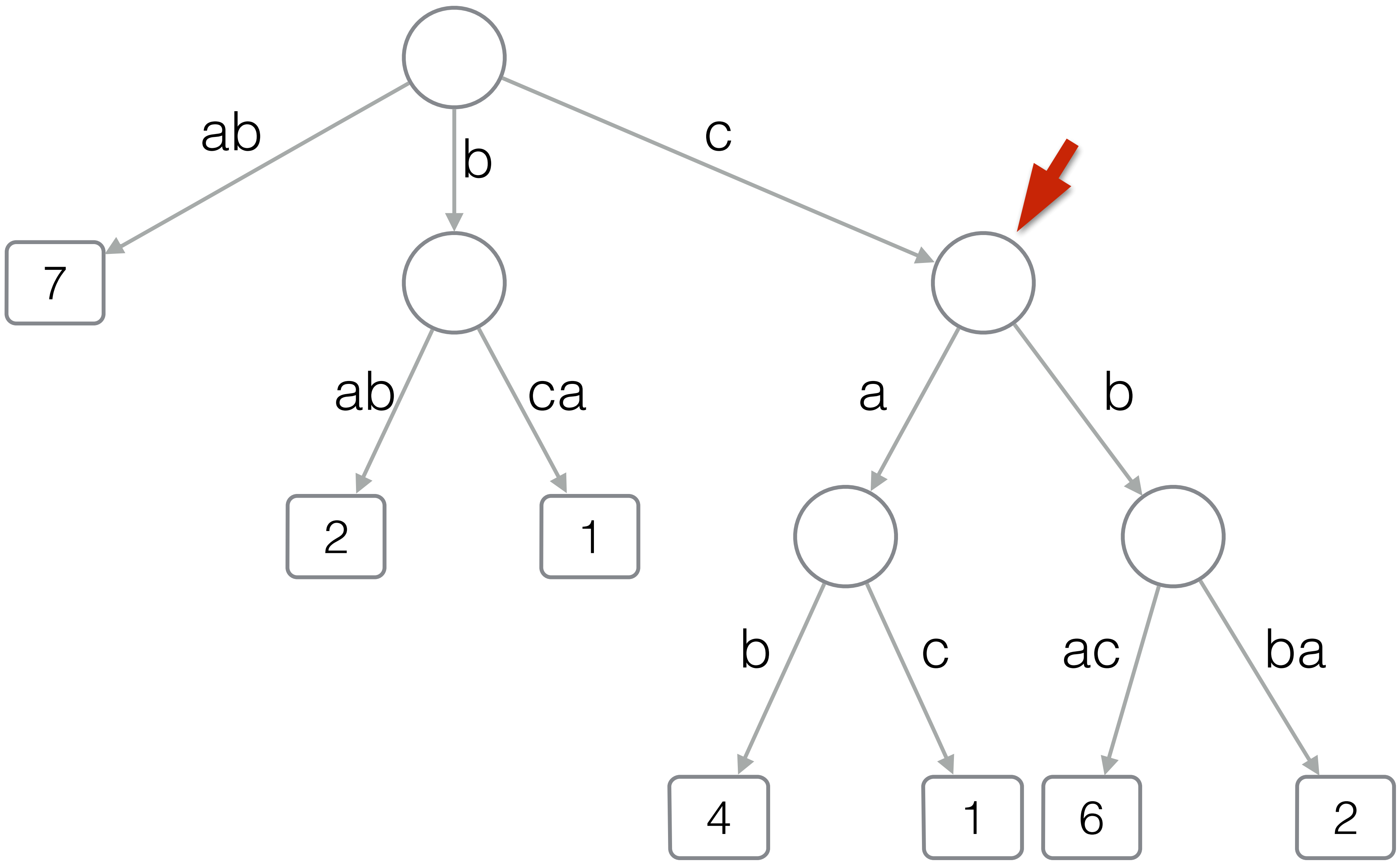
Finding Top-1

P = c



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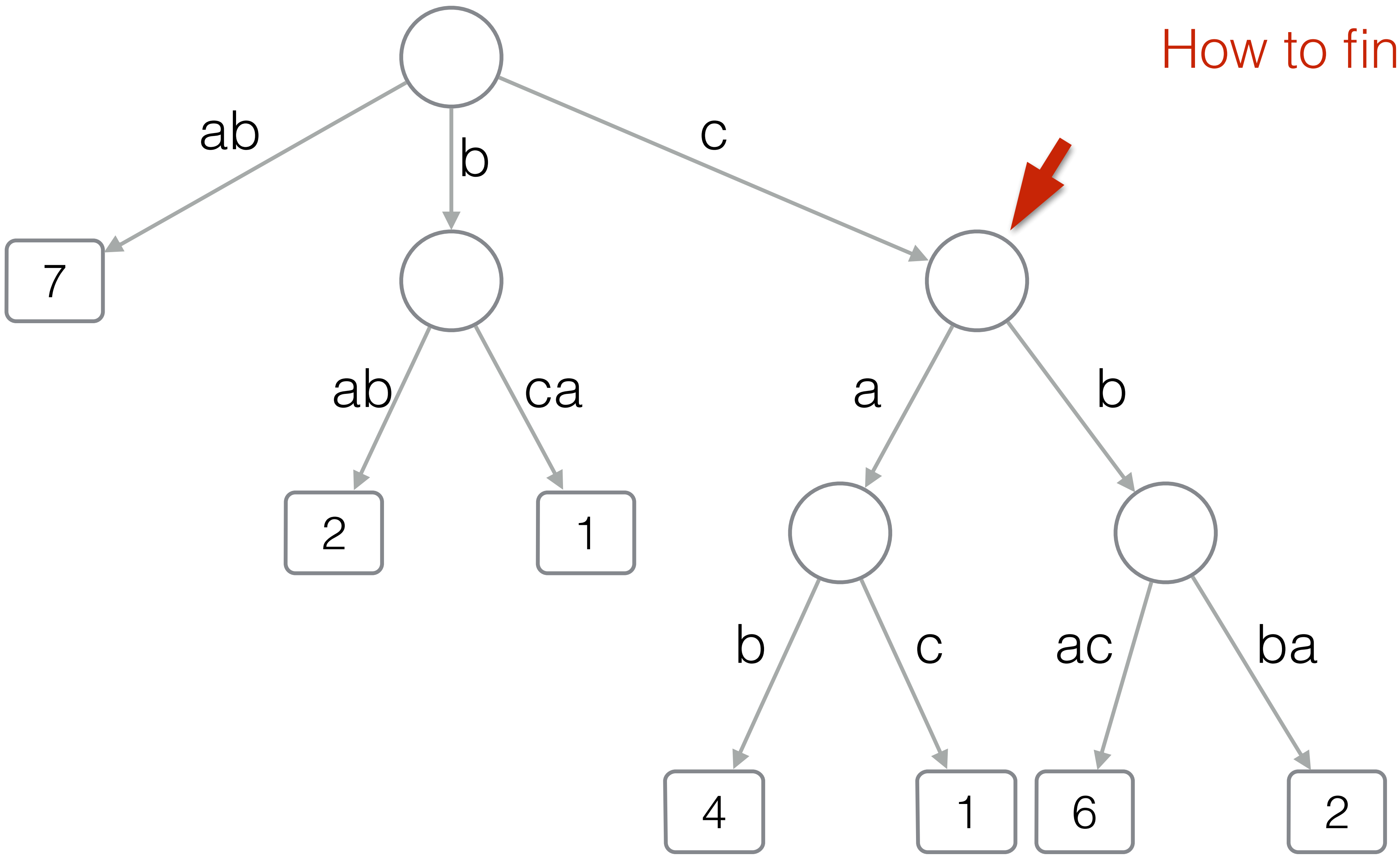
Finding Top-1

P = c

How to find Top-1?



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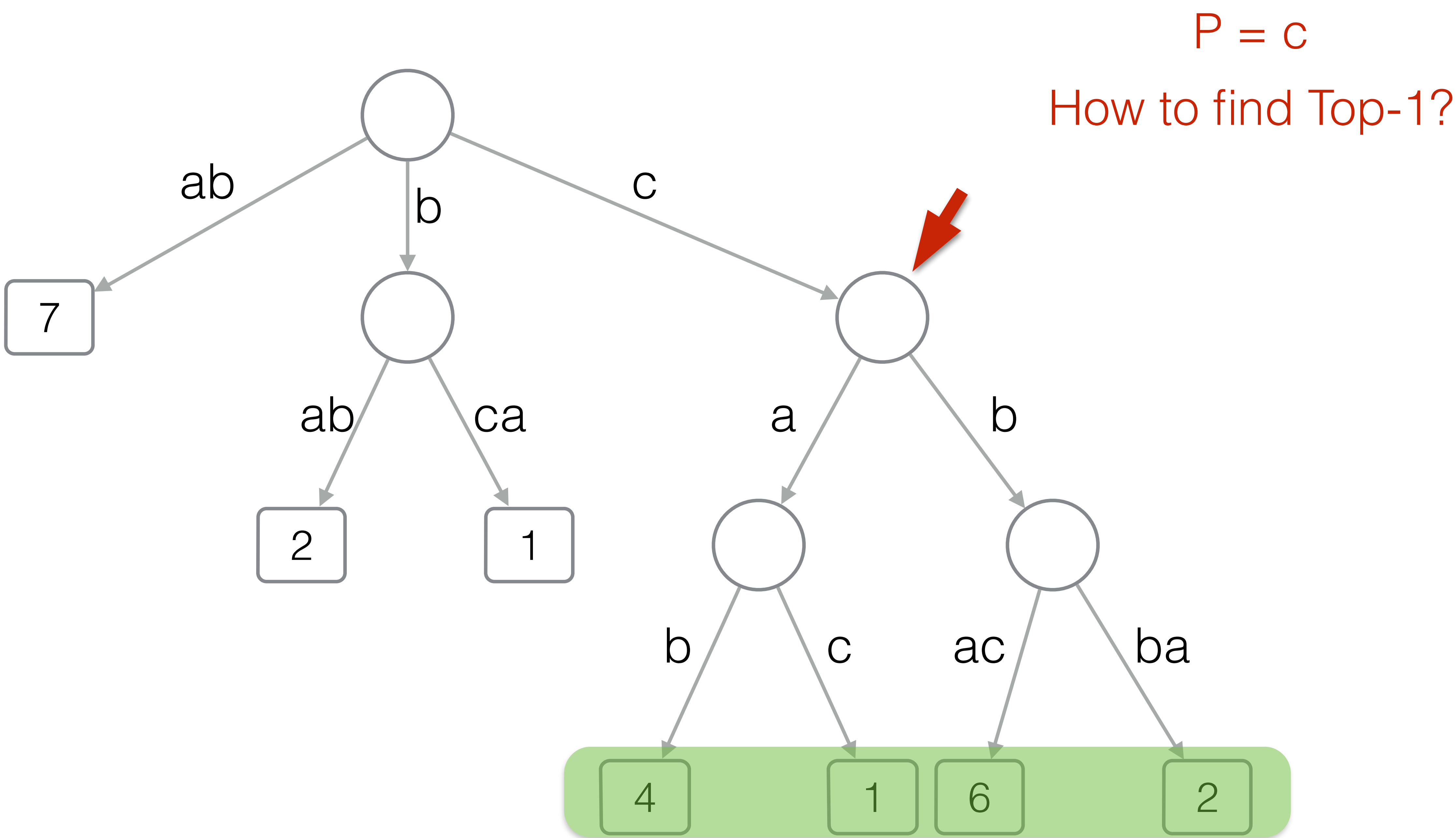
Finding Top-1



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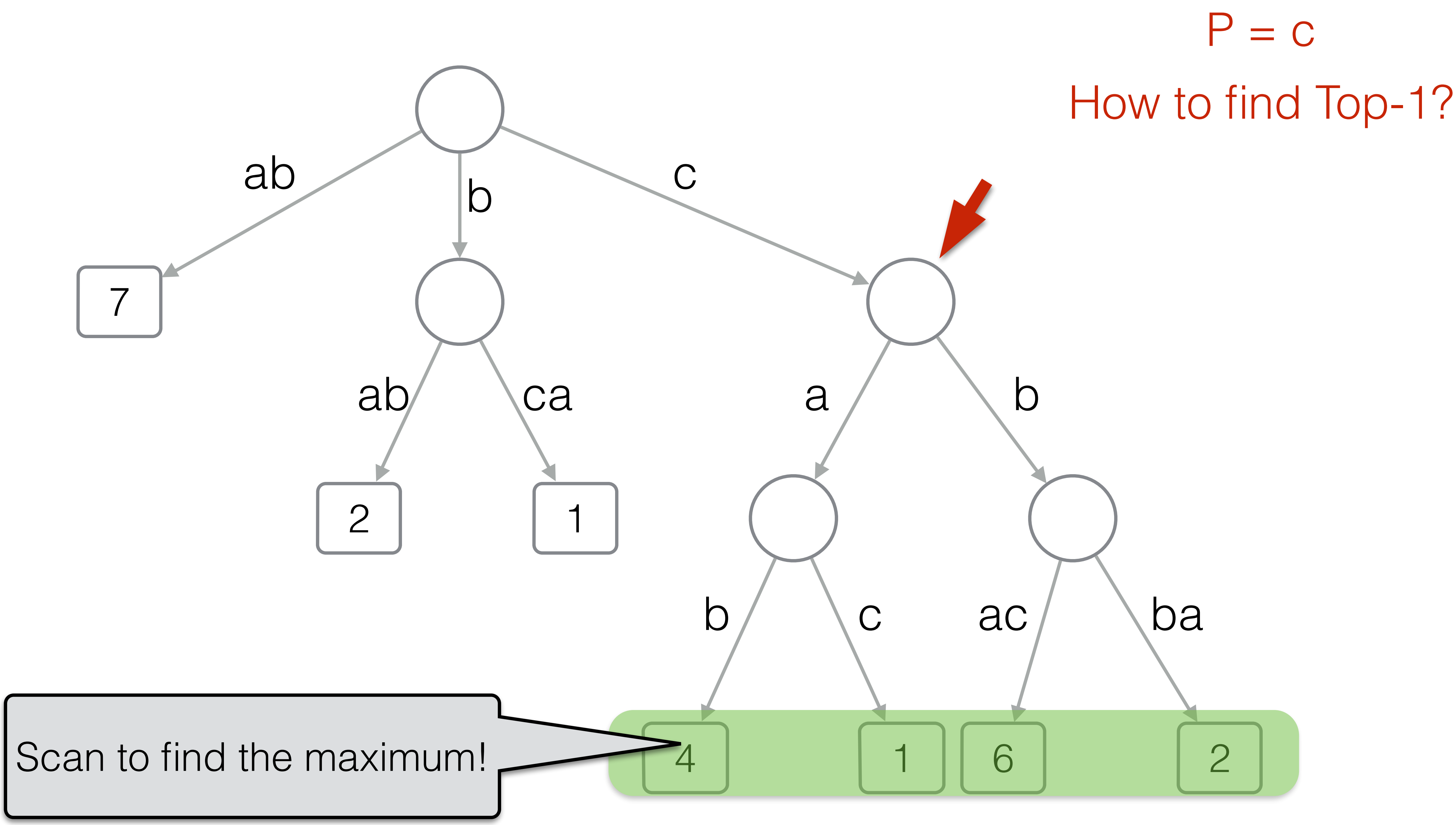
Finding Top-1



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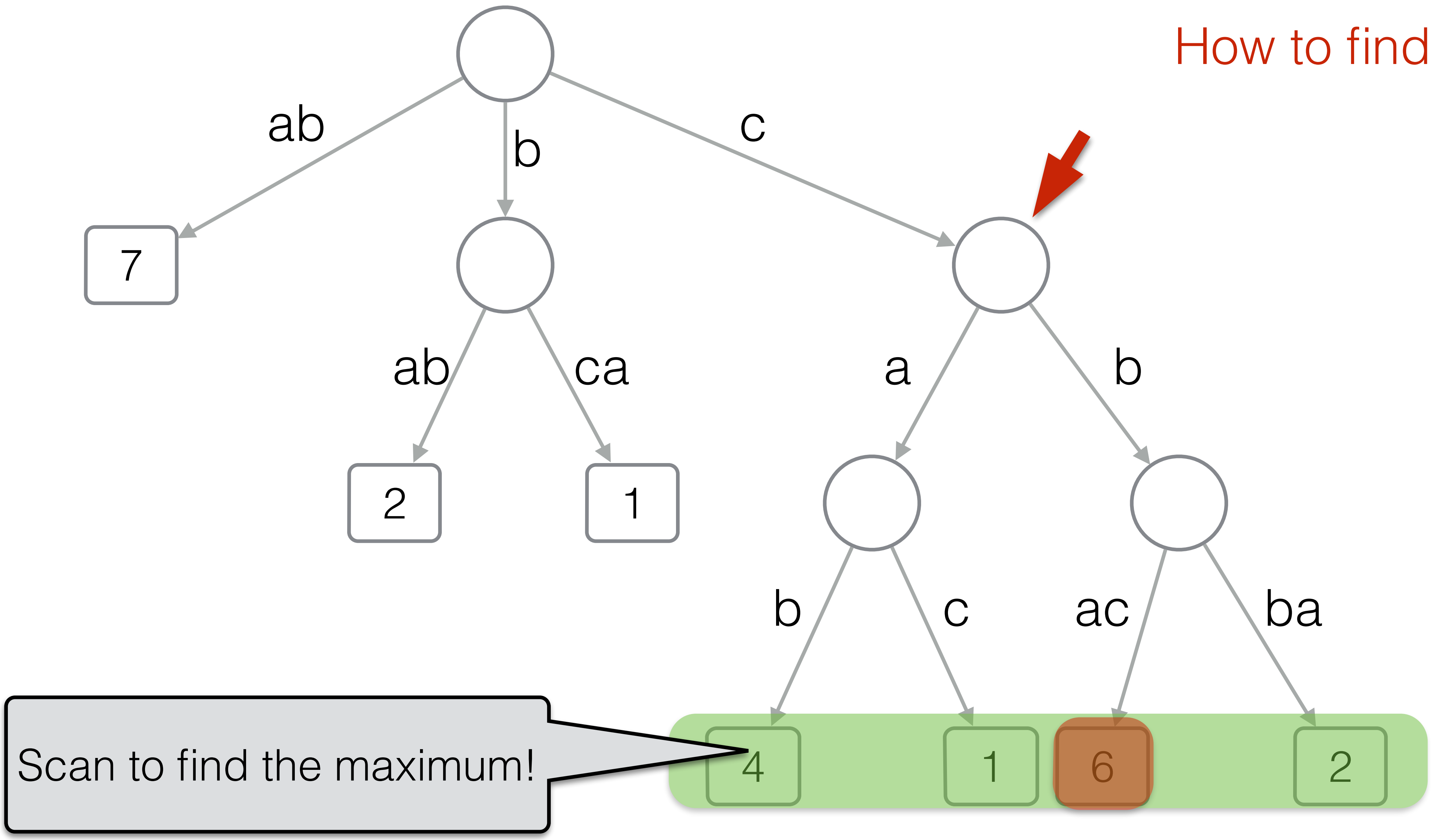

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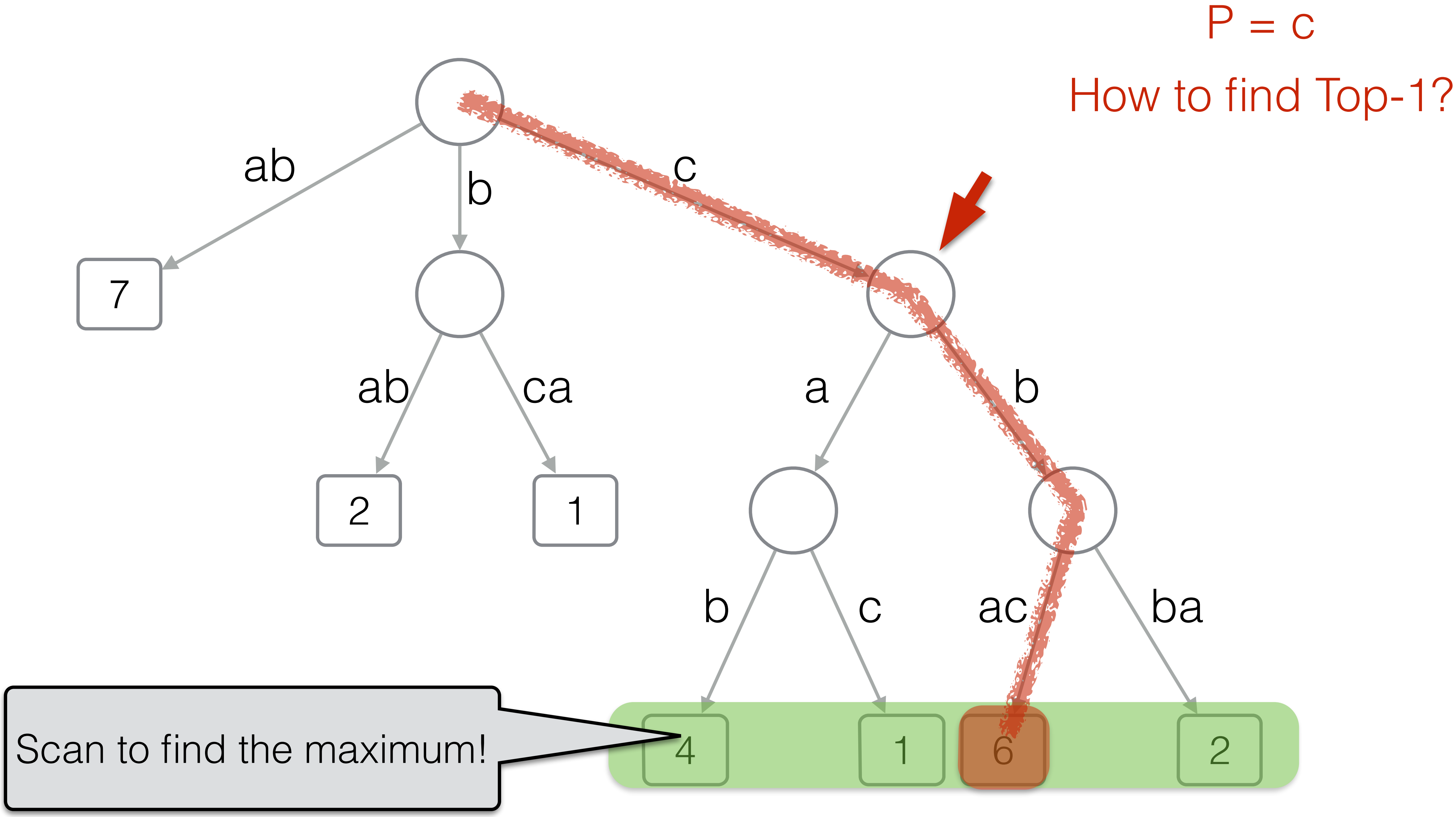

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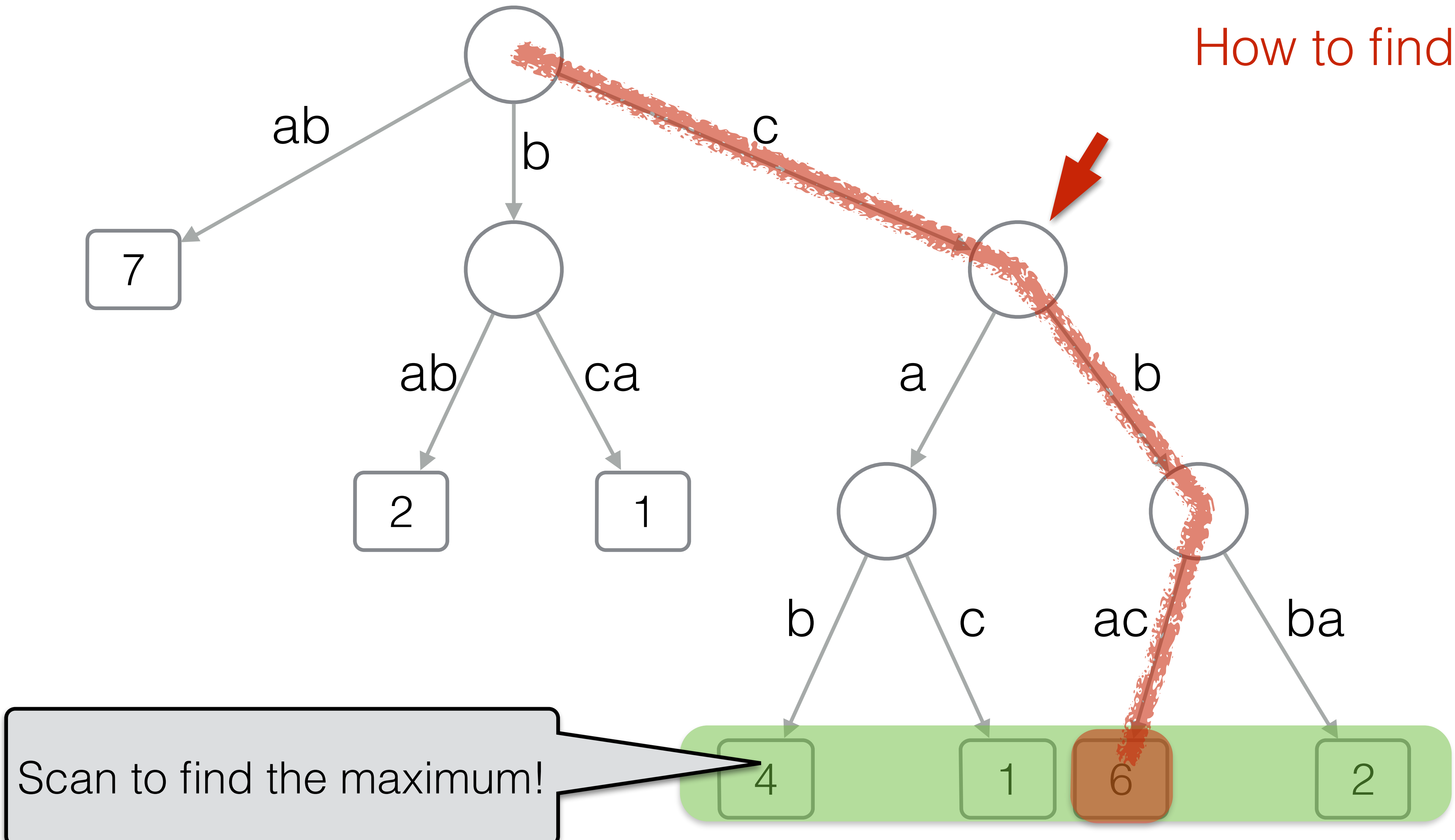

O(n) query time :-(

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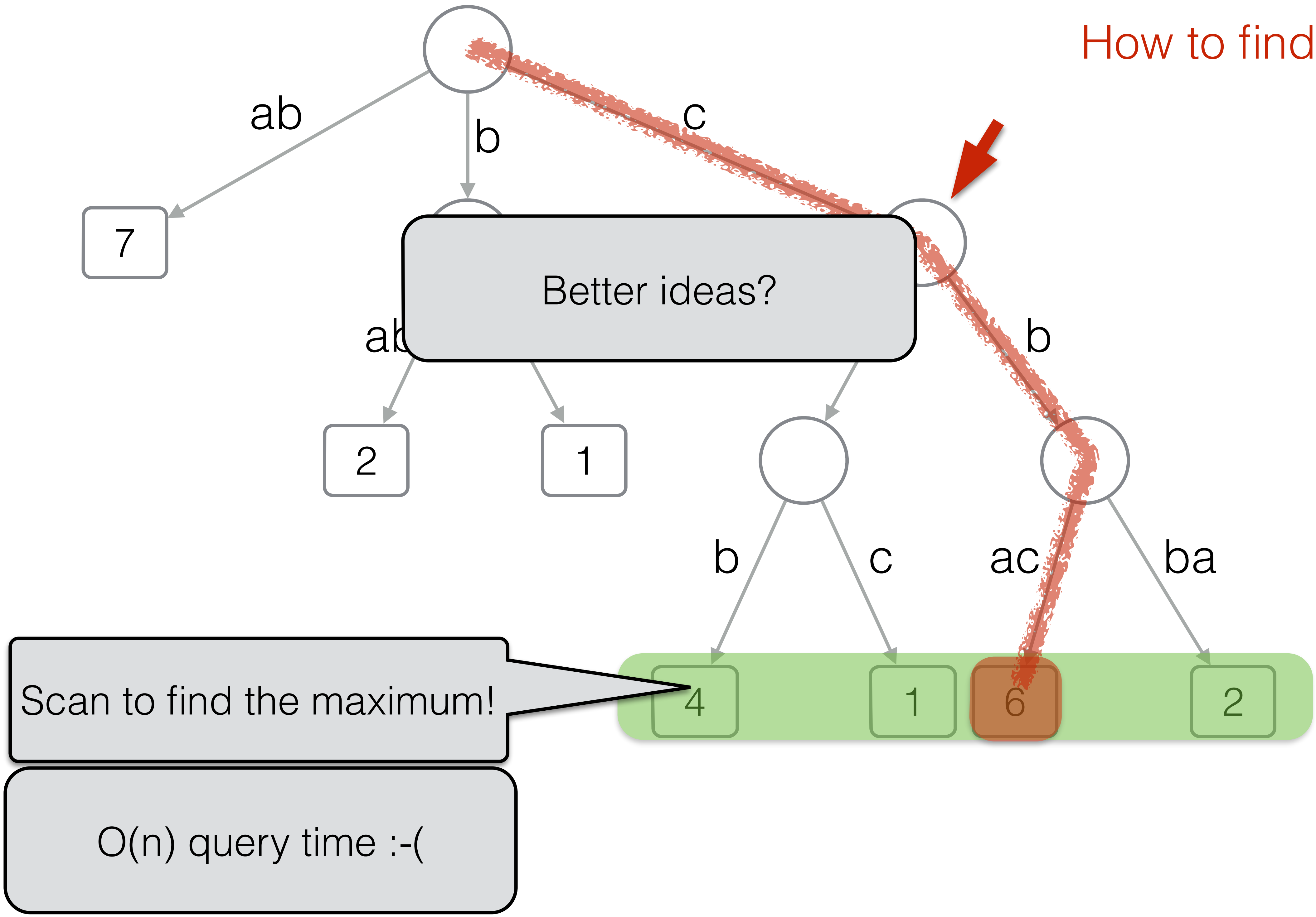

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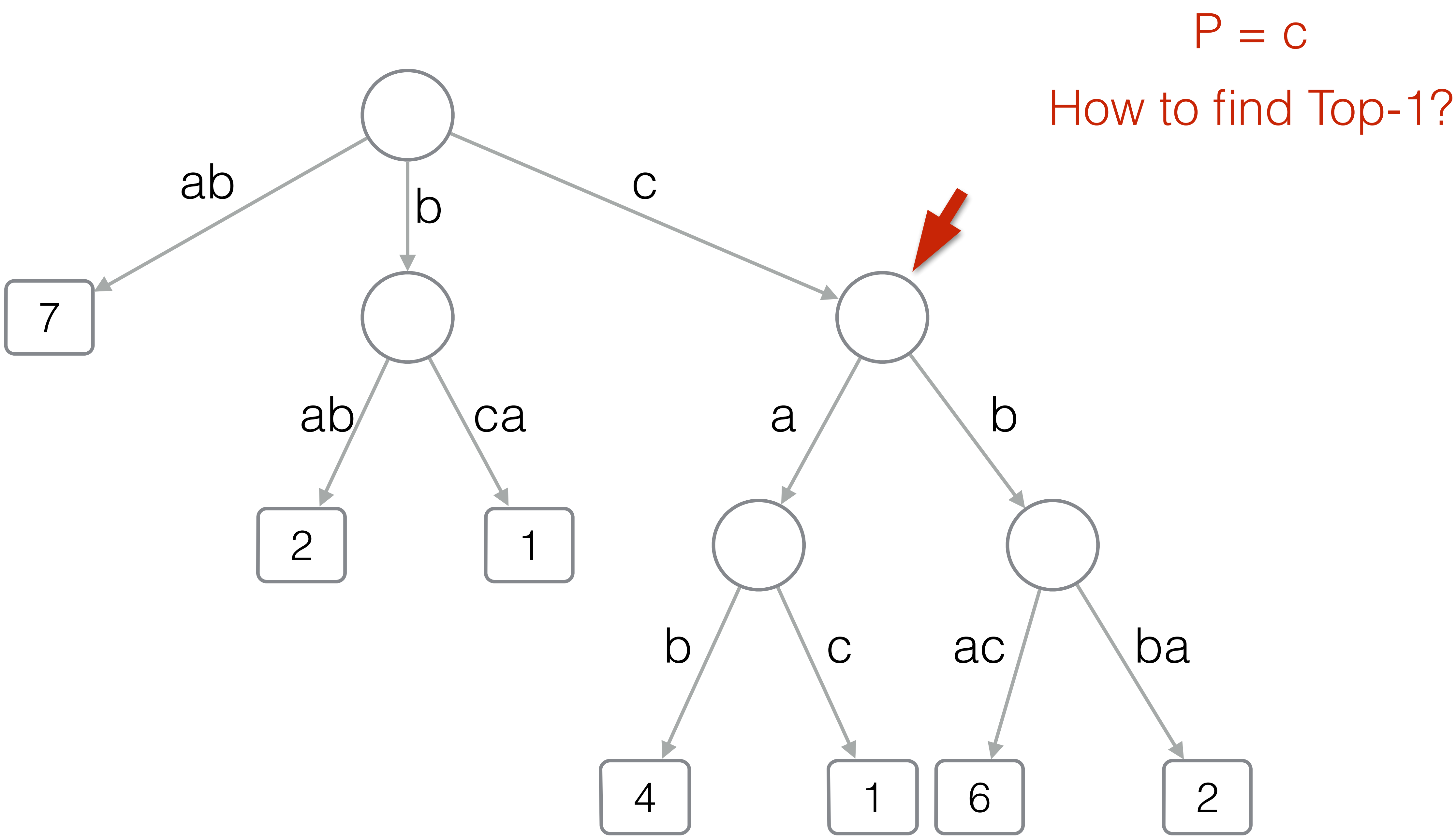

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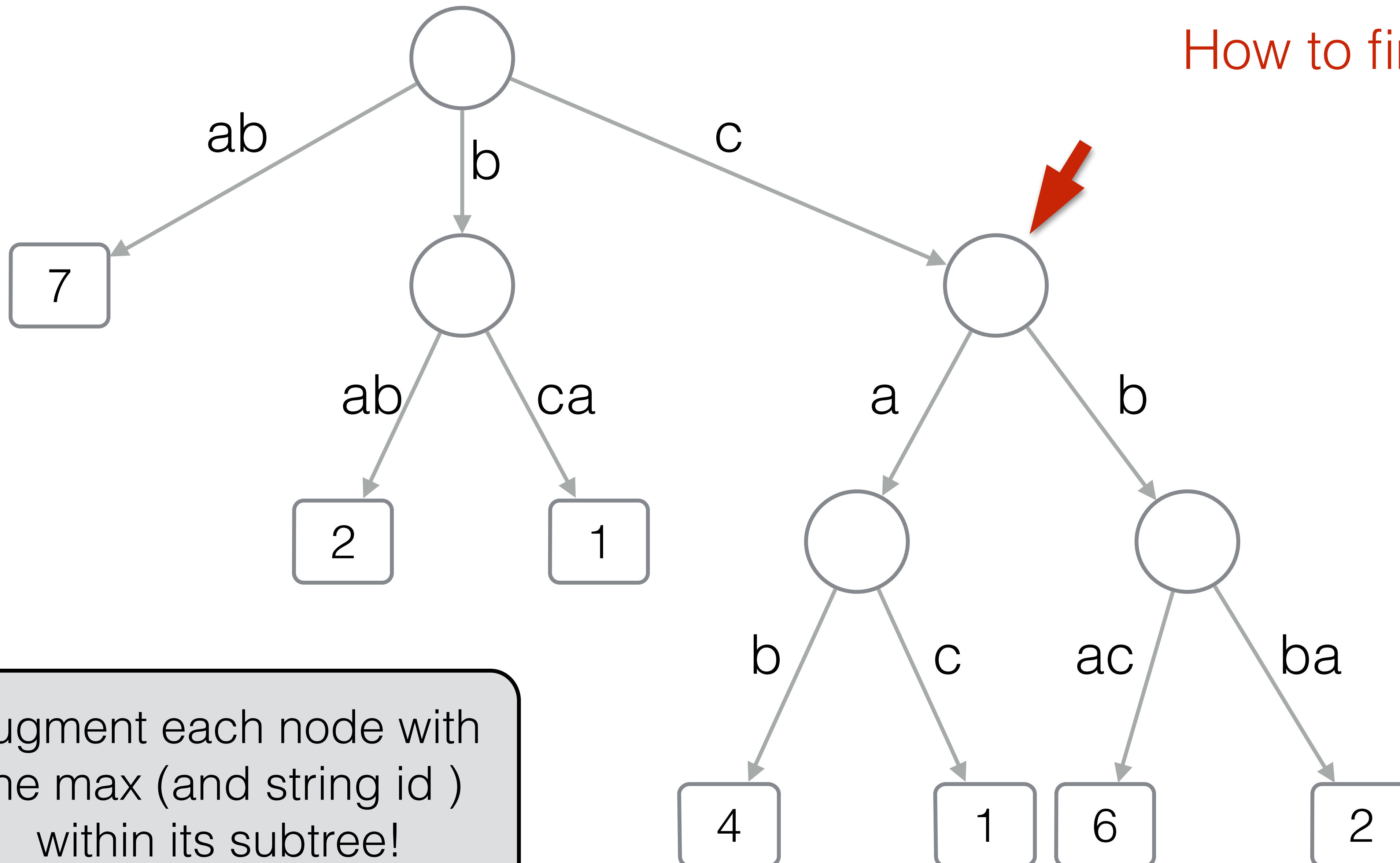

Augment each node with the max (and string id) within its subtree!

D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

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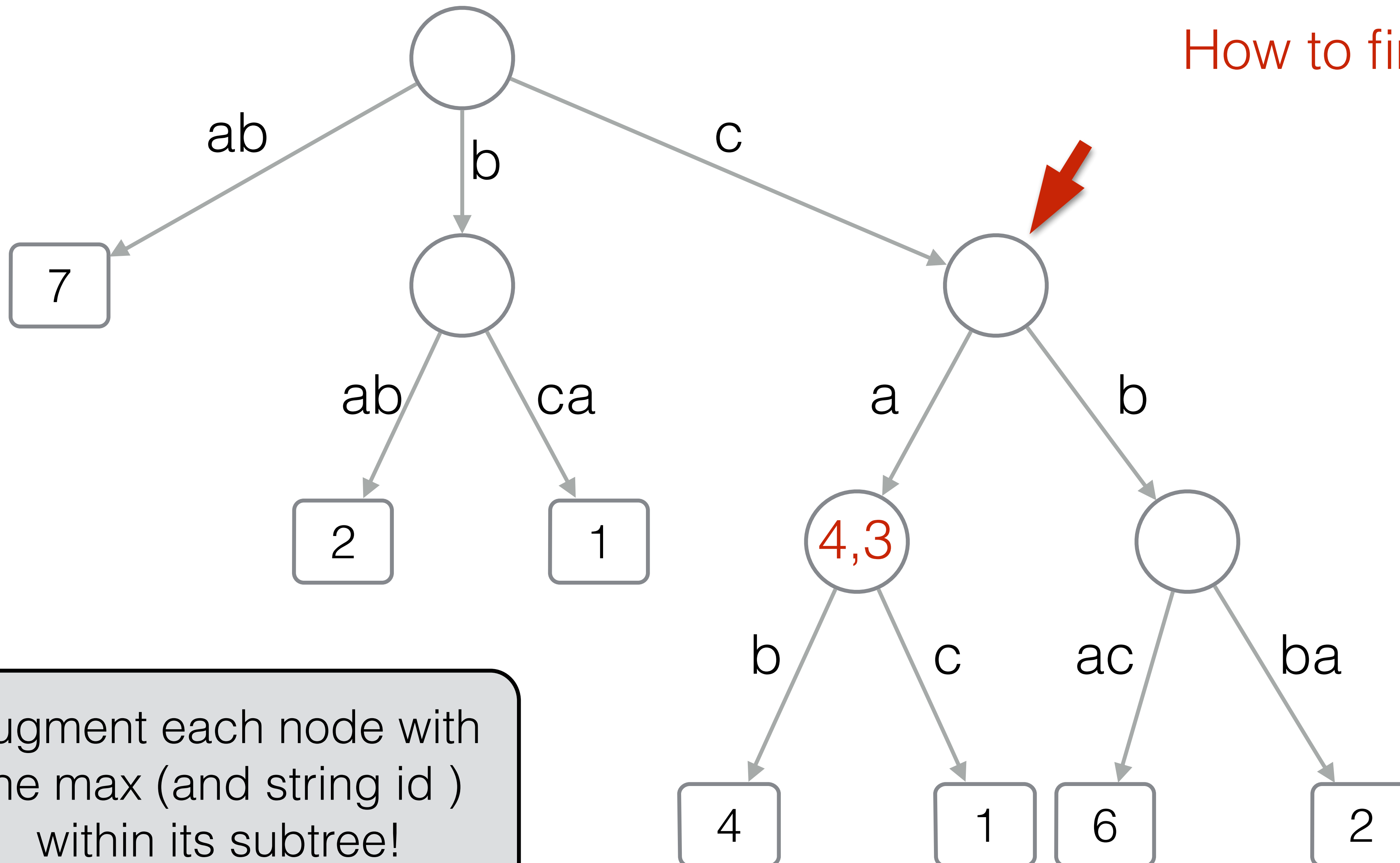

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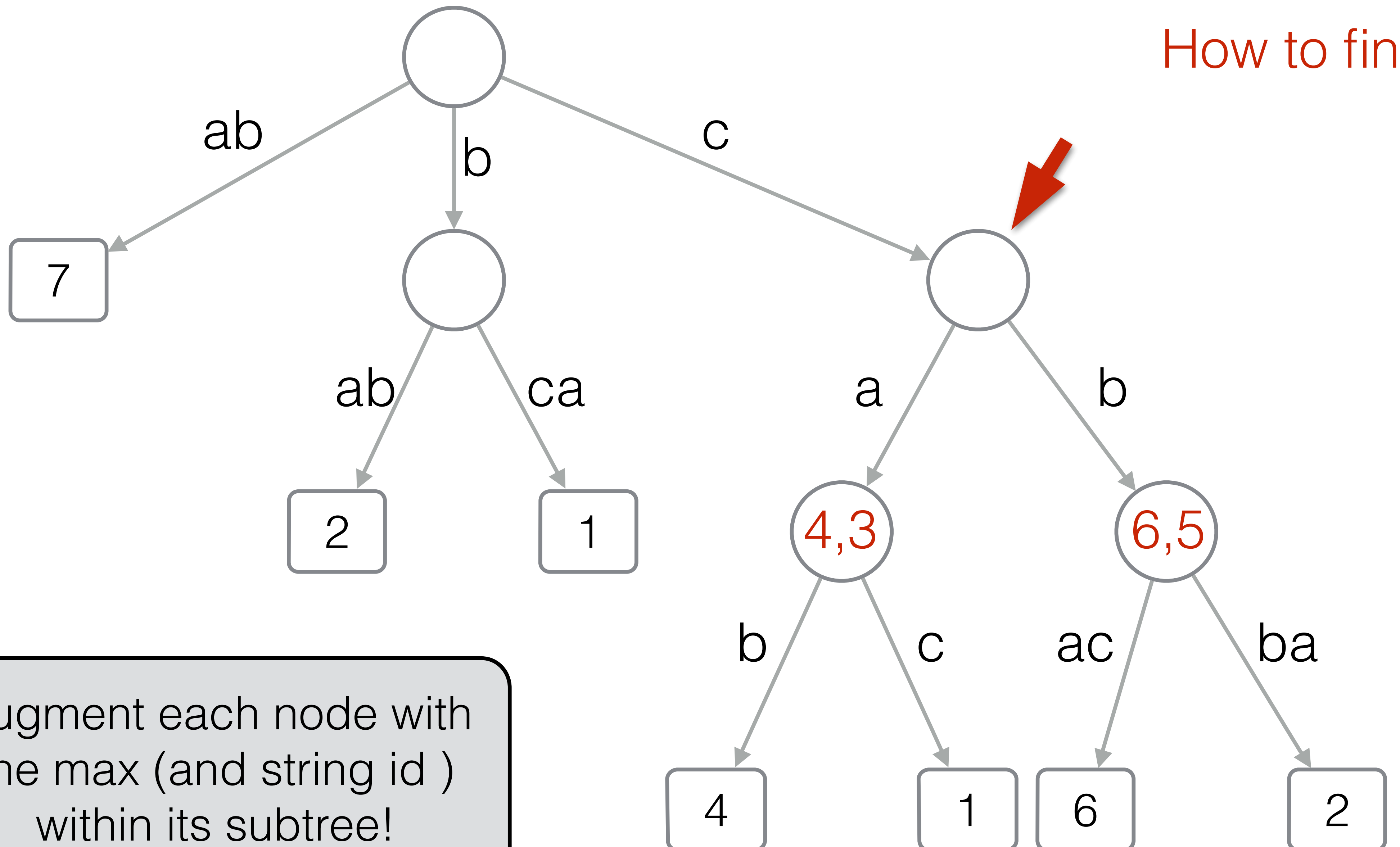

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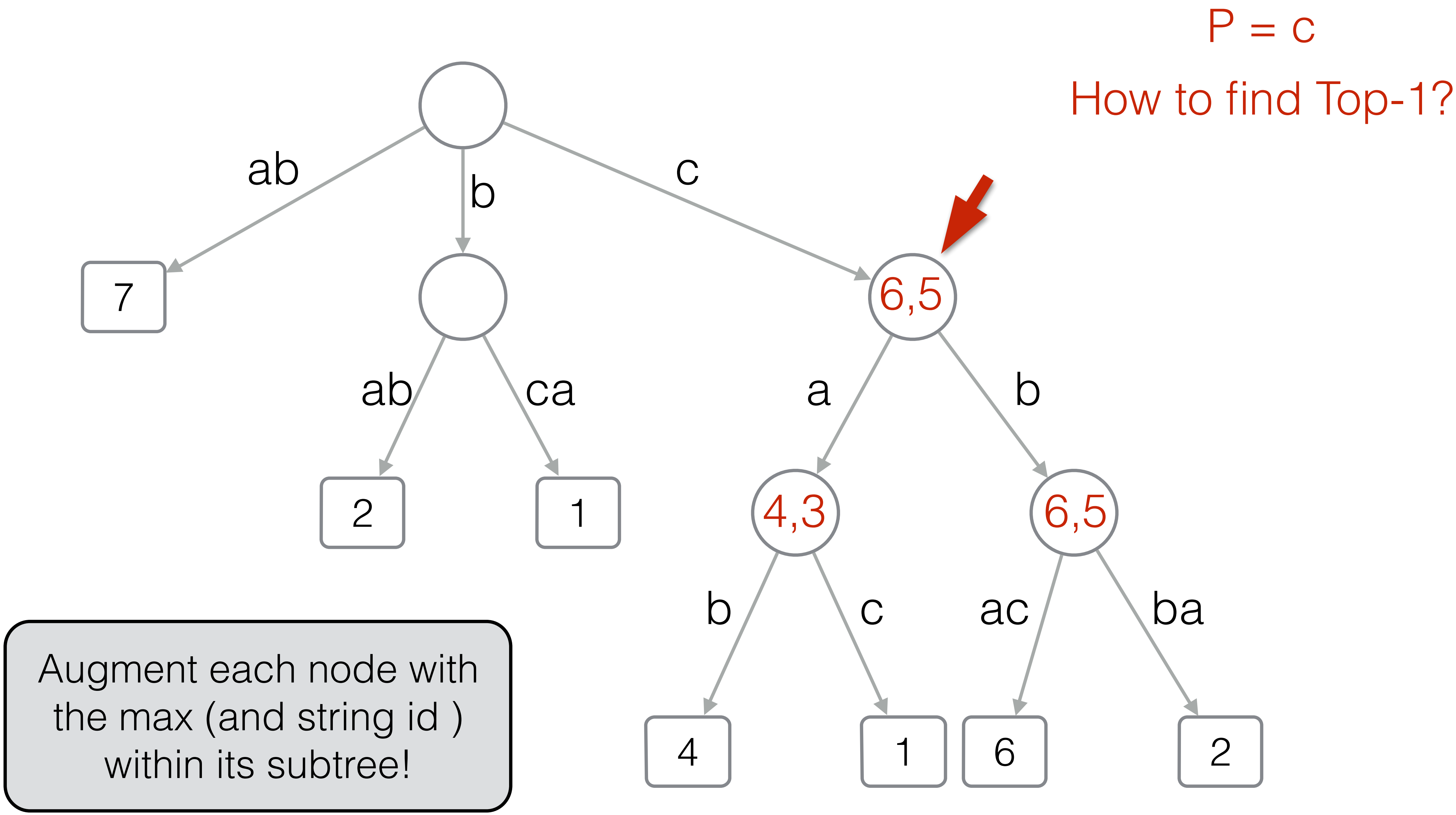

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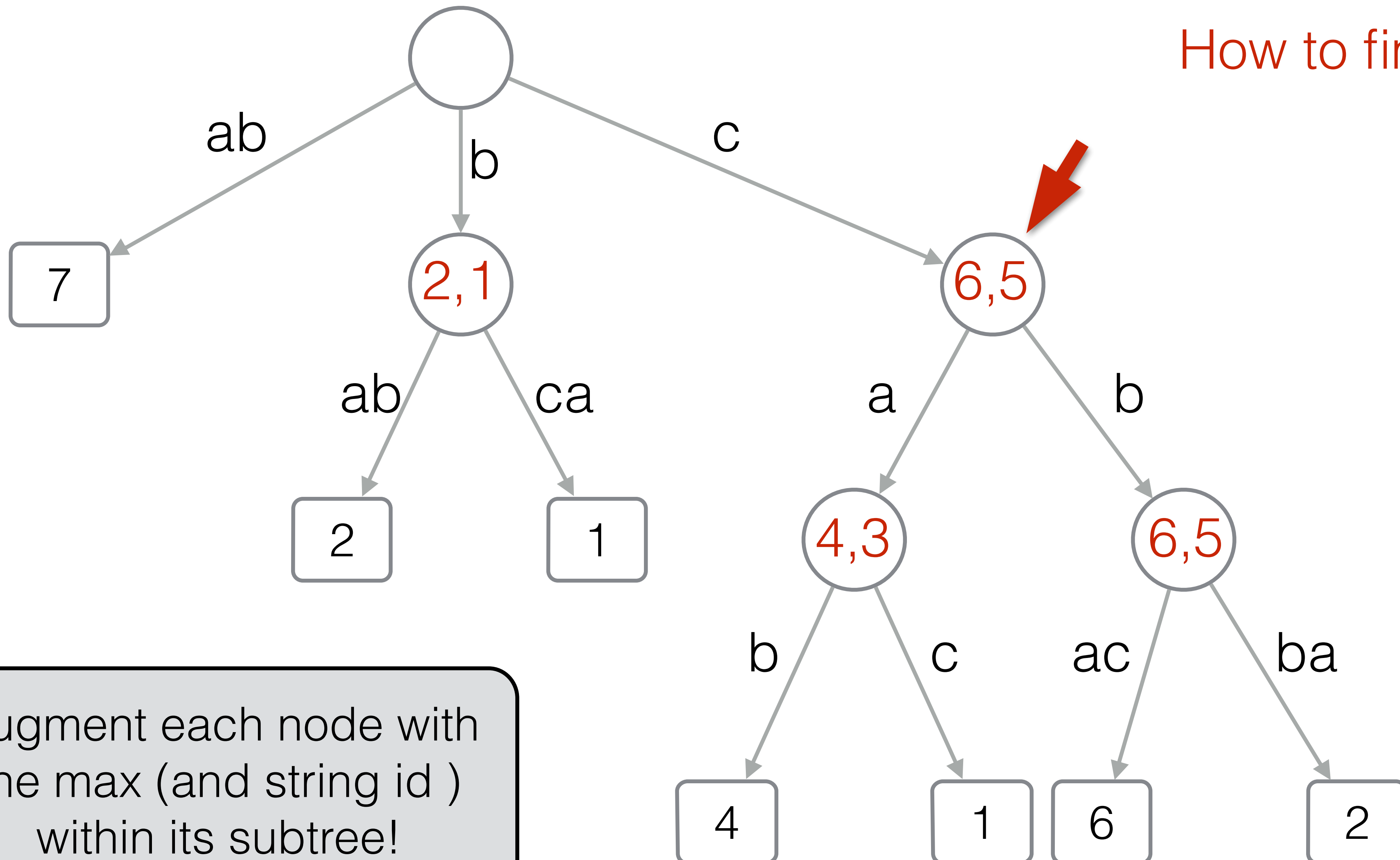

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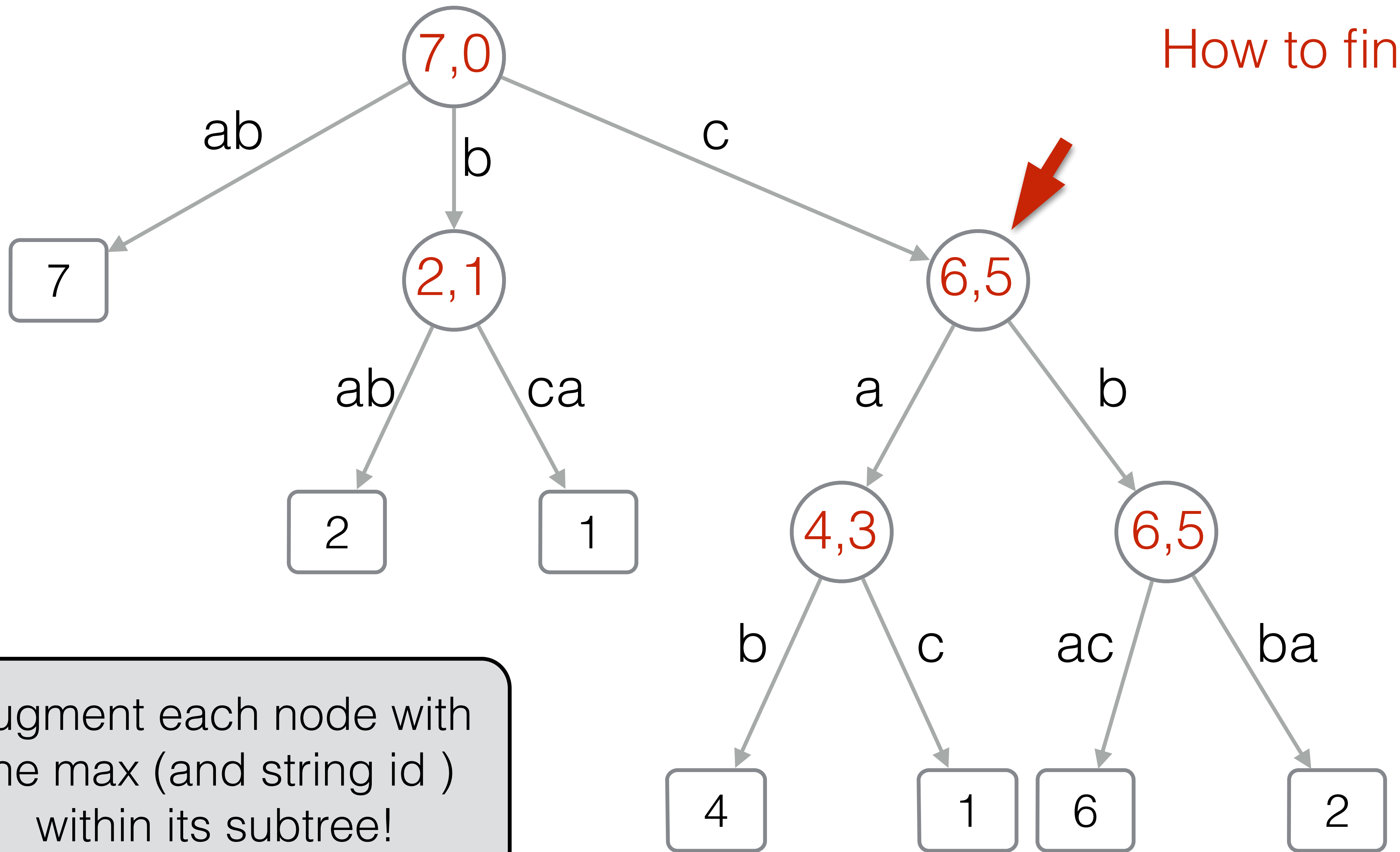

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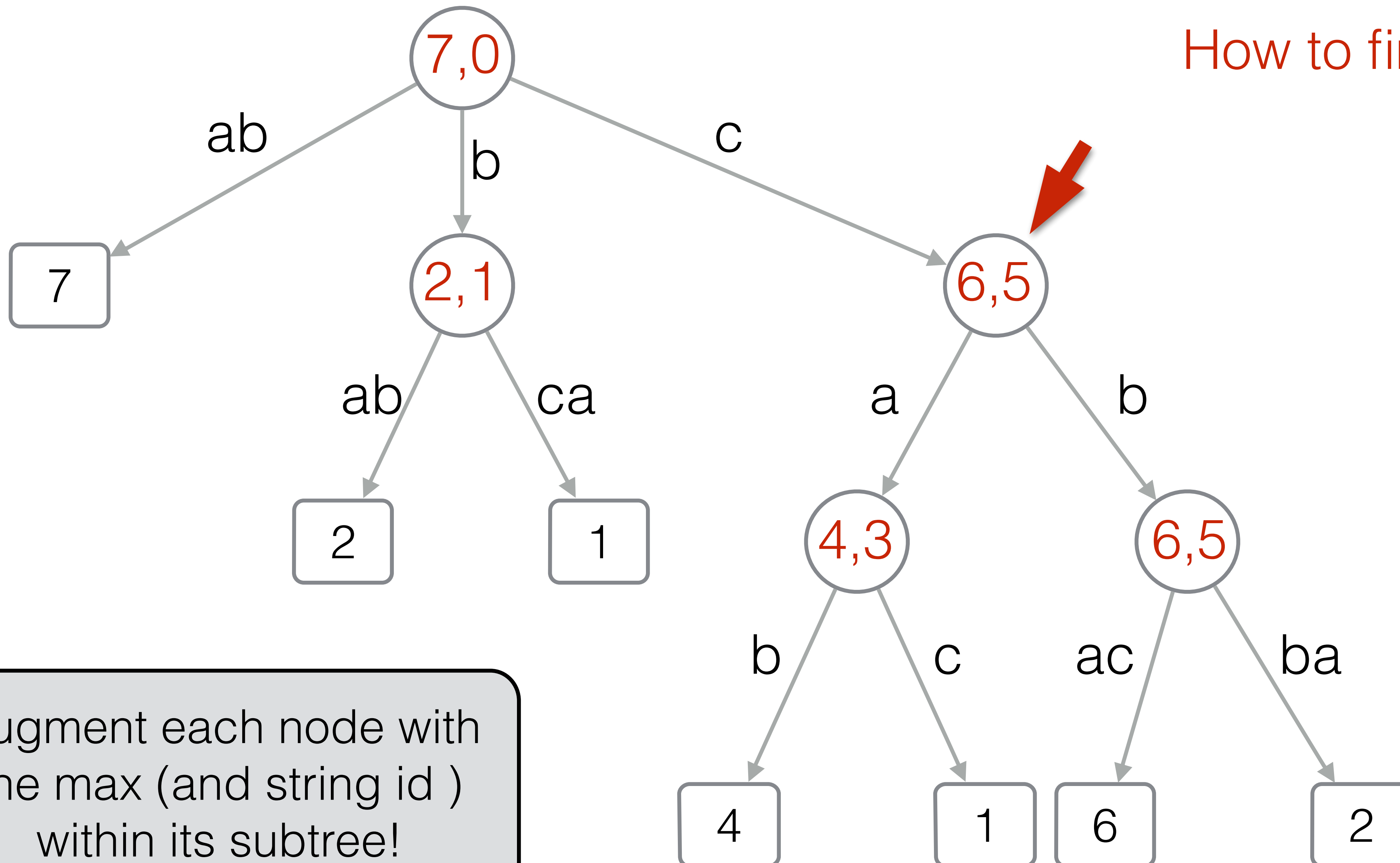


Augment each node with the max (and string id) within its subtree!

Preprocessing time: O(n)

Extra space: O(n log n) bits

Query time: O(1)

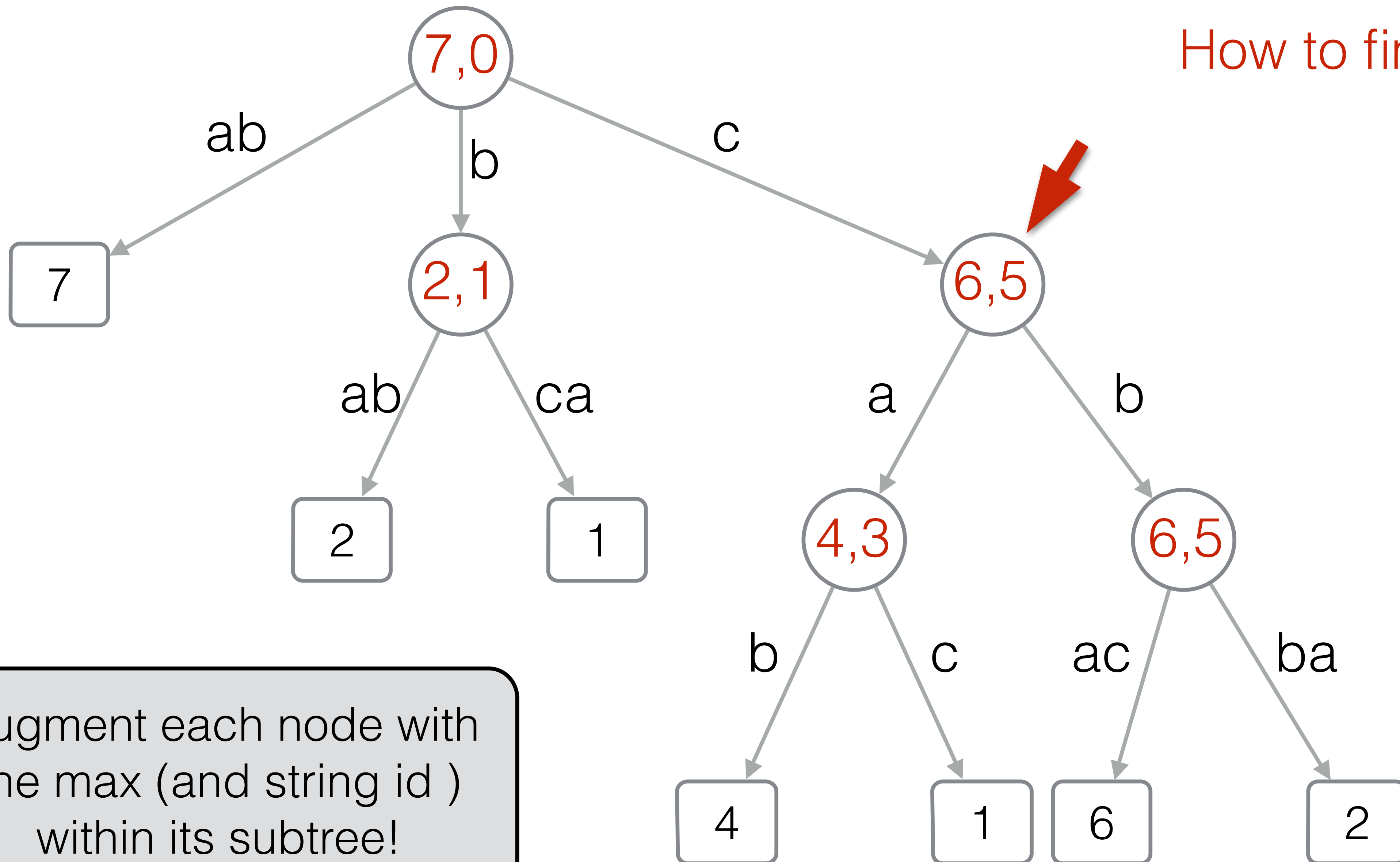
D (1), cab (4), cac (1), cbac (6), cbba (2) }

l length of strings in D

Finding Top-1

P = c

How to find Top-1?



Augment each node with the max (and string id) within its subtree!

Preprocessing time: O(n)

Extra space: O(n log n) bits

Query time: O(1)

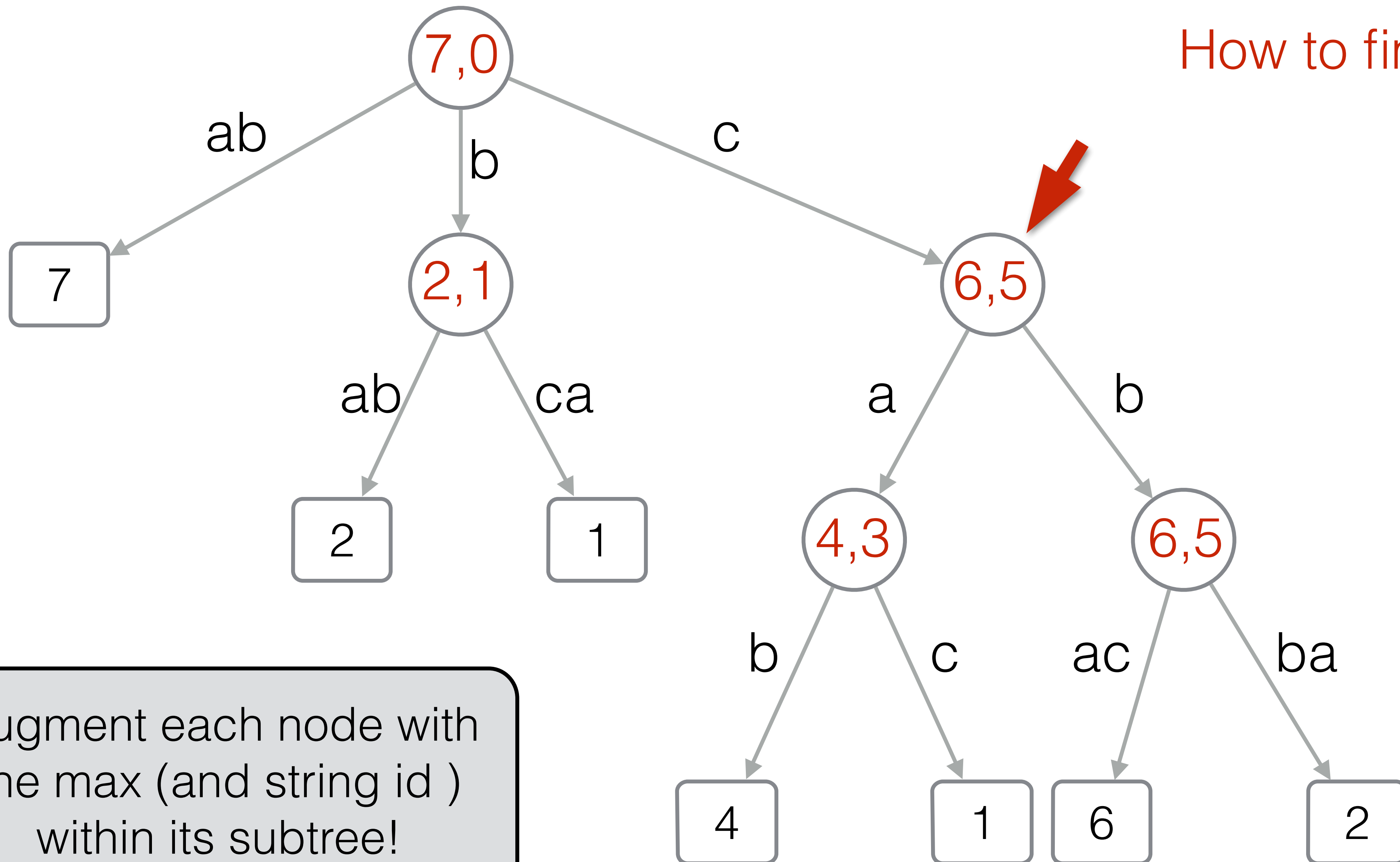
D … (1), ca … l length of strings in D

Solving Top-k?

Finding Top-1

P = c

How to find Top-1?



Augment each node with the max (and string id) within its subtree!

Preprocessing time: O(n)

Extra space: O(n log n) bits

Query time: O(1)

Solving Top-k?

- Extra space: O(k*n*log n) bits :-(
- You must know k at building time! :-(

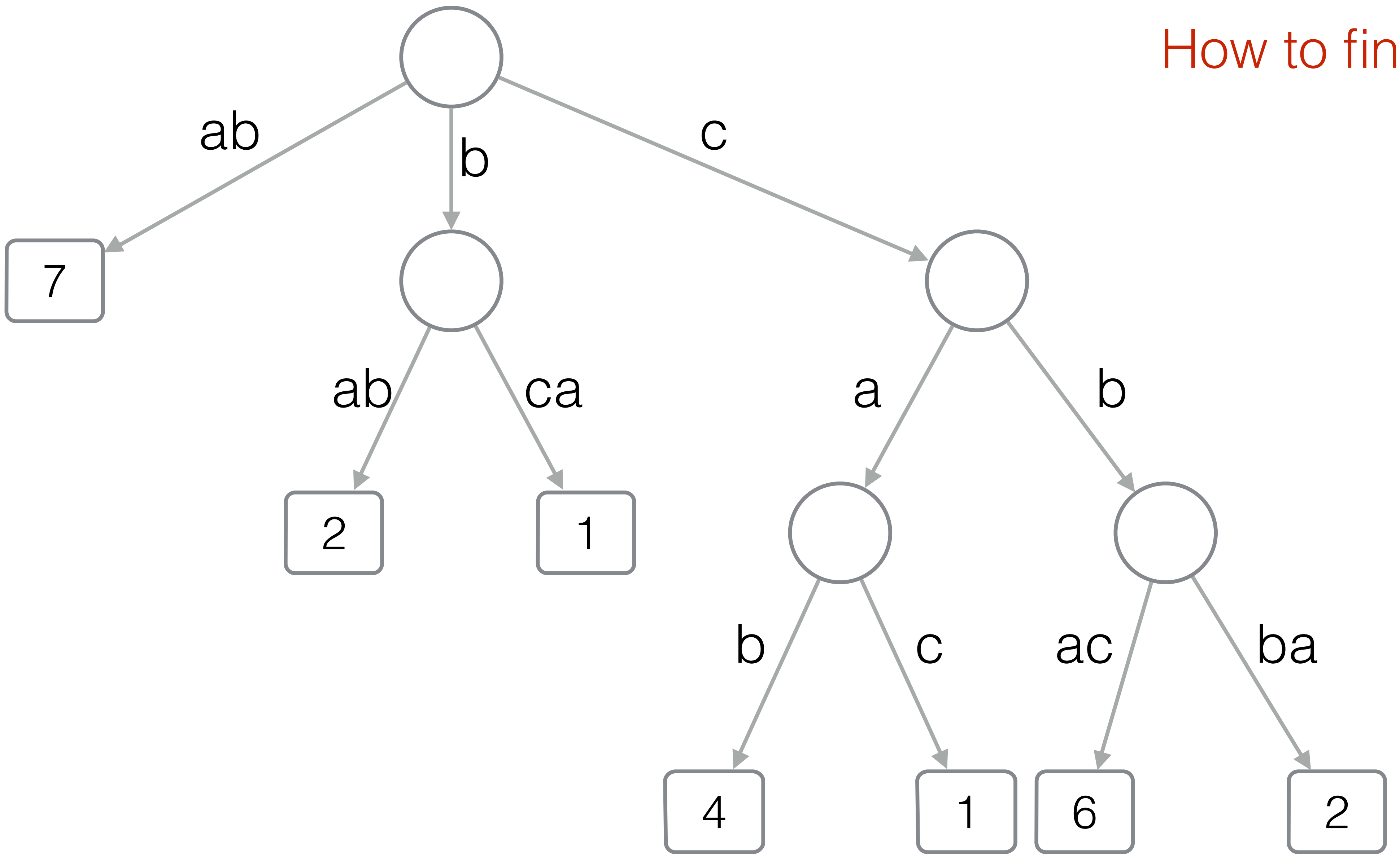
D (1), ca

l length of strings in D

Finding Top-1

P = c

How to find Top-1?



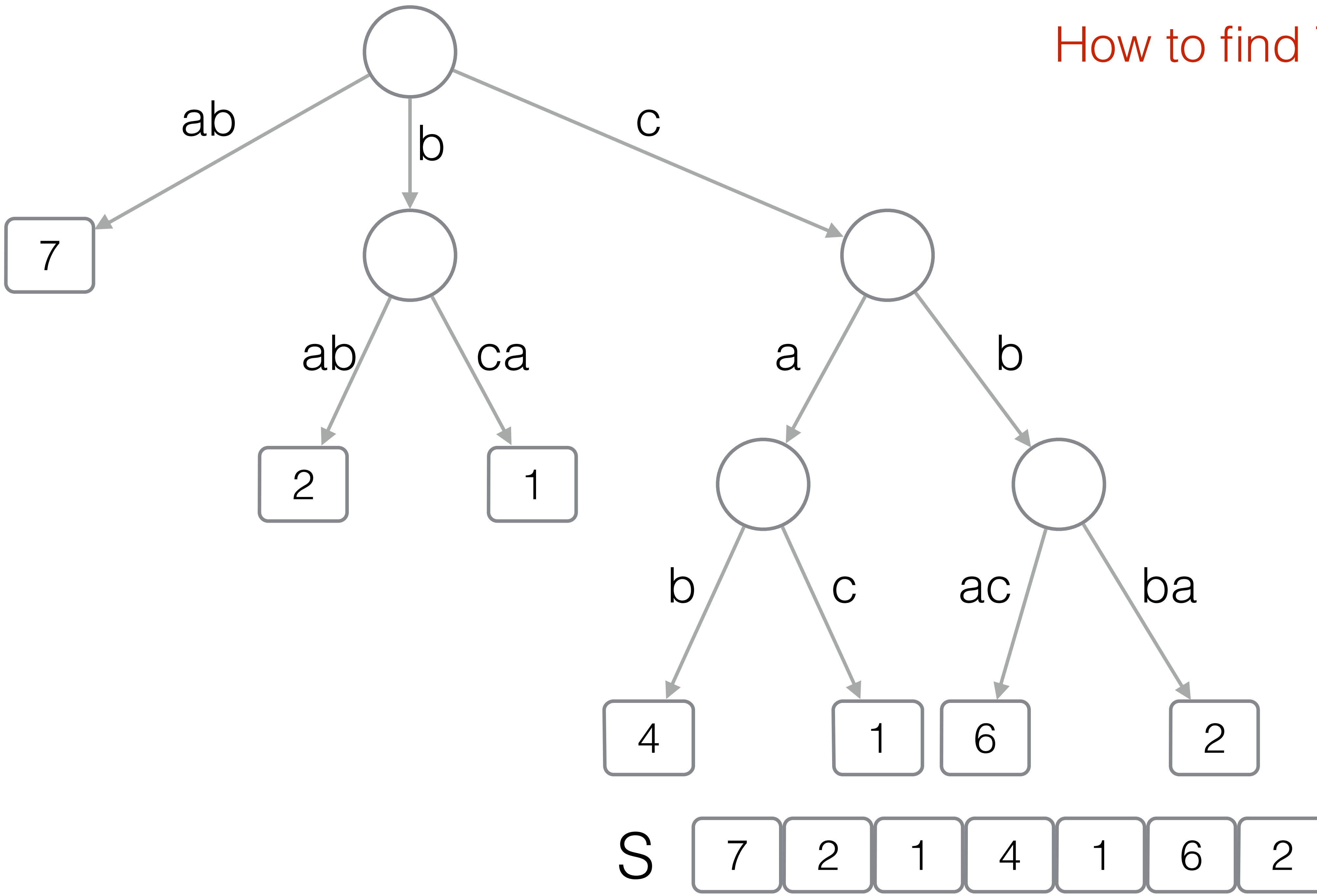
D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Finding Top-1

P = c

How to find Top-1?



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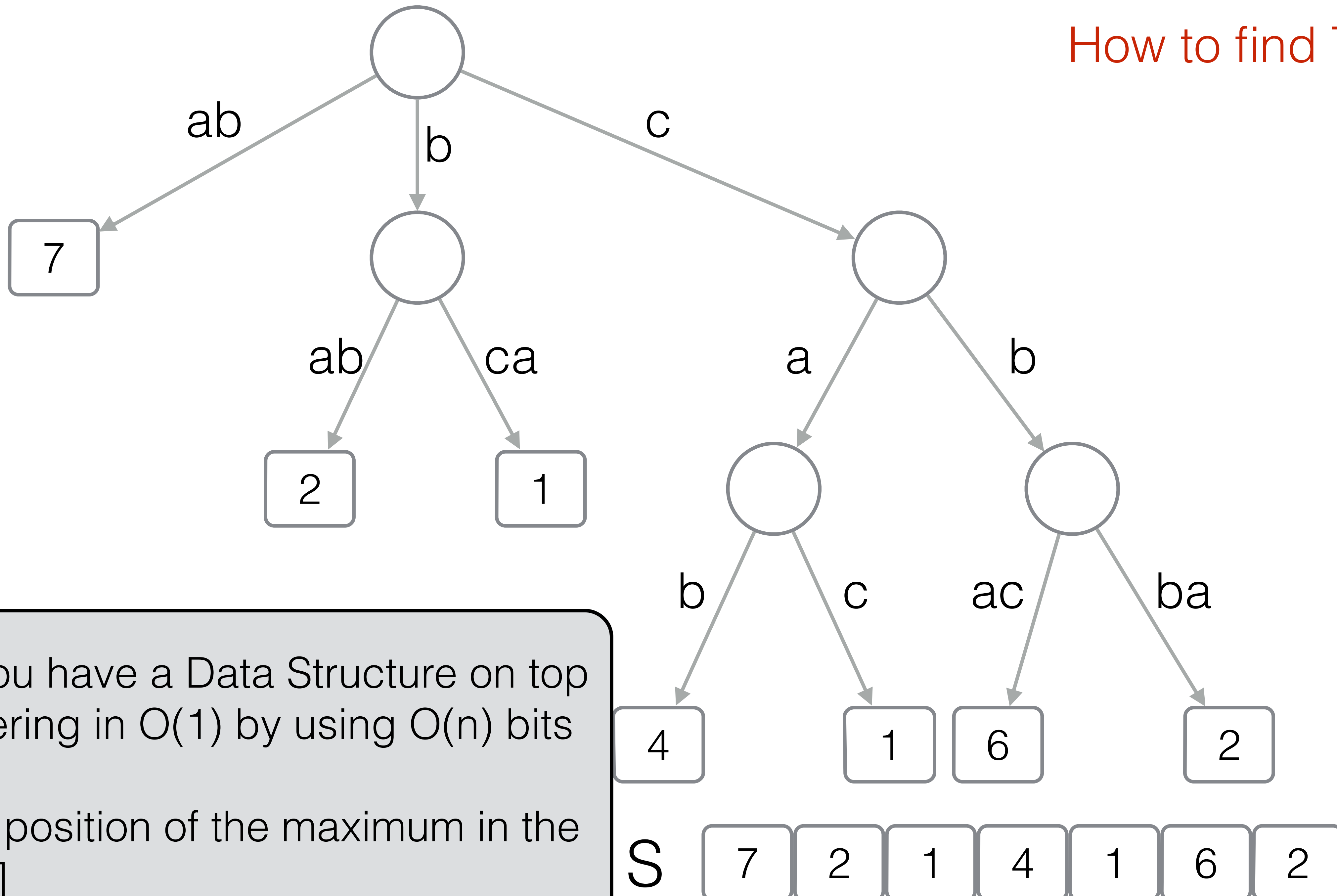

Assume you have a Data Structure on top of S answering in O(1) by using O(n) bits

RMQ(i,j) = position of the maximum in the range S[i,j]

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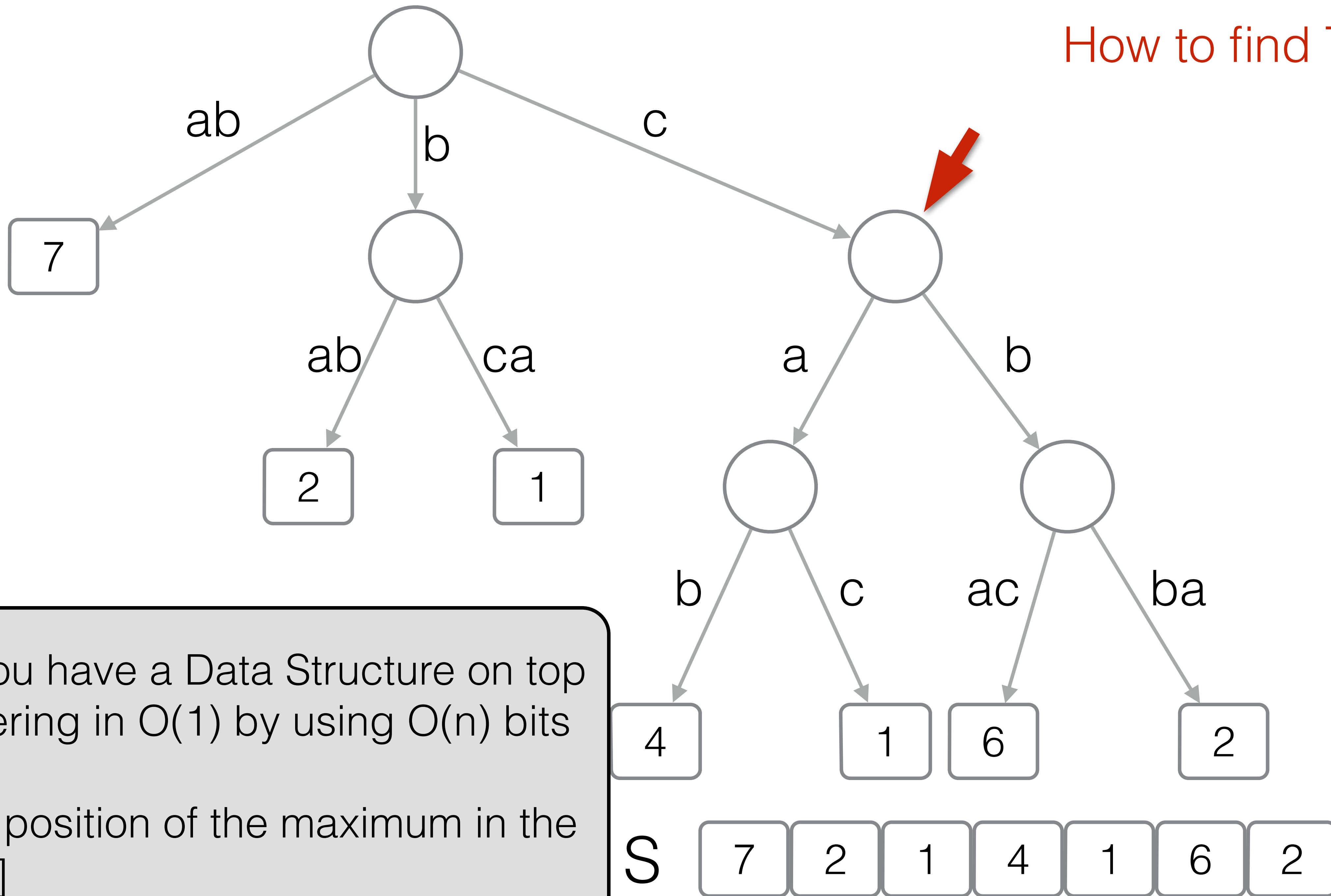

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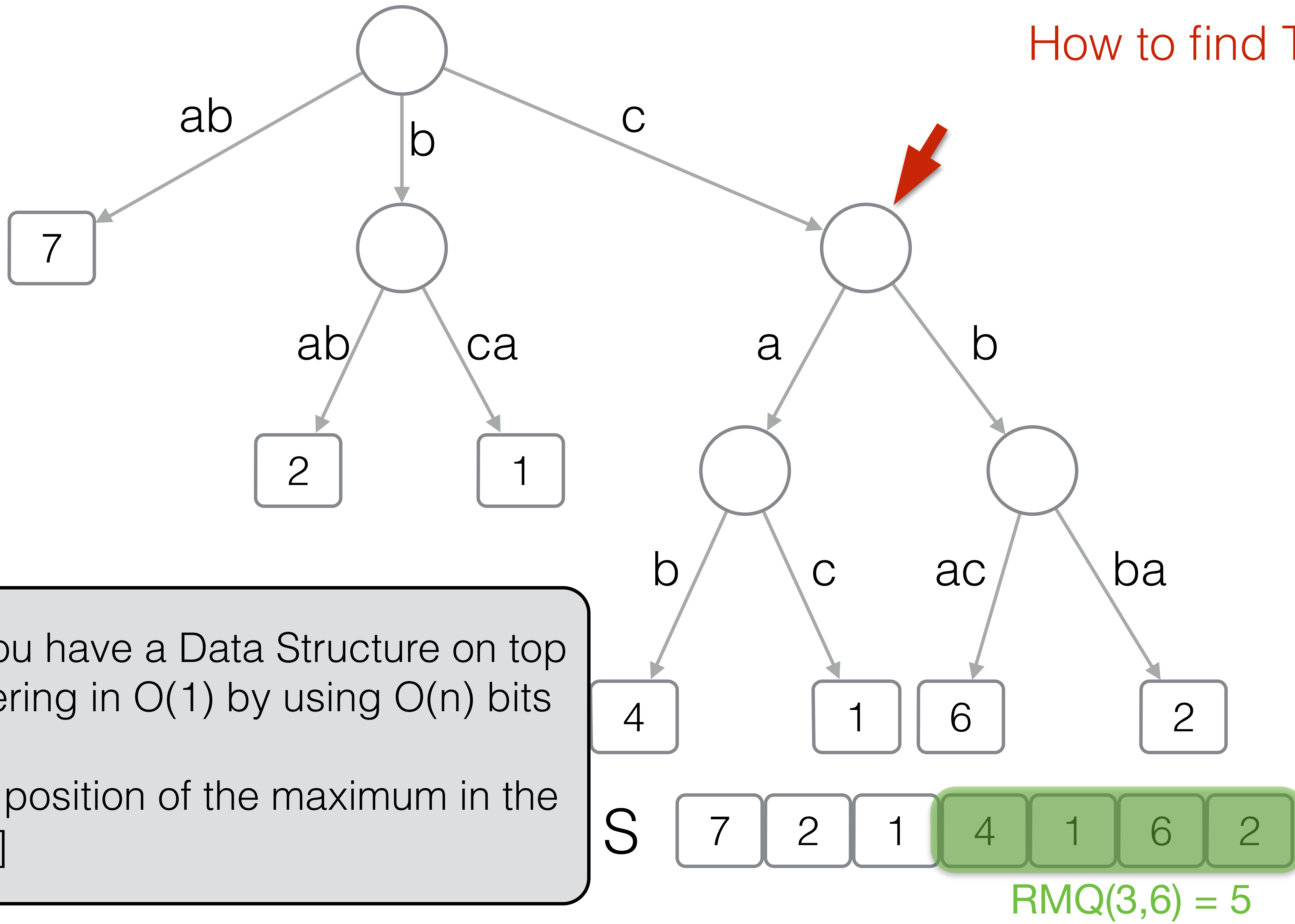

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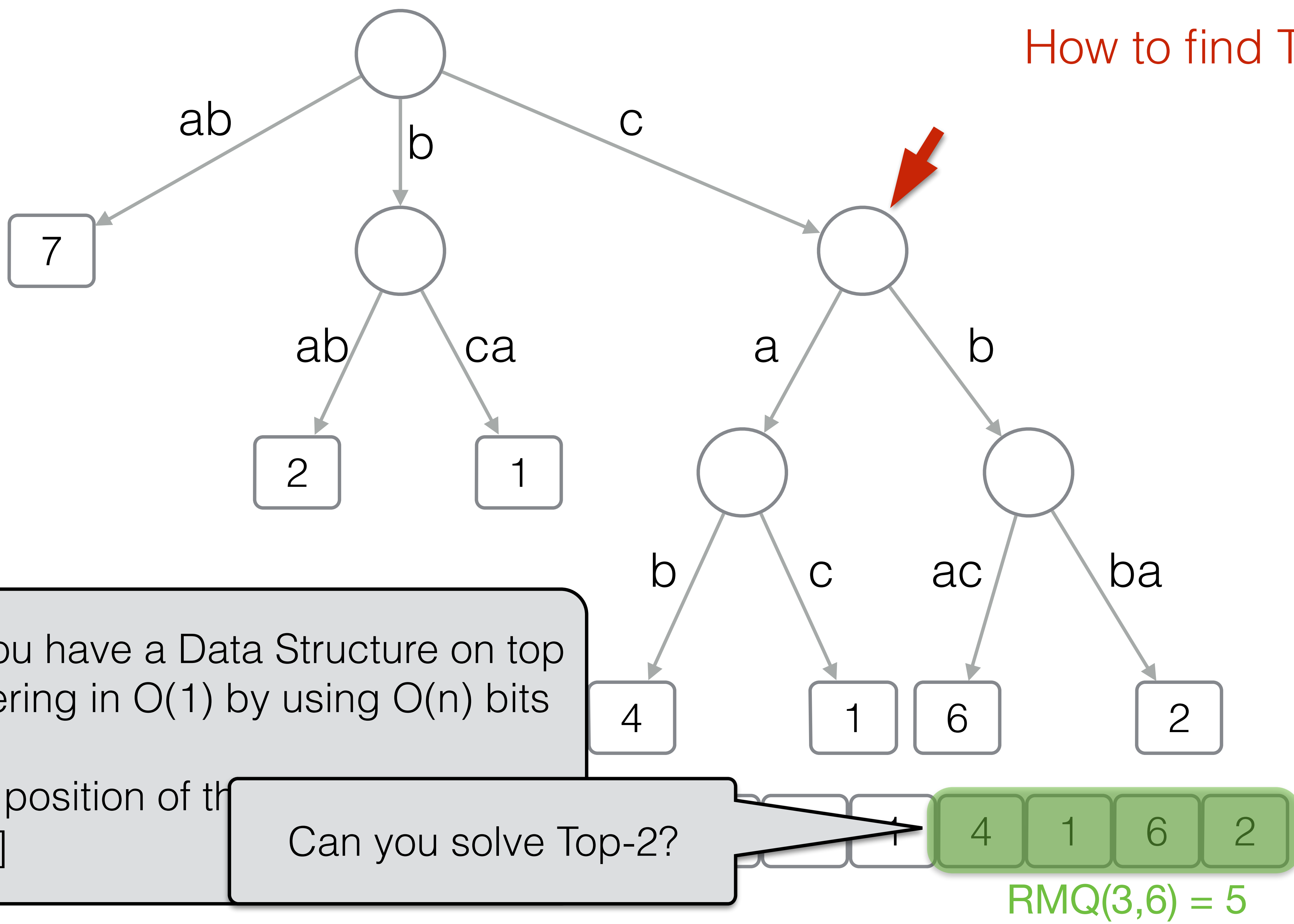


Assume you have a Data Structure on top of S answering in O(1) by using O(n) bits

RMQ(i,j) = position of th
range S[i,j]

Can you solve Top-2?

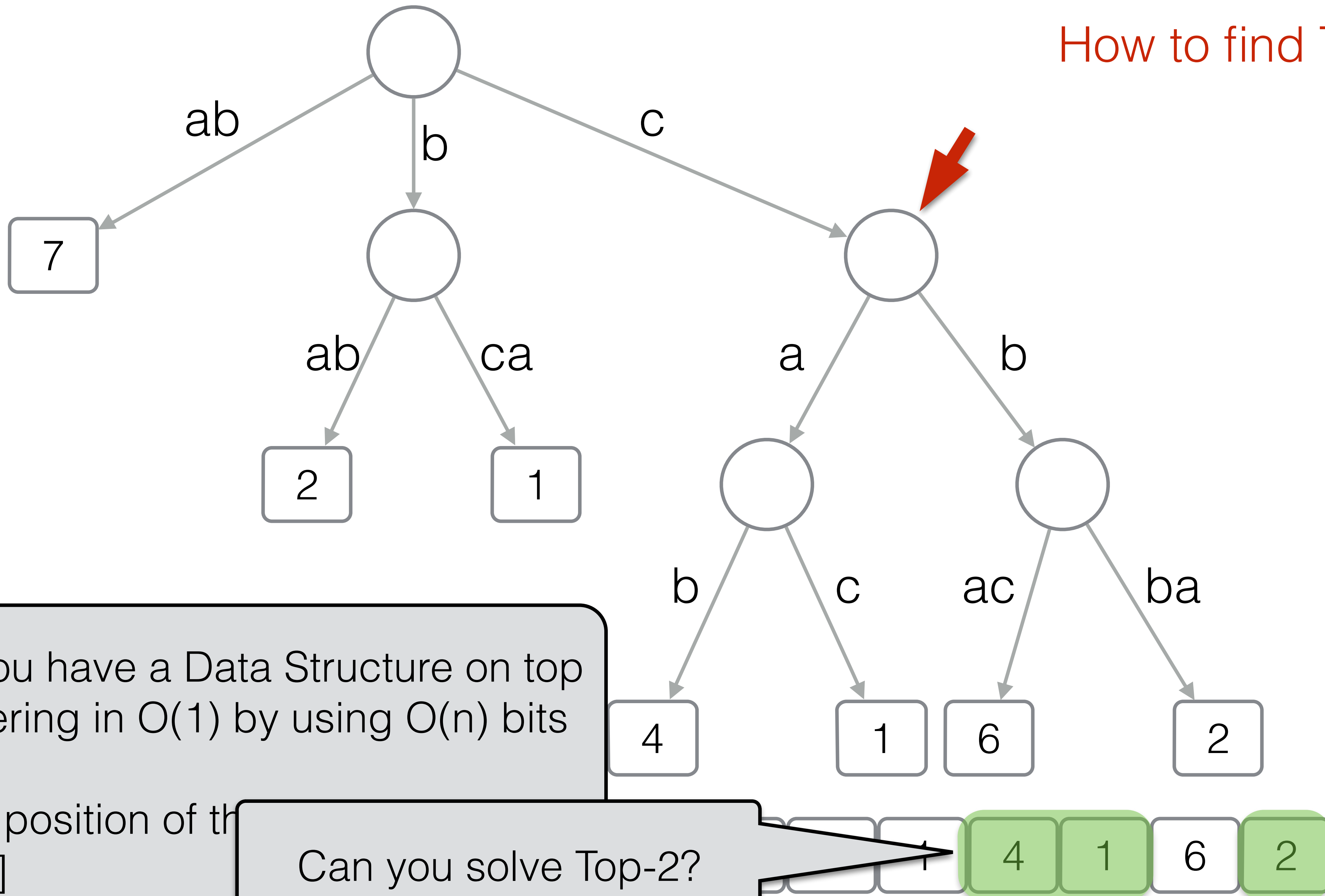
D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Finding Top-1

P = c

How to find Top-1?



Assume you have a Data Structure on top of S answering in O(1) by using O(n) bits

RMQ(i,j) = position of th
range S[i,j]

Can you solve Top-2?

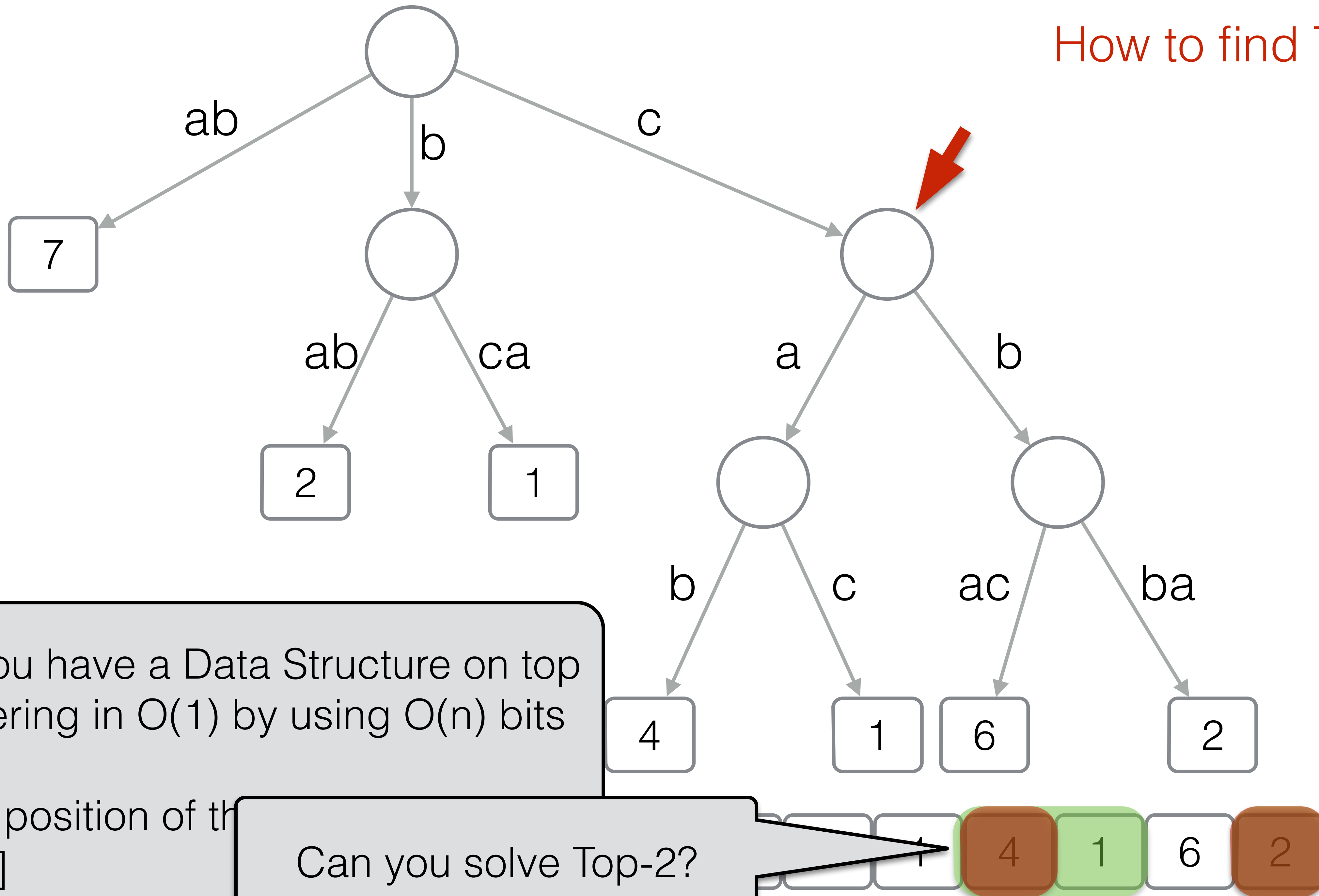
D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Finding Top-1

P = c

How to find Top-1?



Assume you have a Data Structure on top of S answering in O(1) by using O(n) bits

RMQ(i,j) = position of th
range S[i,j]

Can you solve Top-2?

D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

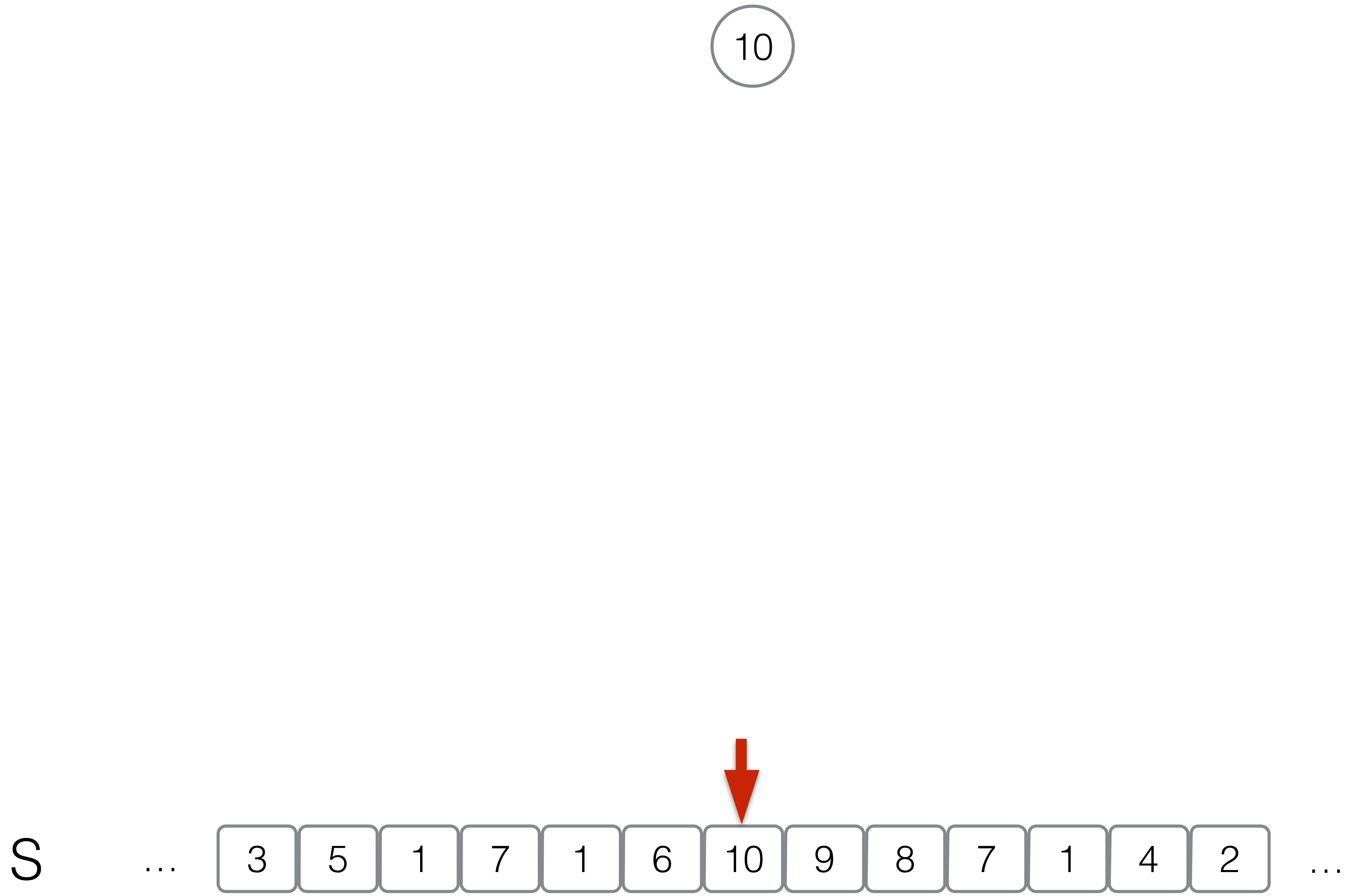
n = |D|, m total length of strings in D

Finding Top-k

Finding Top-k

S … 3 5 1 7 1 6 10 9 8 7 1 4 2 …

Finding Top-k



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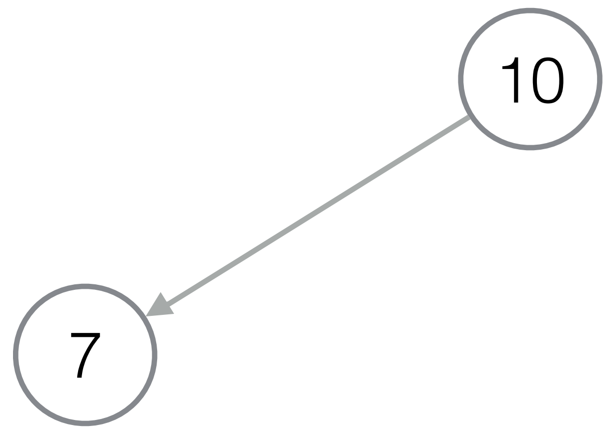
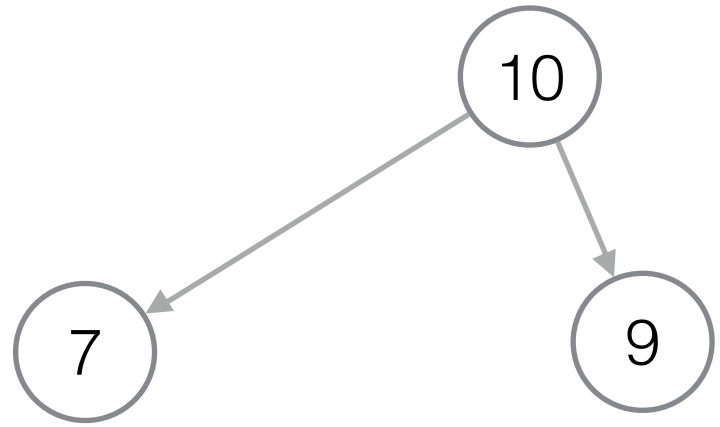


S	…	3	5	1	7	1	6	10	9	8	7	1	4	2	…

Finding Top-k



S … 3 5 1 7 1 6 10 9 8 7 1 4 2 …

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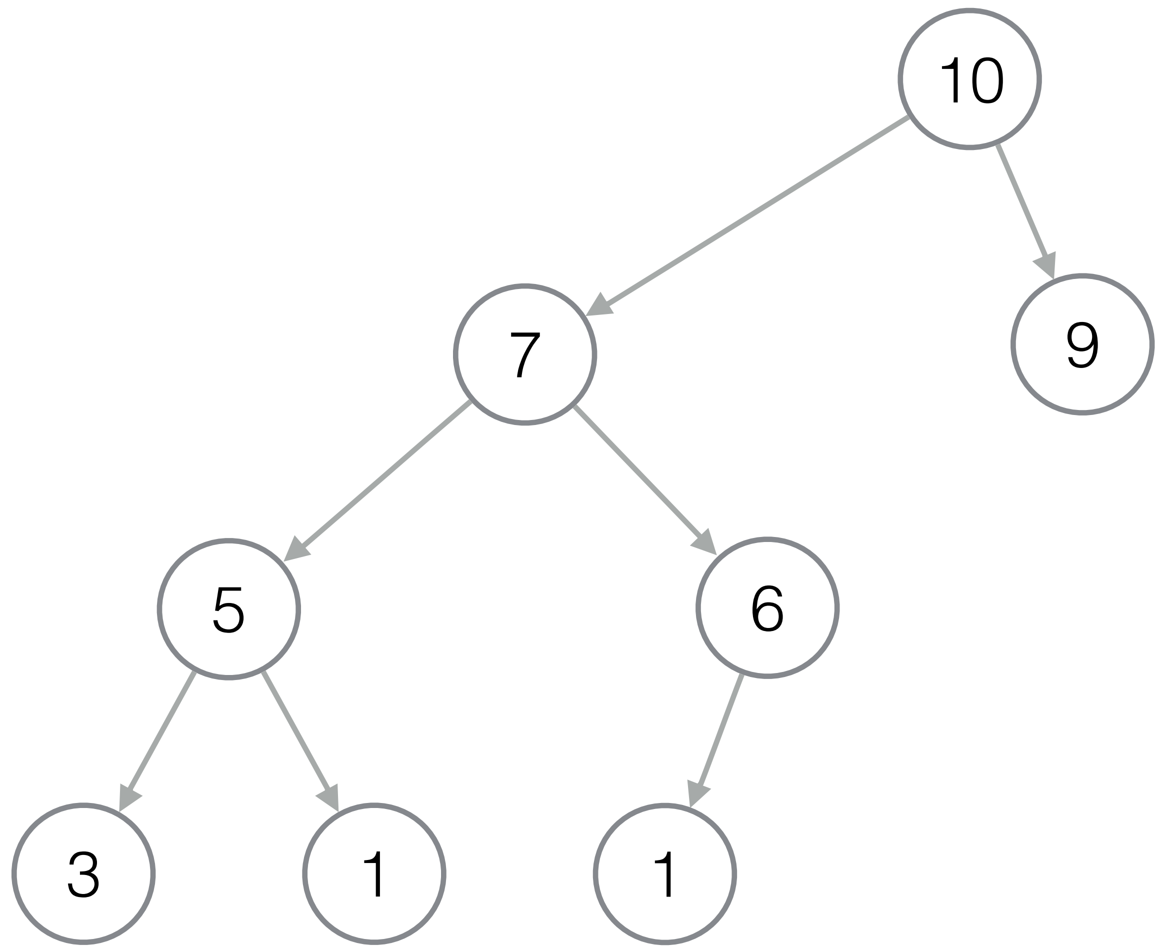

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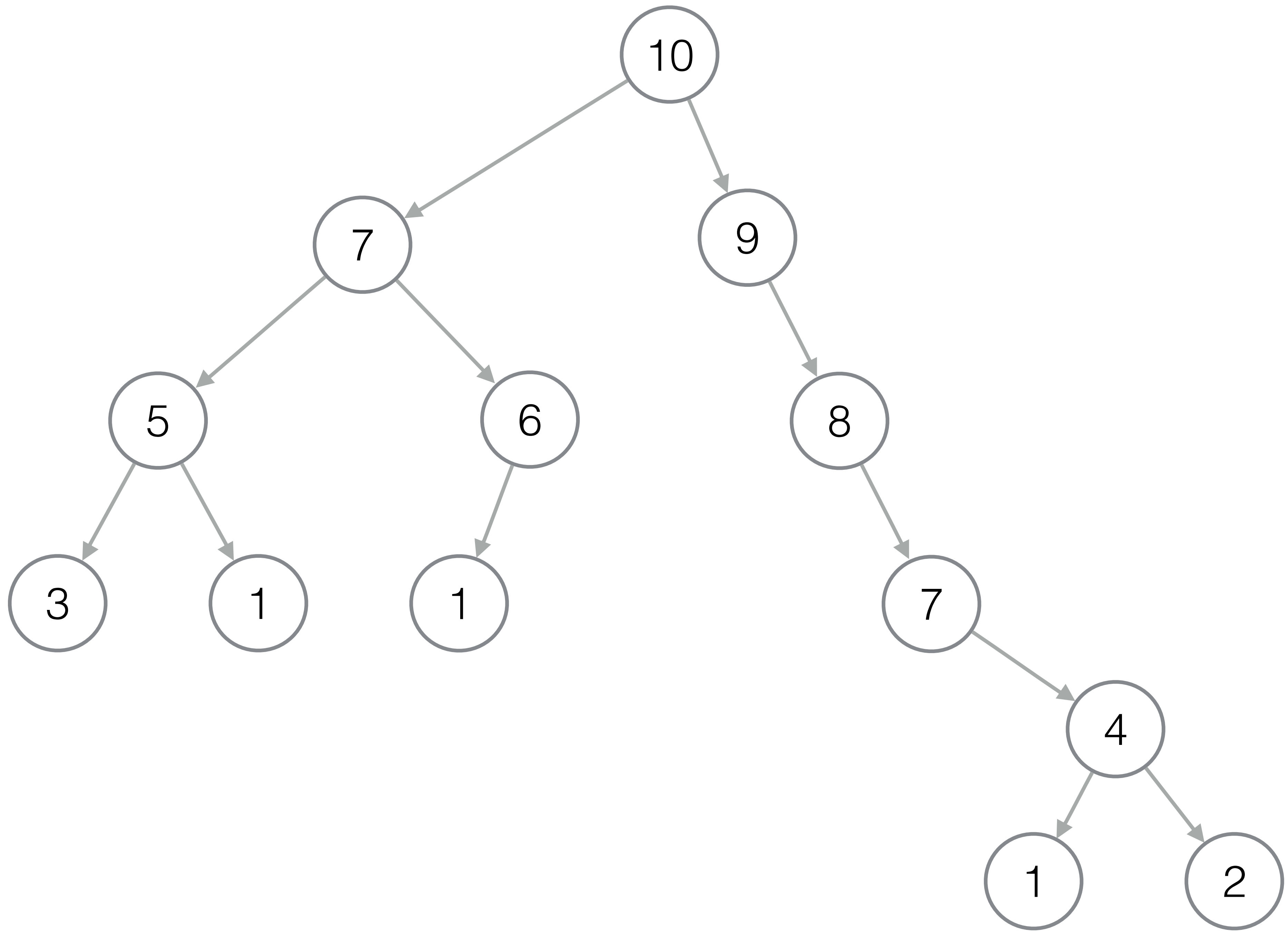
Finding Top-k

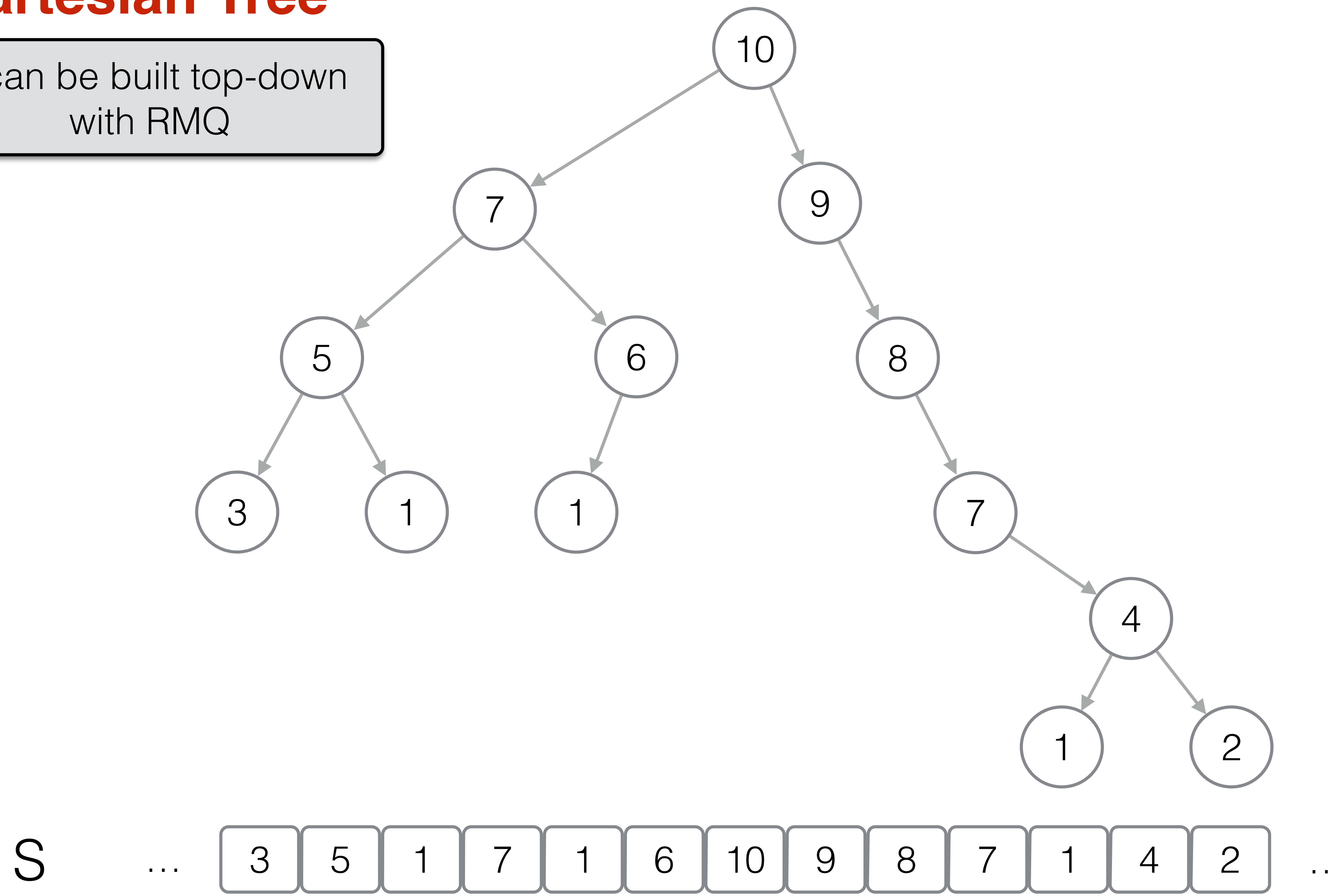


S	…	3	5	1	7	1	6	10	9	8	7	1	4	2	…

Finding Top-k

Cartesian Tree

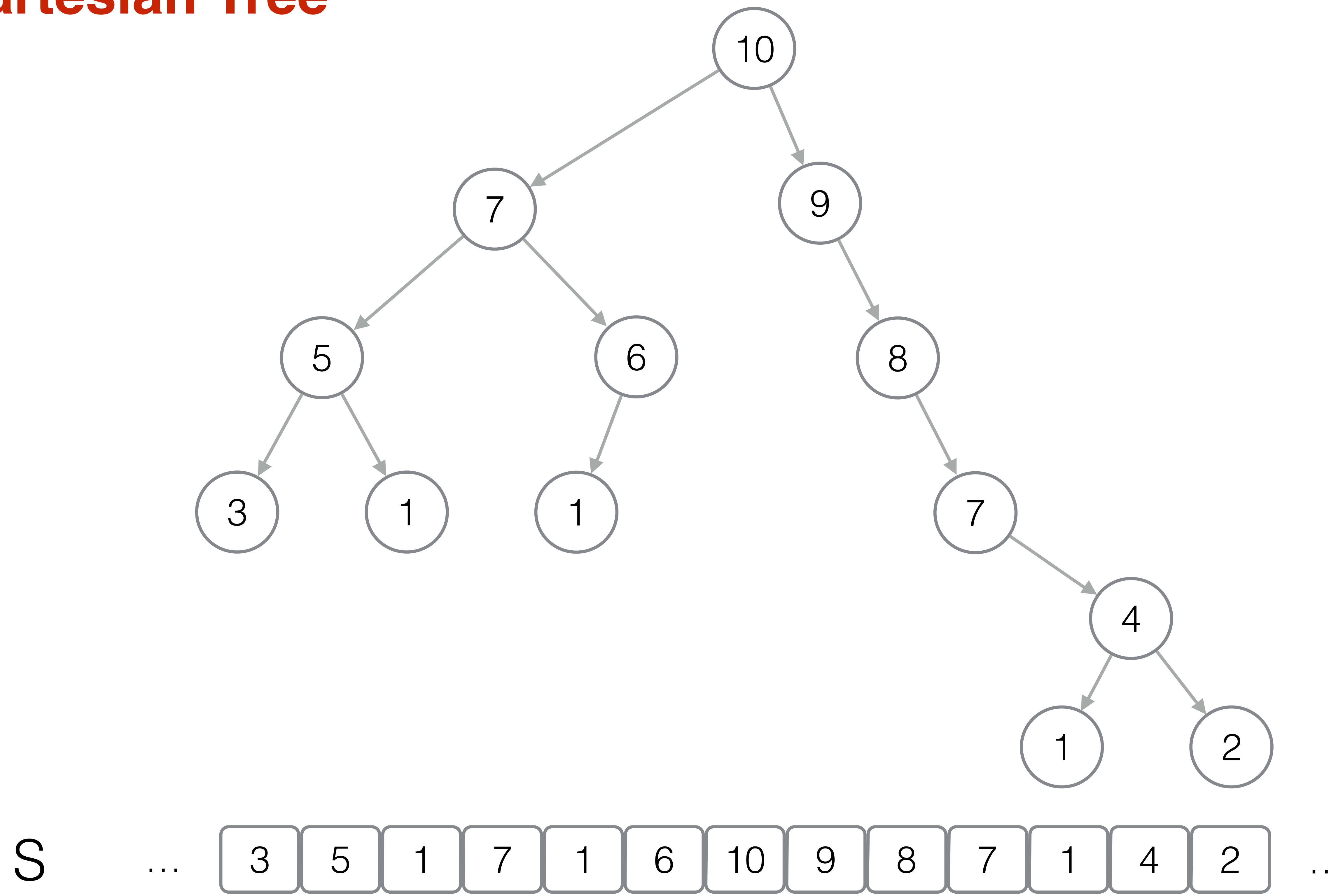
It can be built top-down
with RMQ



Finding Top-k

How to find Top-k?

Cartesian Tree



Finding Top-k

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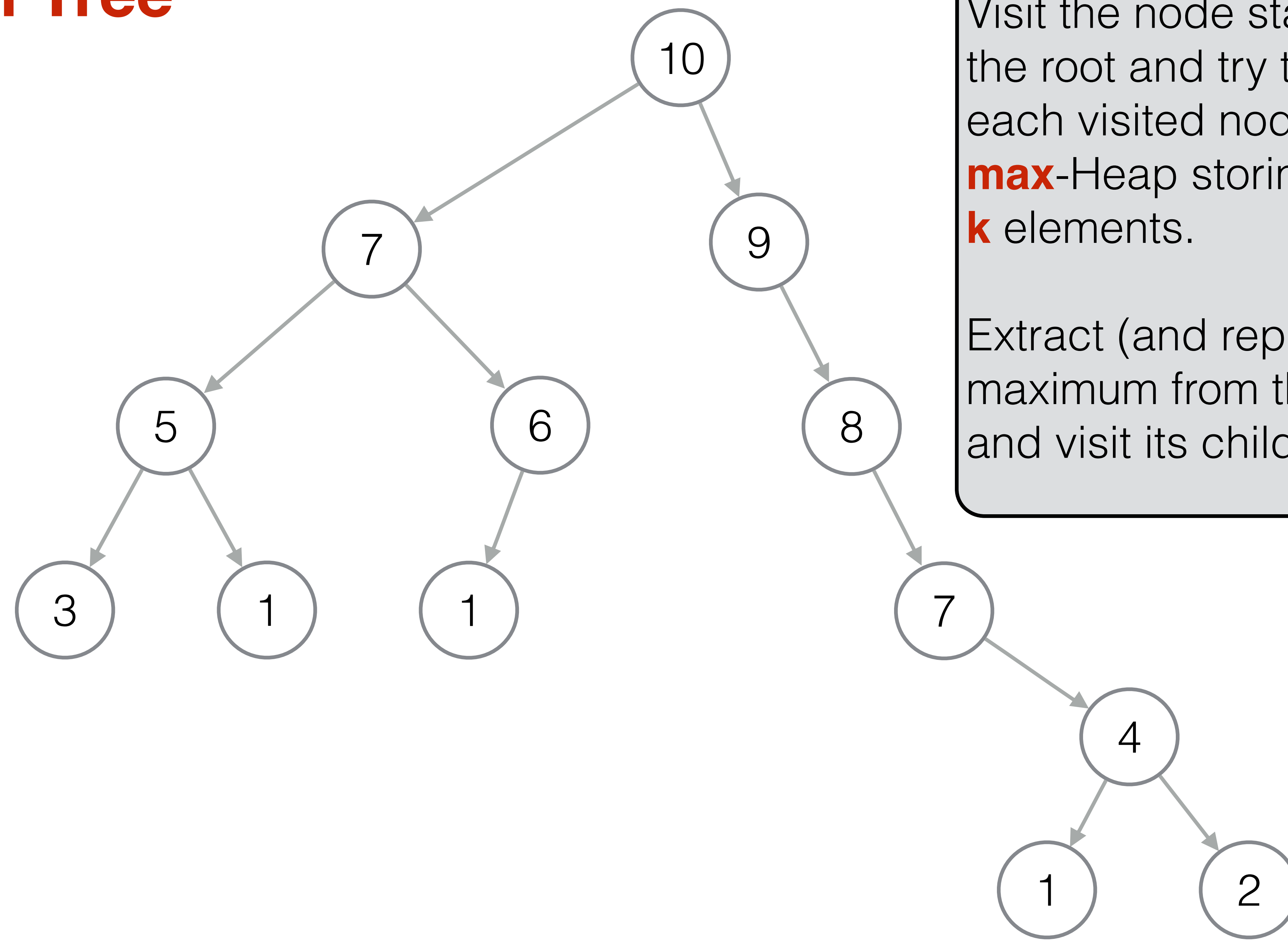

S	…	3	5	1	7	1	6	10	9	8	7	1	4	2	…

Visit the node starting from the root and try to insert each visited node in a **max**-Heap storing at most **k** elements.

Extract (and report) the maximum from the heap and visit its children.

Finding Top-k

How to find Top-k?

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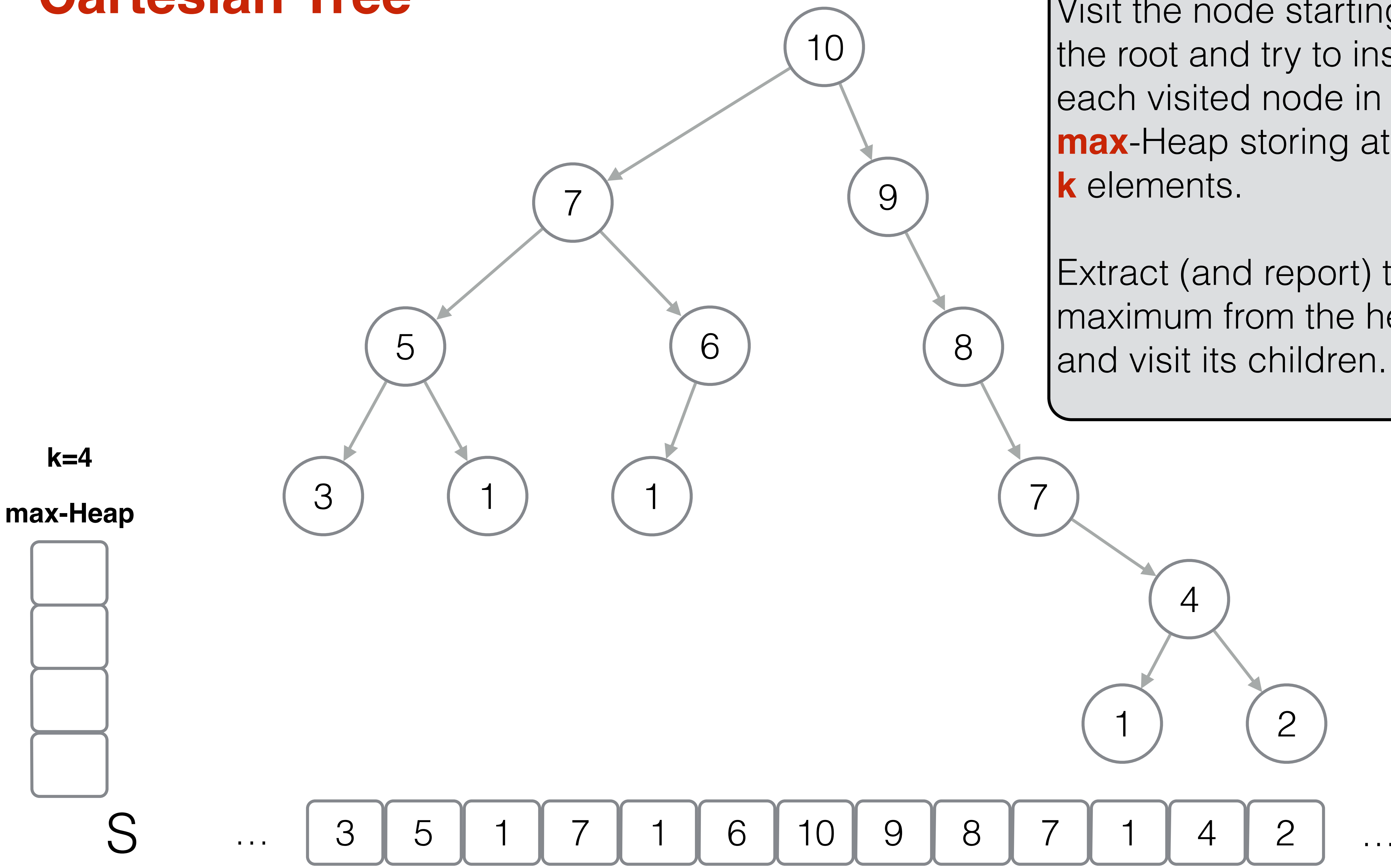
Cartesian Tree



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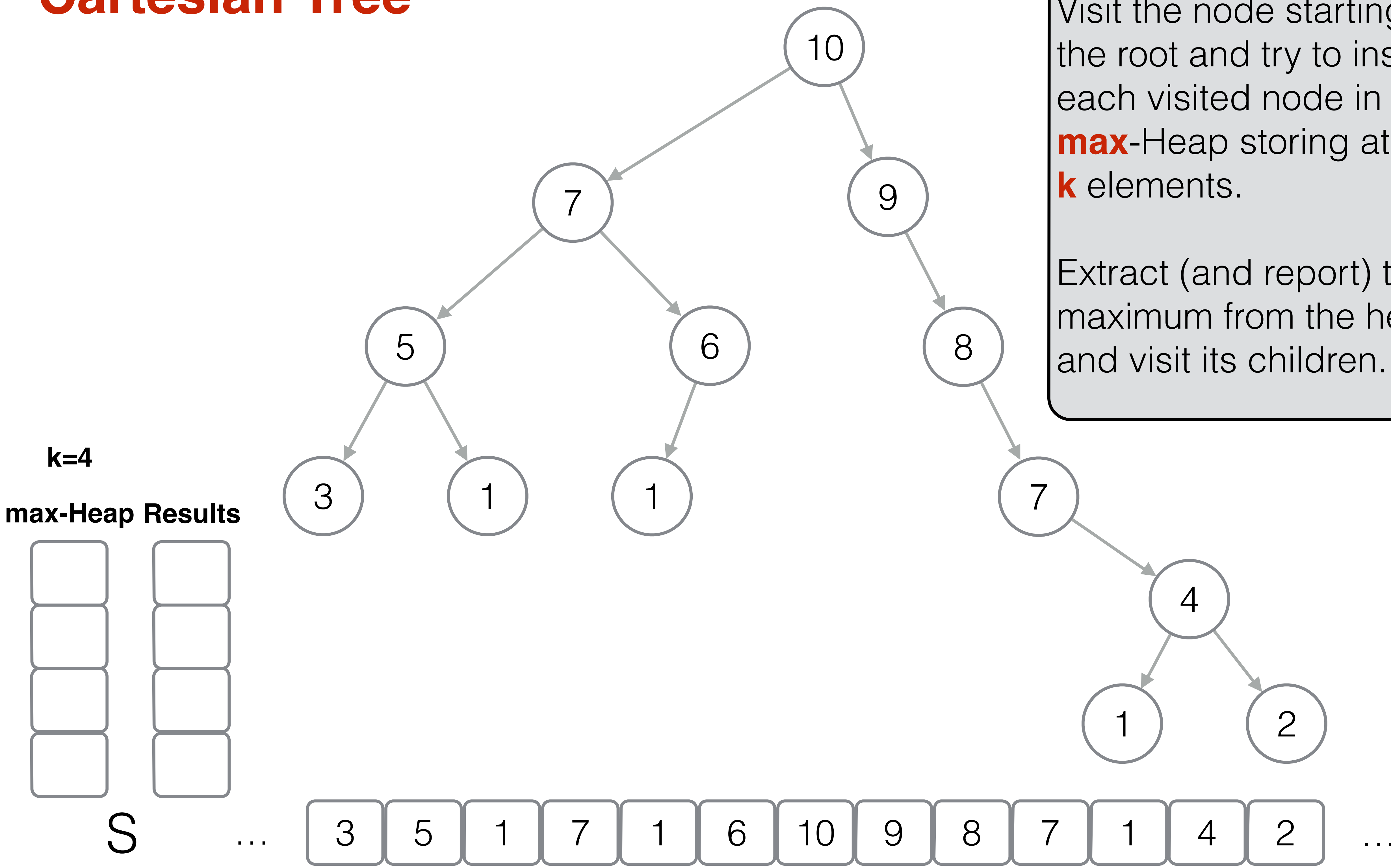
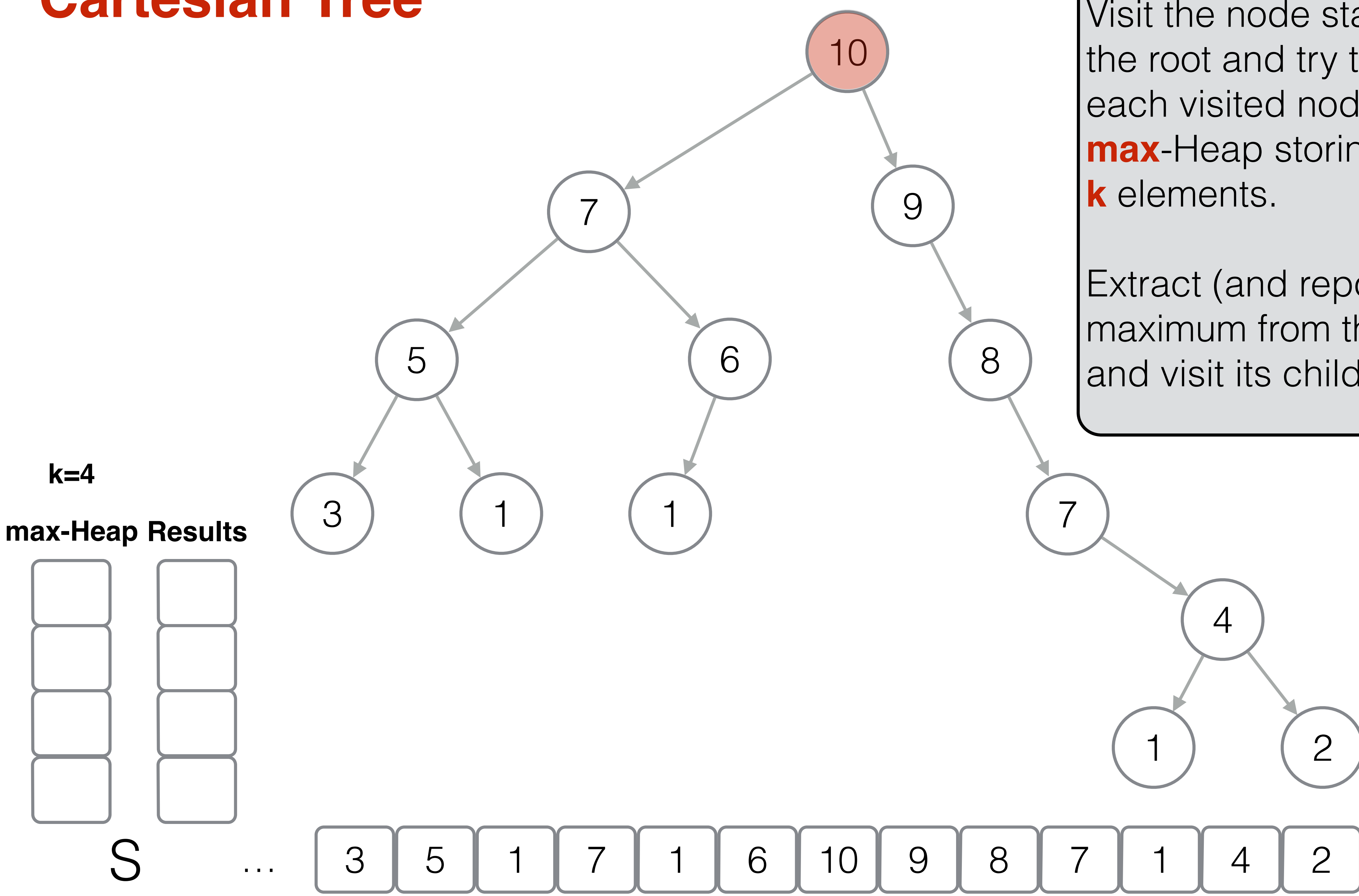


Finding Top-k

Cartesian Tree



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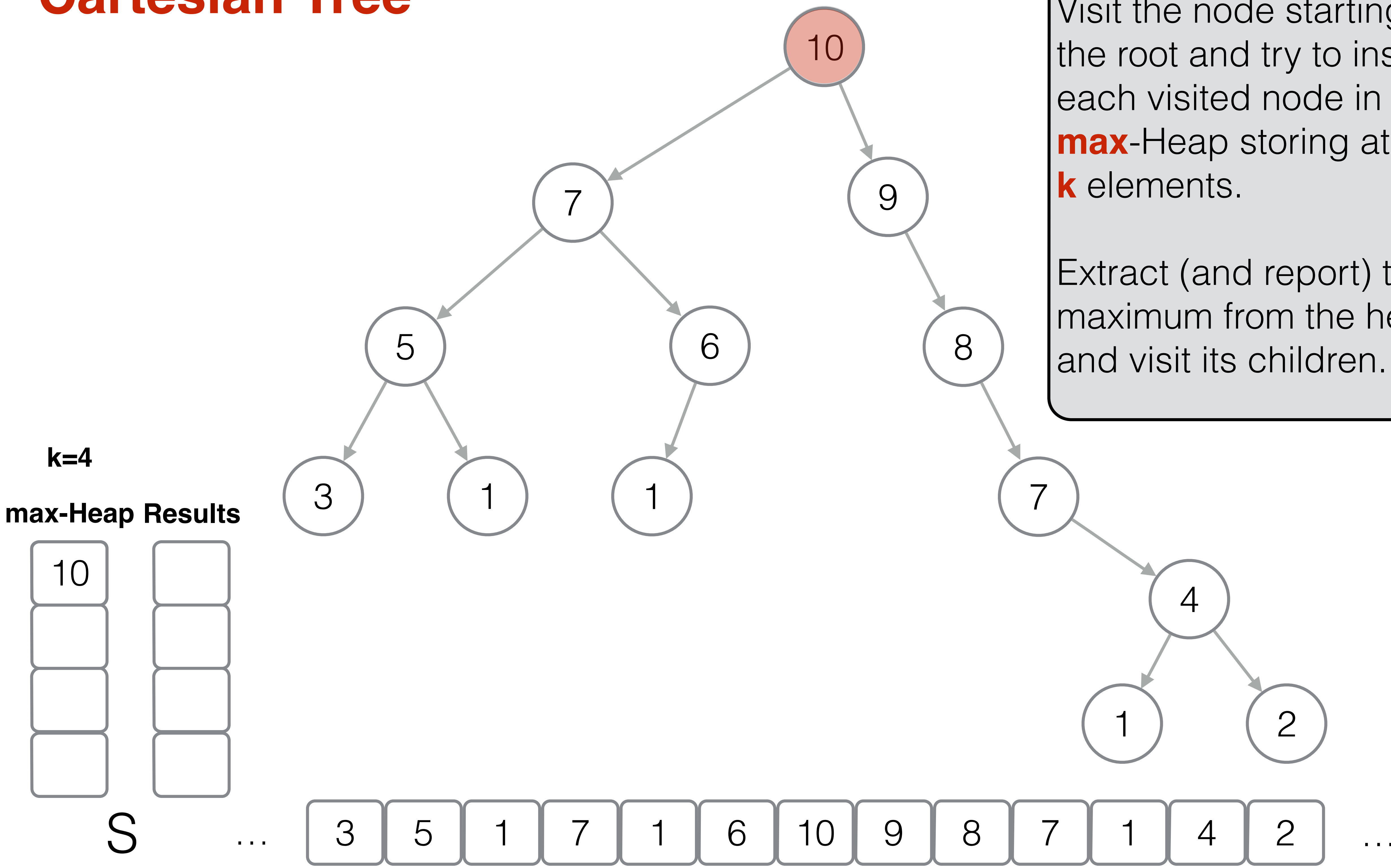

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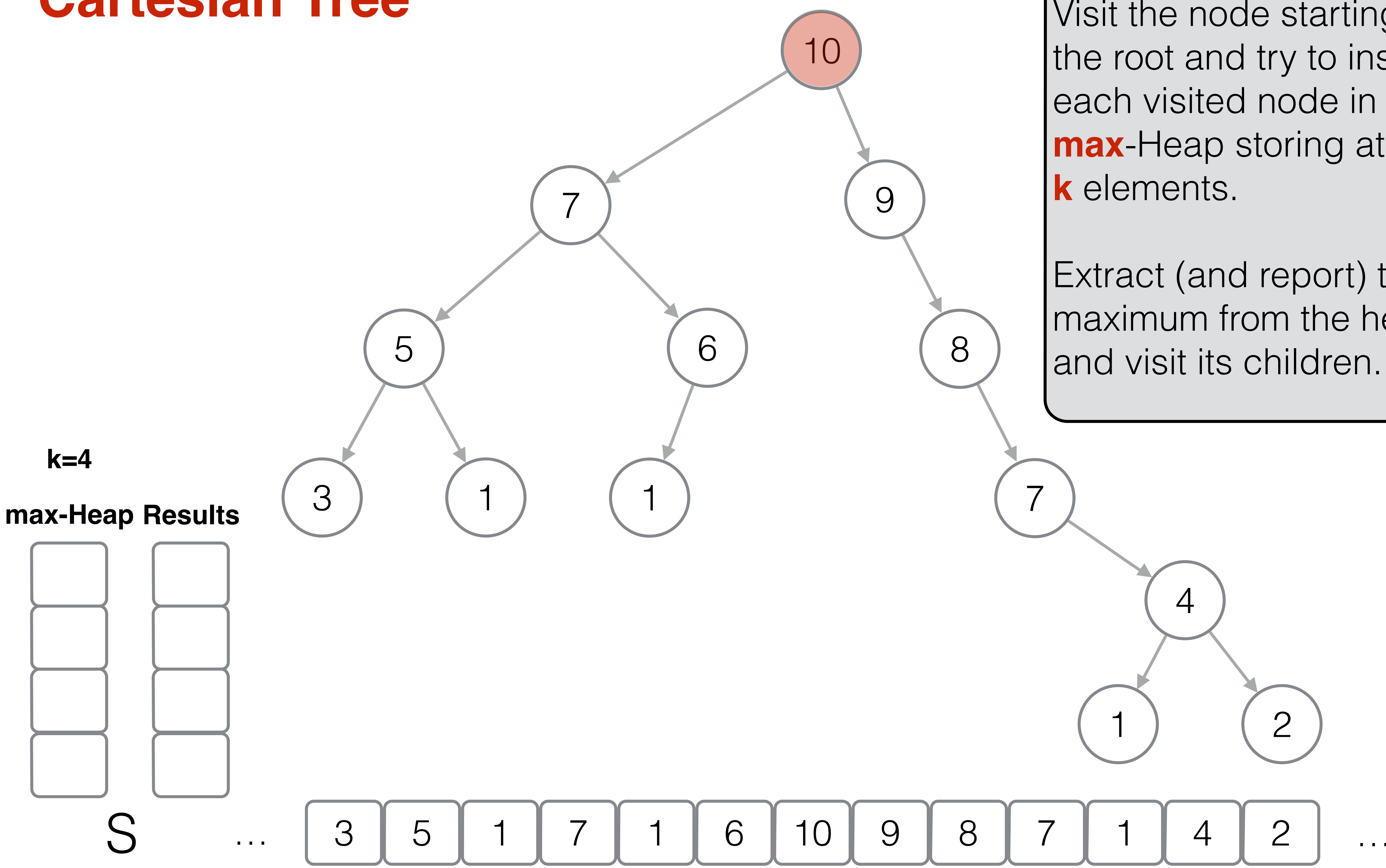
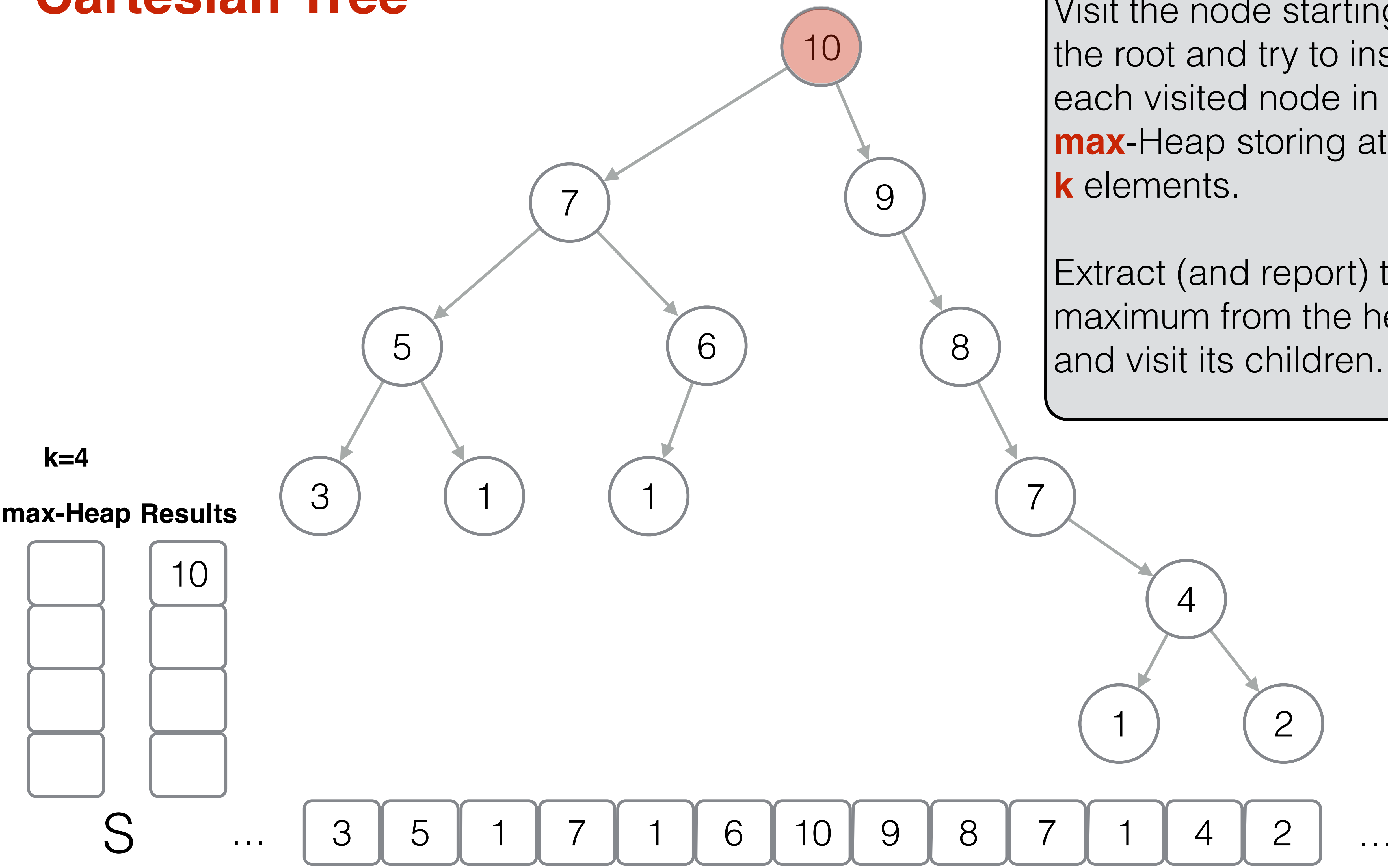


Finding Top-k

Cartesian Tree



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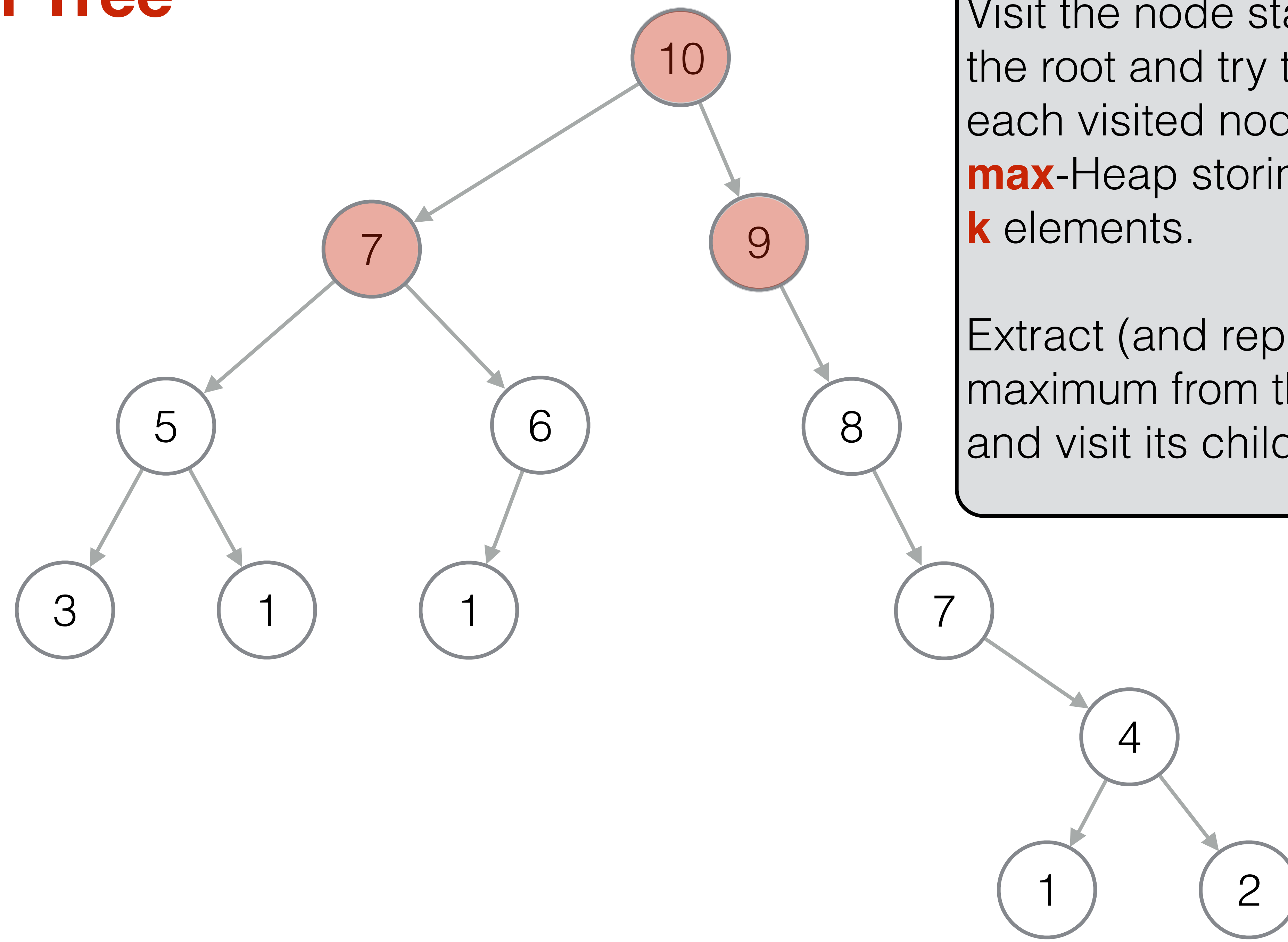
Cartesian Tree



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Extract (and report) the maximum from the heap and visit its children.

k=4

max-Heap	**Results**
	10

S … | 3 | 5 | 1 | 7 | 1 | 6 | 10 | 9 | 8 | 7 | 1 | 4 | 2 | …

Finding Top-k

Cartesian Tree

How to find Top-k?

Visit the node starting from the root and try to insert each visited node in a **max**-Heap storing at most **k** elements.

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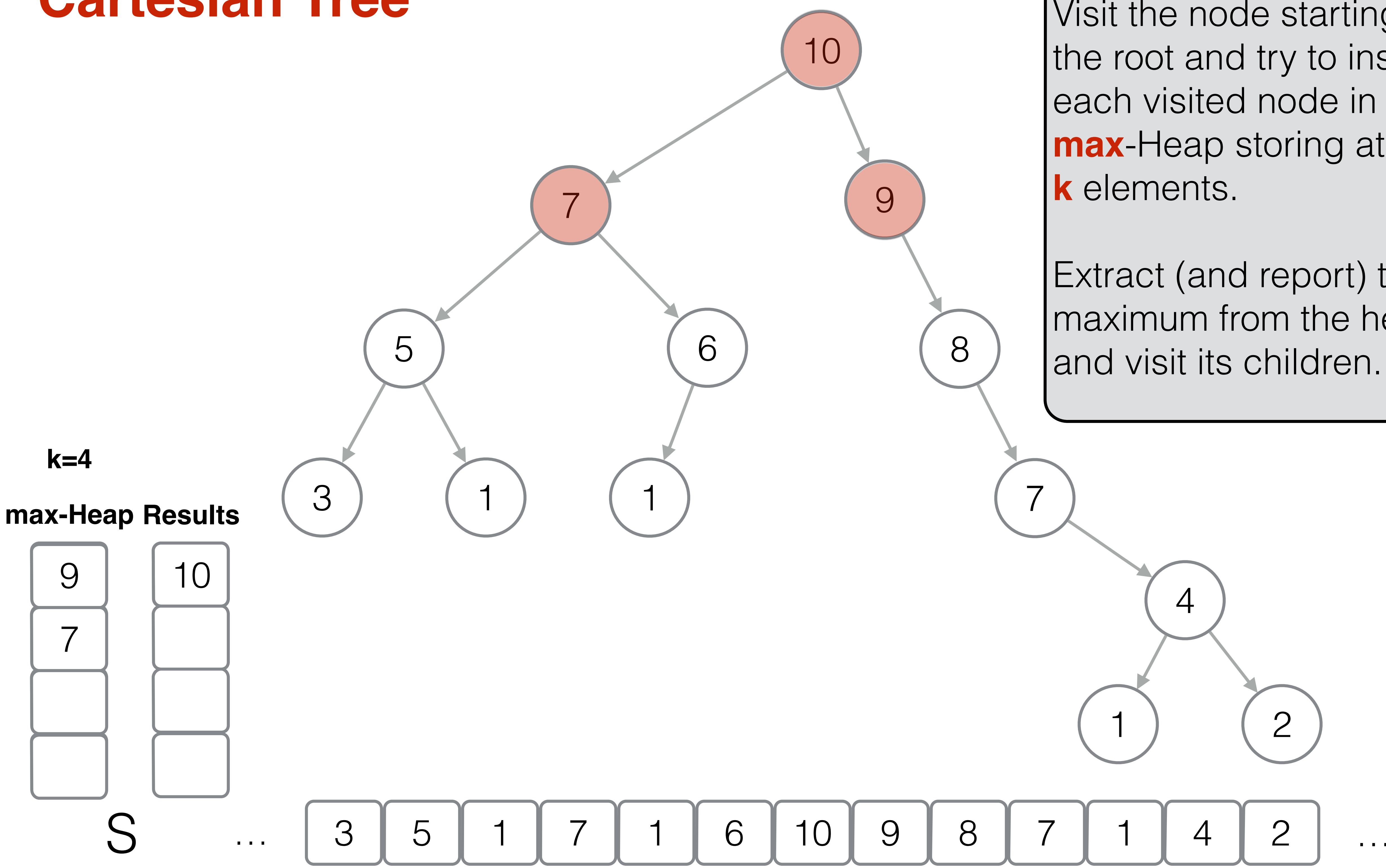
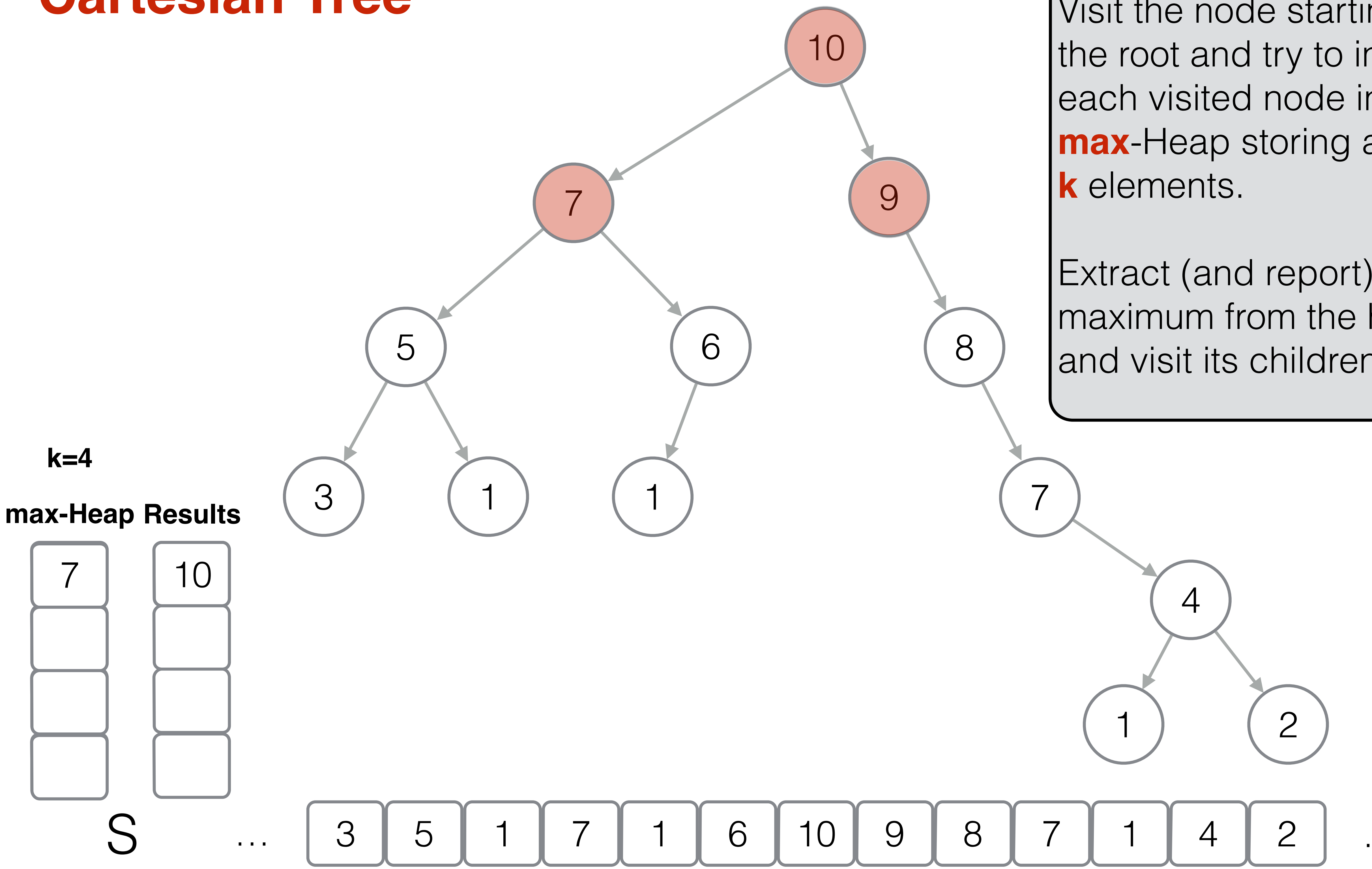


Finding Top-k

Cartesian Tree



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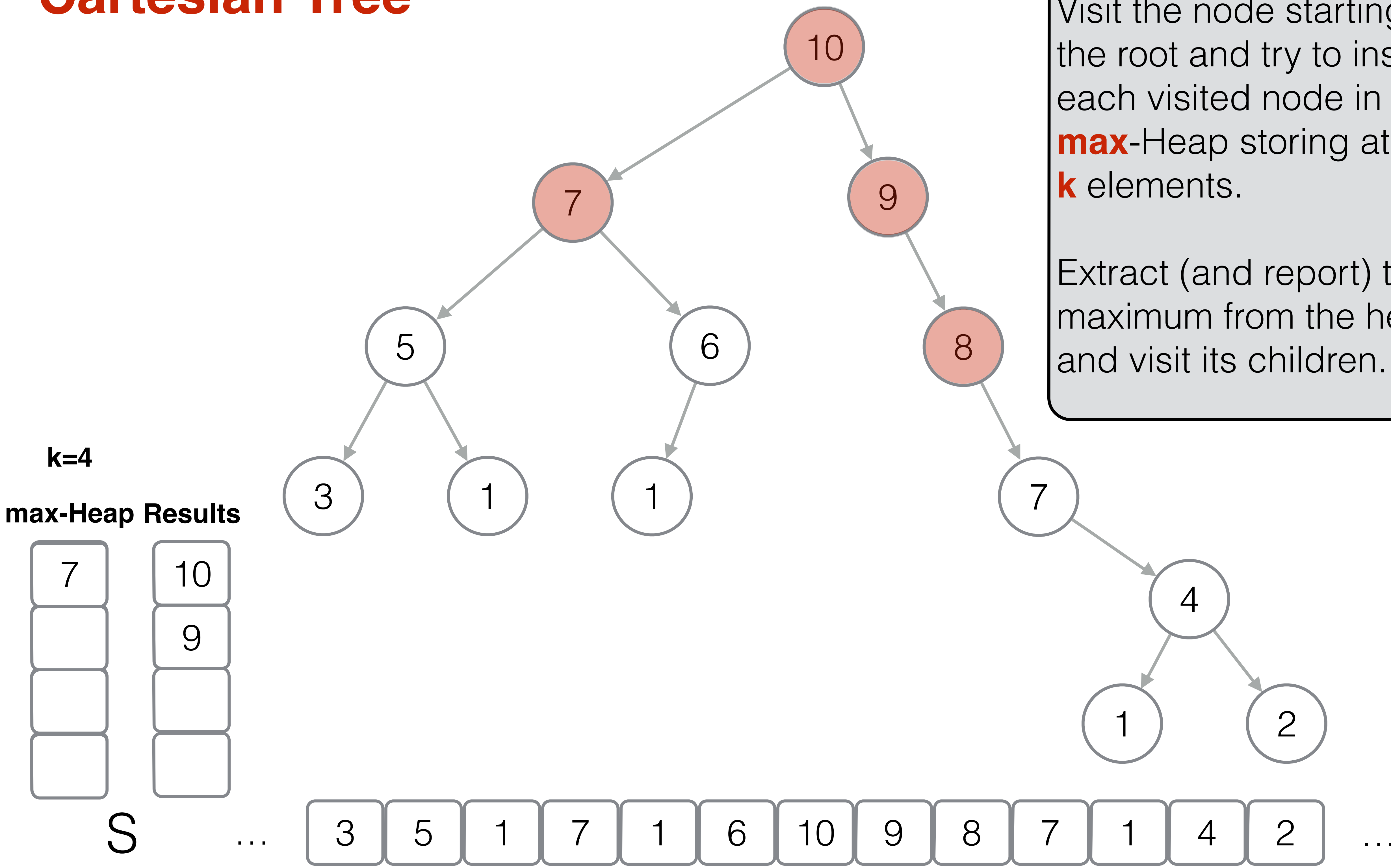

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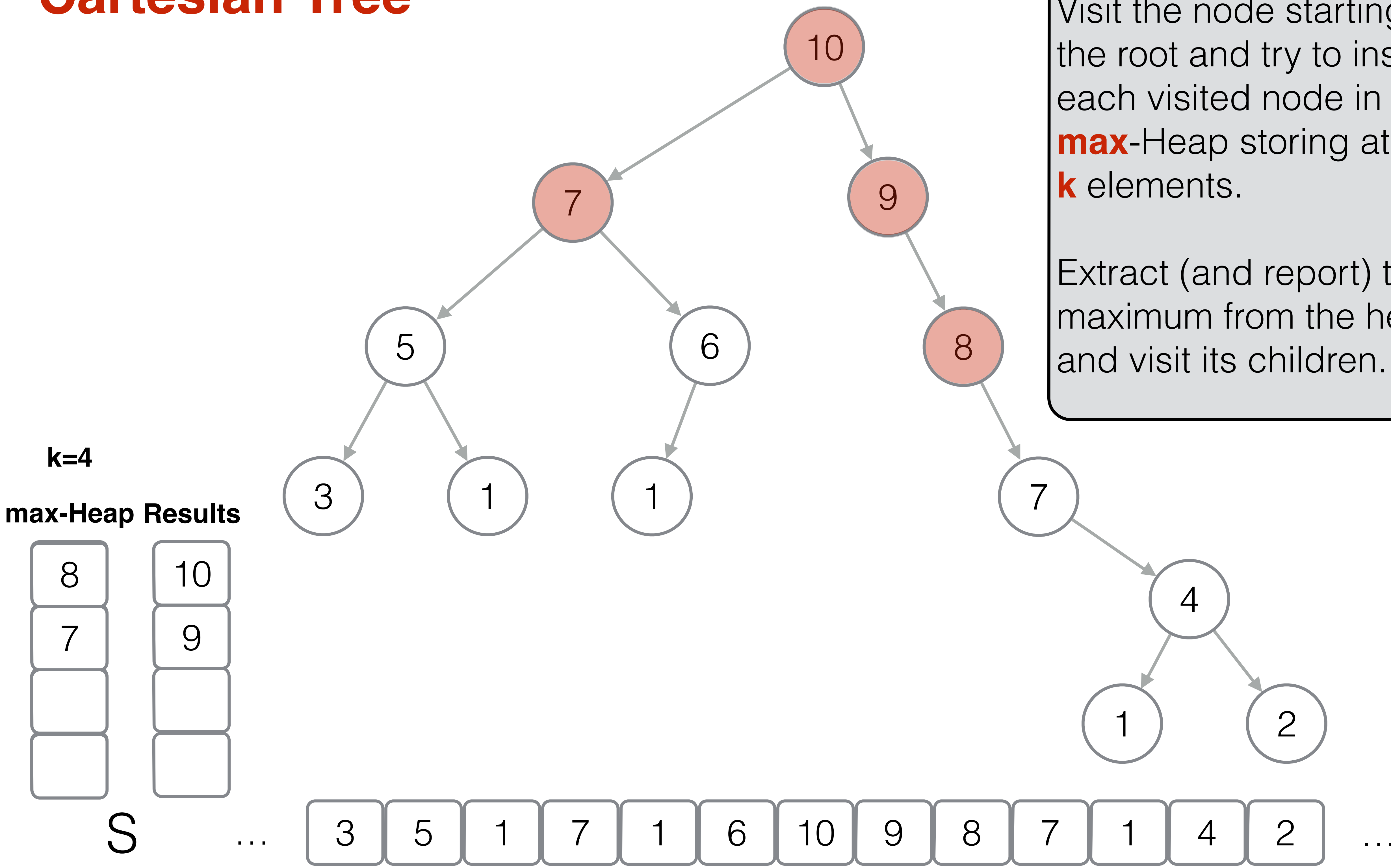

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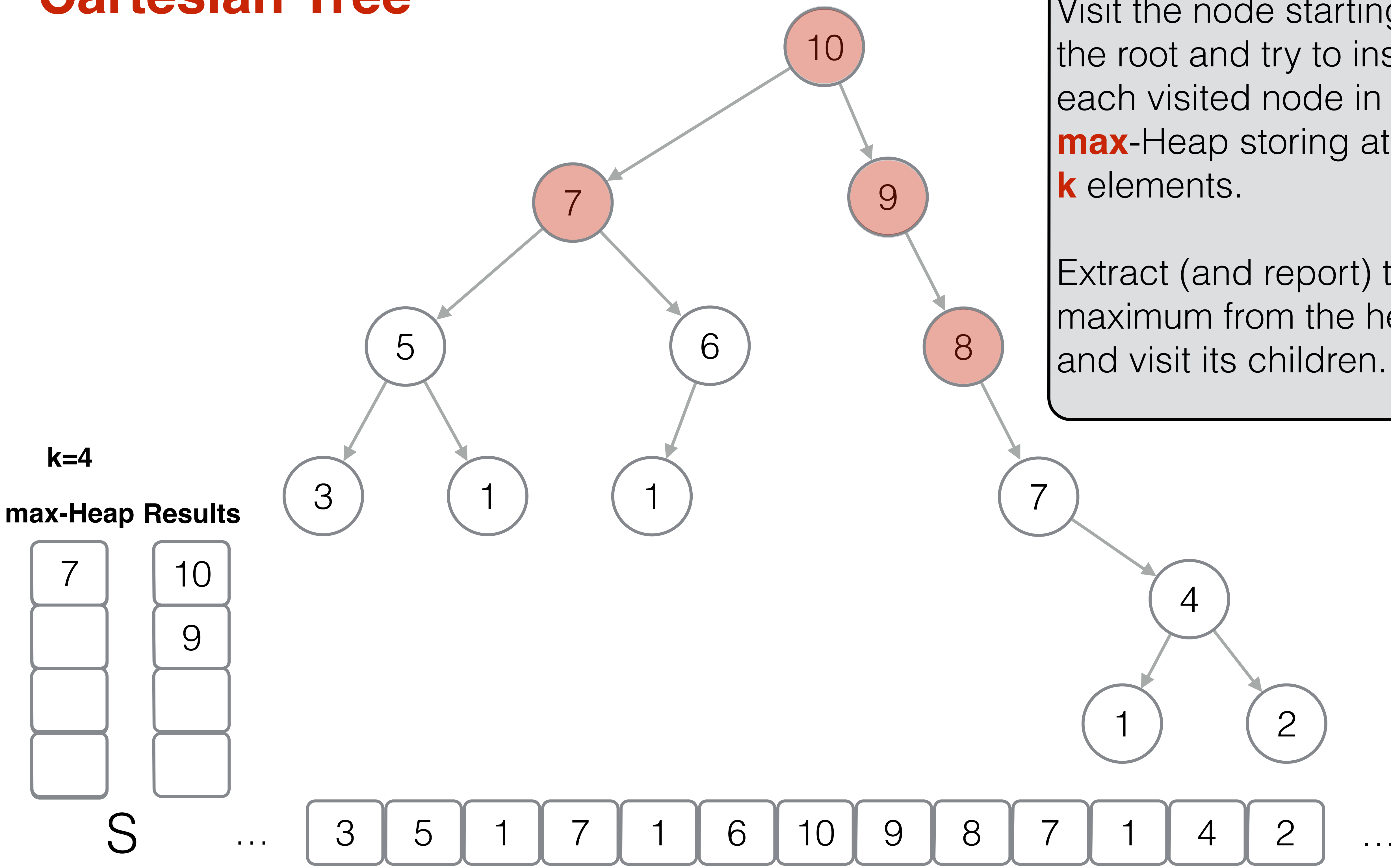
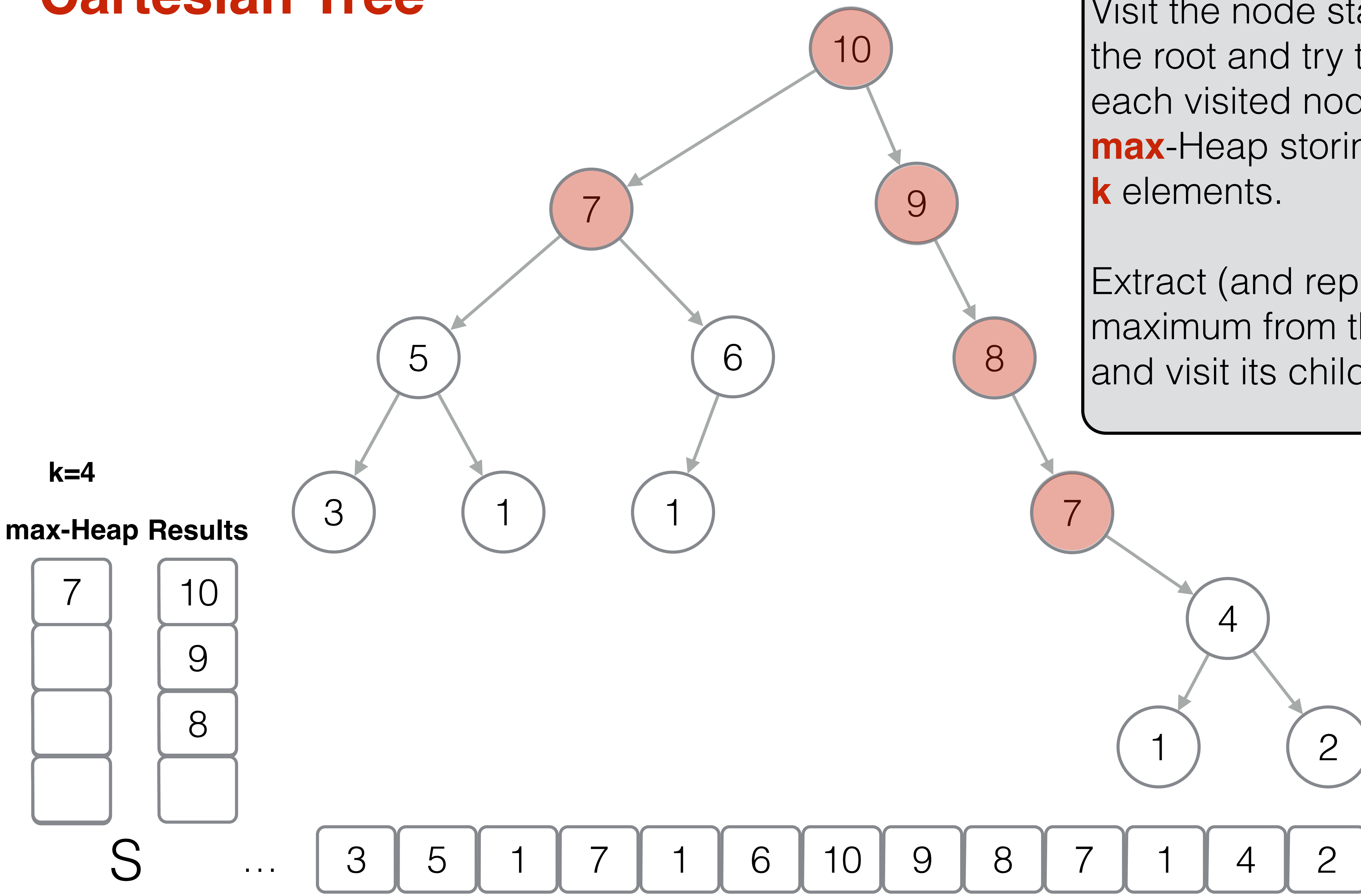


Finding Top-k

Cartesian Tree



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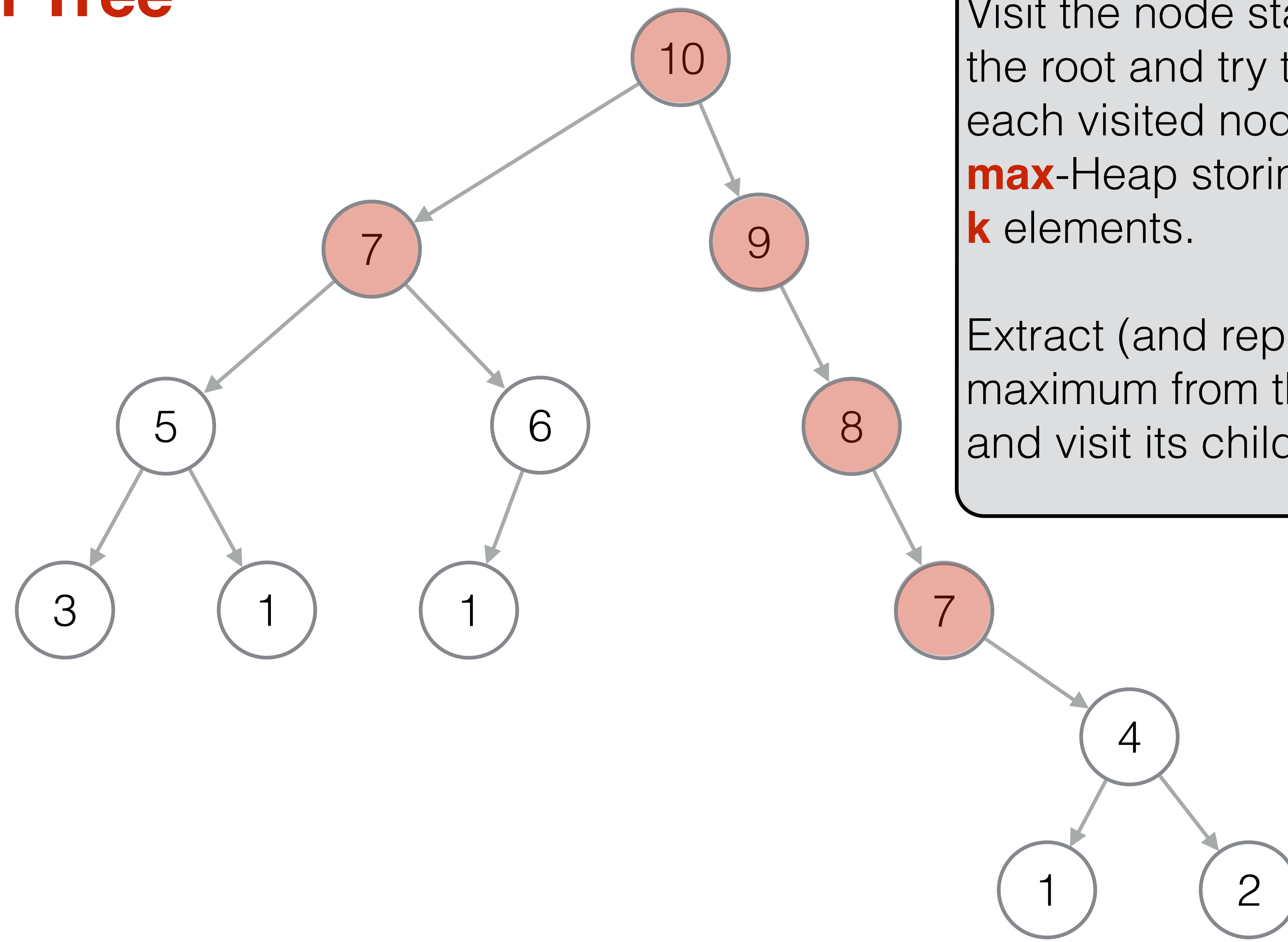
Cartesian Tree



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k=4

max-Heap	**Results**
7	10
7	9
	8

S

…	3	5	1	7	1	6	10	9	8	7	1	4	2	…

Finding Top-k

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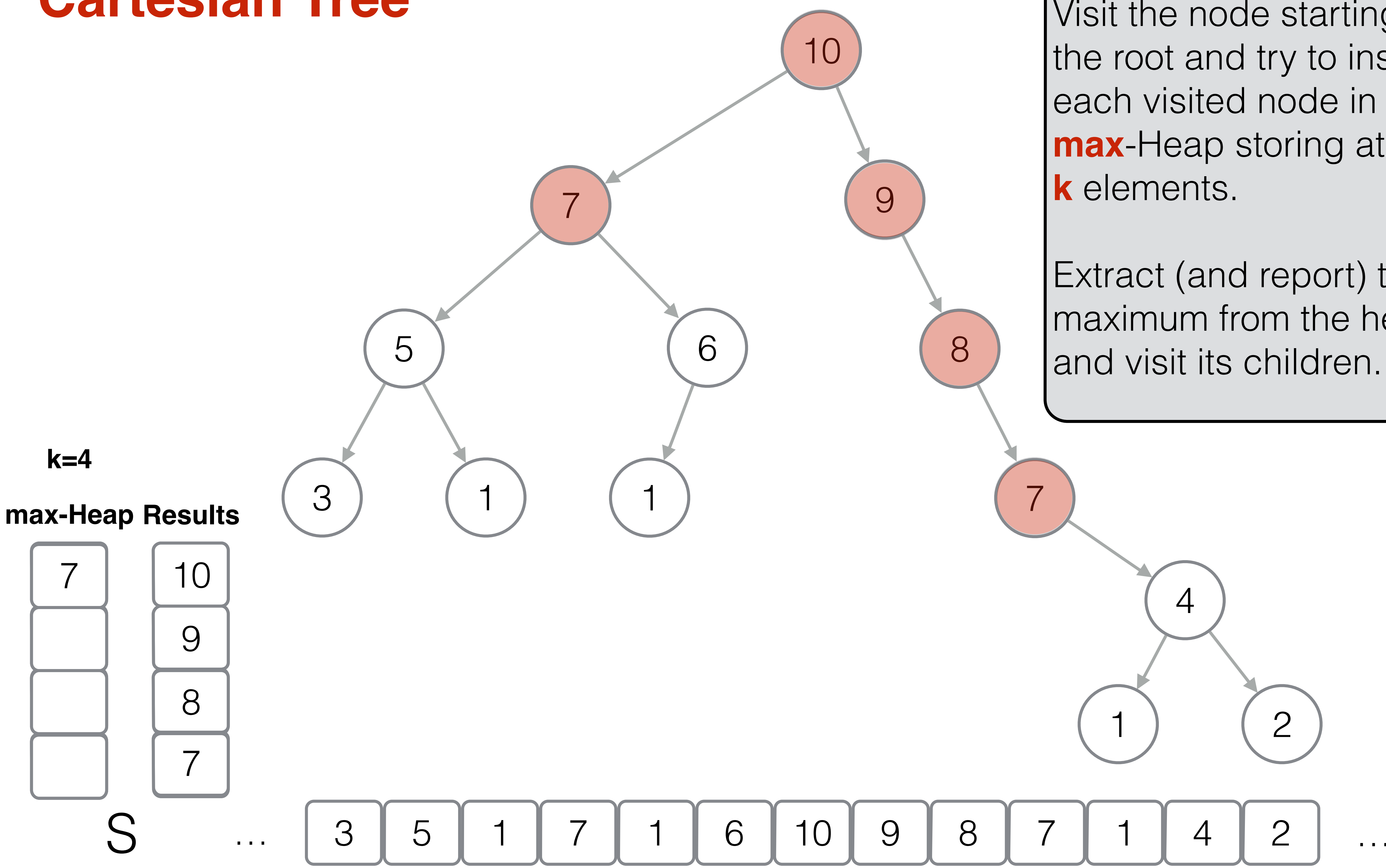

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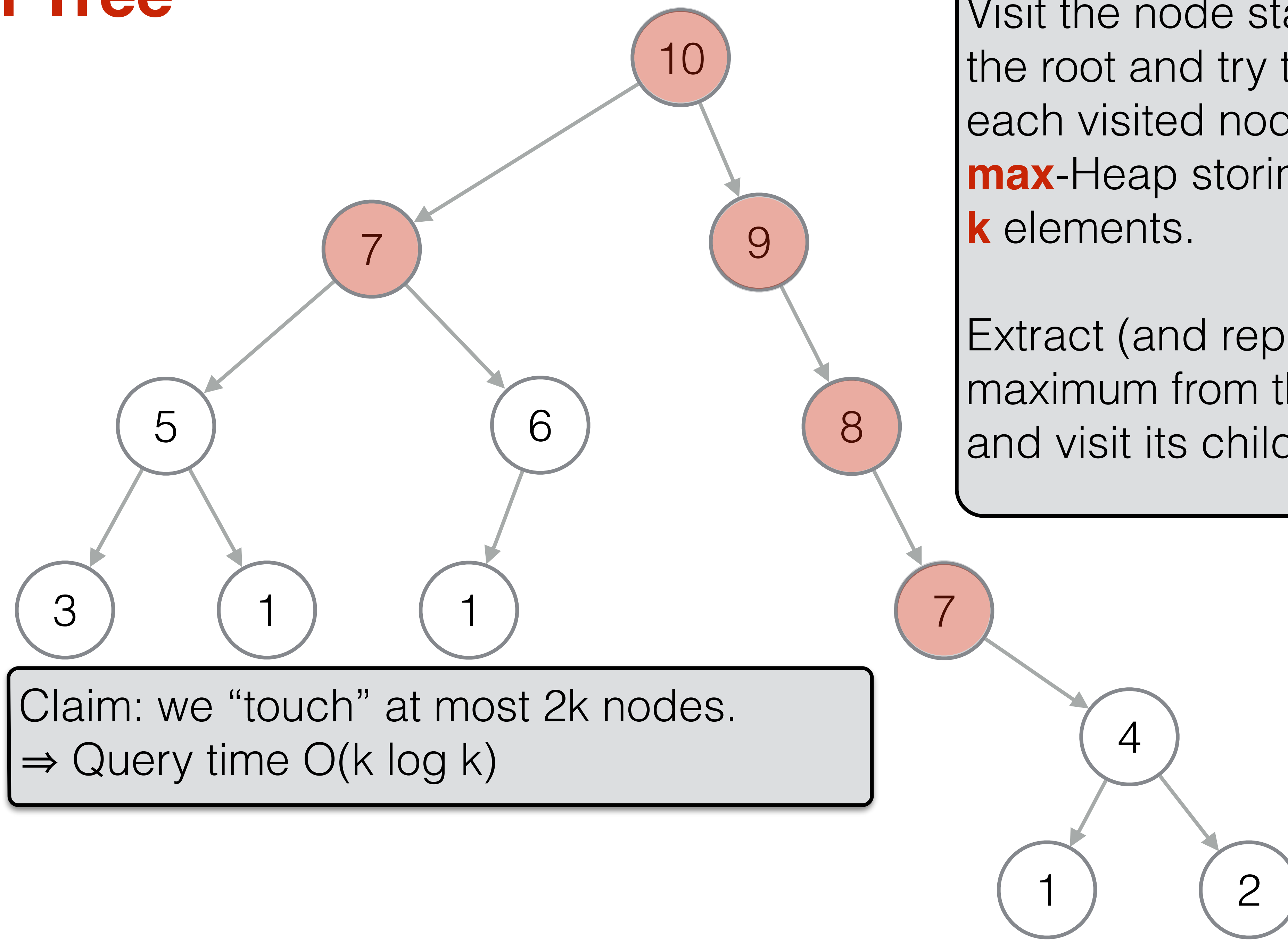


Claim: we "touch" at most 2k nodes.
⇒ Query time O(k log k)

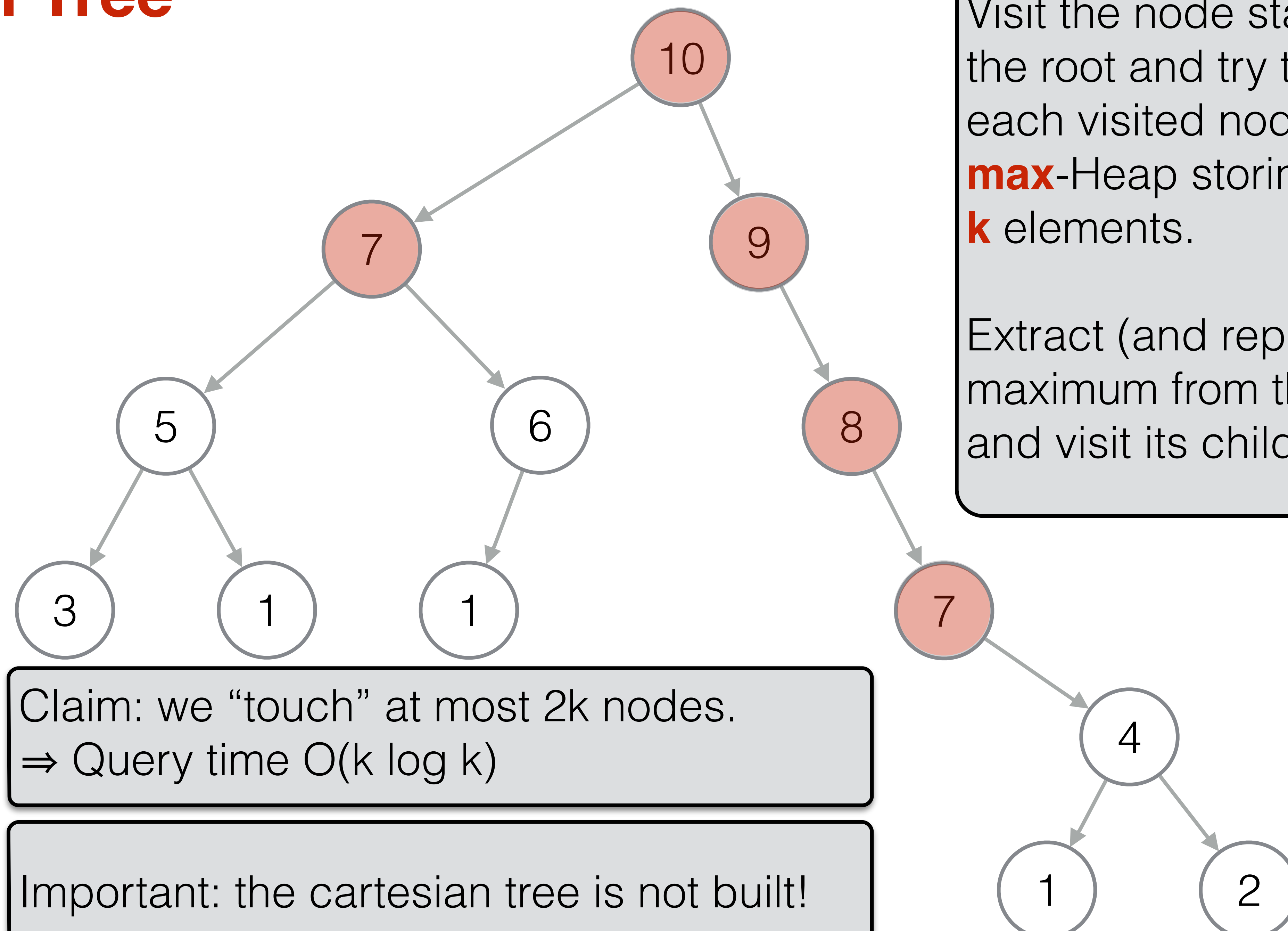
k=4

max-Heap	**Results**
7	10
	9
	8
	7

S … | 3 | 5 | 1 | 7 | 1 | 6 | 10 | 9 | 8 | 7 | 1 | 4 | 2 | …

Finding Top-k

Cartesian Tree



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Important: the cartesian tree is not built!

k=4

max-Heap	**Results**
7	10
	9
	8
	7

S

…	3	5	1	7	1	6	10	9	8	7	1	4	2	…

Finding Top-k

Cartesian Tree

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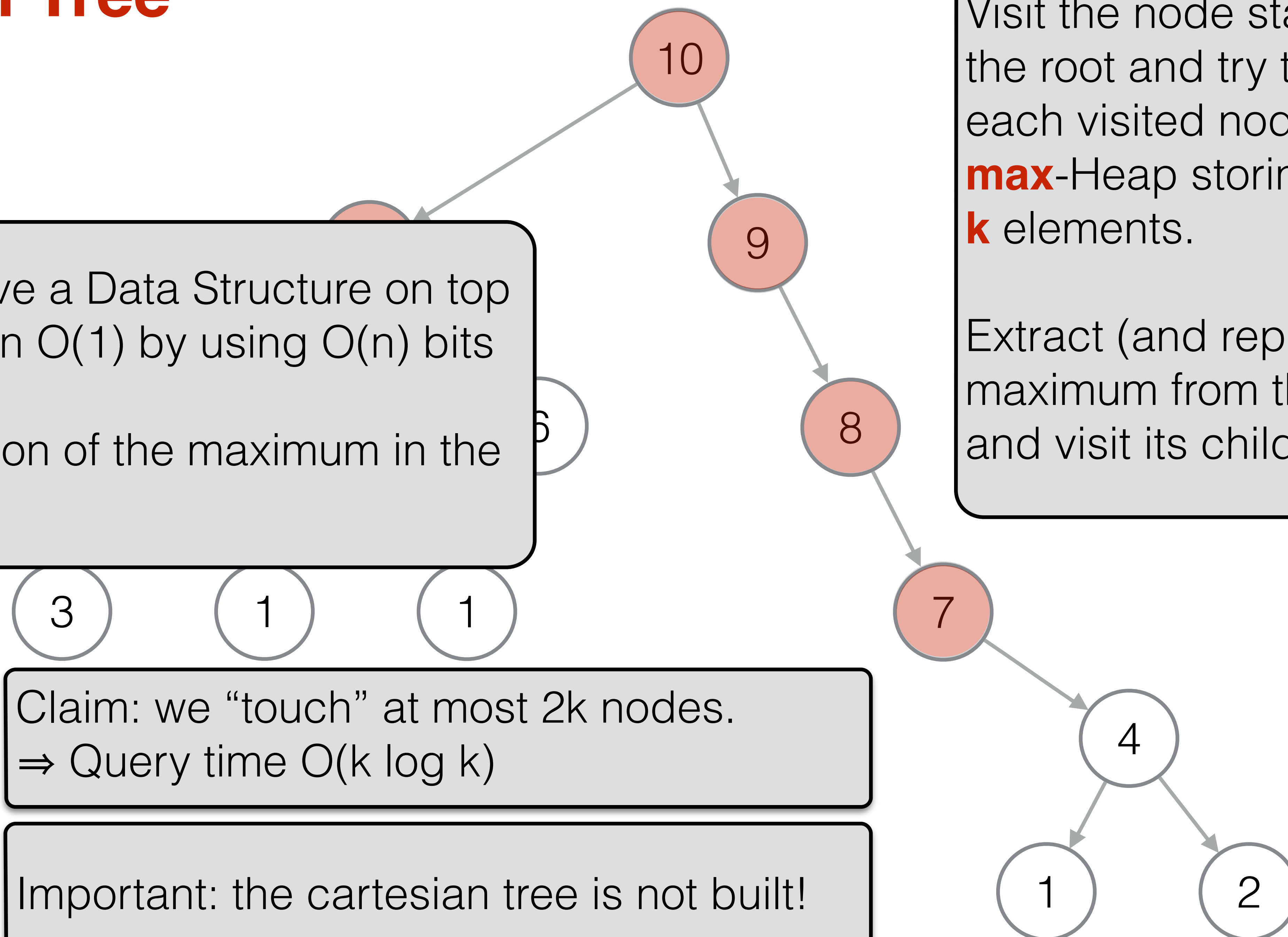

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max-Heap Results

max-Heap	Results
7	10
	9
	8
	7

S … 3 5 1 7 1 6 10 9 8 7 1 4 2 …

Range Maximum Query (1)

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (1)

Space: $O(n^2 \log n)$ bits
Query time: $O(1)$

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (1)

Space: $O(n^2 \log n)$ bits
Query time: $O(1)$

Precompute the answer to any possible query.

There are $O(n^2)$ distinct queries!

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (1)

Space: $O(n^2 \log n)$ bits
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$M[i,j] = RMQ(i,j)$

Precompute the answer to any possible query.

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	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

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M	0	1	2	3	4	5	6	7	8	9	10	11
0												
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

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M

	0	1	2	3	4	5	6	7	8	9	10	11
0												
1												
2						3						
3												
4												
5												
6												
7												
8												
9												
10												
11												

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum in a interval is the max between the maxima of any its subintervals

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum in a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

There are $O(\log n)$ possible intervals starting at any position i.

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

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$M[i,j] = RMQ(i,i+2^j)$

M

	0	1	2	3	4
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

	0	1	2	3	4	5	6	7	8	9	10	11
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	0	1	2	3	4
0					
1				?	
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

	0	1	2	3	4	5	6	7	8	9	10	11
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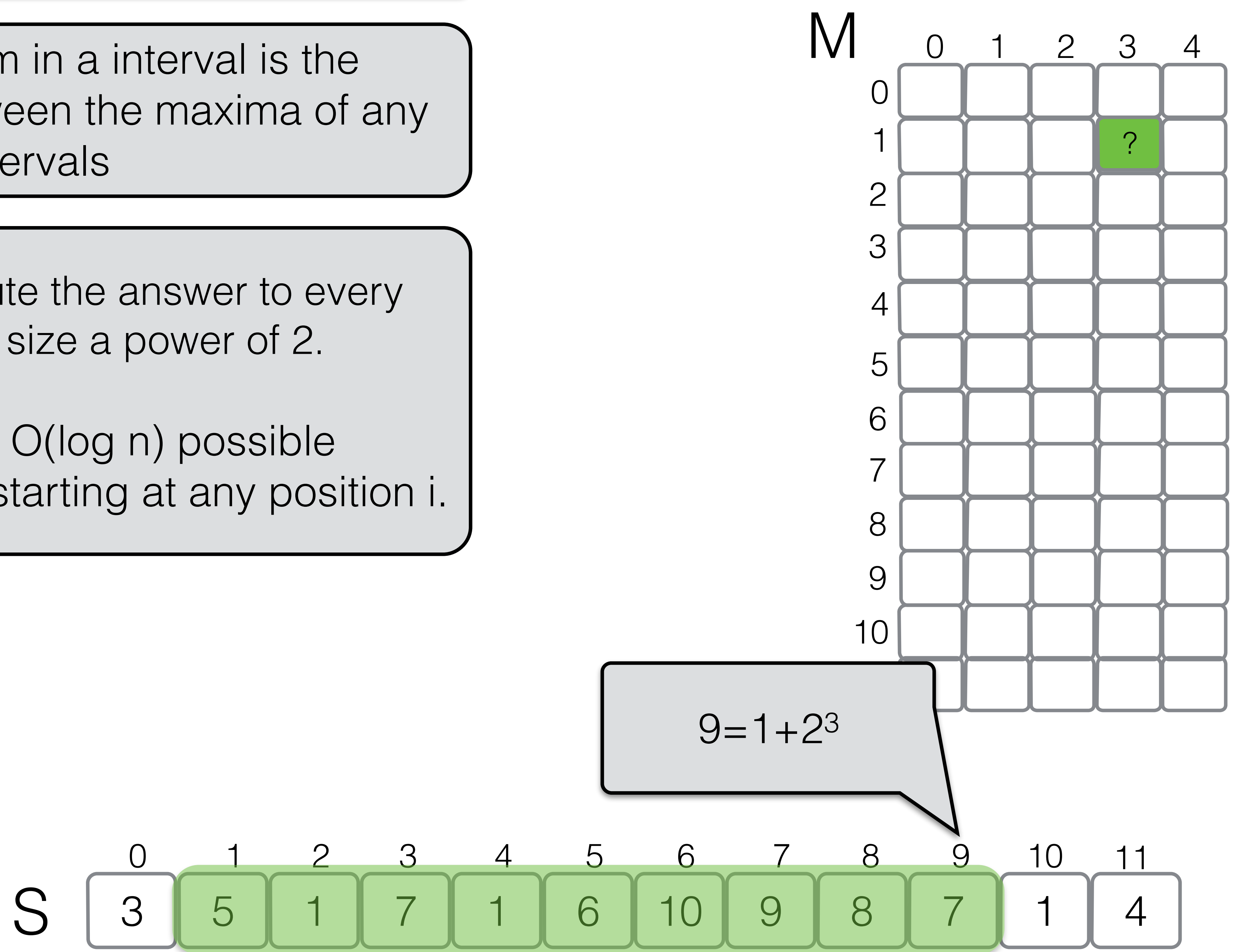
$$M[i,j] = RMQ(i,i+2^j)$$



Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum in a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

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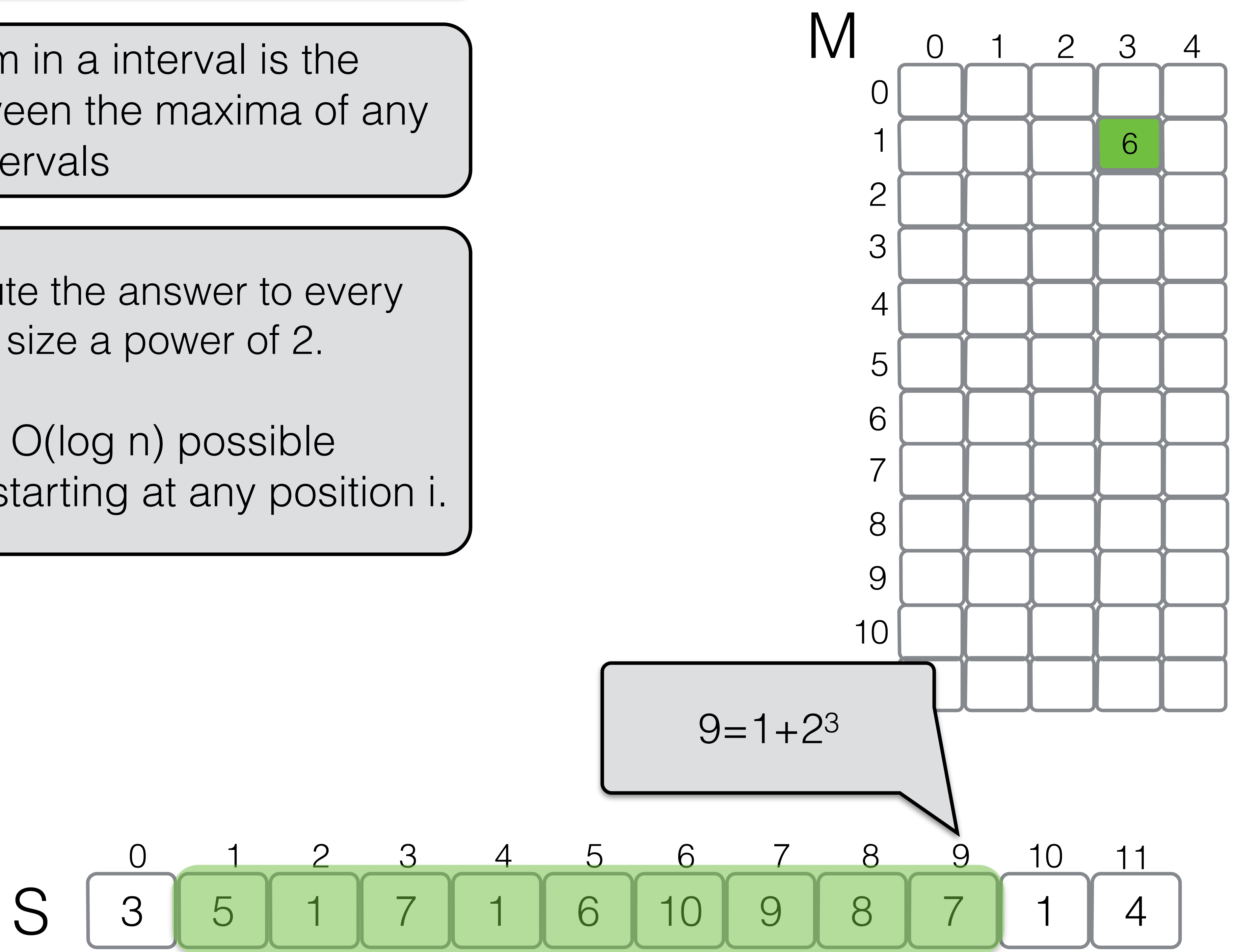

Range Maximum Query (2)

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Maximum of a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

There are $O(\log n)$ possible intervals starting at any position i.

$$M[i,j] = RMQ(i,i+2^j)$$

M

	0	1	2	3	4
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

S

0	1	2	3	4	5	6	7	8	9	10	11
3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
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Maximum of a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

There are $O(\log n)$ possible intervals starting at any position i.

RMQ(1,7) =

$$M[i,j] = RMQ(i,i+2^j)$$

M

	0	1	2	3	4
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

S

0	1	2	3	4	5	6	7	8	9	10	11
3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

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M

	0	1	2	3	4
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

$RMQ(1,7) = \text{argmax}(S[M[1,1+2^2]], S[M[7-2^2,7]]) = 6$

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

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M

	0	1	2	3	4
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

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	0	1	2	3	4	5	6	7	8	9	10	11
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M

	0	1	2	3	4
0					
1			3		
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

$RMQ(1,7) = \text{argmax}(S[M[1,1+2^2]], S[M[7-2^2,7]]) = 6$

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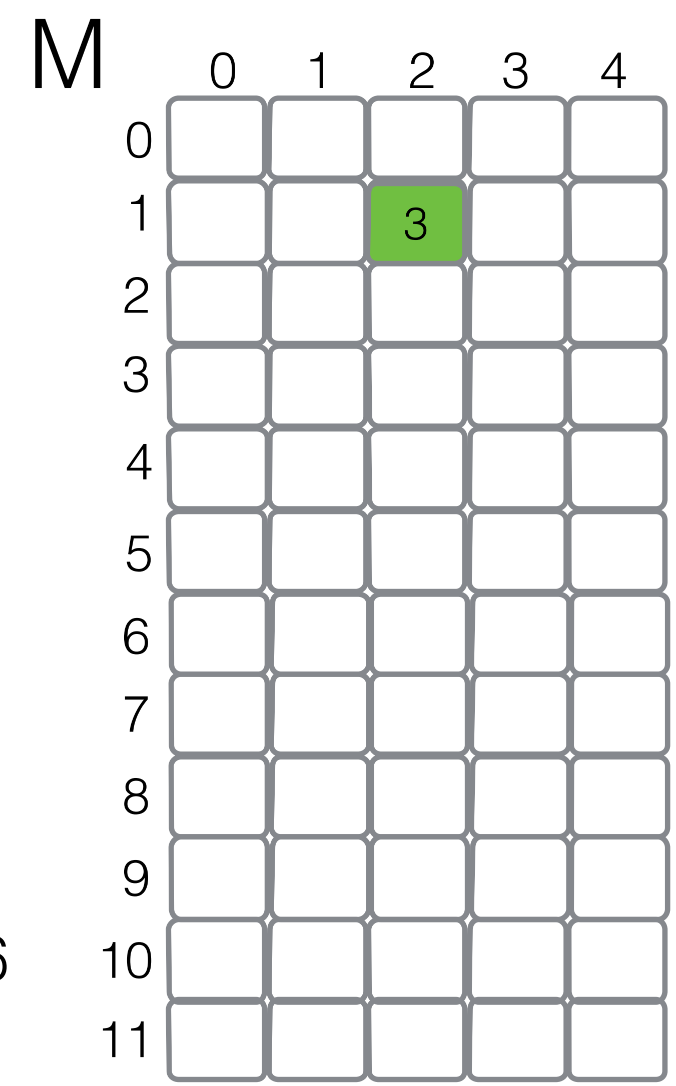
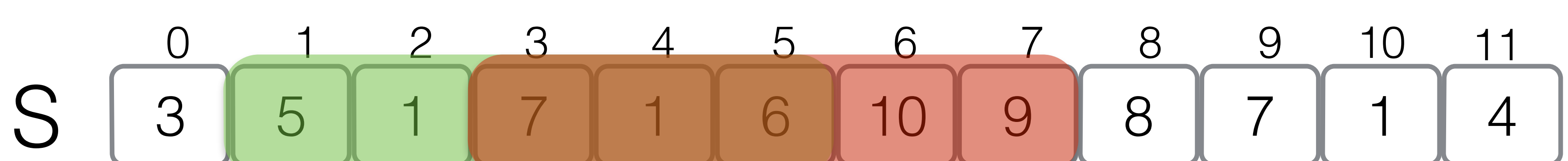


$$RMQ(1,7) = \text{argmax}(S[M[1,1+2^2]], S[M[7-2^2,7]]) = 6$$



Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum of a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

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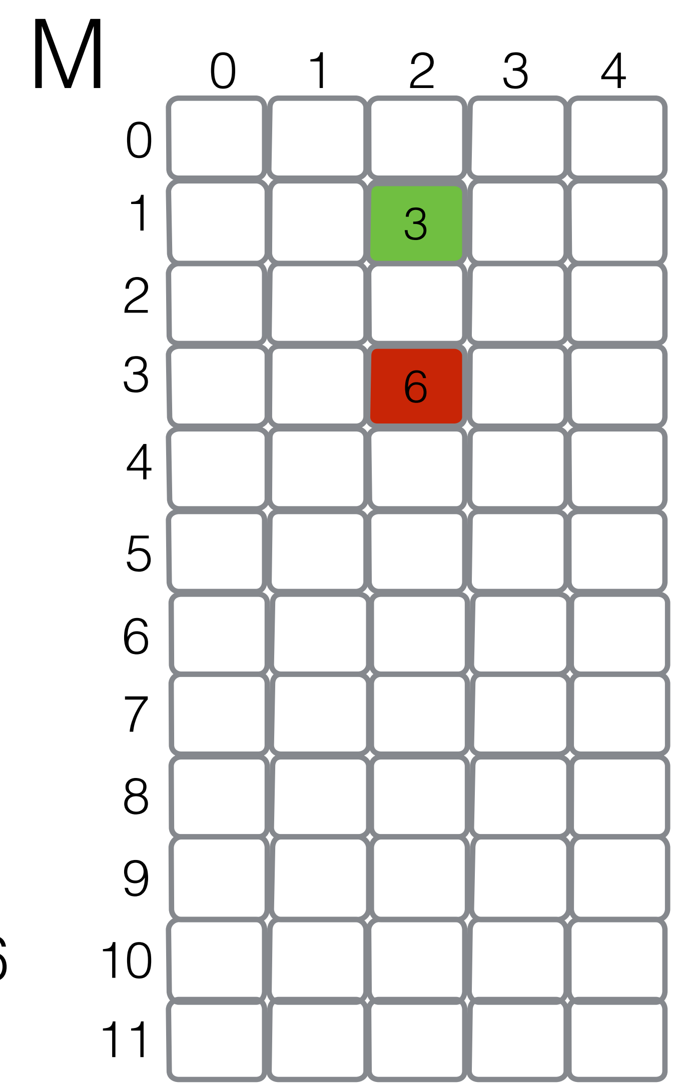
$$M[i,j] = RMQ(i,i+2^j)$$



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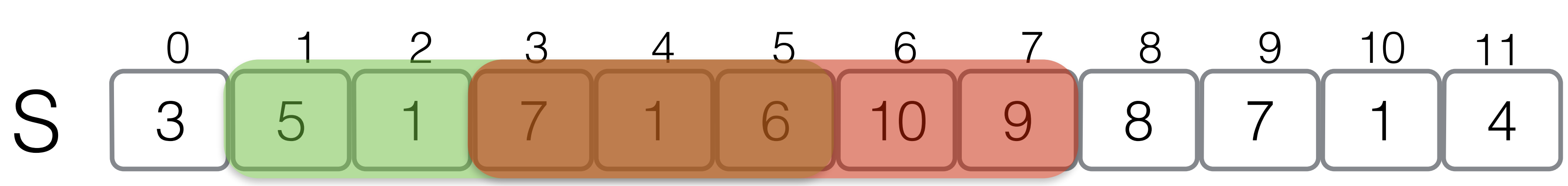

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

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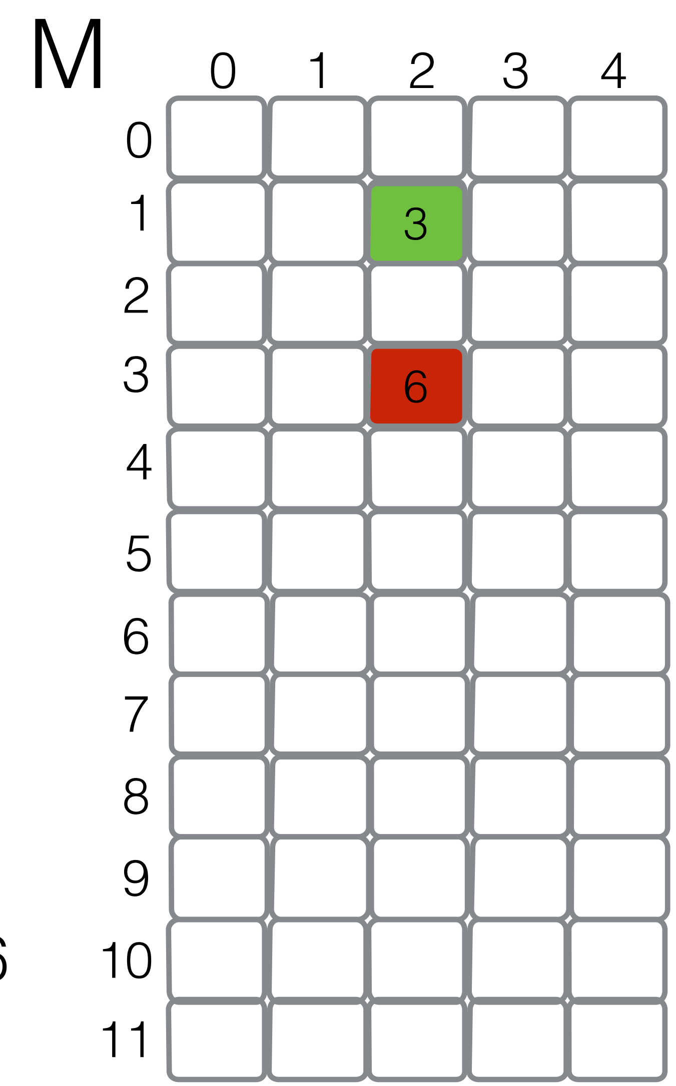
There are $O(\log n)$ possible intervals starting at any position i.

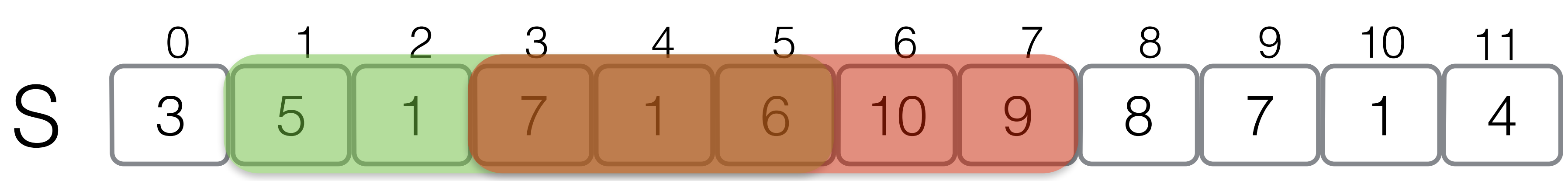
$$M[i,j] = RMQ(i,i+2^j)$$



$$RMQ(1,7) = \text{argmax}(S[M[1,1+2^2]], S[M[7-2^2,7]]) = 6$$

$$RMQ(i,j) = \text{argmax}(S[M[i,i+2^{len}]], S[M[j-2^{len},j]])$$

where $len = \lfloor \log (j-i+1) \rfloor$



Range Maximum Query (3)

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

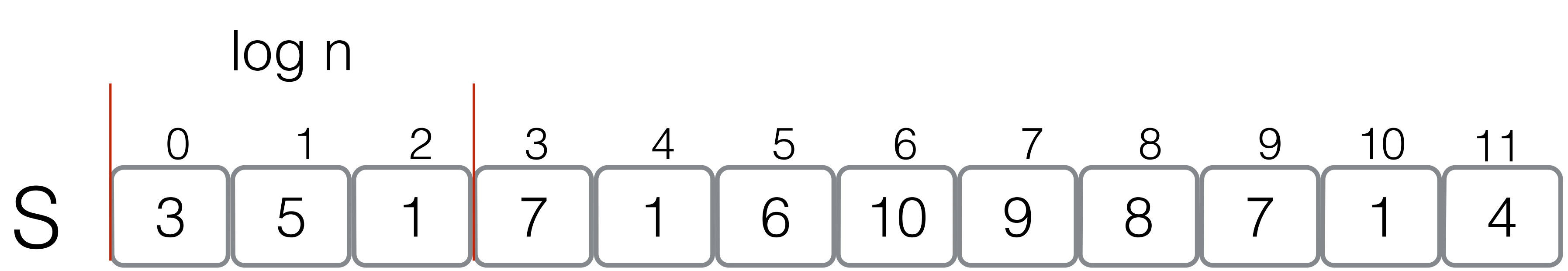
Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)



Range Maximum Query (3)

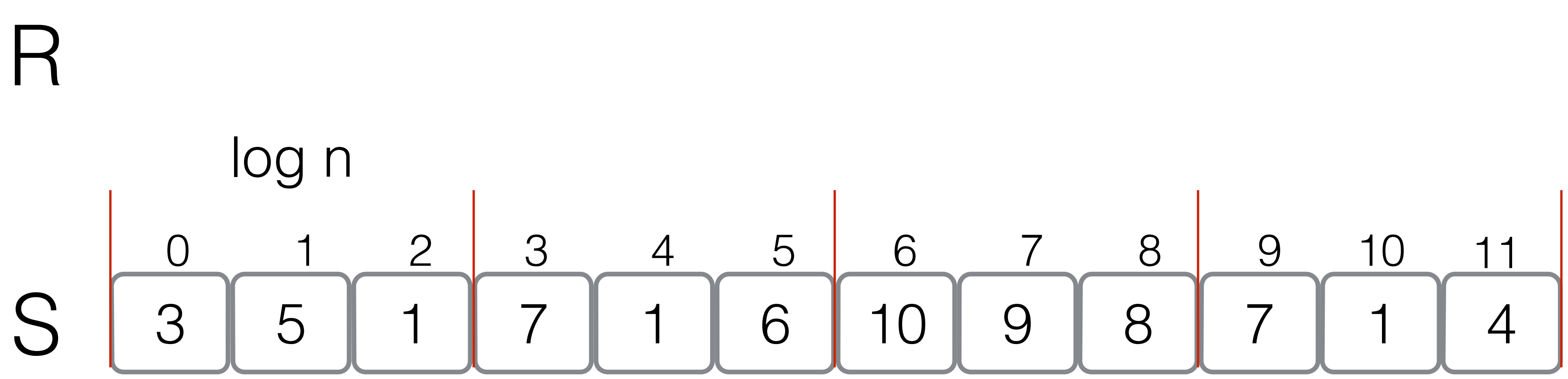
Space: O(n log n) bits
Query time: O(log n)

log n

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

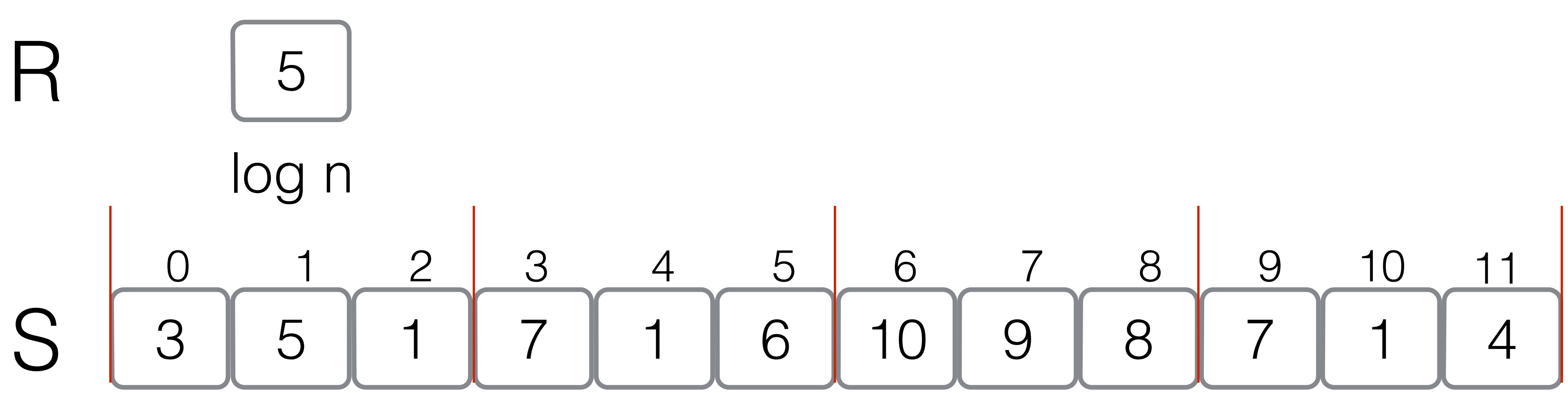
Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)



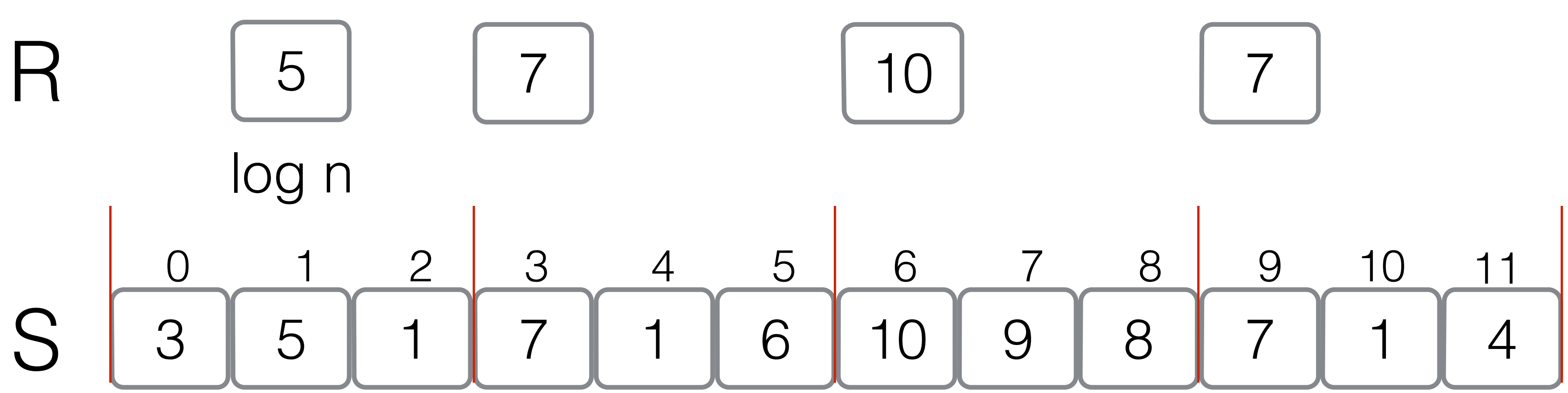
Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)



Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

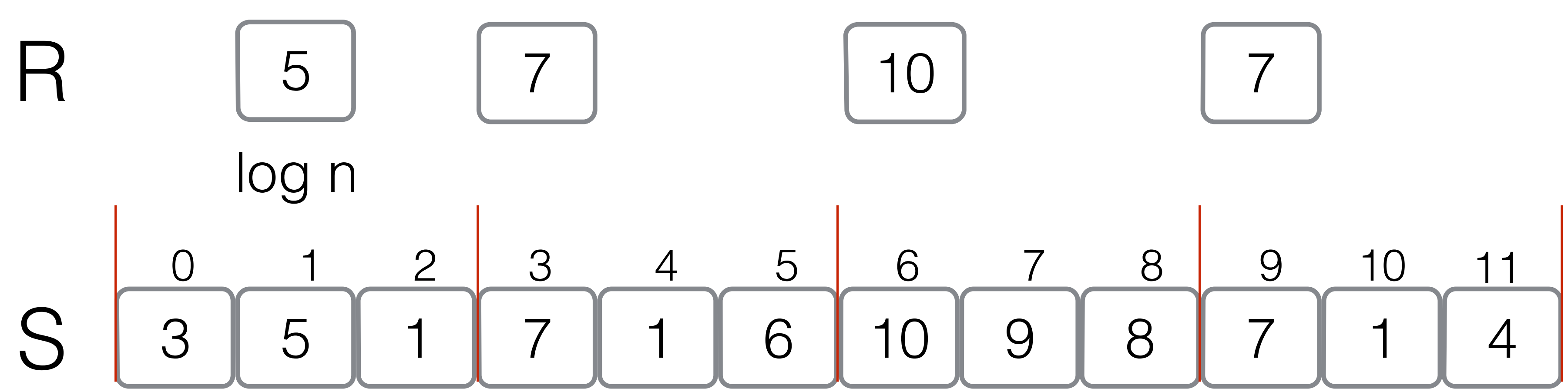


Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

Use the previous solution on R!

Space: ? bits
Query time: O(1)

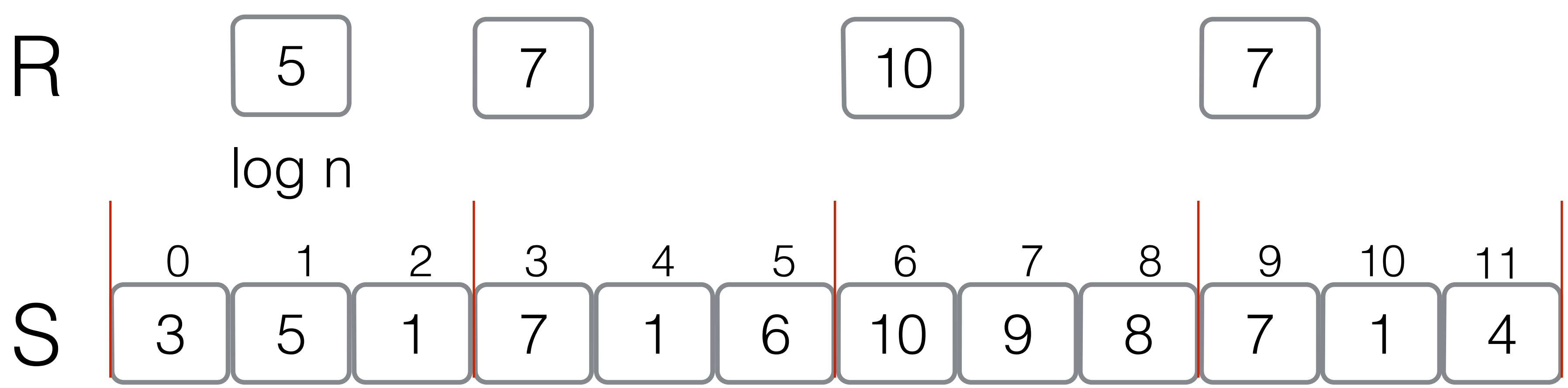


Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

Use the previous solution on R!

Space: O(n log n) bits
Query time: O(1)



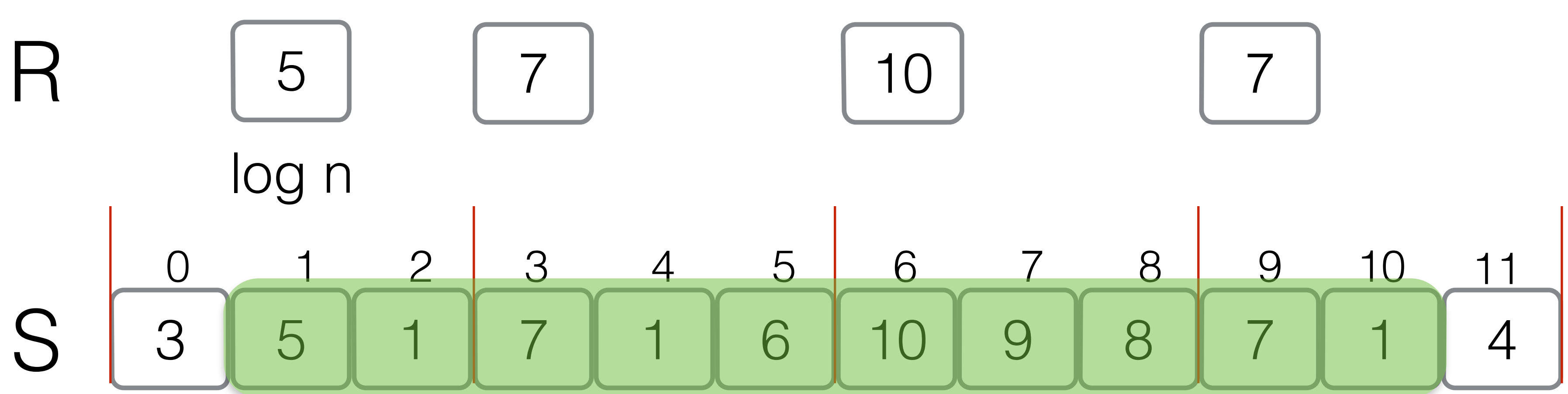
Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

Use the previous solution on R!

Space: O(n log n) bits
Query time: O(1)

RMQ(1,10) = ?



Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

Use the previous solution on R!

Space: O(n log n) bits
Query time: O(1)

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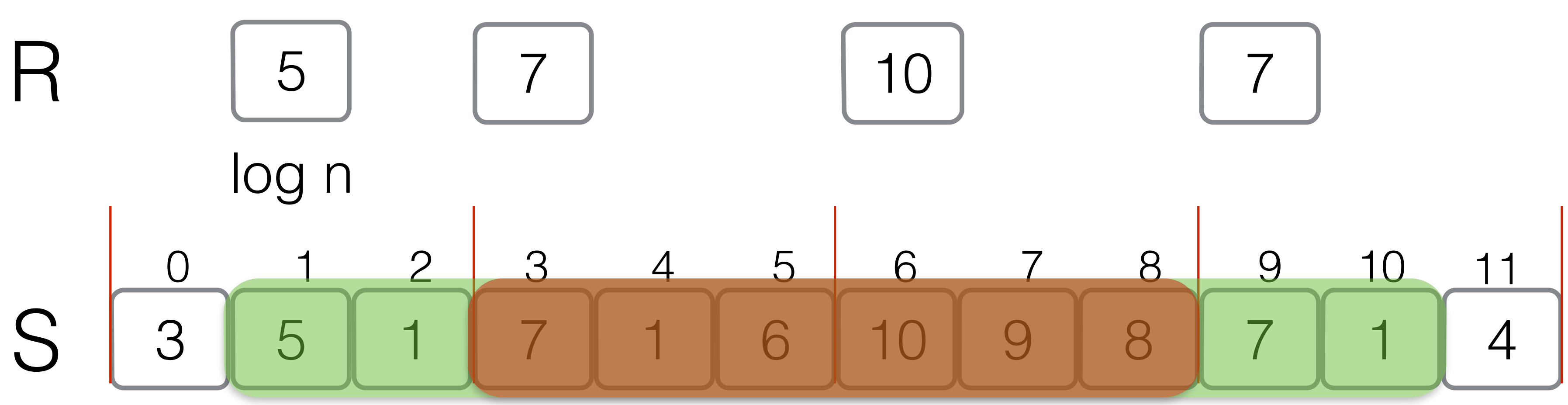

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Space: O(n log n) bits
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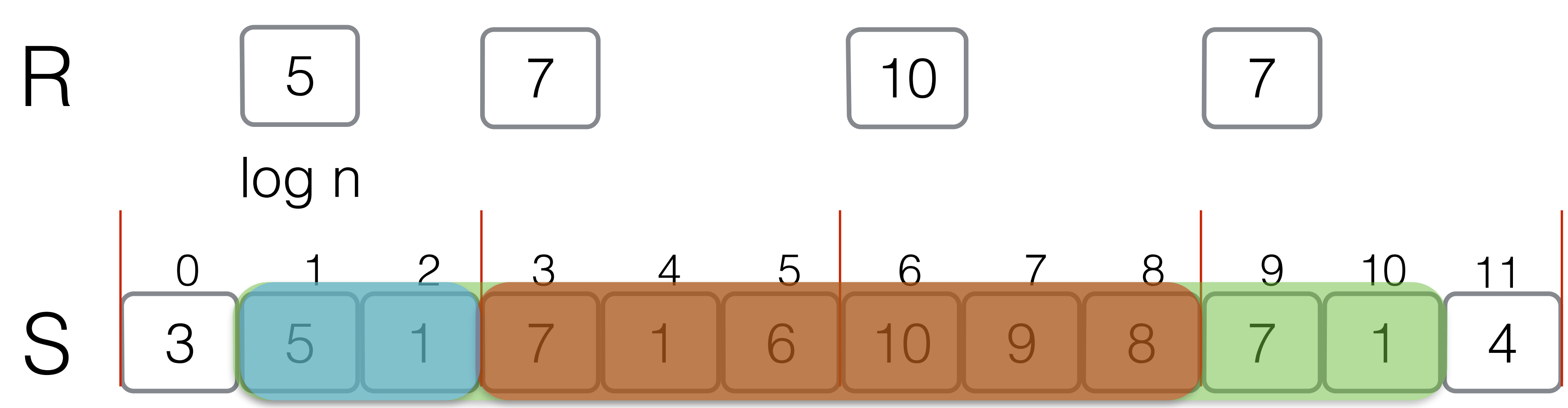

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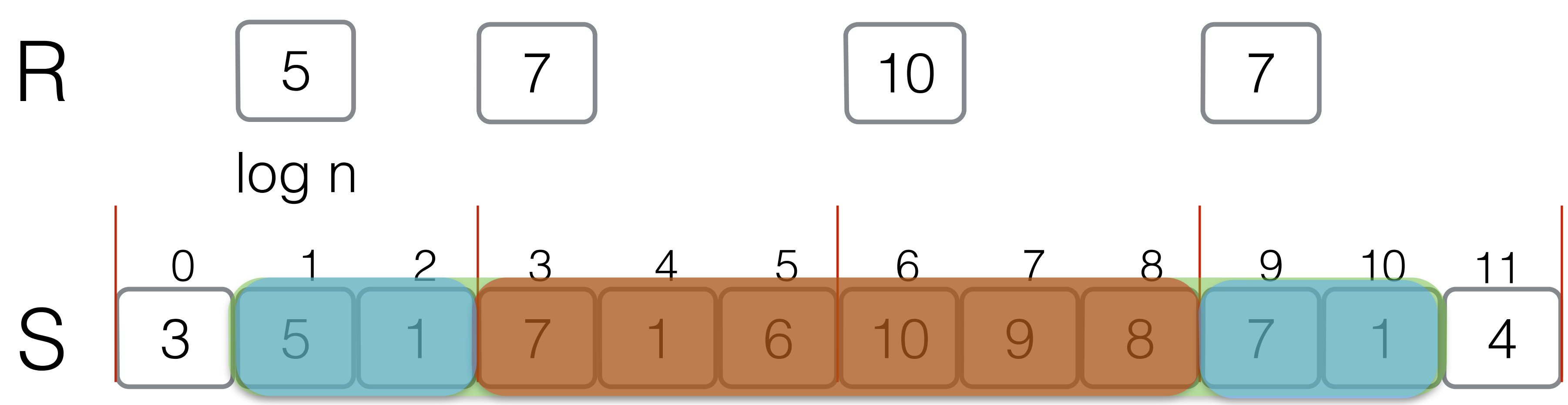

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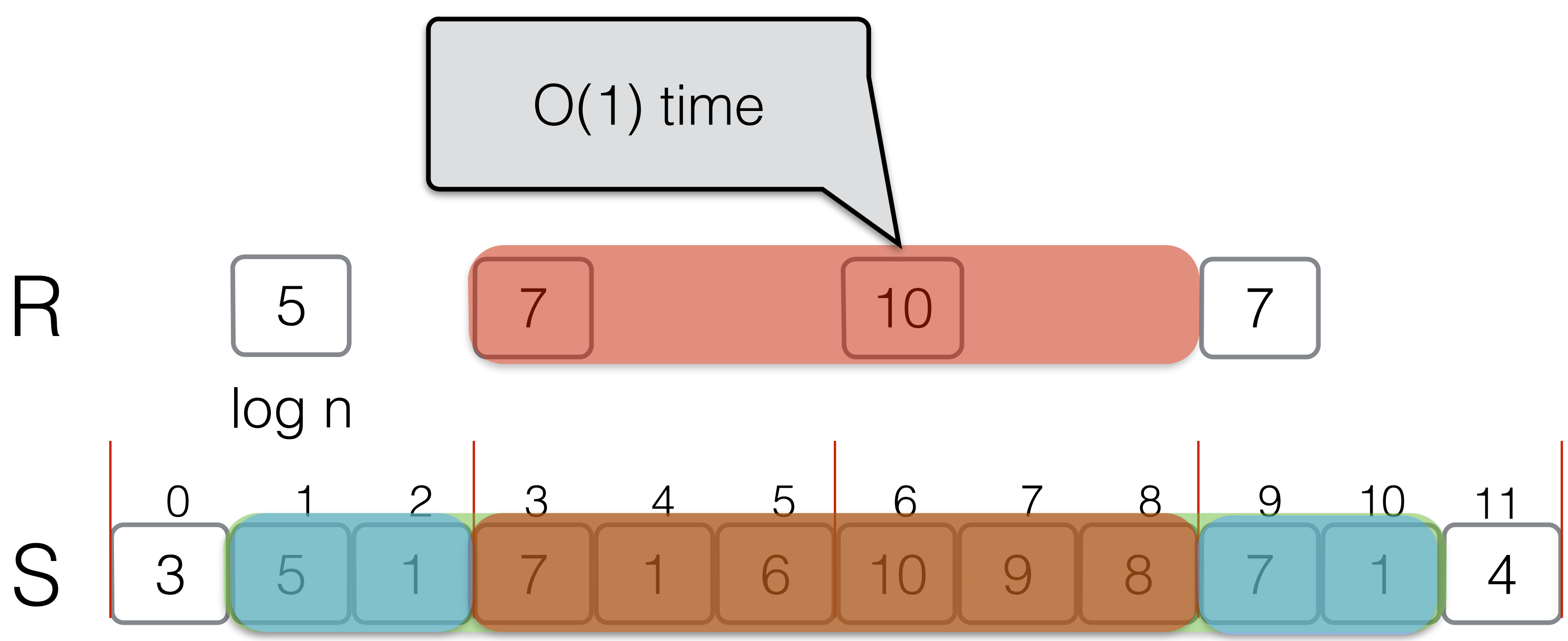

Range Maximum Query (3)

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Space: O(n log n) bits
Query time: O(1)

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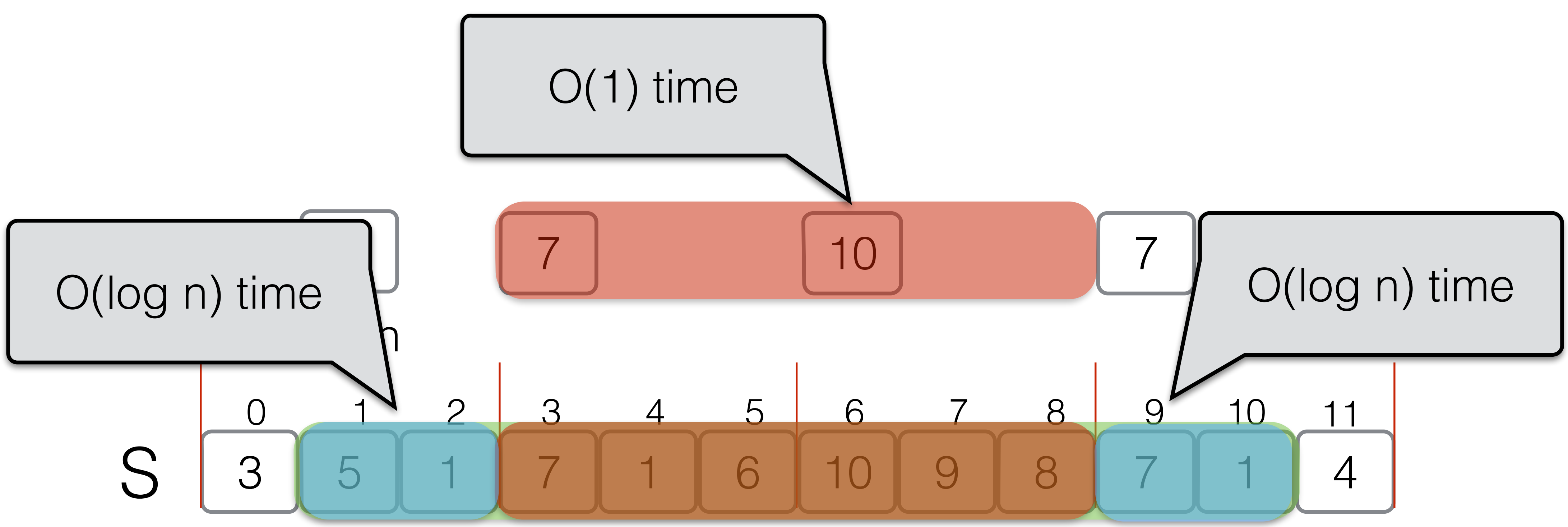

Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

Space: O(n log n) bits
Query time: O(1)

Use the previous solution on R!

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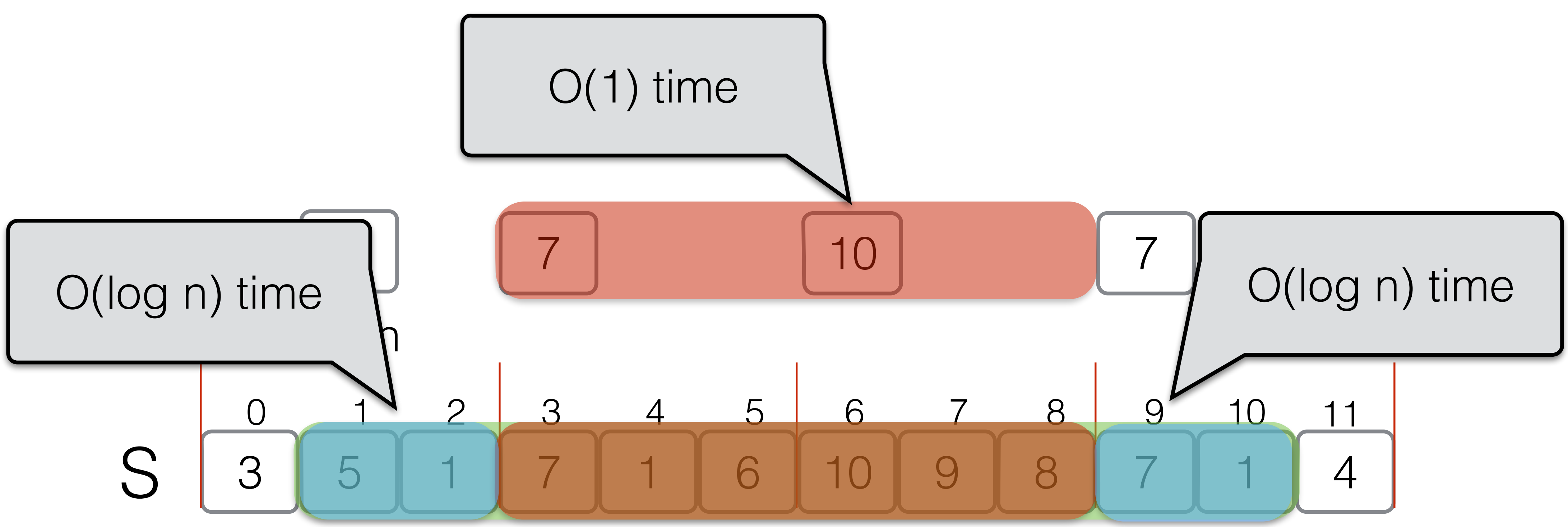
RMQ(1,10) = ?



Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

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Query time: O(1)

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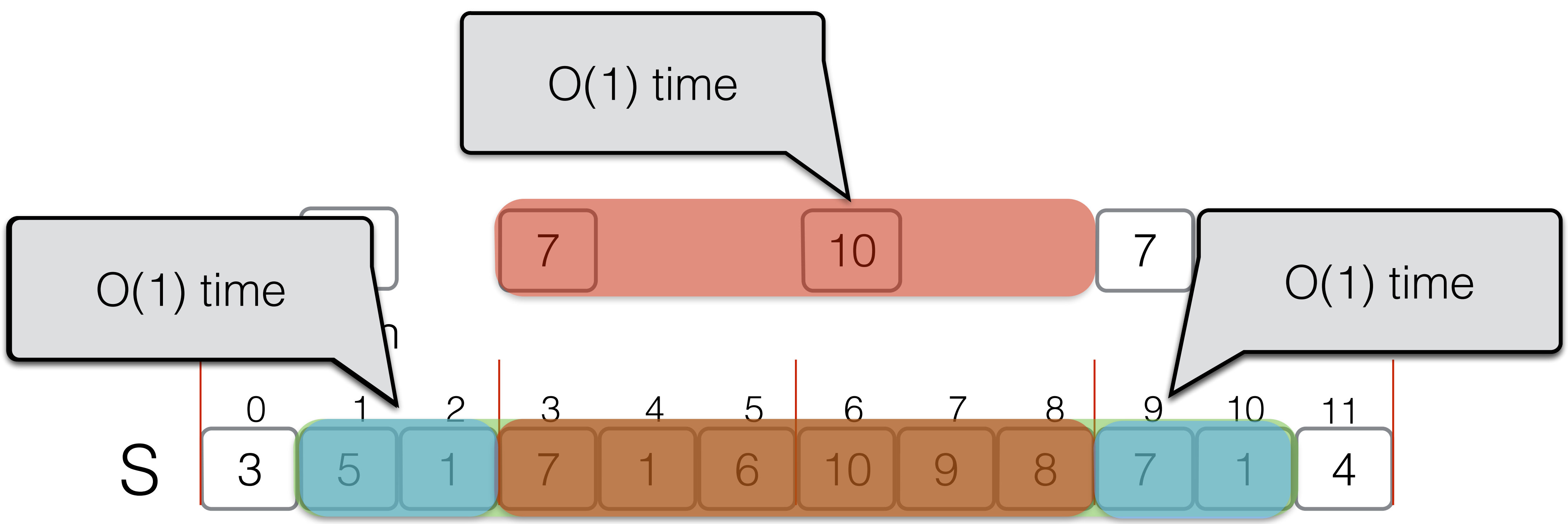
Space: O(n log n) bits
Query time: O(1)

RMQ(1,10) = ?



Range Maximum Query (3)

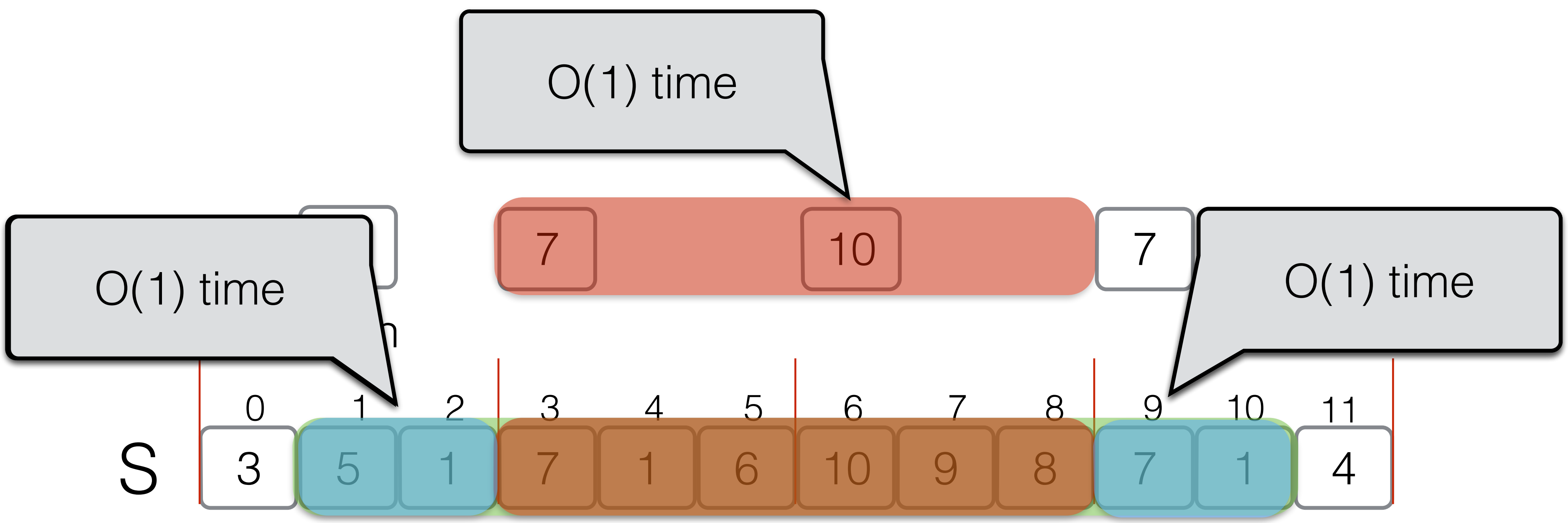
Space: O(n log n) bits
Query time: O(log n)

Space: O(n ~~log~~ n) bits
Query time: O(1)

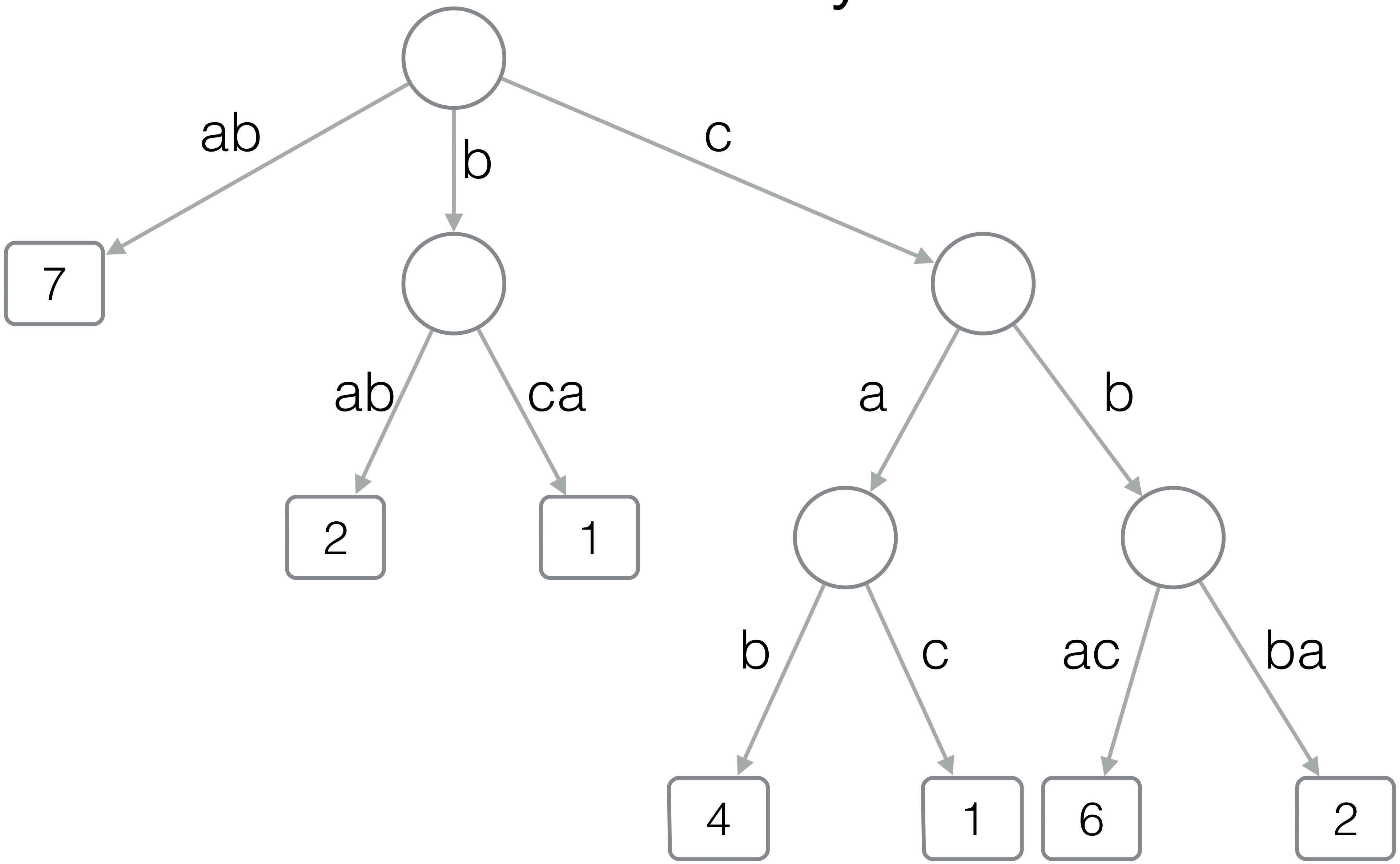
Use the previous solution on R!

Space: O(n log n) bits
Query time: O(1)

RMQ(1,10) = ?



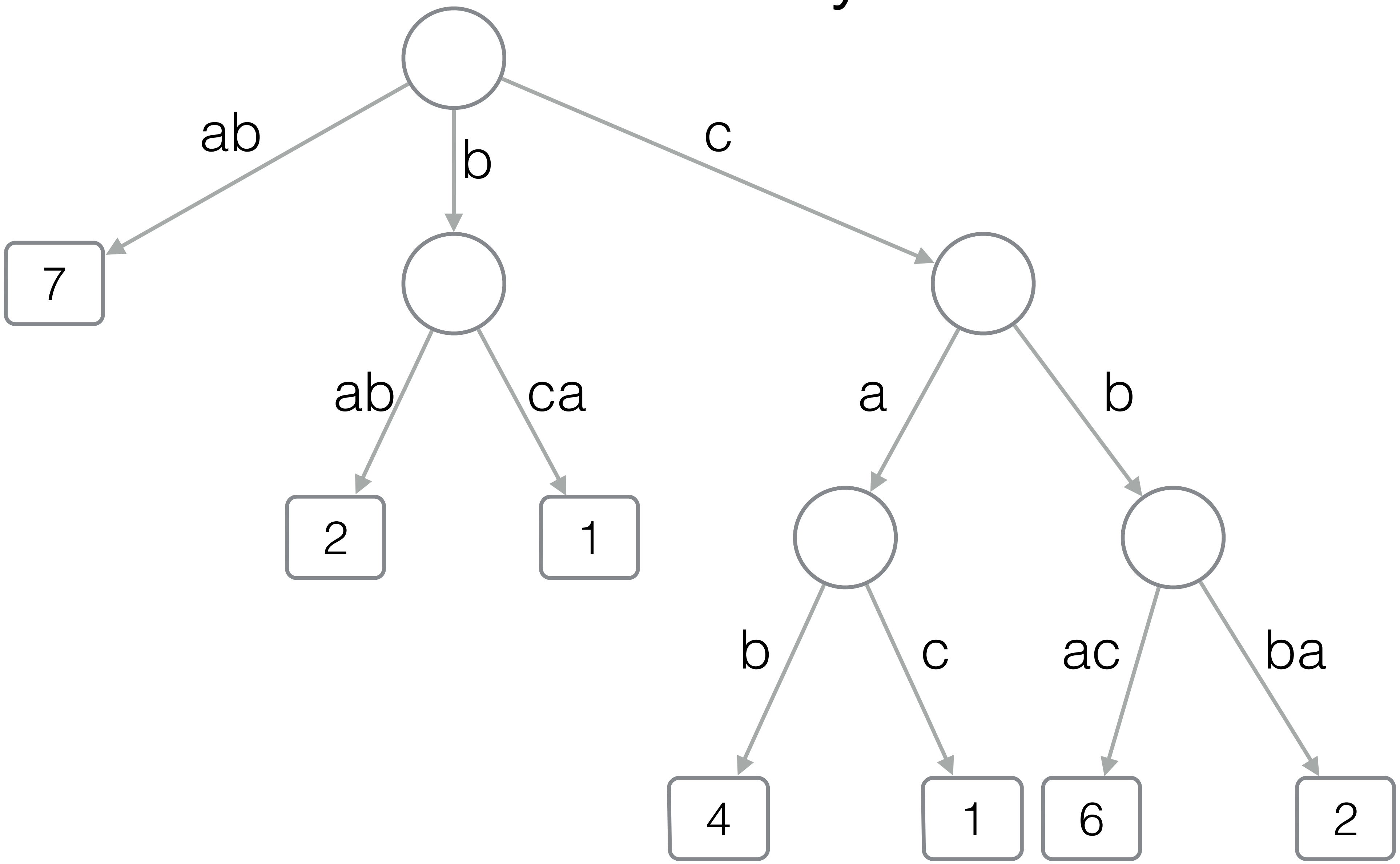
Summary



D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Summary

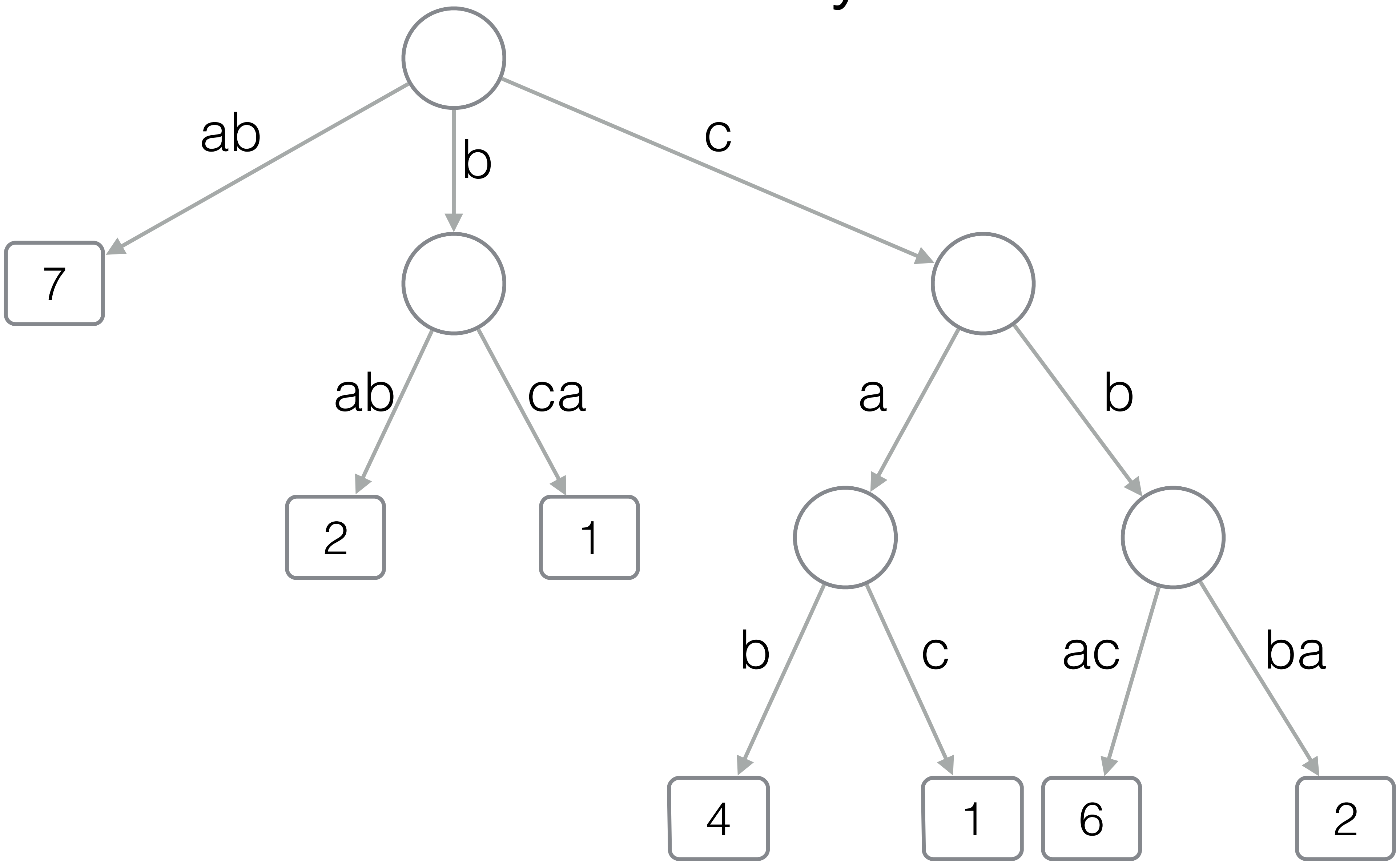


Find the node "prefixed" by P

D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Summary



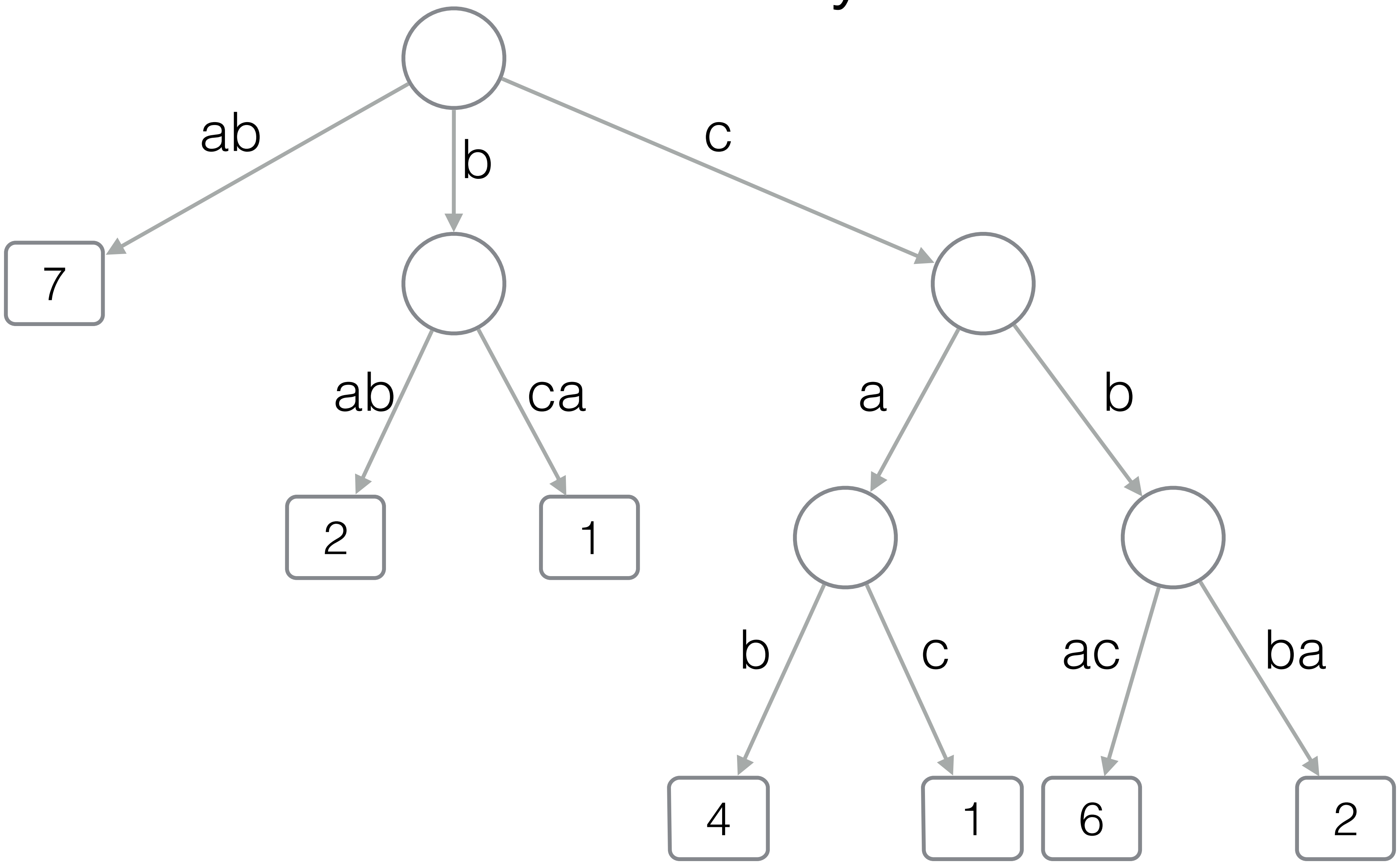
Find the node "prefixed" by P

O(|P|) time

D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Summary

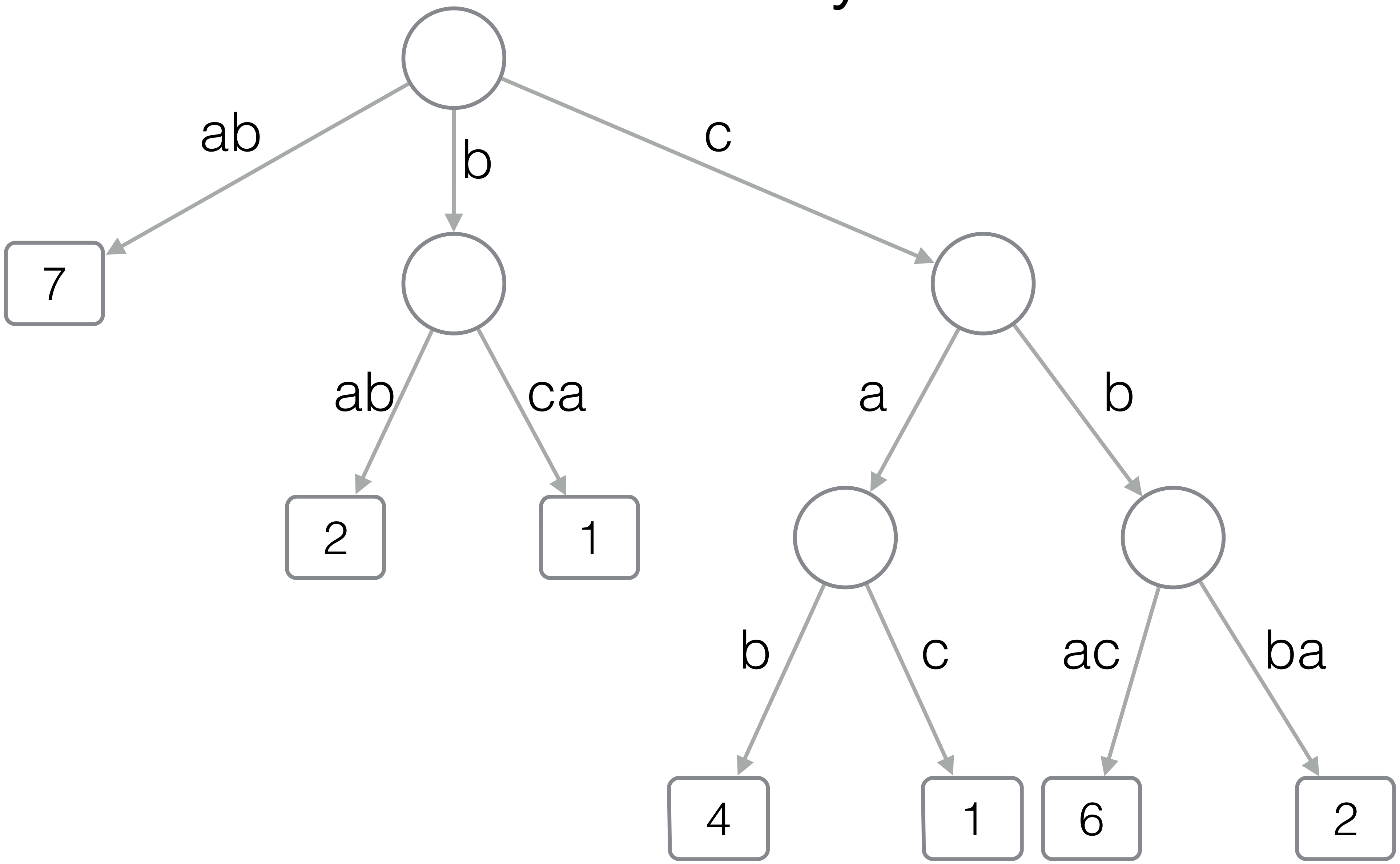


Find the node "prefixed" by P	O(|P|) time	O(m log σ + n log m) bits

D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Summary



Find the node "prefixed" by P

O(|P|) time

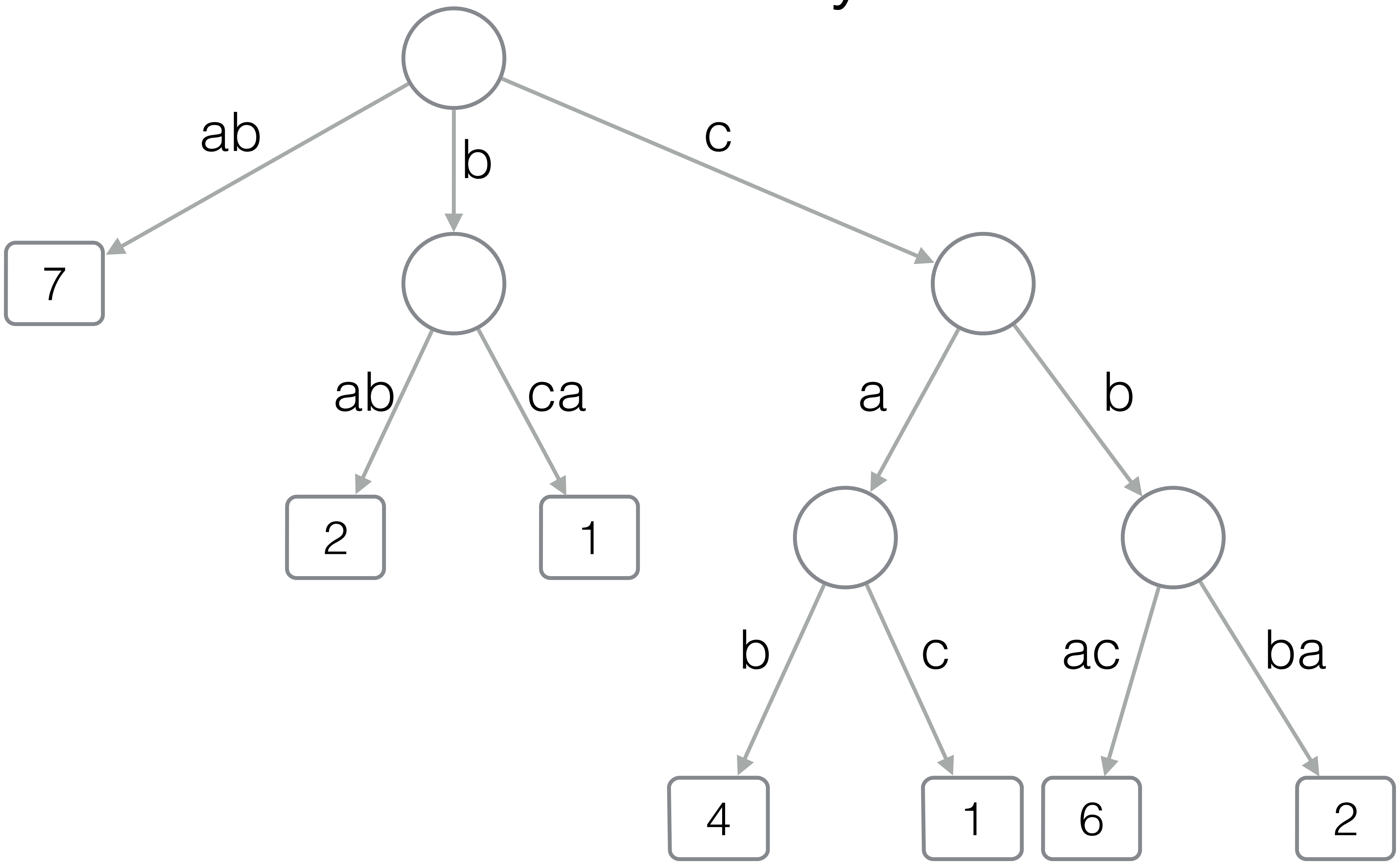
O(m log σ + n log m) bits

Compute the top-k strings

a (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Summary



Find the node “prefixed” by P

O(|P|) time

O(m log σ + n log m) bits

Compute the top-k strings

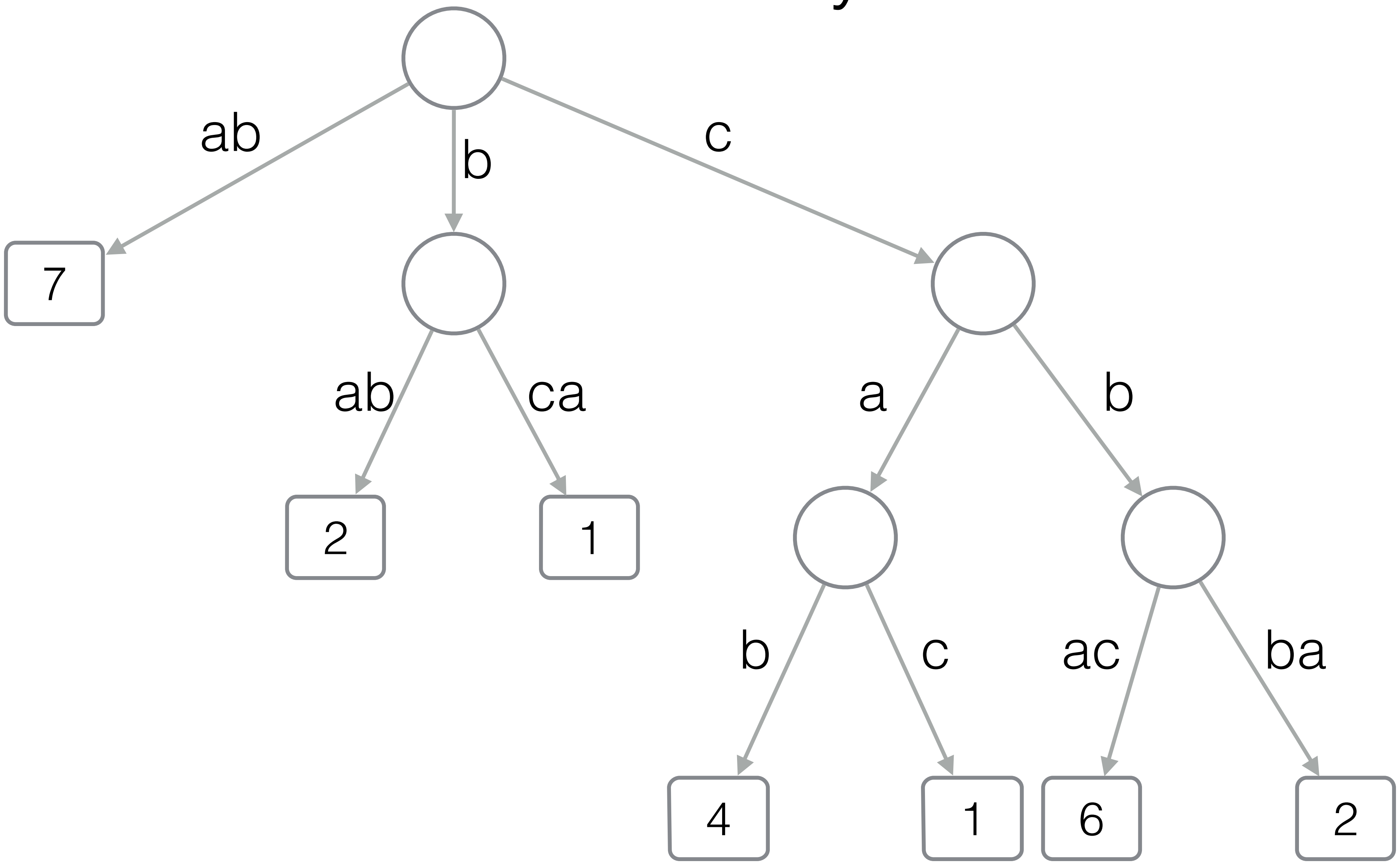
a

O(k log k) time

cbac (6), cbba (2) }

n = |D|, m total length of strings in D

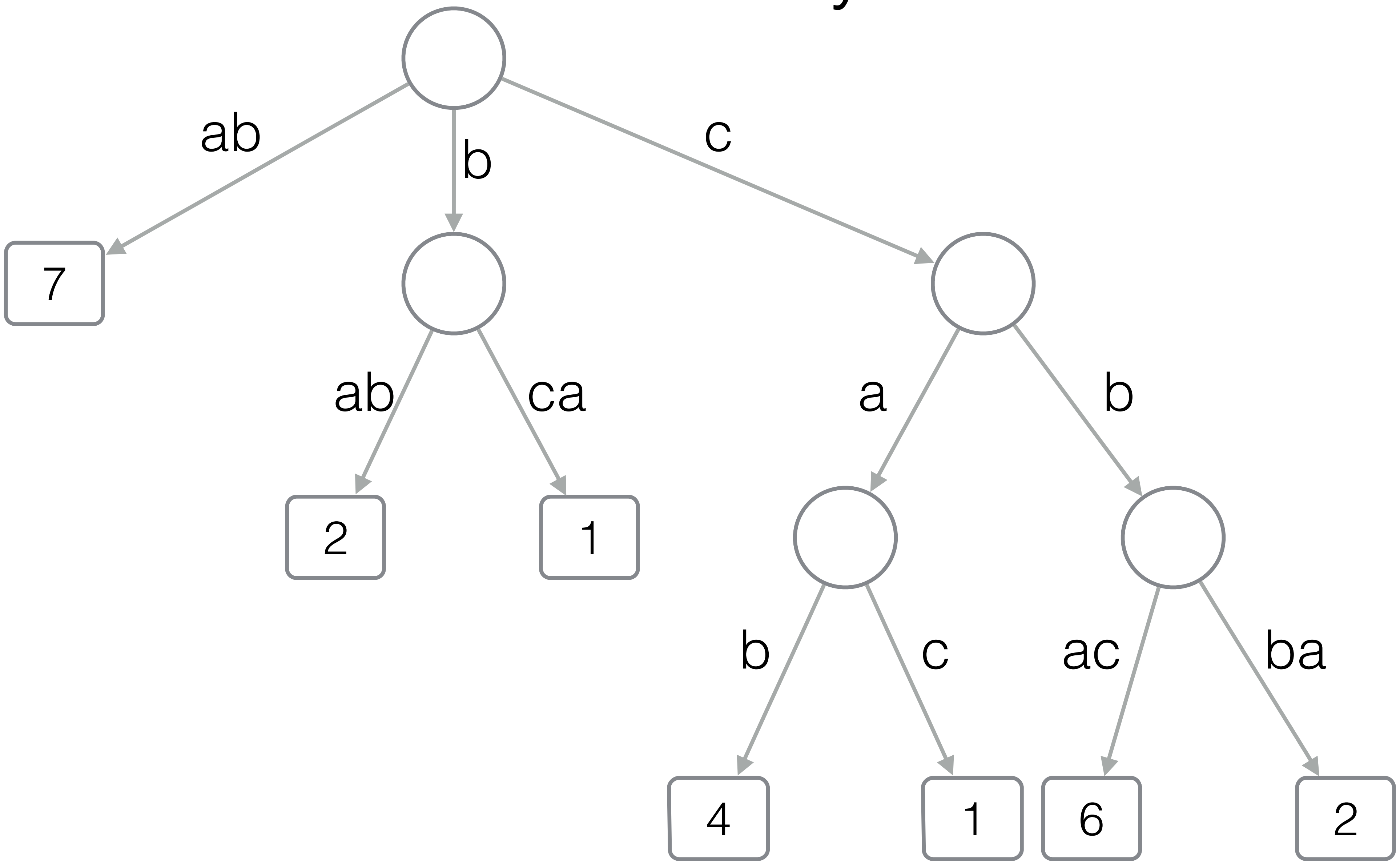
Summary



Find the node “prefixed” by P	O(|P|) time	O(m log σ + n log m) bits
Compute the top-k strings	O(k log k) time	O(n) bits

n = |D|, m total length of strings in D

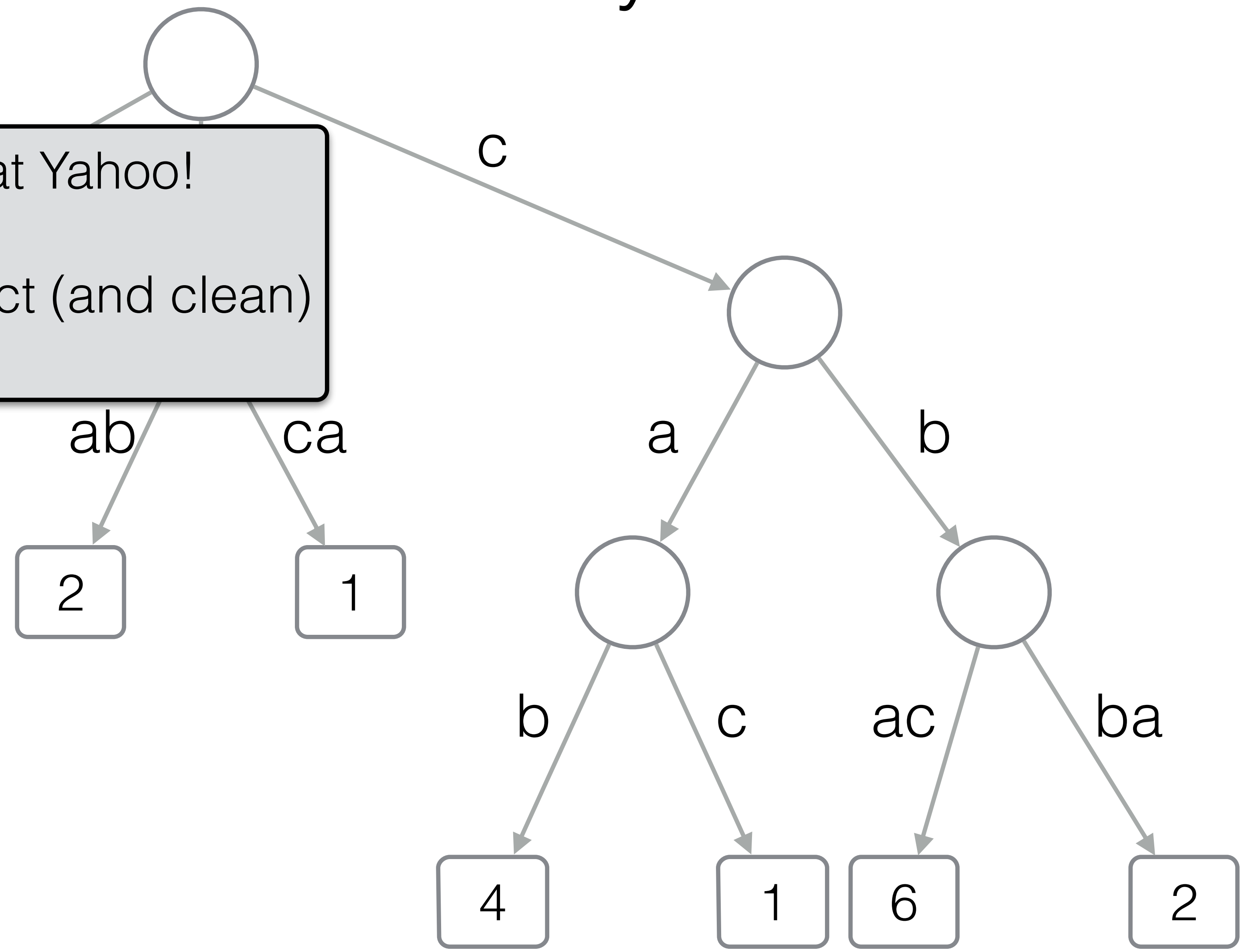
Summary



Find the node "prefixed" by P	O(|P|) time	O(m log σ + n log m) bits
Compute the top-k strings	O(k log k) time	O(n) bits

n = |D|, m total length of strings in D

Summary



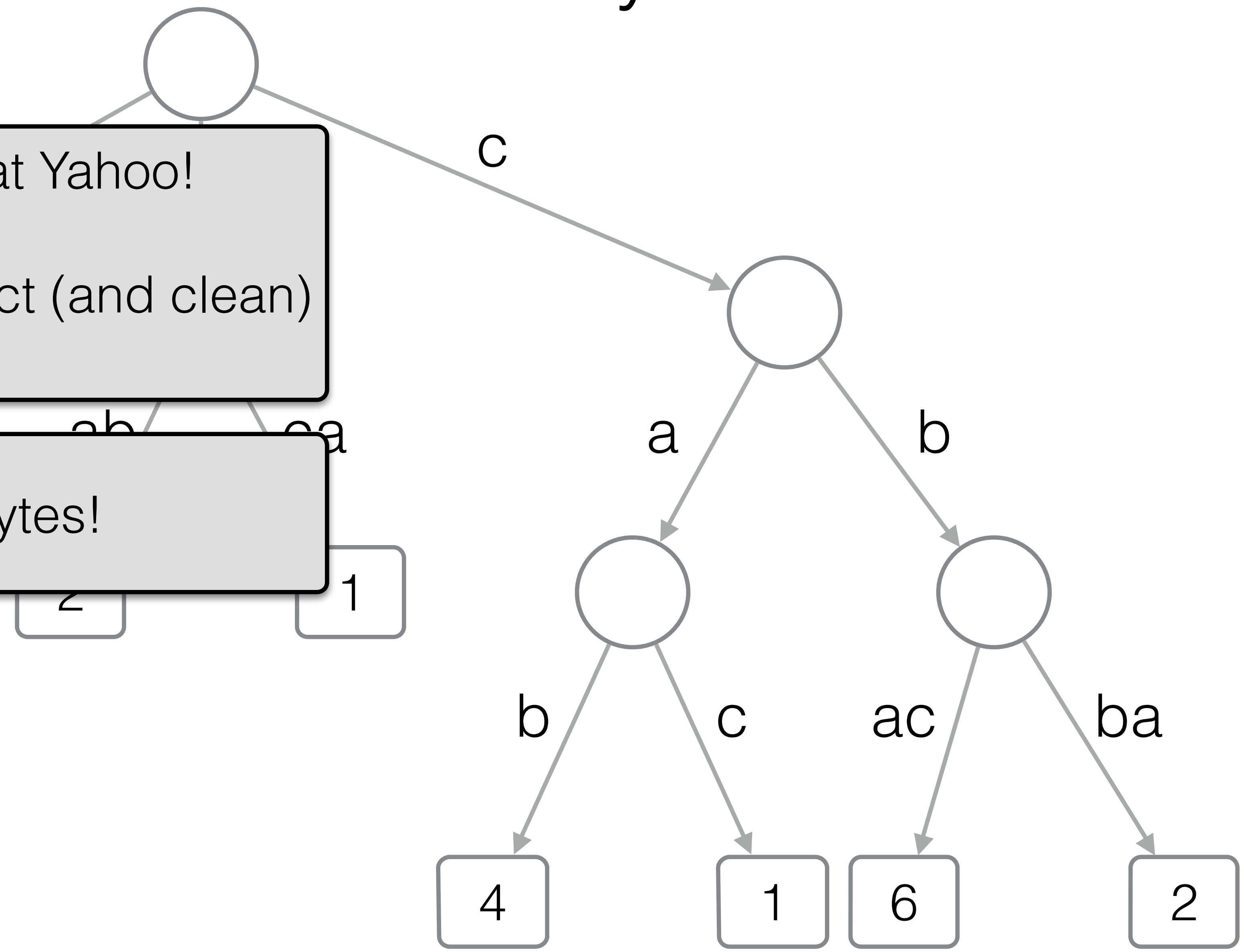
3 months query log at Yahoo!

≈600 million of distinct (and clean) queries

Find the node “prefixed” by P	O(|P|) time	O(m log σ + n log m) bits
Compute the top-k strings	O(k log k) time	O(n) bits

n = \|D\|, m total length of strings in D

Summary



3 months query log at Yahoo!

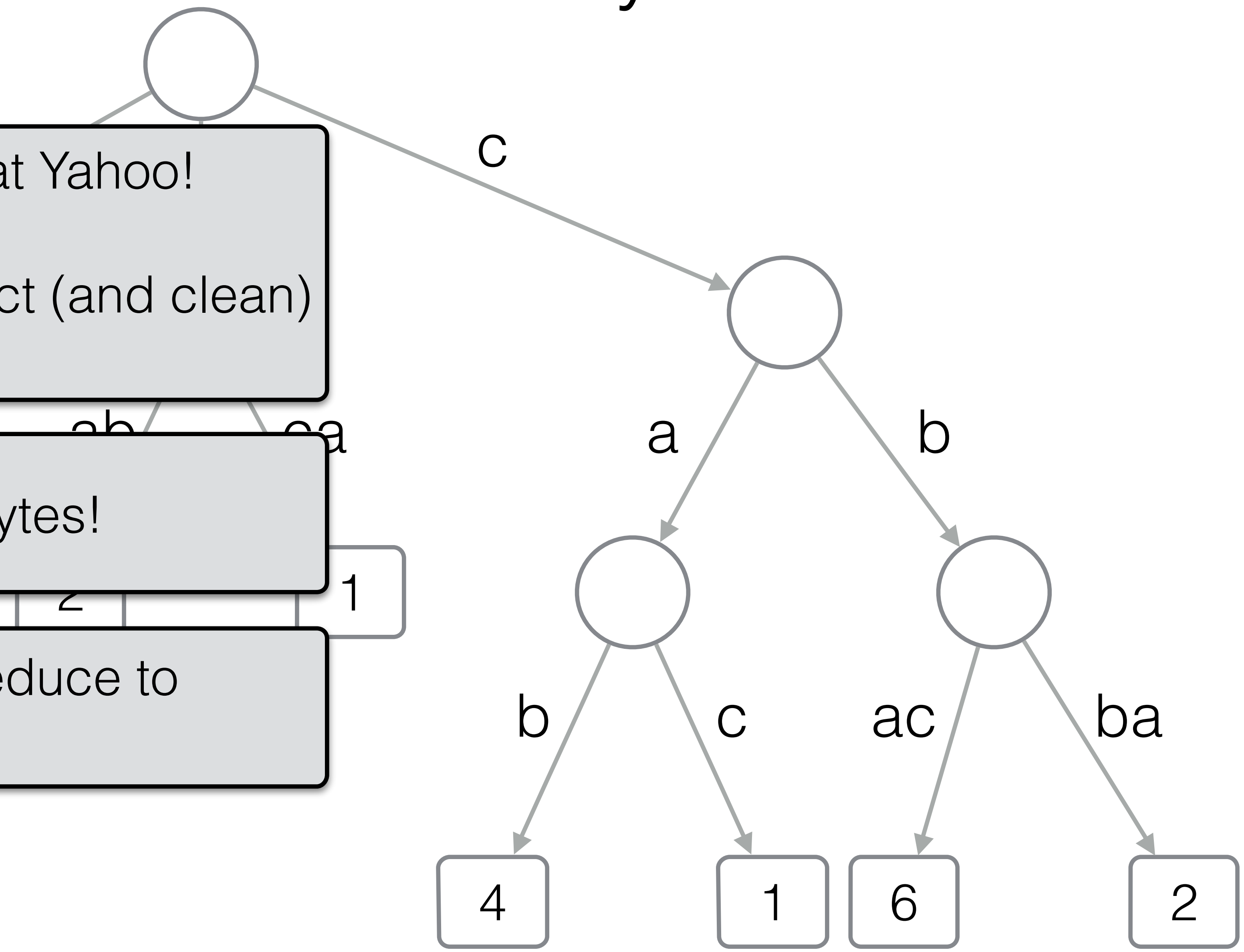
≈600 million of distinct (and clean) queries

Trie requires ≈50 Gbytes!

Find the node "prefixed" by P	O(|P|) time	O(m log σ + n log m) bits
Compute the top-k strings	O(k log k) time	O(n) bits

n = |D|, m total length of strings in D

Summary



3 months query log at Yahoo!

≈600 million of distinct (and clean) queries

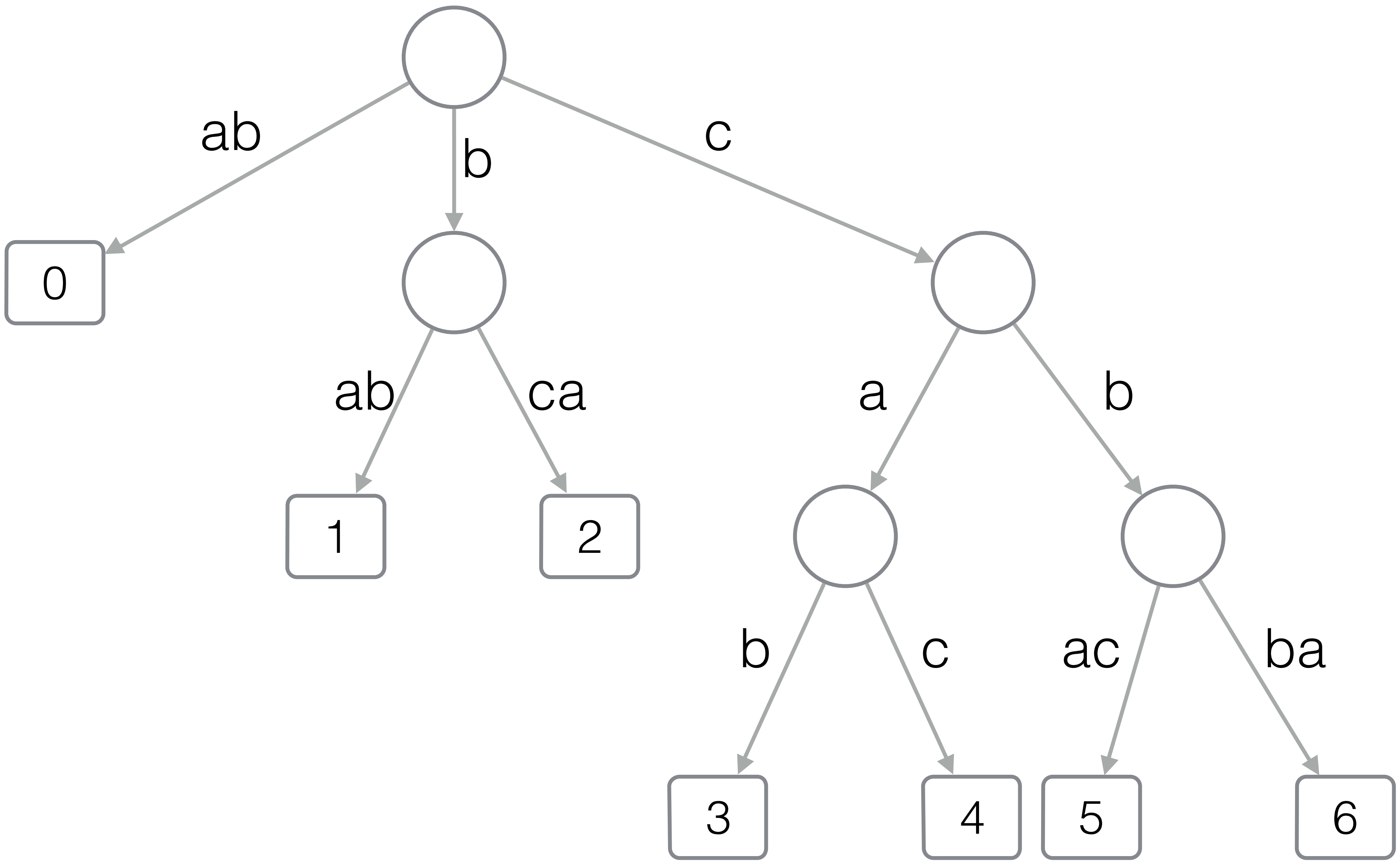
Trie requires ≈50 Gbytes!

We will see how to reduce to ≈5 Gbytes!

Find the node "prefixed" by P	O(|P|) time	O(m log σ + n log m) bits
Compute the top-k strings	O(k log k) time	O(n) bits

n = |D|, m total length of strings in D

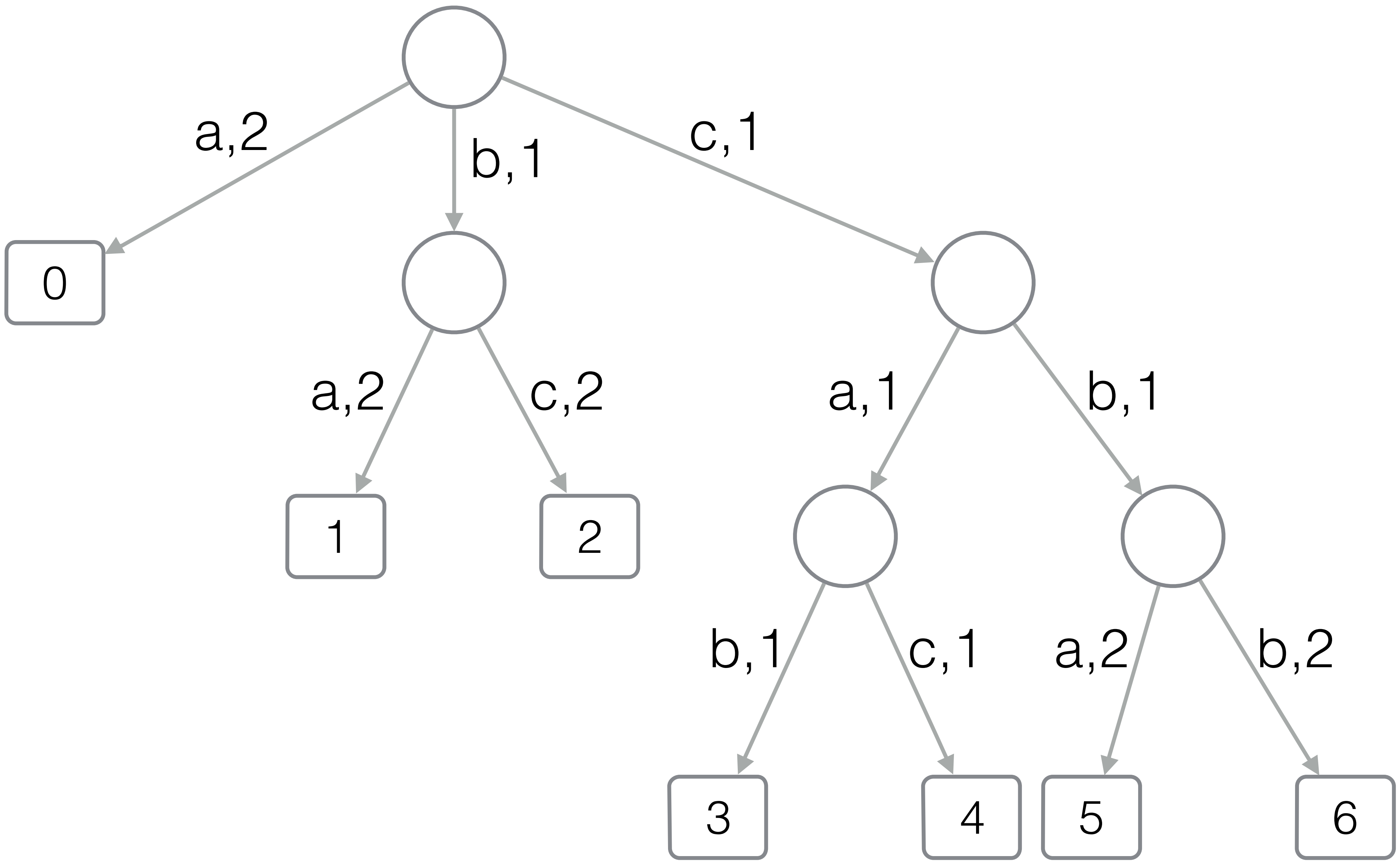
Patricia trie



D = { ab, bab, bca, cab, cac, cbac, cbba }

n = |D|, m total length of strings in D

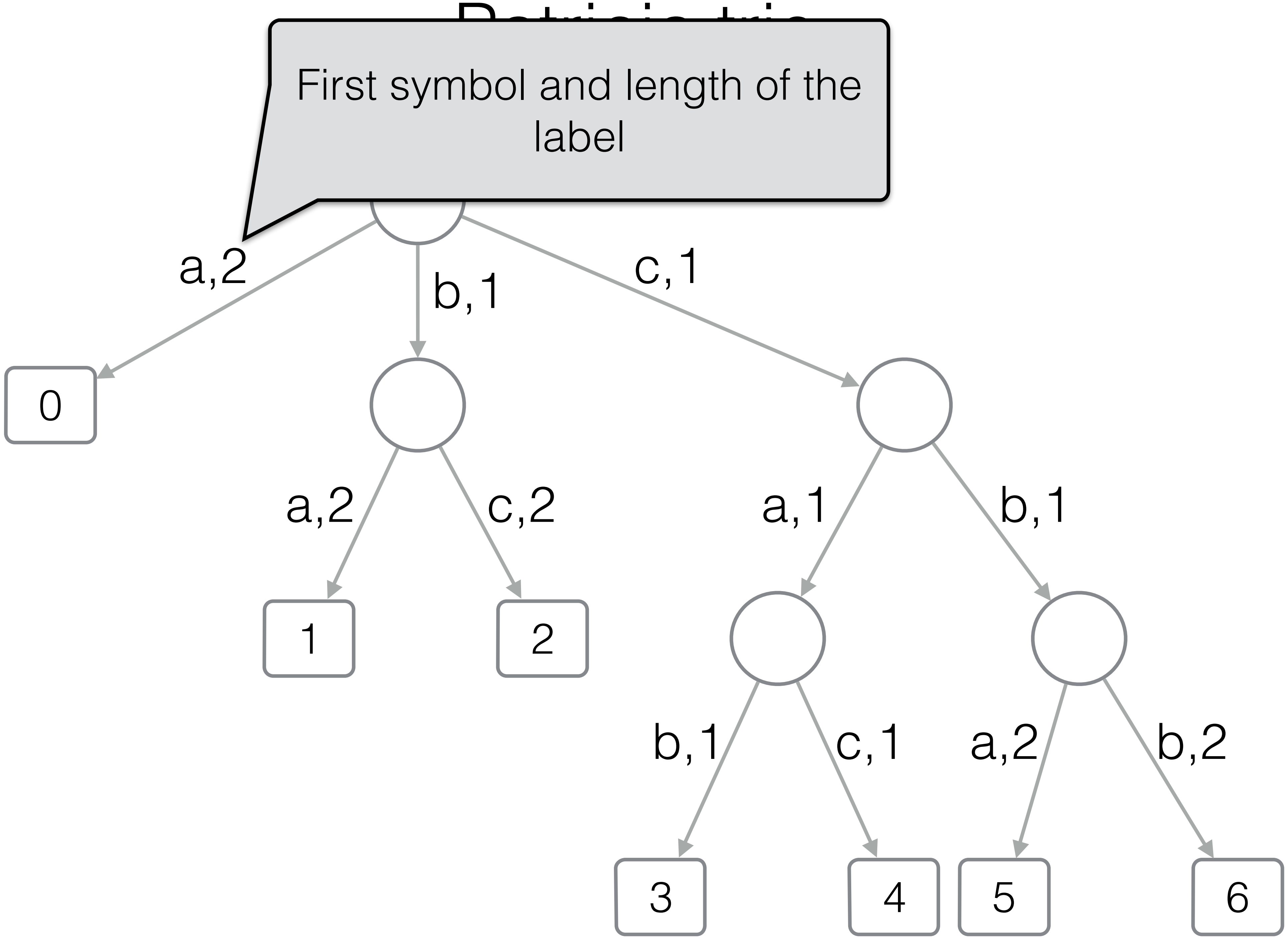
Patricia trie



D = { ab, bab, bca, cab, cac, cbac, cbba }

n = |D|, m total length of strings in D

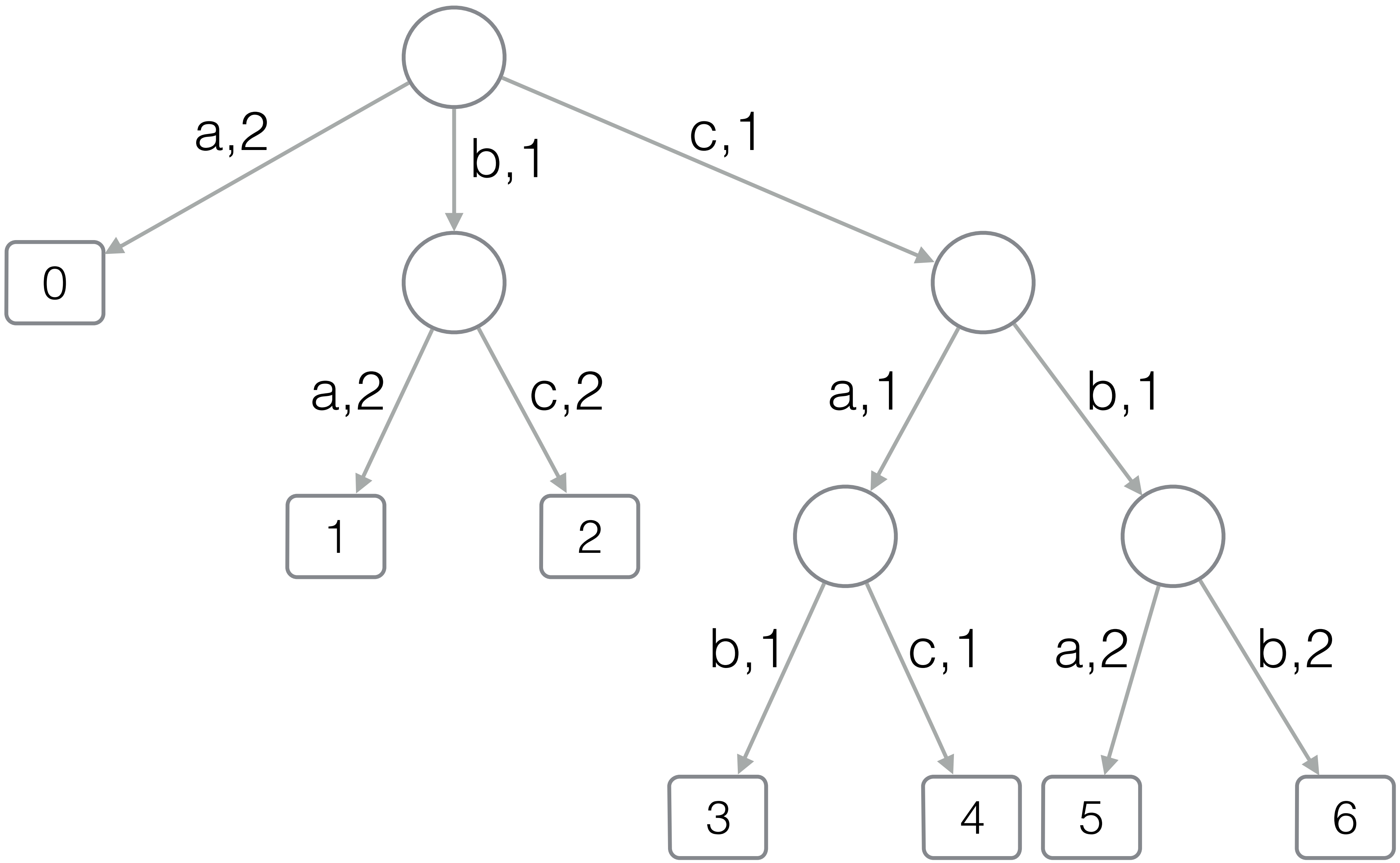
Patricia trie



D = { ab, bab, bca, cab, cac, cbac, cbba }

n = |D|, m total length of strings in D

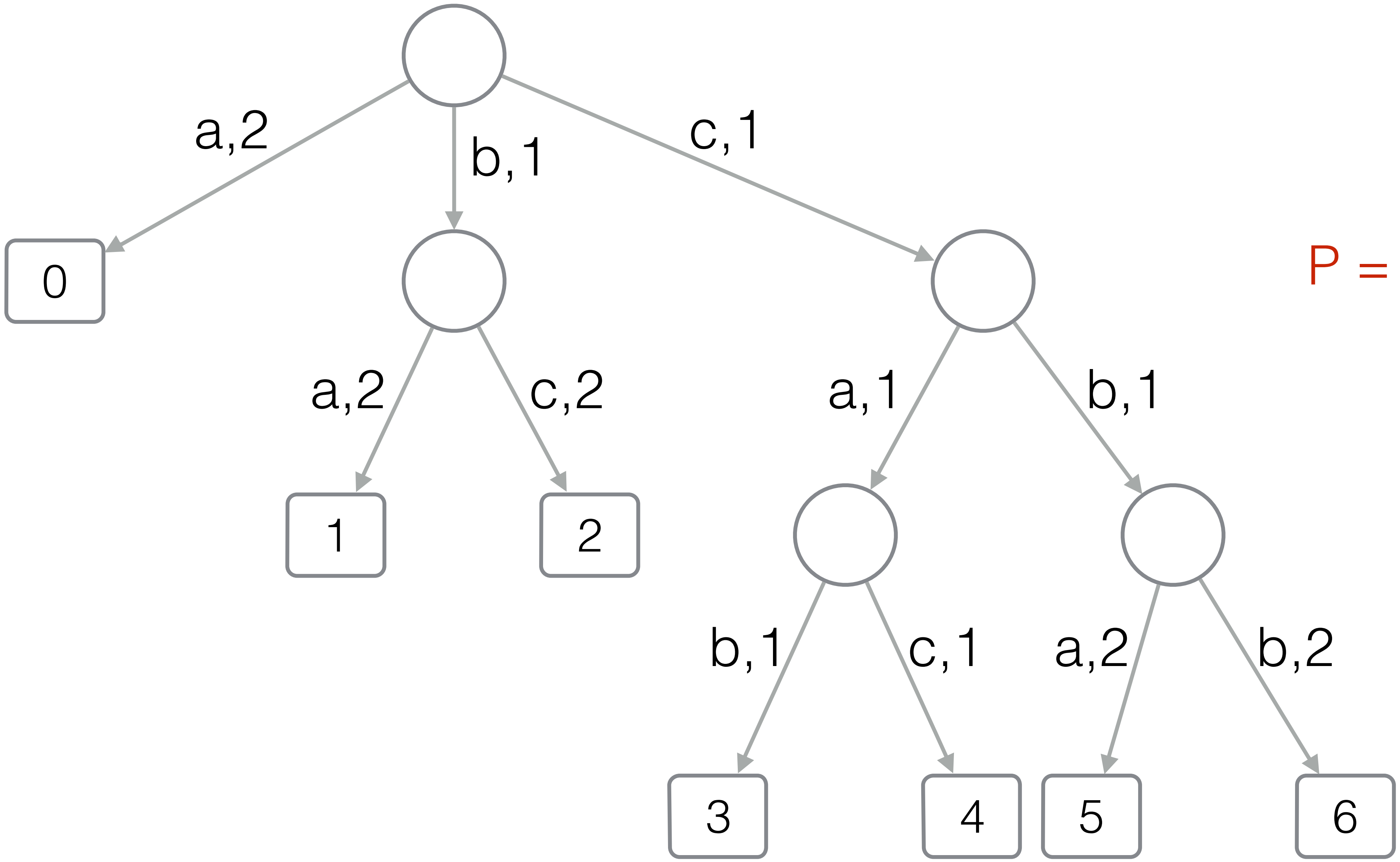
Patricia trie



D = { ab, bab, bca, cab, cac, cbac, cbba }

n = |D|, m total length of strings in D

Patricia trie

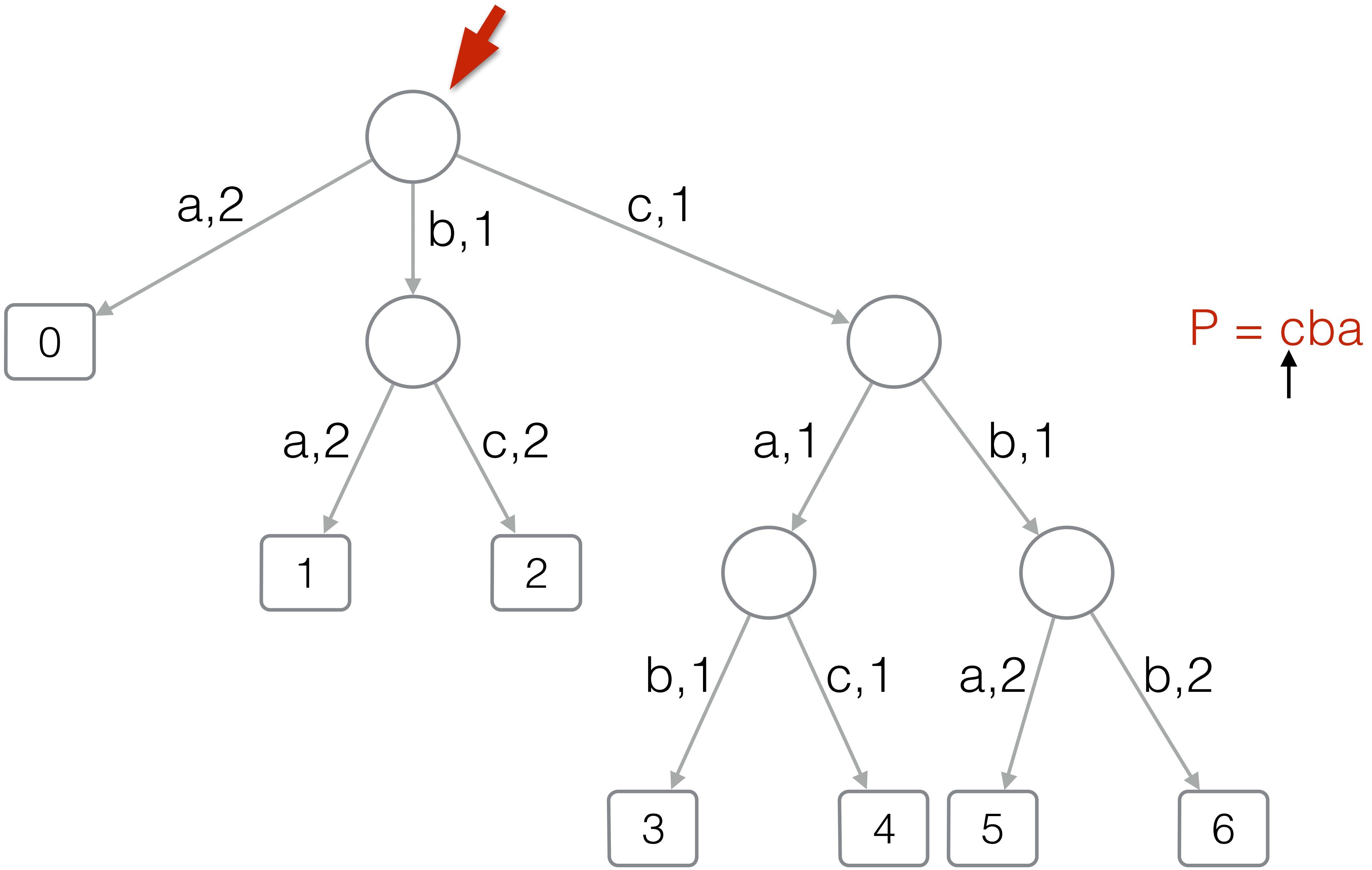


P = cba

D = { ab, bab, bca, cab, cac, cbac, cbba }

n = |D|, m total length of strings in D

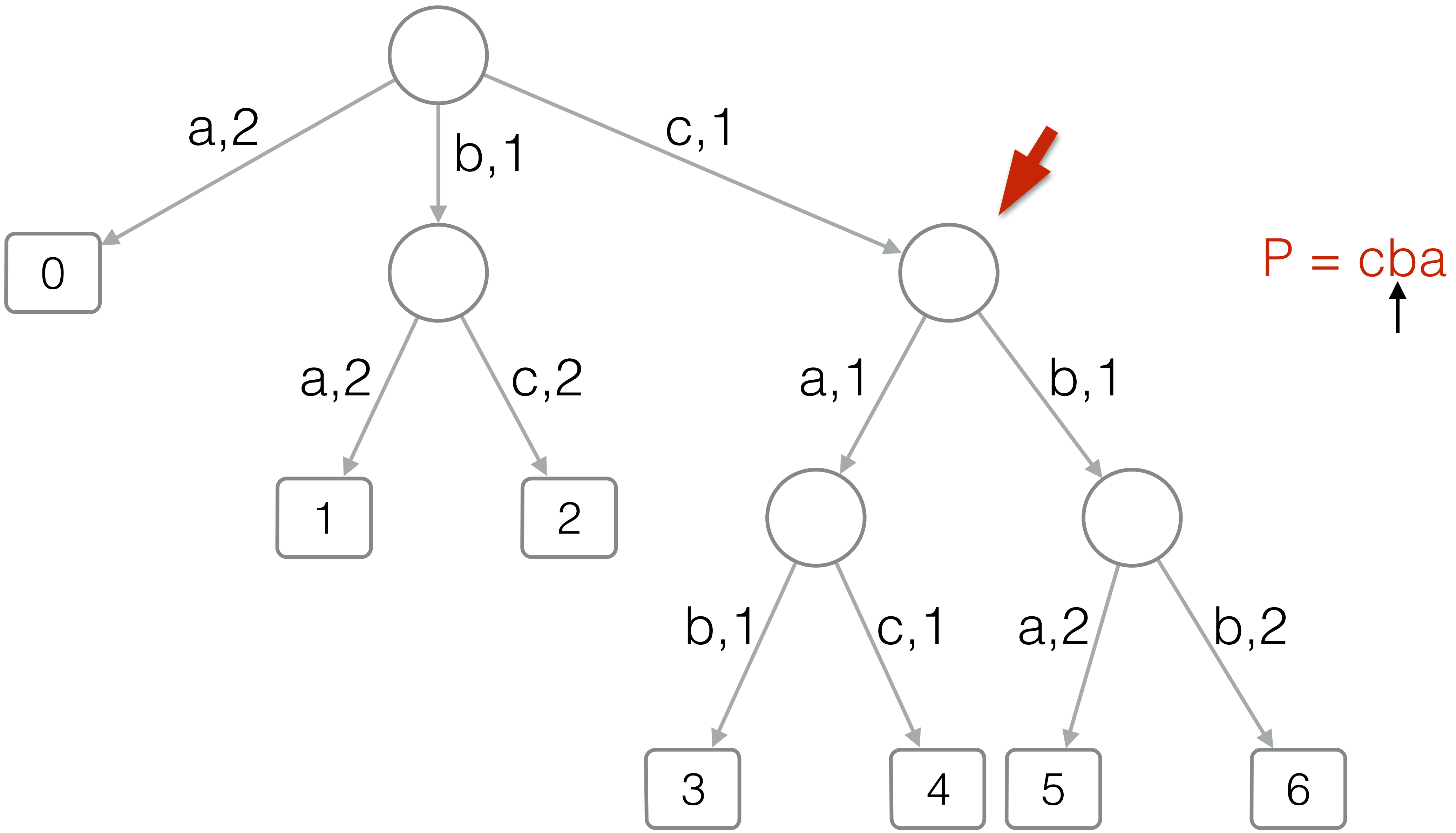
Patricia trie



D = { ab, bab, bca, cab, cac, cbac, cbba }

n = |D|, m total length of strings in D

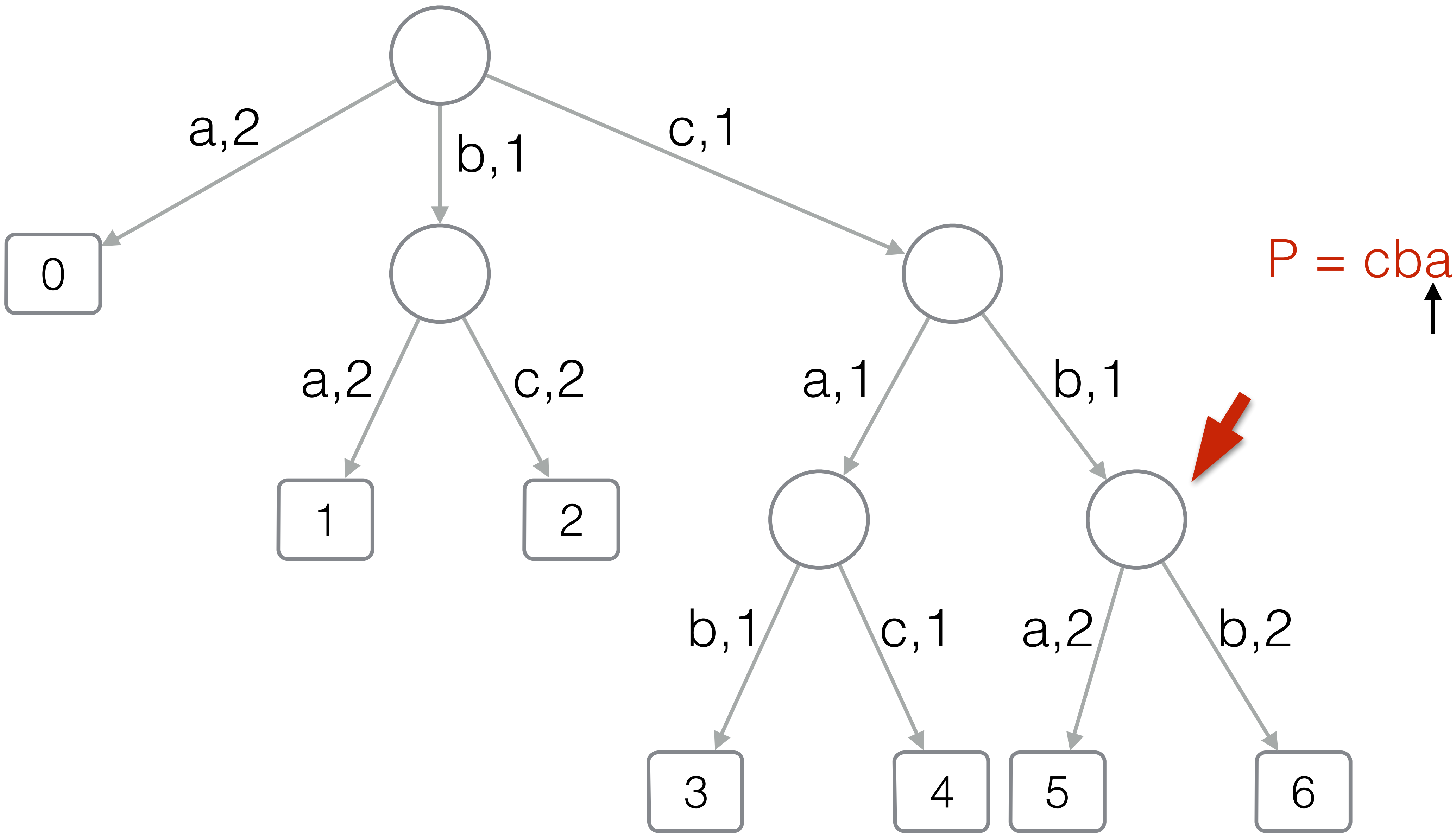
Patricia trie



D = { ab, bab, bca, cab, cac, cbac, cbba }

n = |D|, m total length of strings in D

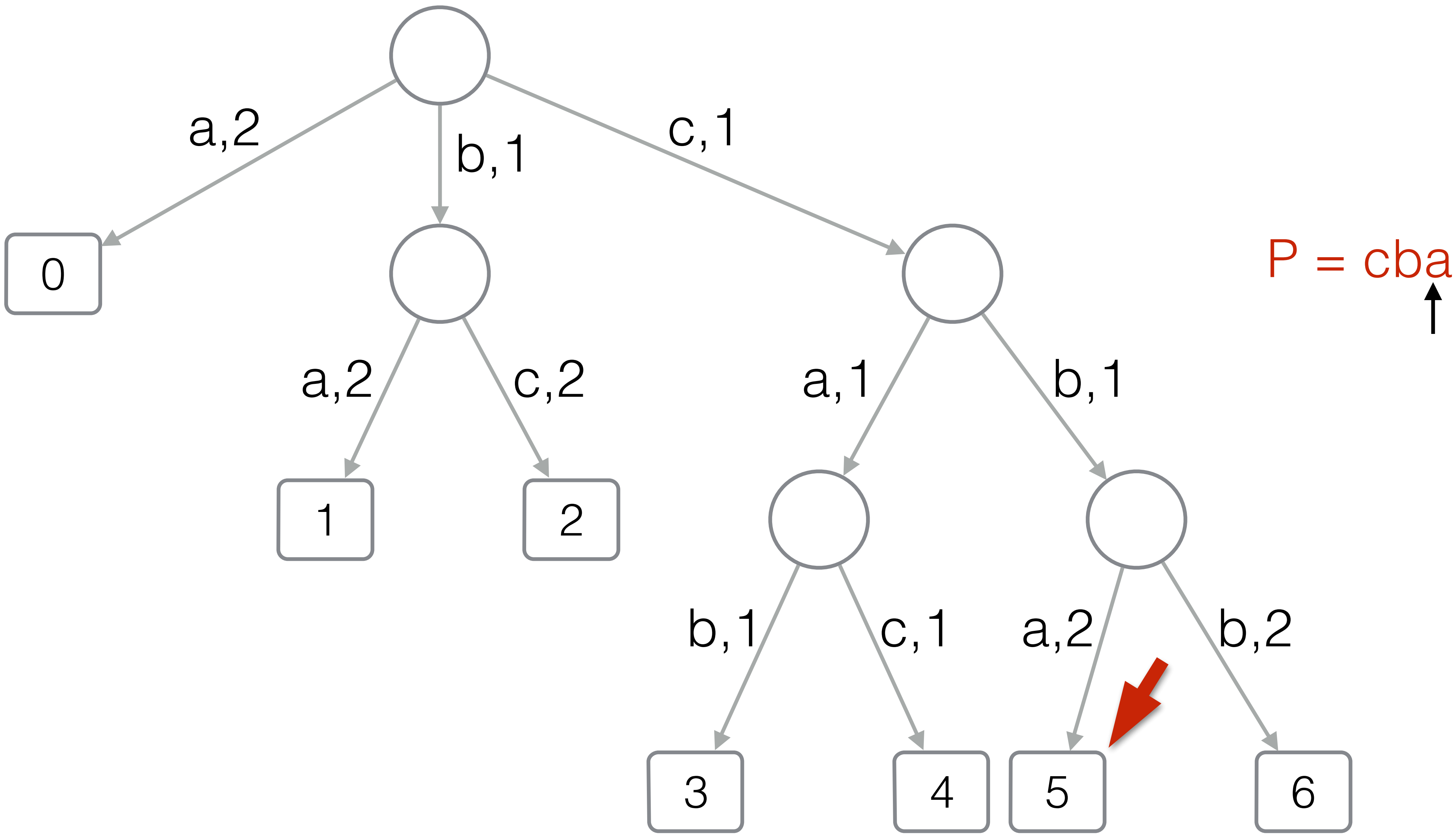
Patricia trie



D = { ab, bab, bca, cab, cac, cbac, cbba }

n = |D|, m total length of strings in D

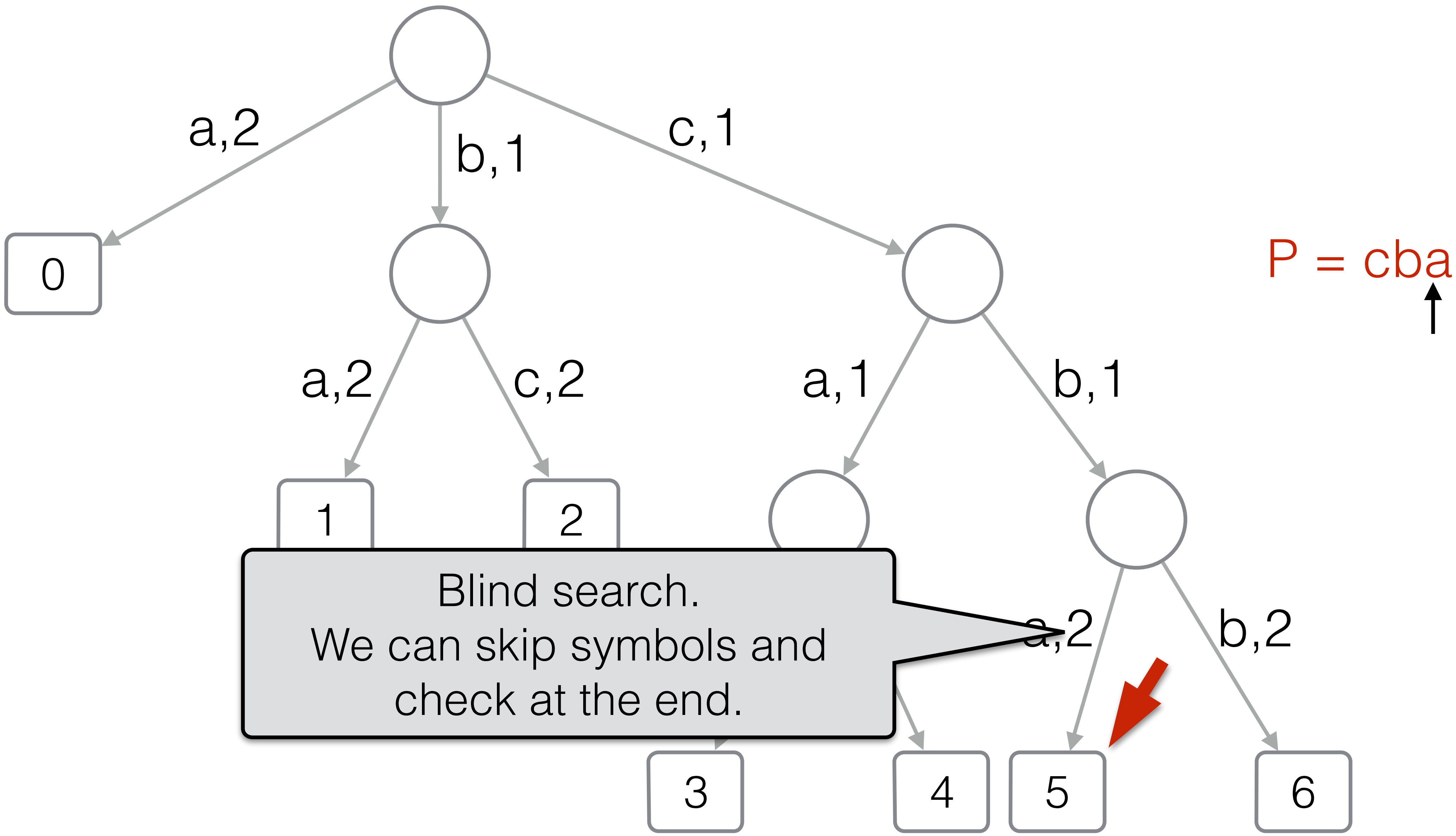
Patricia trie



D = { ab, bab, bca, cab, cac, cbac, cbba }

n = |D|, m total length of strings in D

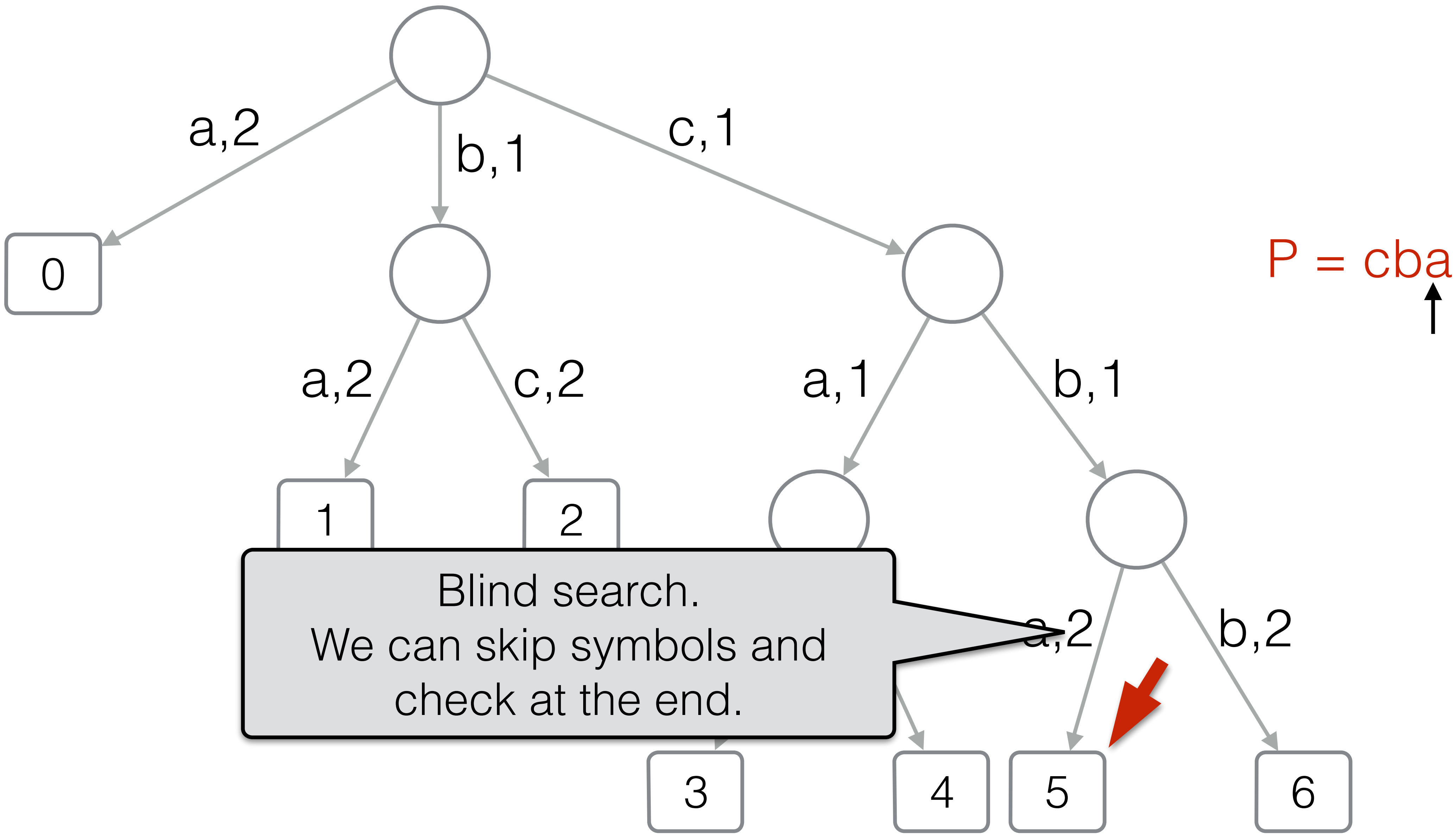
Patricia trie



D = { ab, bab, bca, cab, cac, cbac, cbba }

n = |D|, m total length of strings in D

Patricia trie



O(|P|) time

O(n log m + m log σ) bits

n = |D|, m total length of strings in D

Rank/Select queries

	0	1	2	3	4	5	6	7	8	9	10	11
B	0	0	1	1	1	0	1	0	1	1	0	1

Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

	0	1	2	3	4	5	6	7	8	9	10	11
B	0	0	1	1	1	0	1	0	1	1	0	1

Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

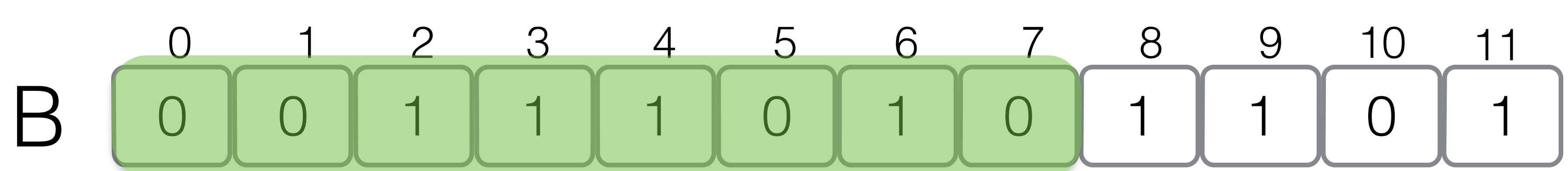
$Rank_0(7)$ =

	0	1	2	3	4	5	6	7	8	9	10	11
B	0	0	1	1	1	0	1	0	1	1	0	1

Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_0(7) = 4$

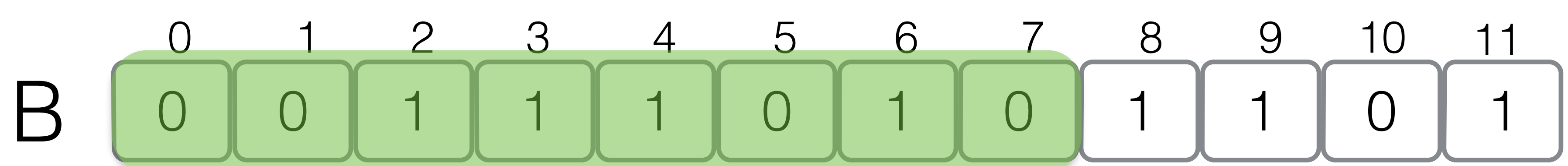


Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

$Rank_0(7) = 4$



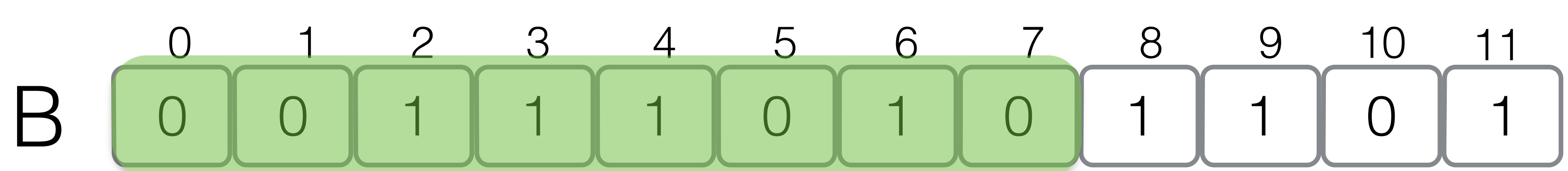
Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

$Rank_0(7) = 4$

$Rank_1(7) = 8 - Rank_0(7) = 4$



Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Select_0(j)$ = position of the j-th 0 in B

$Rank_1(j)$ = # of 1 in B[0,j]

$Rank_0(7) = 4$

$Rank_1(7) = 8 - Rank_0(7) = 4$

	0	1	2	3	4	5	6	7	8	9	10	11
B	0	0	1	1	1	0	1	0	1	1	0	1

Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

$Select_0(j)$ = position of the j-th 0 in B

$Select_1(j)$ = position of the j-th 0 in B

$Rank_0(7) = 4$

$Rank_1(7) = 8 - Rank_0(7) = 4$

	0	1	2	3	4	5	6	7	8	9	10	11
B	0	0	1	1	1	0	1	0	1	1	0	1

Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

$Rank_0(7) = 4$

$Rank_1(7) = 8 - Rank_0(7) = 4$

$Select_0(j)$ = position of the j-th 0 in B

$Select_1(j)$ = position of the j-th 0 in B

$Select_1(4) =$

	0	1	2	3	4	5	6	7	8	9	10	11
B	0	0	1	1	1	0	1	0	1	1	0	1

Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

$Rank_0(7) = 4$

$Rank_1(7) = 8 - Rank_0(7) = 4$

$Select_0(j)$ = position of the j-th 0 in B

$Select_1(j)$ = position of the j-th 0 in B

$Select_1(4) = 6$

	0	1	2	3	4	5	6	7	8	9	10	11
B	0	0	1	1	1	0	1	0	1	1	0	1

Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

$Select_0(j)$ = position of the j-th 0 in B

$Select_1(j)$ = position of the j-th 0 in B

(4) = 6

Space: n + O(n log log n/log n) bits
Query time: O(1)

	0	1	2	3	4	5	6	7	8	9	10	11
B	0	0	1	1	1	0	1	0	1	1	0	1

Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

$Select_0(j)$ = position of the j-th 0 in B

$Select_1(j)$ = position of the j-th 0 in B

(4) = 6

Space: n + O(n log log n/log n) bits
Query time: O(1)

	0	1	2	3	4	5	6	7	8	9	10	11
B	0	0	1	1	1	0	1	0	1	1	0	1

Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

Space: n + O(n log log n/log n) bits
Query time: O(1)

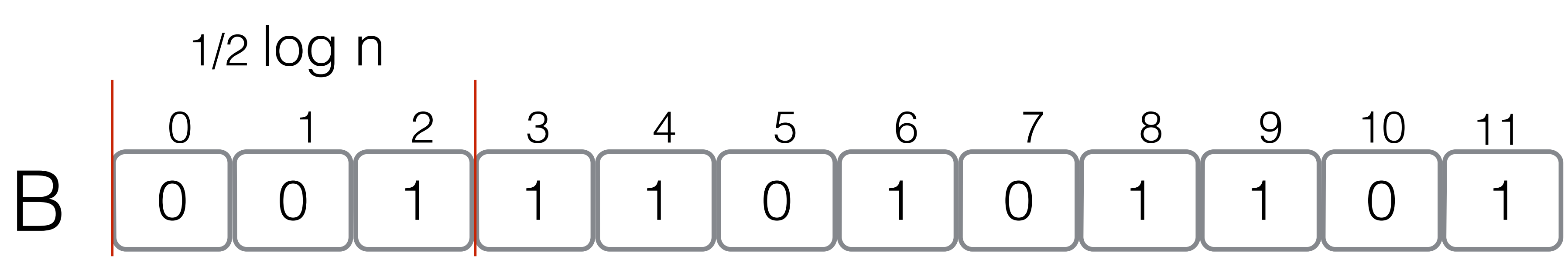
	0	1	2	3	4	5	6	7	8	9	10	11
B	0	0	1	1	1	0	1	0	1	1	0	1

Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

Space: n + O(n log log n/log n) bits
Query time: O(1)



Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

> Space: n + O(n log log n/log n) bits
> Query time: O(1)

1/2 log n

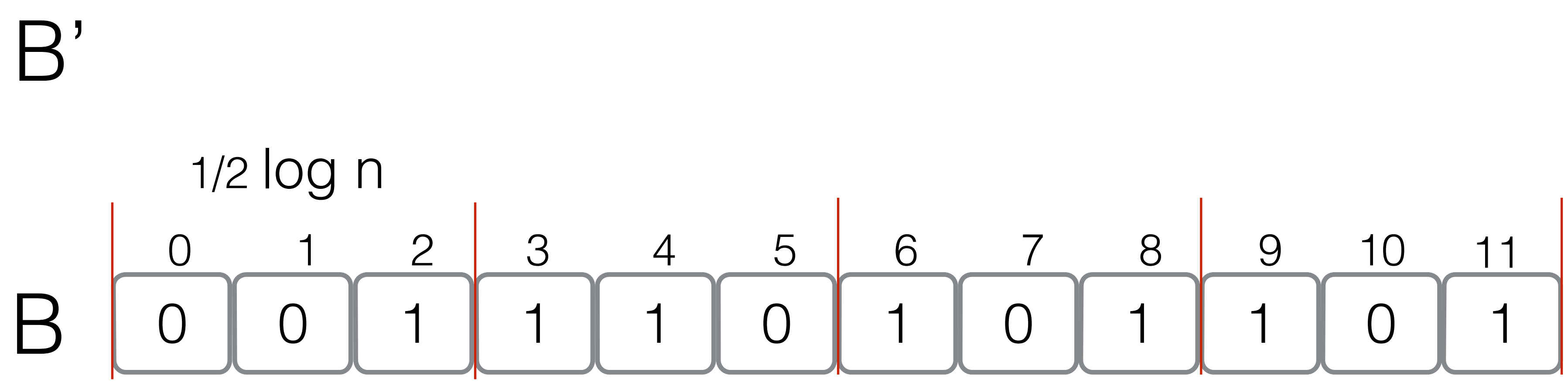
	0	1	2	3	4	5	6	7	8	9	10	11
B	0	0	1	1	1	0	1	0	1	1	0	1

Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

Space: n + O(n log log n/log n) bits
Query time: O(1)

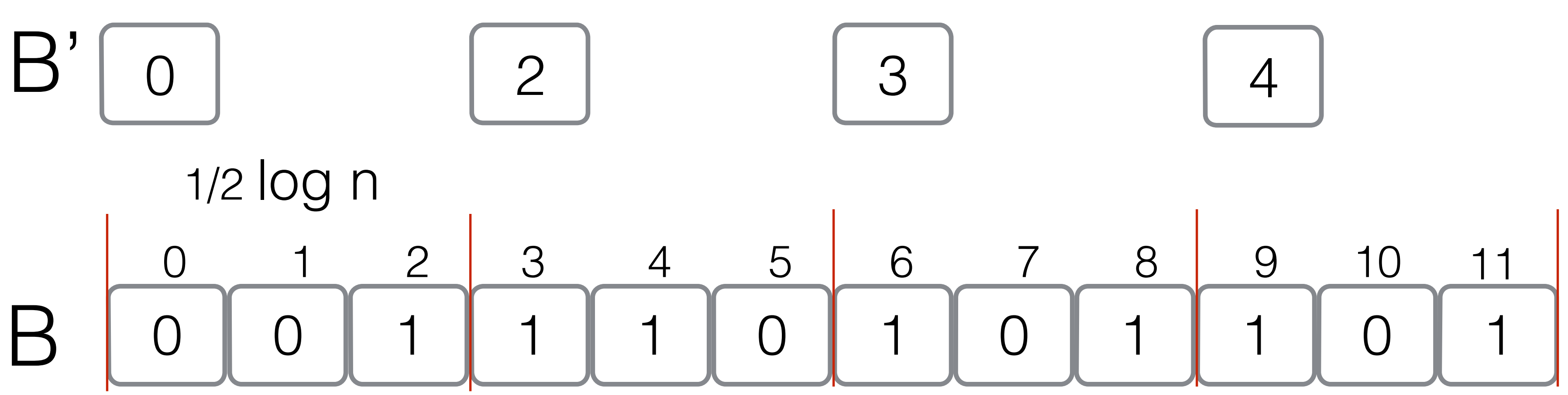


Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

Space: n + O(n log log n/log n) bits
Query time: O(1)



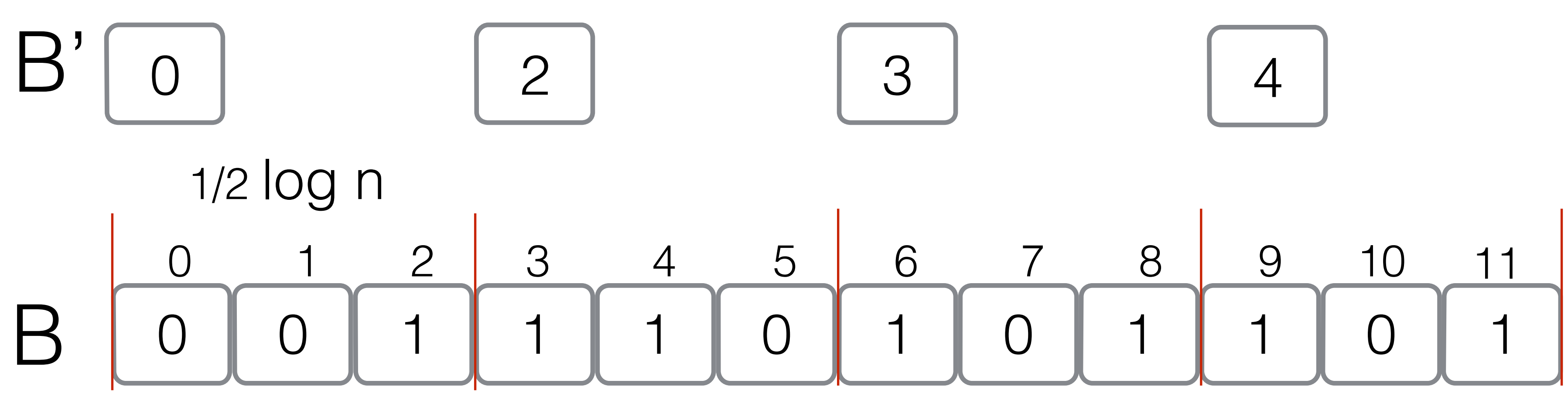
Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

Space: n + O(n log log n/log n) bits
Query time: O(1)

$Rank_0(7)$ =



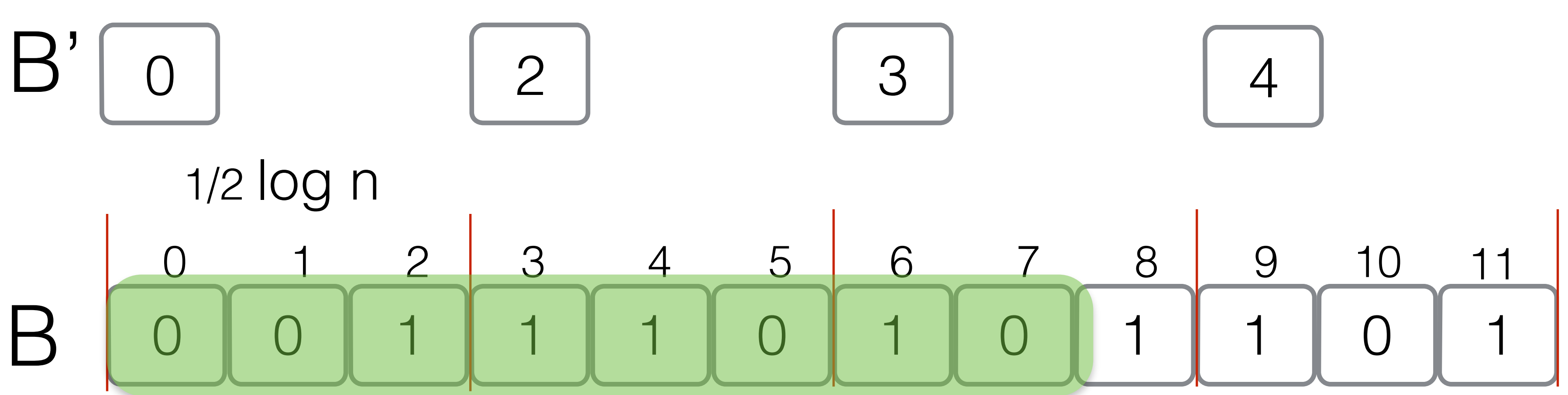
Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

Space: $n + O(n \log \log n / \log n)$ bits
Query time: $O(1)$

$Rank_0(7)$ =



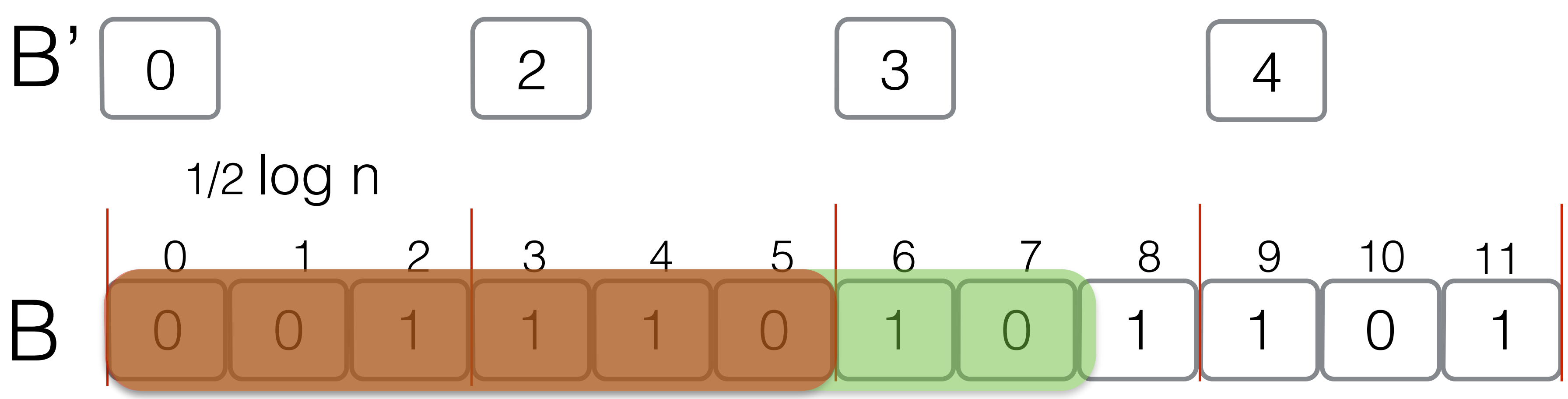
Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

Space: $n + O(n \log\log n / \log n)$ bits
Query time: O(1)

$Rank_0(7)$ =



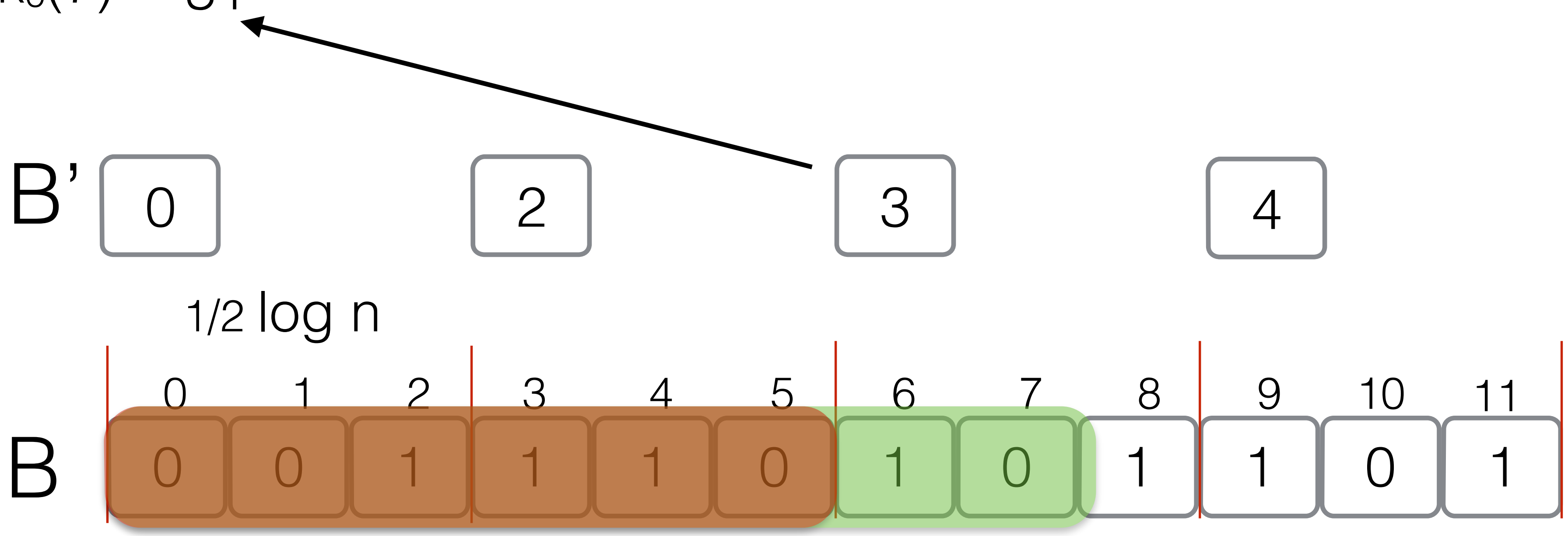
Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

Space: n + O(n log log n/log n) bits
Query time: O(1)

$Rank_0(7) = 3+$



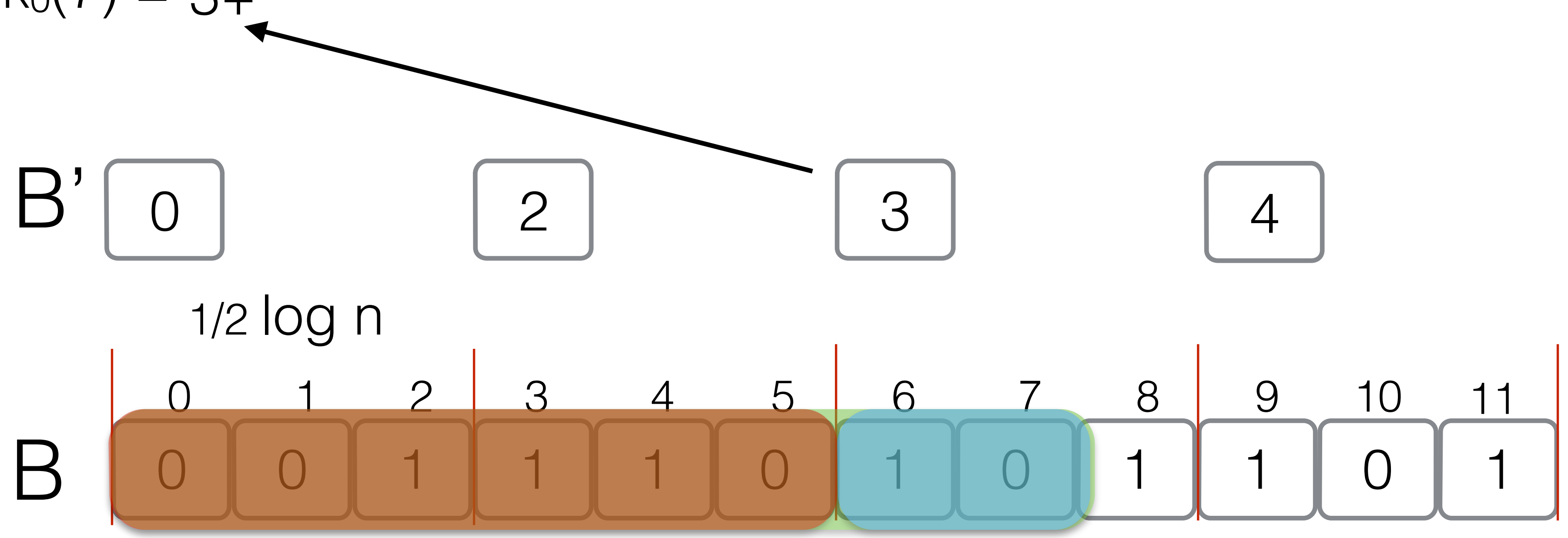
Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

Space: $n + O(n \log \log n / \log n)$ bits
Query time: $O(1)$

$Rank_0(7) = 3+$

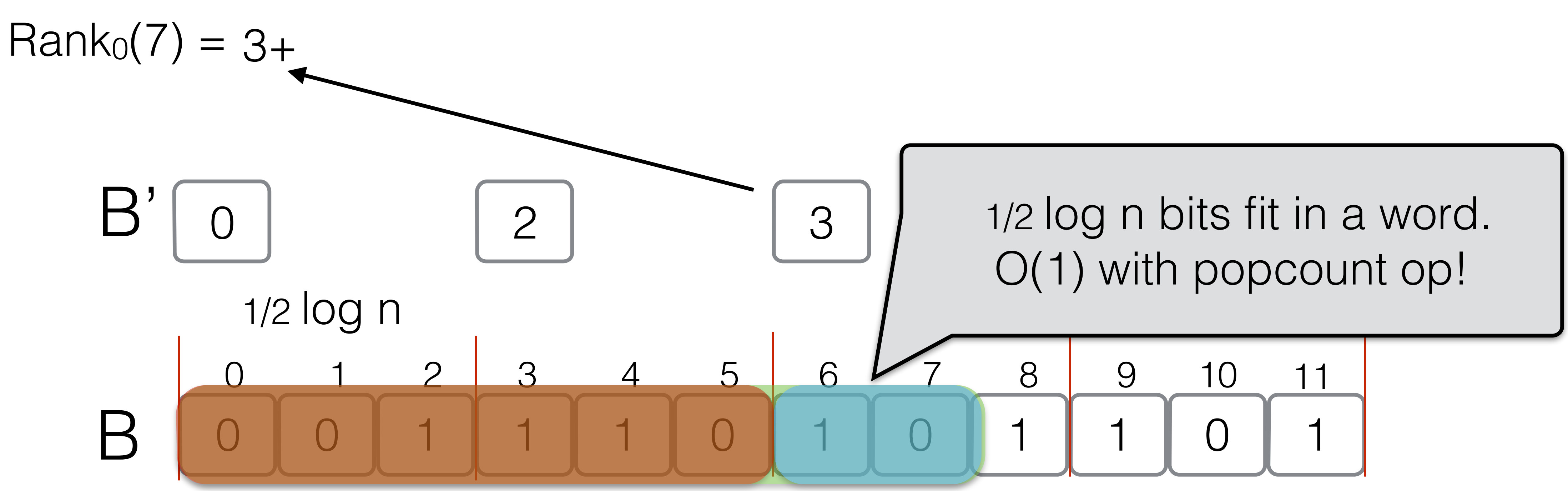


Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

Space: n + O(n log log n/log n) bits
Query time: O(1)



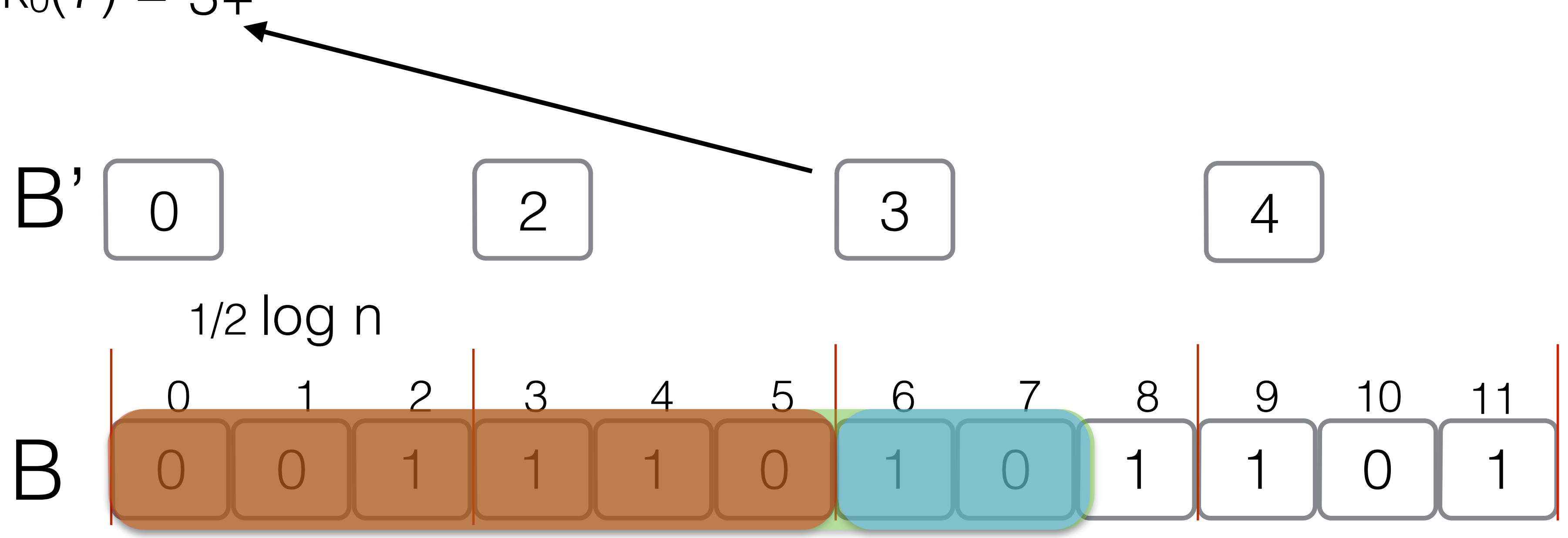
Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

Space: n + O(n log log n/log n) bits
Query time: O(1)

$Rank_0(7)$ = 3+



Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

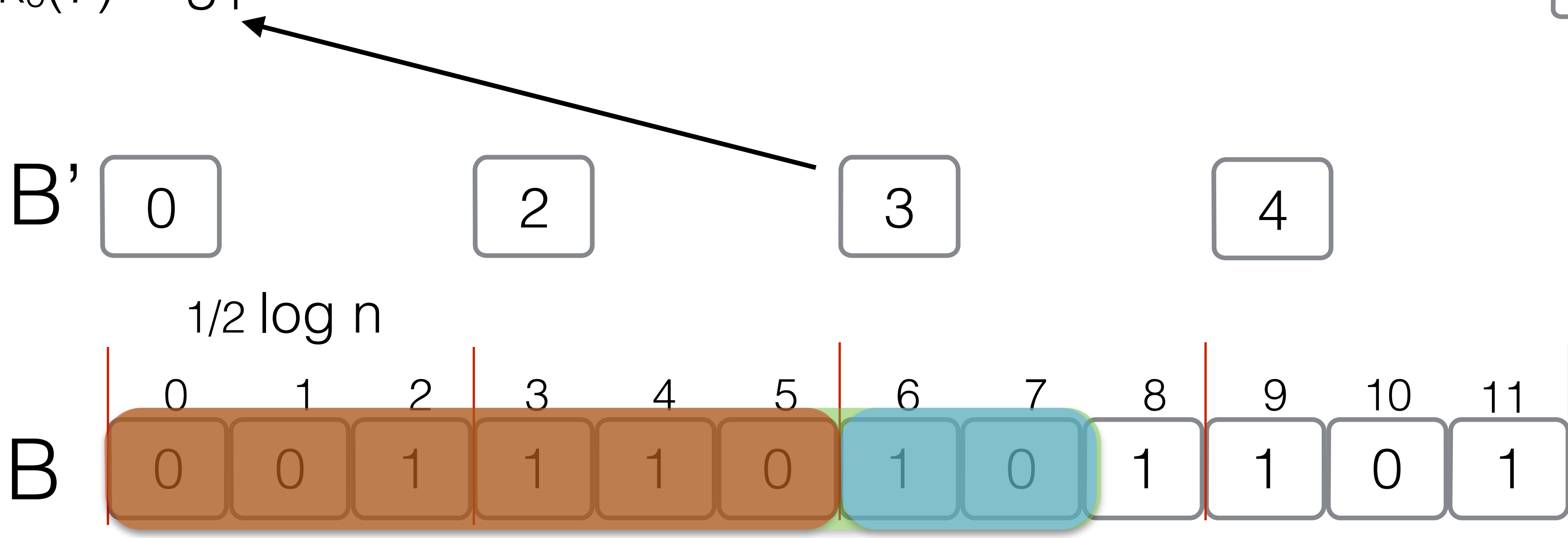
$Rank_1(j)$ = # of 1 in B[0,j]

> Space: n + O(n log log n/log n) bits
> Query time: O(1)

M

	1	2	3
000	1	2	3
001	1	2	2
010	1	1	2
011	1	1	1
100	0	1	2
101	0	1	1
110	0	0	1
111	0	0	0

$Rank_0(7)$ = 3+



Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

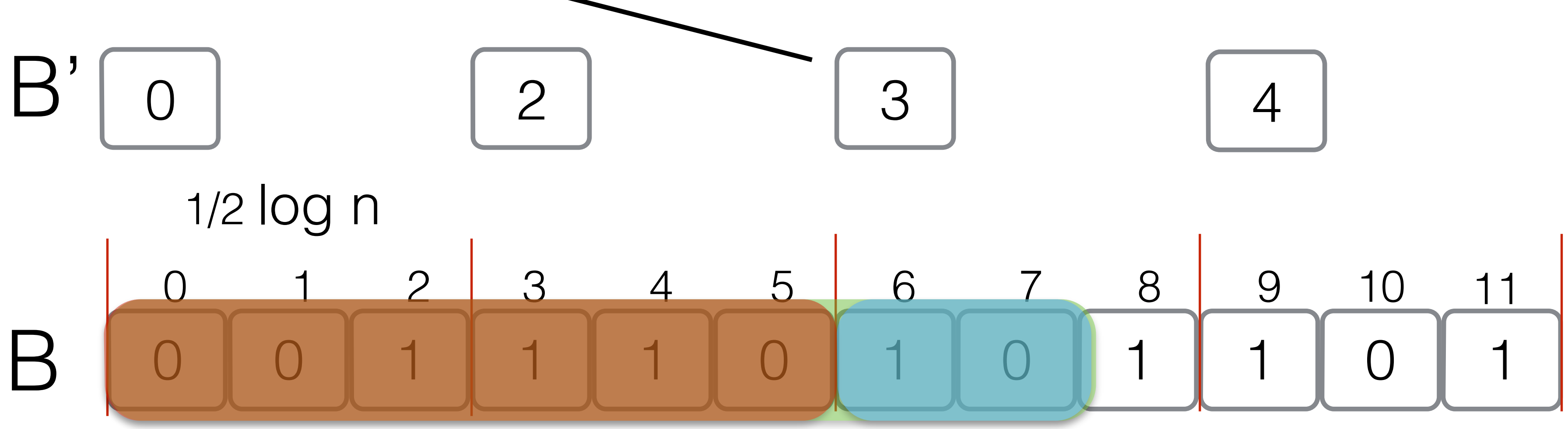
$Rank_1(j)$ = # of 1 in B[0,j]

> Space: $n + O(n \log \log n / \log n)$ bits
> Query time: $O(1)$

M

	1	2	3
000	1	2	3
001	1	2	2
010	1	1	2
011	1	1	1
100	0	1	2
101	0	1	1
110	0	0	1
111	0	0	0

$Rank_0(7) = 3+$



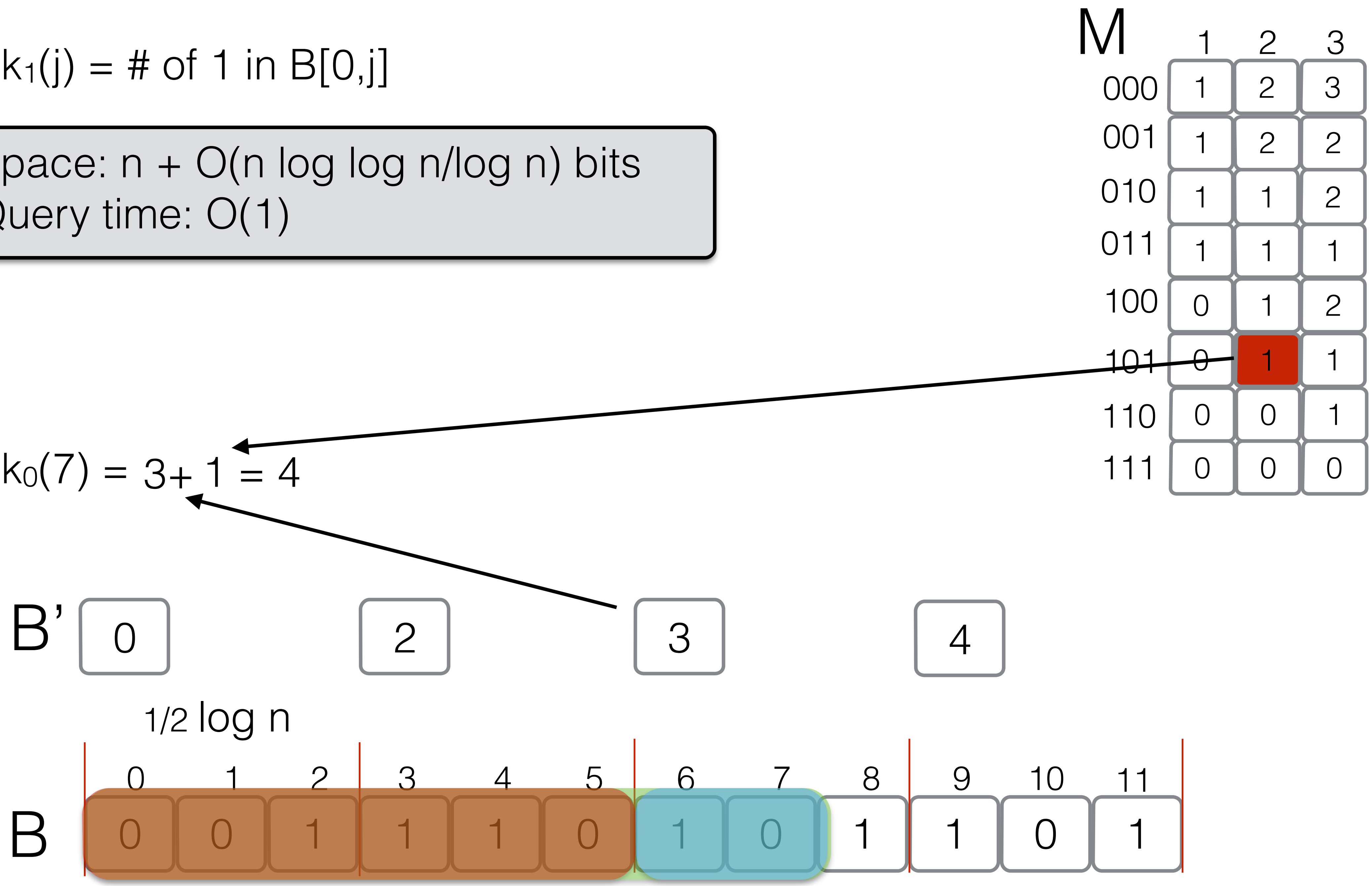
Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

> Space: $n + O(n \log \log n / \log n)$ bits
> Query time: $O(1)$

$Rank_0(7) = 3 + 1 = 4$



Rank/Select queries

How much space?

$Rank_0(j)$ = # of 0 in B[0,j]

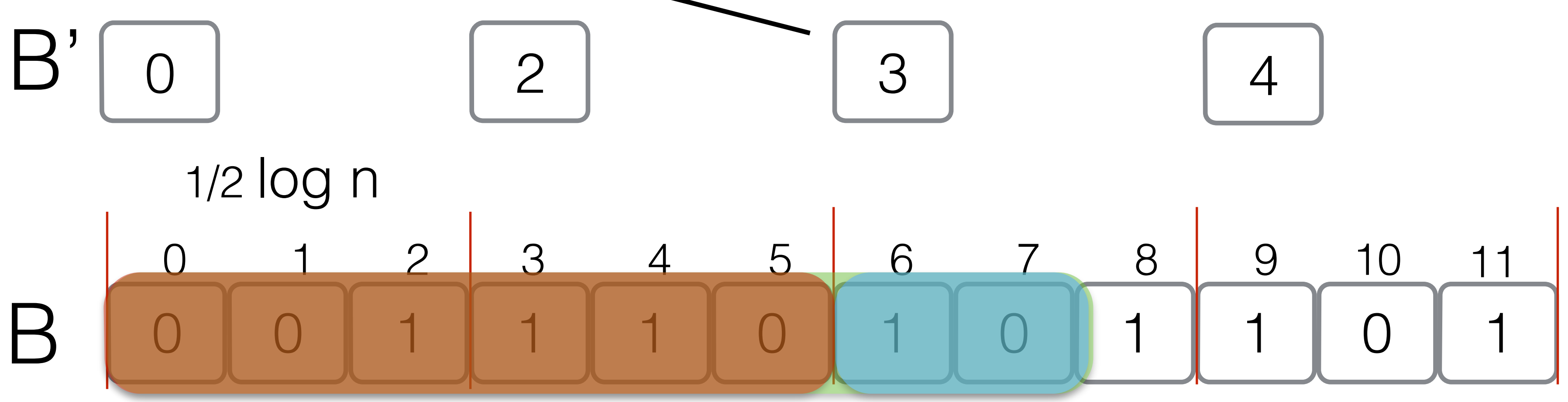
$Rank_1(j)$ = # of 1 in B[0,j]

Space: n + O(n log log n/log n) bits
Query time: O(1)

M

	1	2	3
000	1	2	3
001	1	2	2
010	1	1	2
011	1	1	1
100	0	1	2
101	0	1	1
110	0	0	1
111	0	0	0

$Rank_0(7) = 3 + 1 = 4$



Rank/Select queries

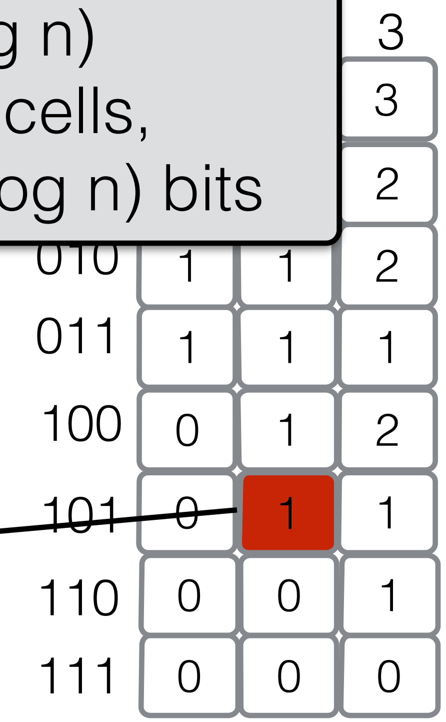
How much space?

$O(2^{1/2 \log n} \log n)$
$= O(\sqrt{n} \log n)$ cells,
each uses $O(\log \log n)$ bits

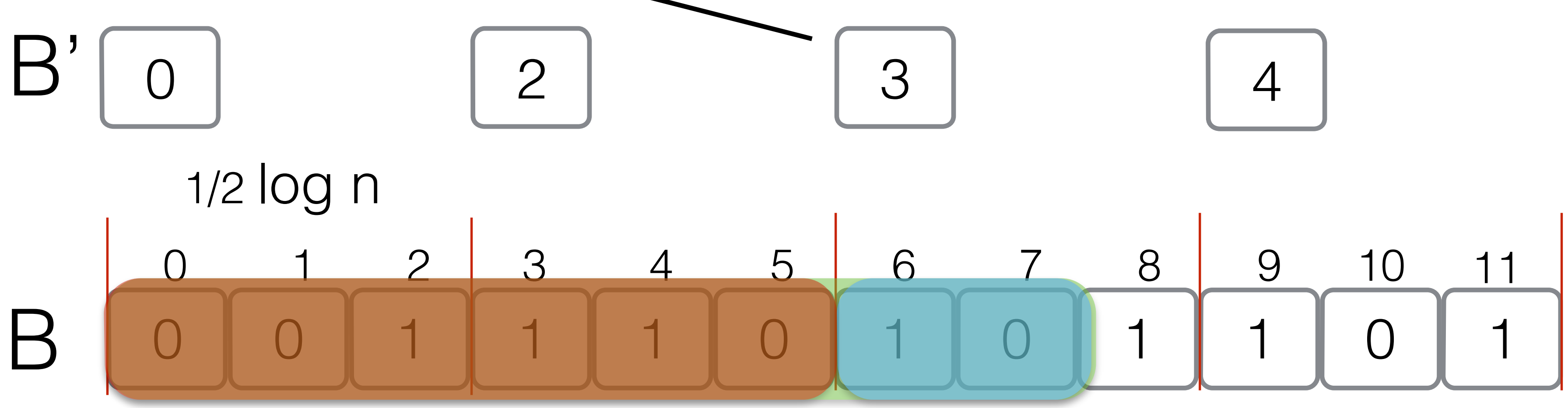
$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

Space: $n + O(n \log \log n / \log n)$ bits
Query time: $O(1)$



$Rank_0(7) = 3 + 1 = 4$



Rank/Select queries

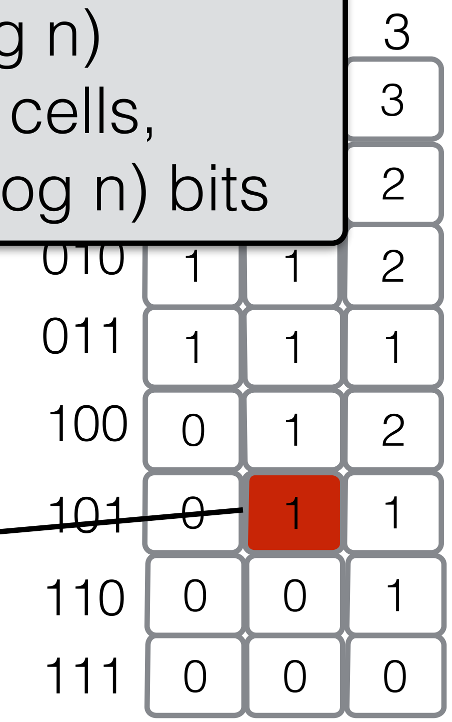
$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

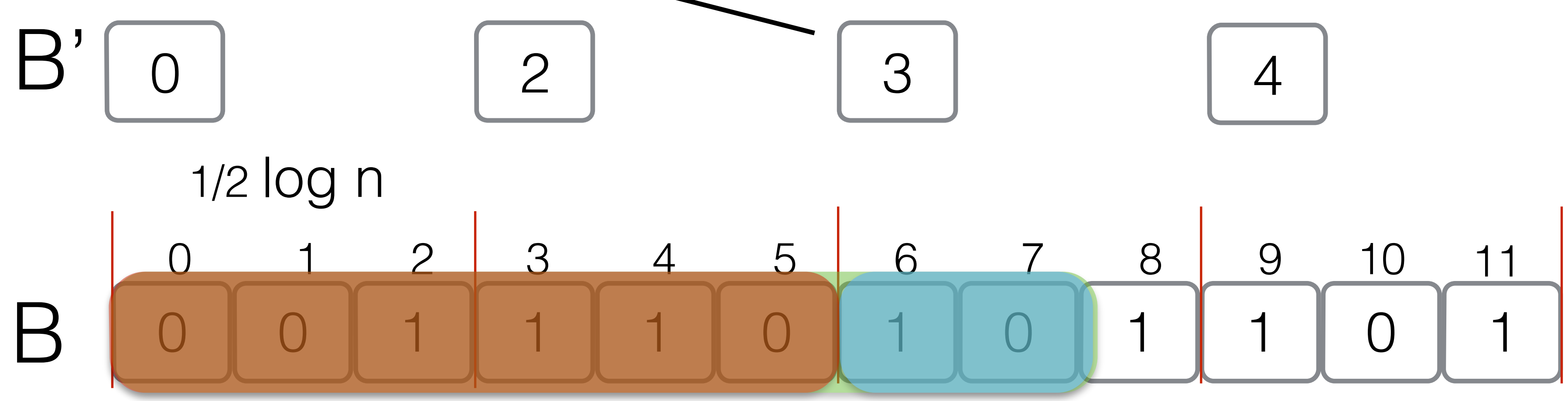
Space: n + O(n log log n/log n) bits
Query time: O(1)

How much space?

$O(2^{1/2 \log n} \log n)$
$= O(\sqrt{n} \log n)$ cells,
each uses O(log log n) bits



How much space?



Rank/Select queries

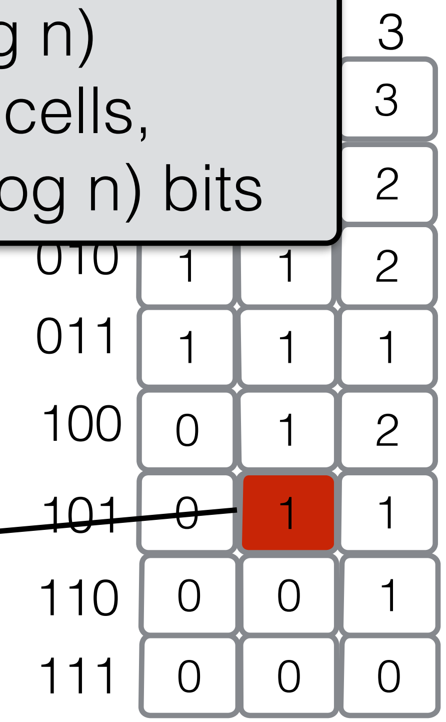
$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

Space: $n + O(n \log\log n / \log n)$ bits
Query time: $O(1)$

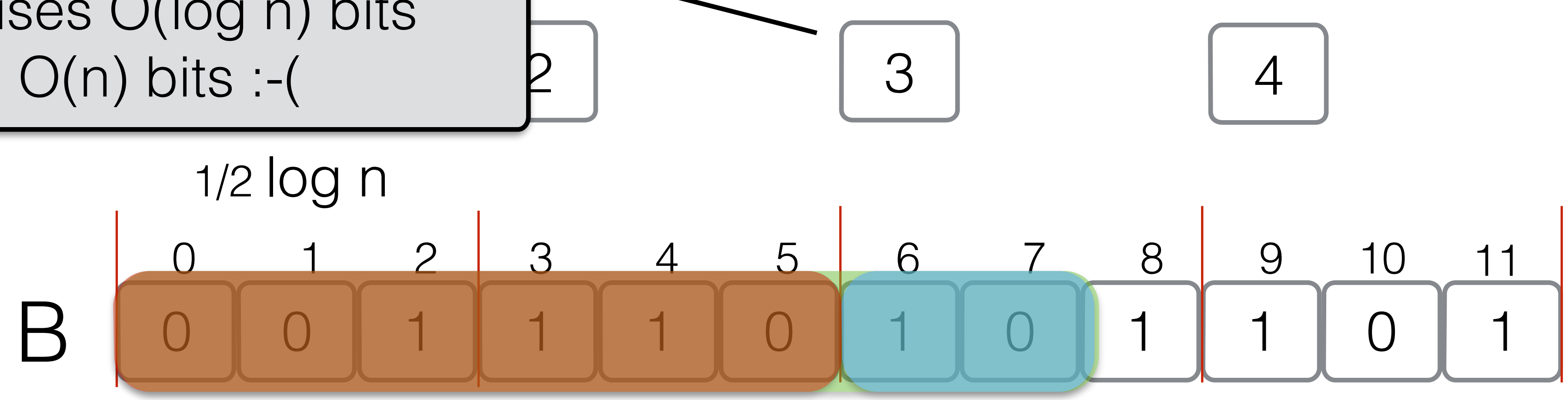
How much space?

$O(2^{1/2 \log n} \log n)$
$= O(\sqrt{n} \log n)$ cells,
each uses $O(\log\log n)$ bits



How much space?

$O(n/\log n)$ entries,
each uses $O(\log n)$ bits
$\Rightarrow O(n)$ bits :-(



Rank/Select queries

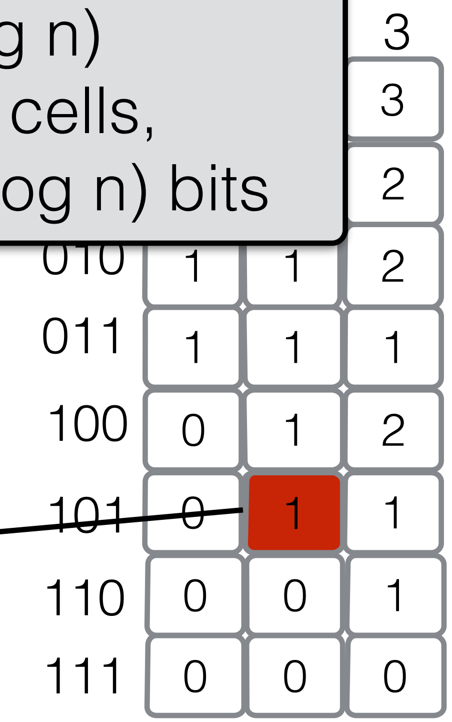
How much space?

$O(2^{1/2 \log n} \log n)$
$= O(\sqrt{n} \log n)$ cells,
each uses $O(\log \log n)$ bits

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

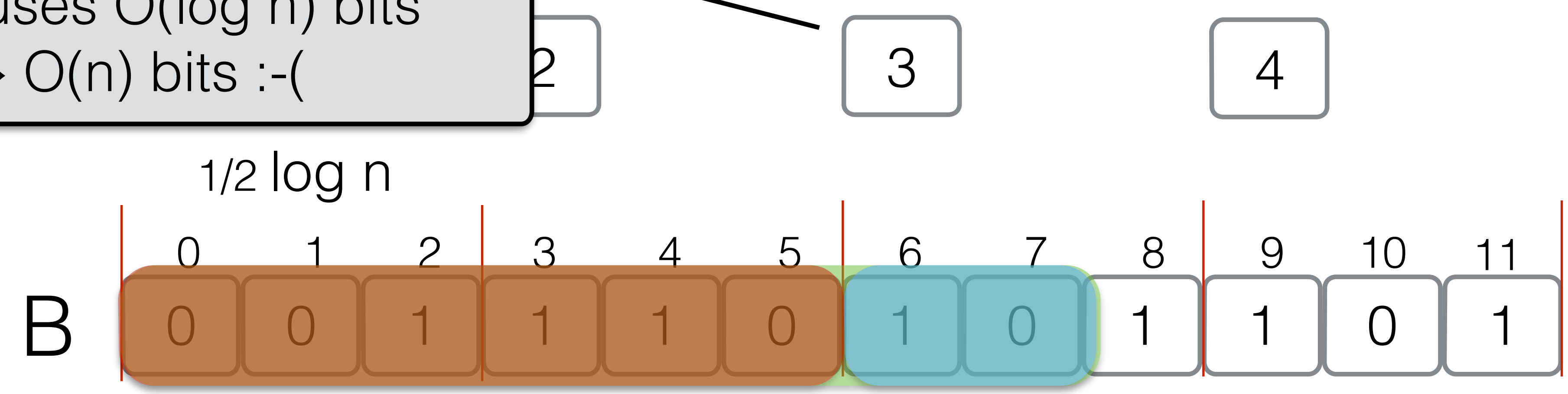
Space: $n + O(n \log \log n / \log n)$ bits
Query time: $O(1)$



How much space?

each uses $O(\log n)$ bits
$\Rightarrow O(n)$ bits :-(

Space: $O(n) + o(n)$ bits
Query time: $O(1)$



Rank/Select queries

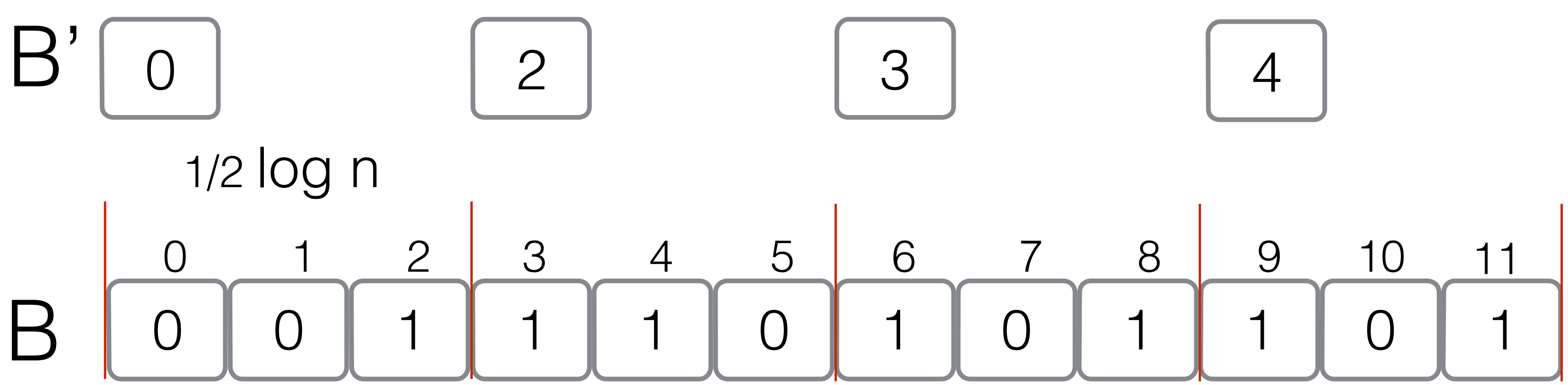
$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

Space: n + O(n log log n/log n) bits
Query time: O(1)

M

	1	2	3
000	1	2	3
001	1	2	2
010	1	1	2
011	1	1	1
100	0	1	2
101	0	1	1
110	0	0	1
111	0	0	0

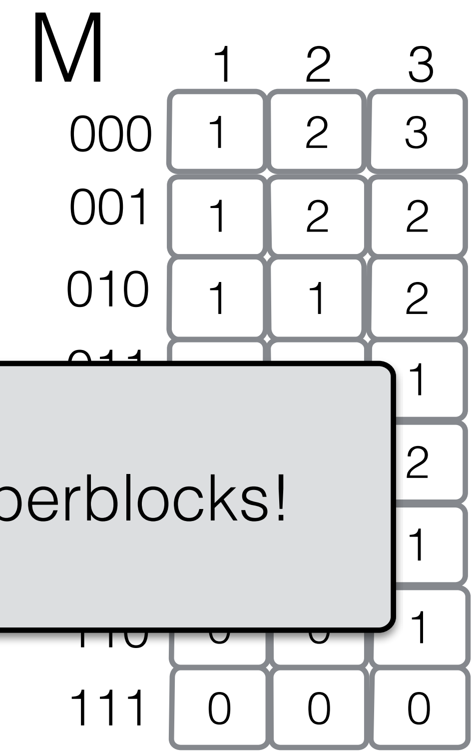


Rank/Select queries

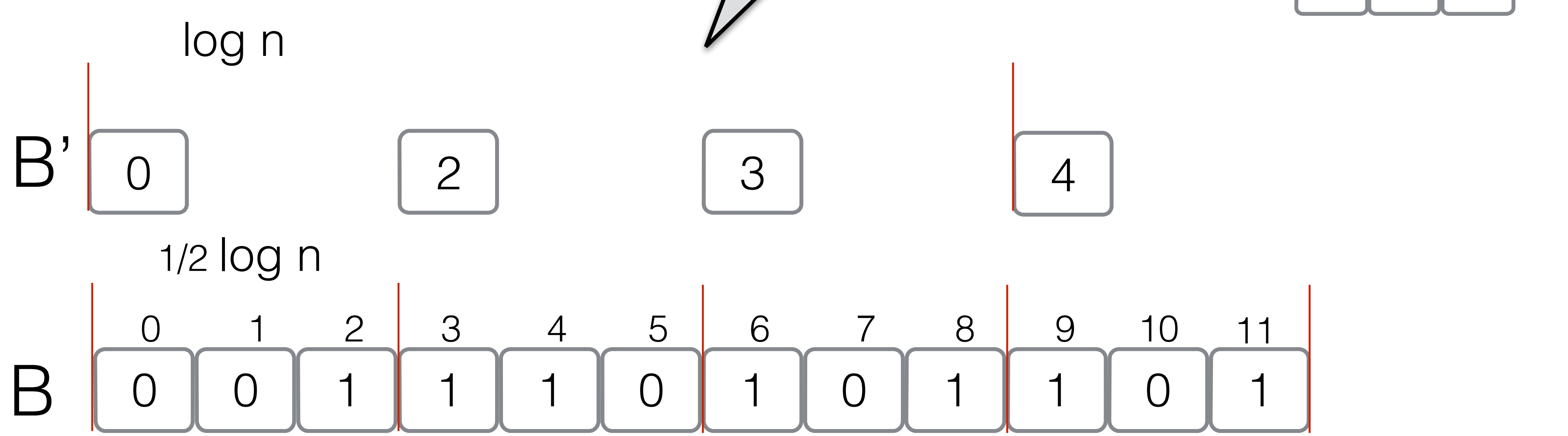
$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

Space: n + O(n log log n/log n) bits
Query time: O(1)



Groups into superblocks!



Rank/Select queries

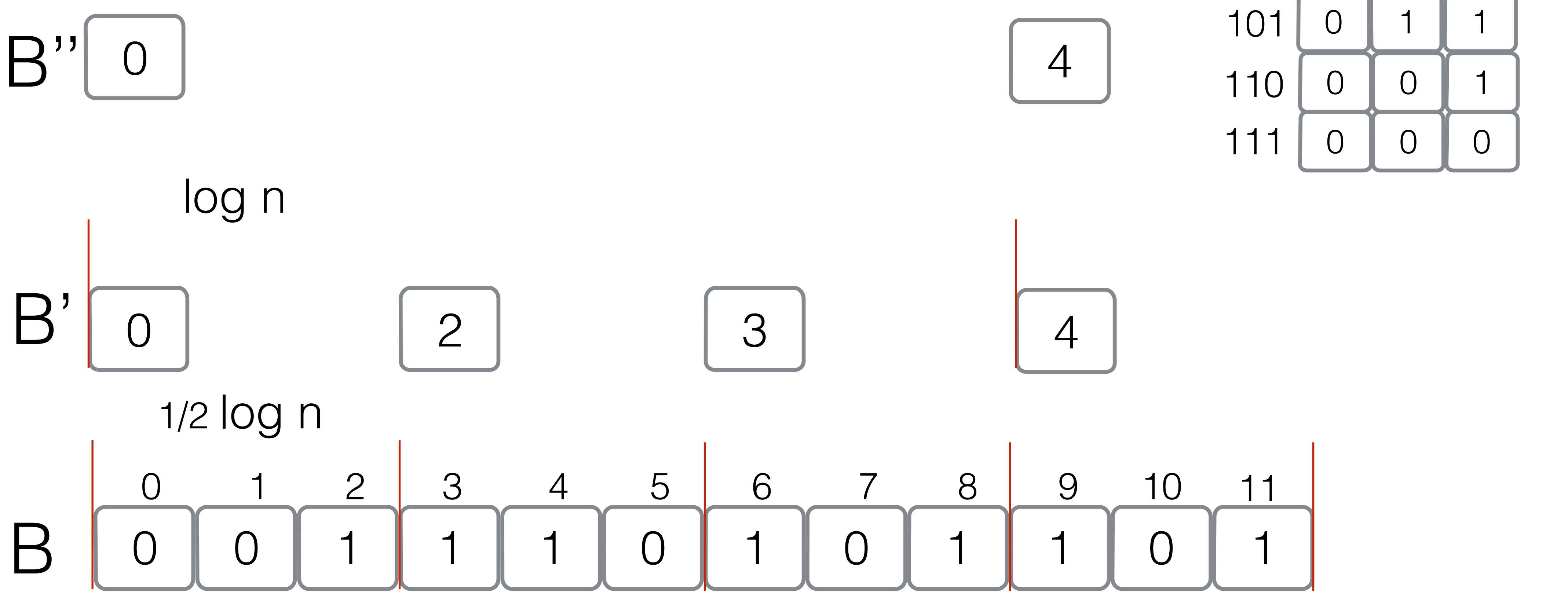
$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

Space: n + O(n log log n/log n) bits
Query time: O(1)

M

	1	2	3
000	1	2	3
001	1	2	2
010	1	1	2
011	1	1	1
100	0	1	2
101	0	1	1
110	0	0	1
111	0	0	0



Rank/Select queries

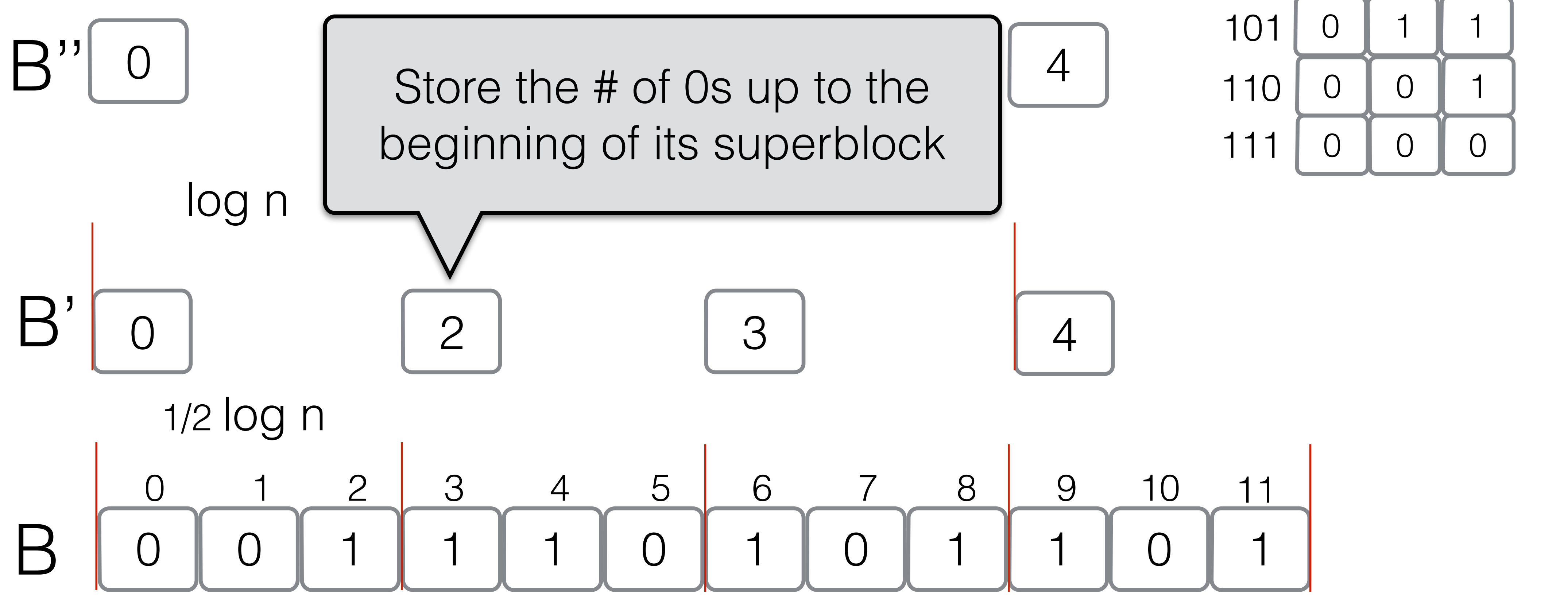
$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

Space: n + O(n log log n/log n) bits
Query time: O(1)

M

	1	2	3
000	1	2	3
001	1	2	2
010	1	1	2
011	1	1	1
100	0	1	2
101	0	1	1
110	0	0	1
111	0	0	0



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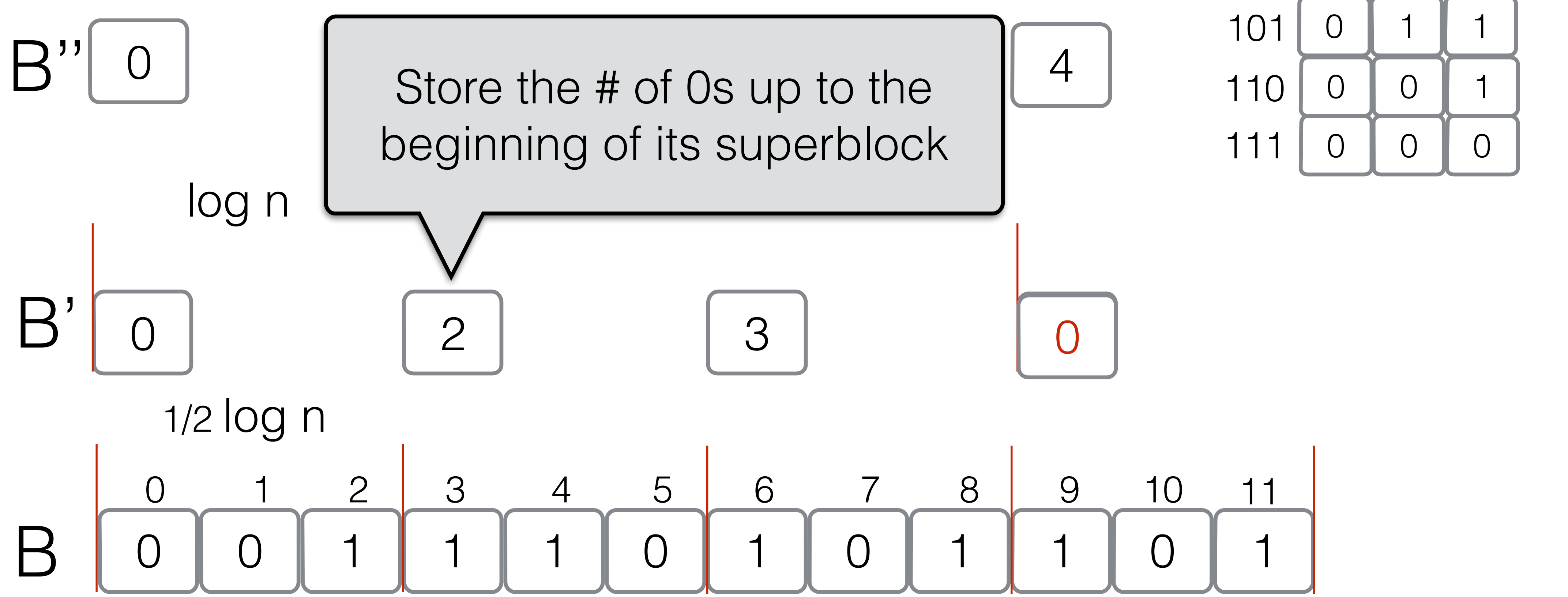

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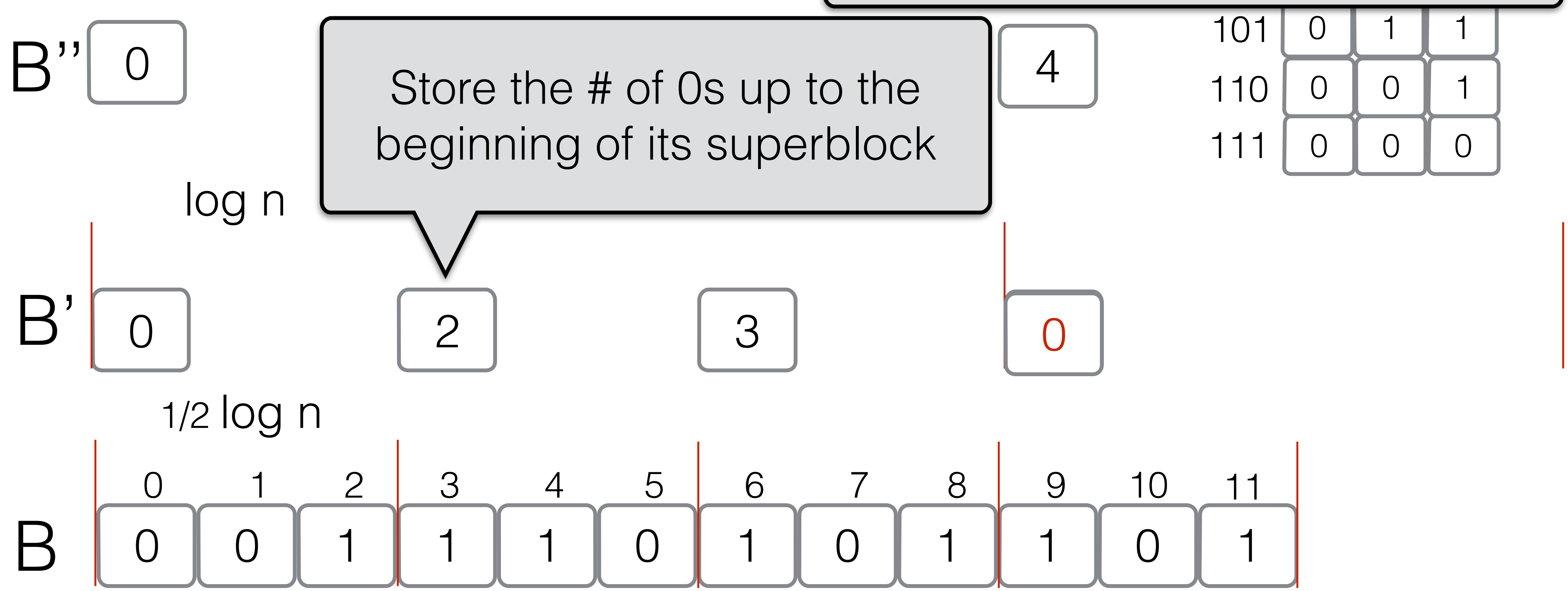
$Rank_1(j)$ = # of 1 in B[0,j]

Space: n + O(n log log n/log n) bits
Query time: O(1)

$Rank_0(j)$ is split into 3 parts:

- # of 0s up to the superblock of j
- # of 0s up to the block of j
- # of 0s within the block of j



Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

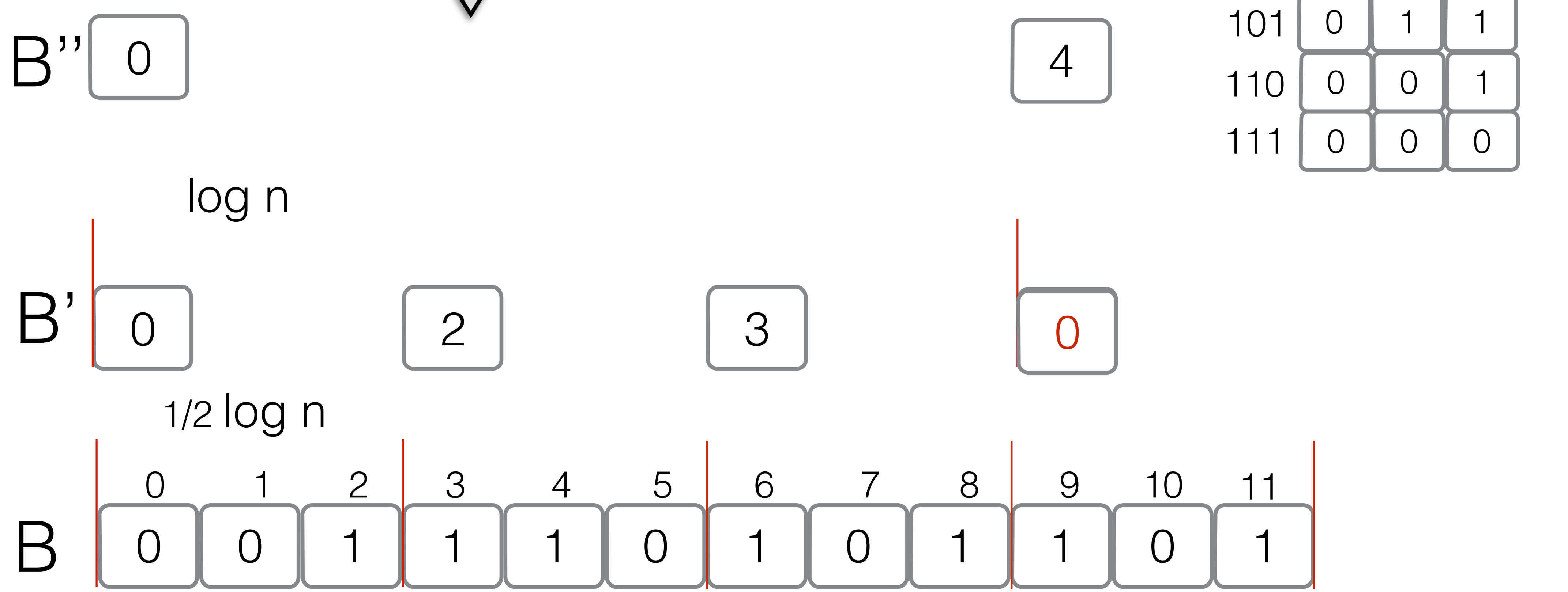
$Rank_1(j)$ = # of 1 in B[0,j]

) bits

How much space?

M

	1	2	3
000	1	2	3
001	1	2	2
010	1	1	2
011	1	1	1
100	0	1	2
101	0	1	1
110	0	0	1
111	0	0	0



Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

) bits

How much space?

O(n/log² n) entries,
each uses O(log n) bits
⇒ O(n/log n) bits :-)

M

	1	2	3
000	1	2	3
001	1	2	2
010	1	1	2
011	1	1	1
100	0	1	2
101	0	1	1
110	0	0	1
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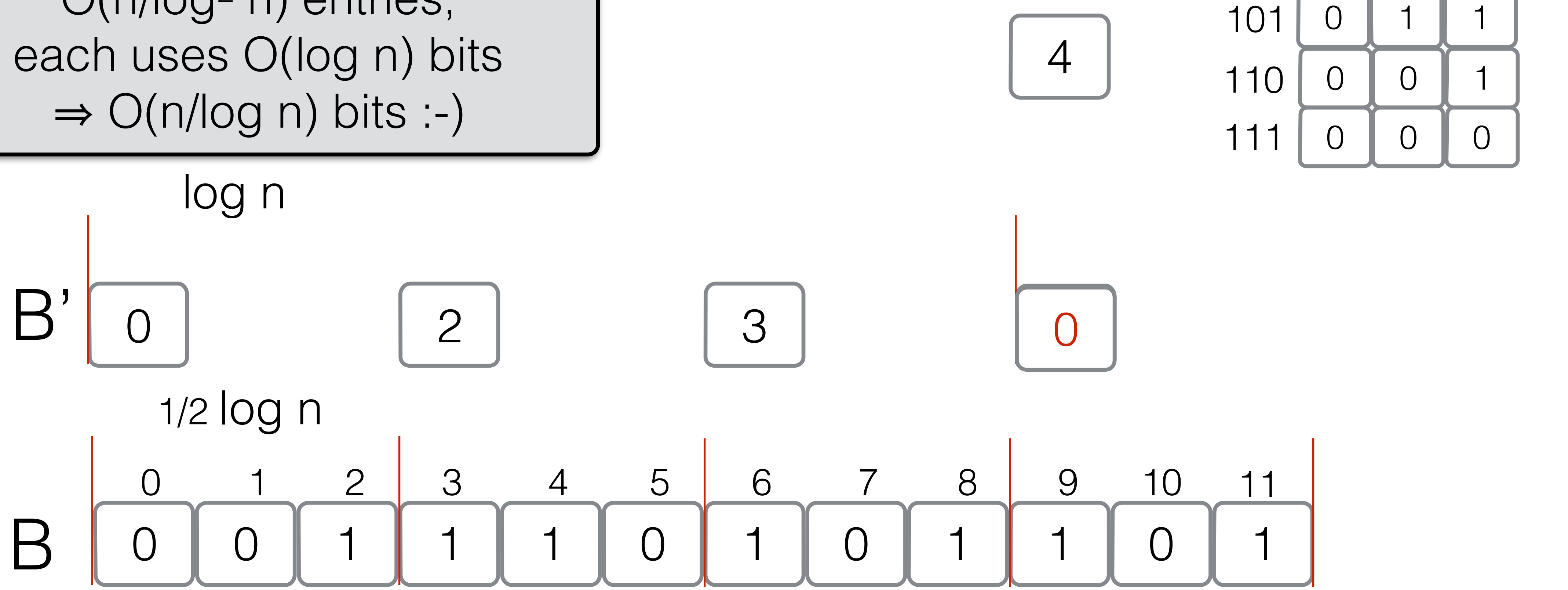

Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

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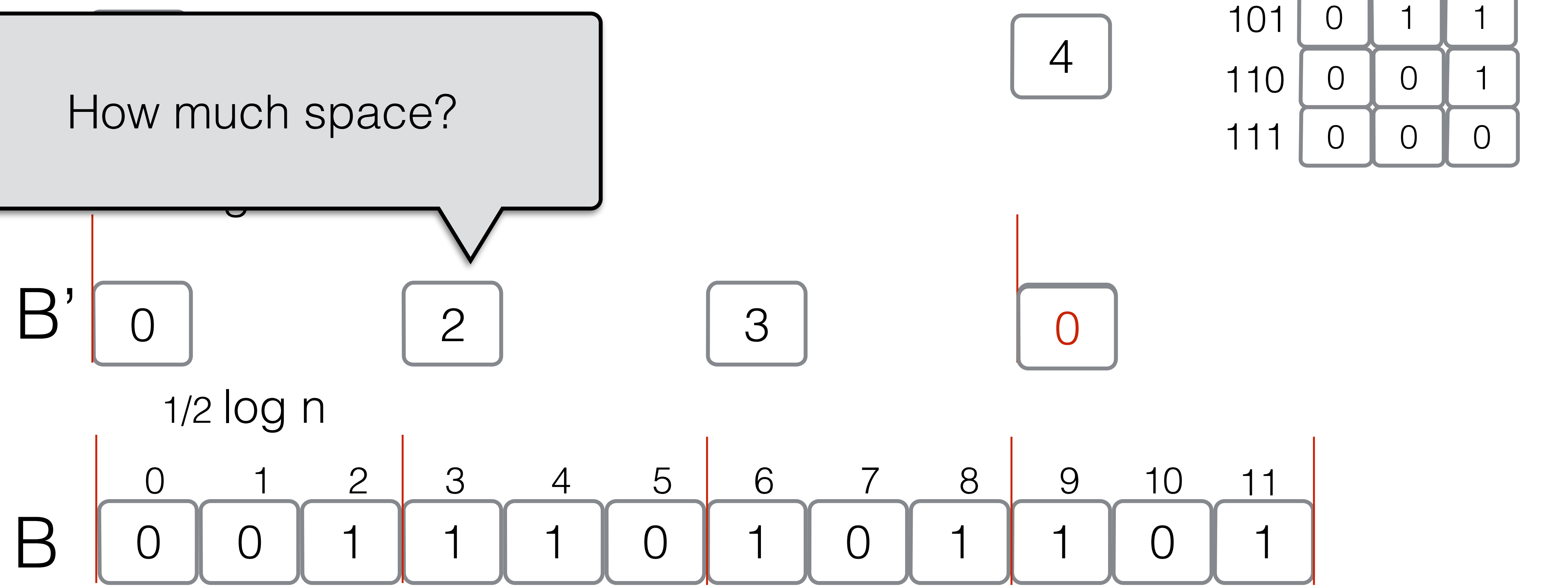
Space: n + O(n log log n/log n) bits
Query time: O(1)

How much space?

M

	1	2	3
000	1	2	3
001	1	2	2
010	1	1	2
011	1	1	1
100	0	1	2
101	0	1	1
110	0	0	1
111	0	0	0



Rank/Select queries

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> How much space?

> O(n/log n) entries,
> each uses O(log log n) bits
> ⇒ O(n log log n/log n) bits :-)

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	1	2	3
000	1	2	3
001	1	2	2
010	1	1	2
011	1	1	1
100	0	1	2
101	0	1	1
110	0	0	1
111	0	0	0

4

3

0

B

	0	1	2	3	4	5	6	7	8	9	10	11
B	0	0	1	1	1	0	1	0	1	1	0	1

Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

Space: n + O(n log log n/log n) bits
Query time: O(1)

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	1	2	3
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010	1	1	2
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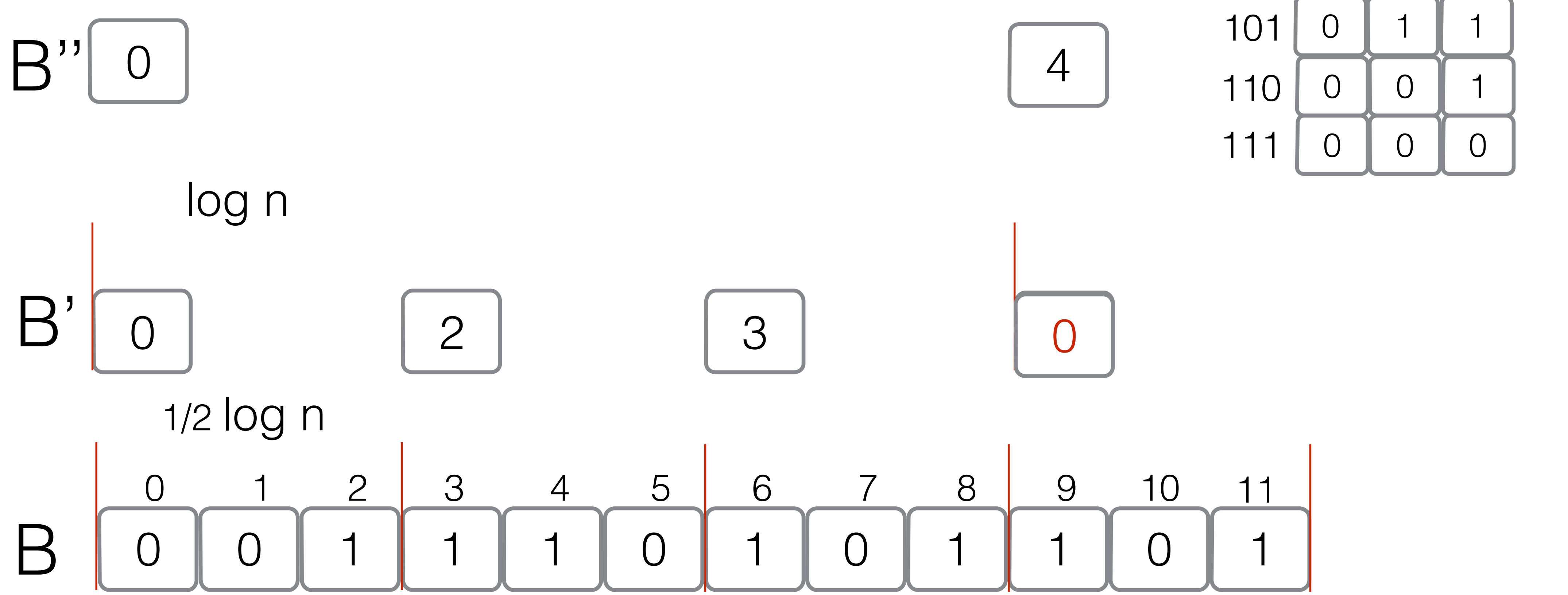
101	0	1	1
110	0	0	1
111	0	0	0



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Space: n + O(n log log n/log n) bits
Query time: O(1)

How to support Select in O(log n) time?

M	1	2	3
000	1	2	3
001	1	2	2
010	1	1	2
011	1	1	1
100	0	1	2
101	0	1	1
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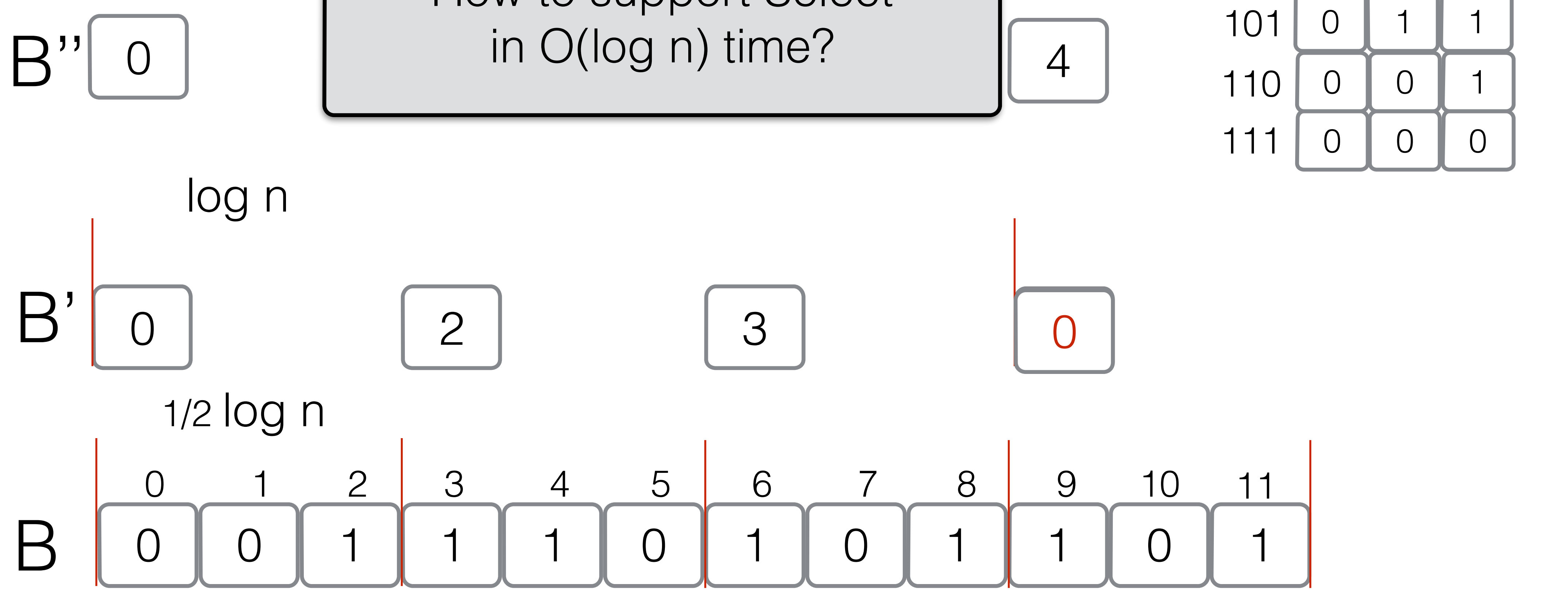
111	0	0	0



Rank/Select queries

$Rank_0(j)$ = # of 0 in B[0,j]

$Rank_1(j)$ = # of 1 in B[0,j]

Space: $n + O(n \log\log n / \log n)$ bits
Query time: $O(1)$

How to support Select in $O(\log n)$ time?

Select can be solved in $O(1)$ time with a more difficult approach

M	1	2	3
000	1	2	3
001	1	2	2
010	1	1	2
011	1	1	1
100	0	1	2
101	0	1	1
110	0	0	1
111	0	0	0

B'': 0 … 4 (log n)

B': 0 … 0 (1/2 log n)

	0	1	2	3	4	5	6	7	8	9	10	11
B	0	0	1	1	1	0	1	0	1	1	0	1

Elias-Fano representation

Given a sequence of n (positive) integers summing up to m

Elias-Fano representation

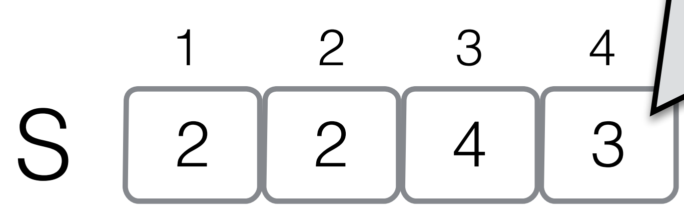
Given a sequence of n (positive) integers summing up to m

	1	2	3	4
S	2	2	4	3

Elias-Fano representation

Given a sequence of n (positive) integers summing up to m

Trivial representation requires
O(n log m) bits :-(
Can we do better?



Elias-Fano representation

Given a sequence of n (positive) integers summing up to m

	1	2	3	4
S	2	2	4	3

Represent the integer x by writing x-1 in unary to obtain B of m bits with n zeros

B

Elias-Fano representation

Given a sequence of n (positive) integers summing up to m

S

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2	2	4	3

Represent the integer x by writing x-1 in unary to obtain B of m bits with n zeros

B

0	1
1	0

Elias-Fano representation

Given a sequence of n (positive) integers summing up to m

	1	2	3	4
S	2	2	4	3

Represent the integer x by writing x-1 in unary to obtain B of m bits with n zeros

	0	1	2	3
B	1	0	1	0

Elias-Fano representation

Given a sequence of n (positive) integers summing up to m

	1	2	3	4
S	2	2	4	3

Represent the integer x by writing x-1 in unary to obtain B of m bits with n zeros

	0	1	2	3	4	5	6	7
B	1	0	1	0	1	1	1	0

Elias-Fano representation

Given a sequence of n (positive) integers summing up to m

S

1	2	3	4
2	2	4	3

Represent the integer x by writing x-1 in unary to obtain B of m bits with n zeros

B

0	1	2	3	4	5	6	7	8	9	10
1	0	1	0	1	1	1	0	1	1	0

Elias-Fano representation

Given a sequence of n (positive) integers summing up to m

S

1	2	3	4
2	2	4	3

The i-th value of S is $Select_0(i)$-$Select_0(i-1)$

Represent the integer x by writing x-1 in unary to obtain B of m bits with n zeros

B

0	1	2	3	4	5	6	7	8	9	10
1	0	1	0	1	1	1	0	1	1	0

Elias-Fano representation

Given a sequence of n (positive) integers summing up to m

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1	0	1	0	1	1	1	0	1	1	0

Elias-Fano representation

Given a sequence of n (positive) integers summing up to m

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S	2	2	4	3

The i-th value of S is $Select_0(i)-Select_0(i-1)$

Represent the integer x by writing x-1 in unary to obtain B of m bits with n zeros

$Select_0(2)=3$ $Select_0(3)=7$

	0	1	2	3	4	5	6	7	8	9	10
B	1	0	1	0	1	1	1	0	1	1	0

Elias-Fano representation

Given a sequence of n (positive) integers summing up to m

Space: n log (m/n) + O(n) bits
$Select_0$ in O(1)

See Lecture 6 (10/10/2013)

	1	2	3	4
S	2	2	4	3

The i-th value of S is $Select_0(i)-Select_0(i-1)$

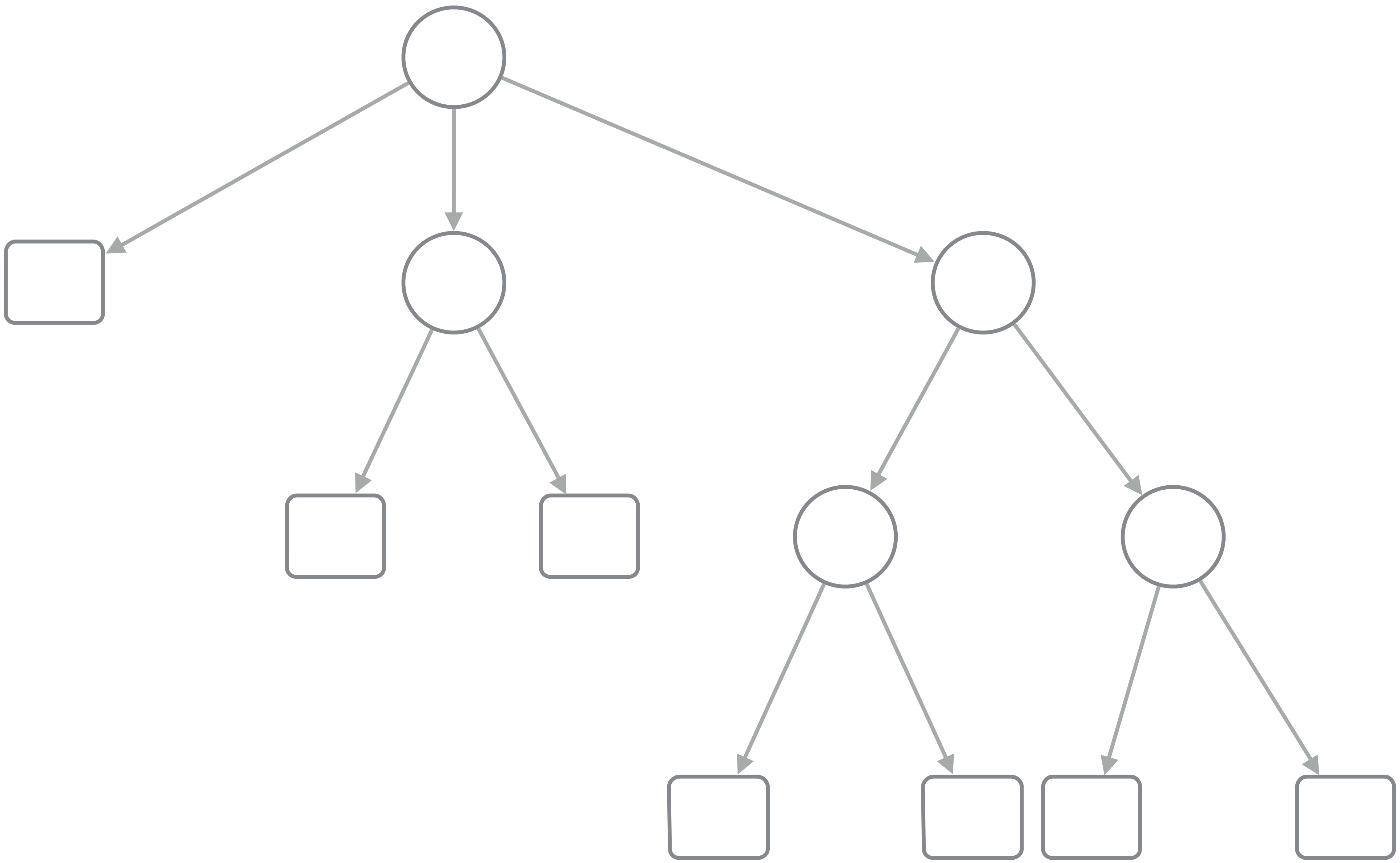
Represent the integer x by writing x-1 in unary to obtain B of m bits with n zeros

$Select_0(2)=3$ $Select_0(3)=7$

	0	1	2	3	4	5	6	7	8	9	10
B	1	0	1	0	1	1	1	0	1	1	0

Succinct representation of trees (1)

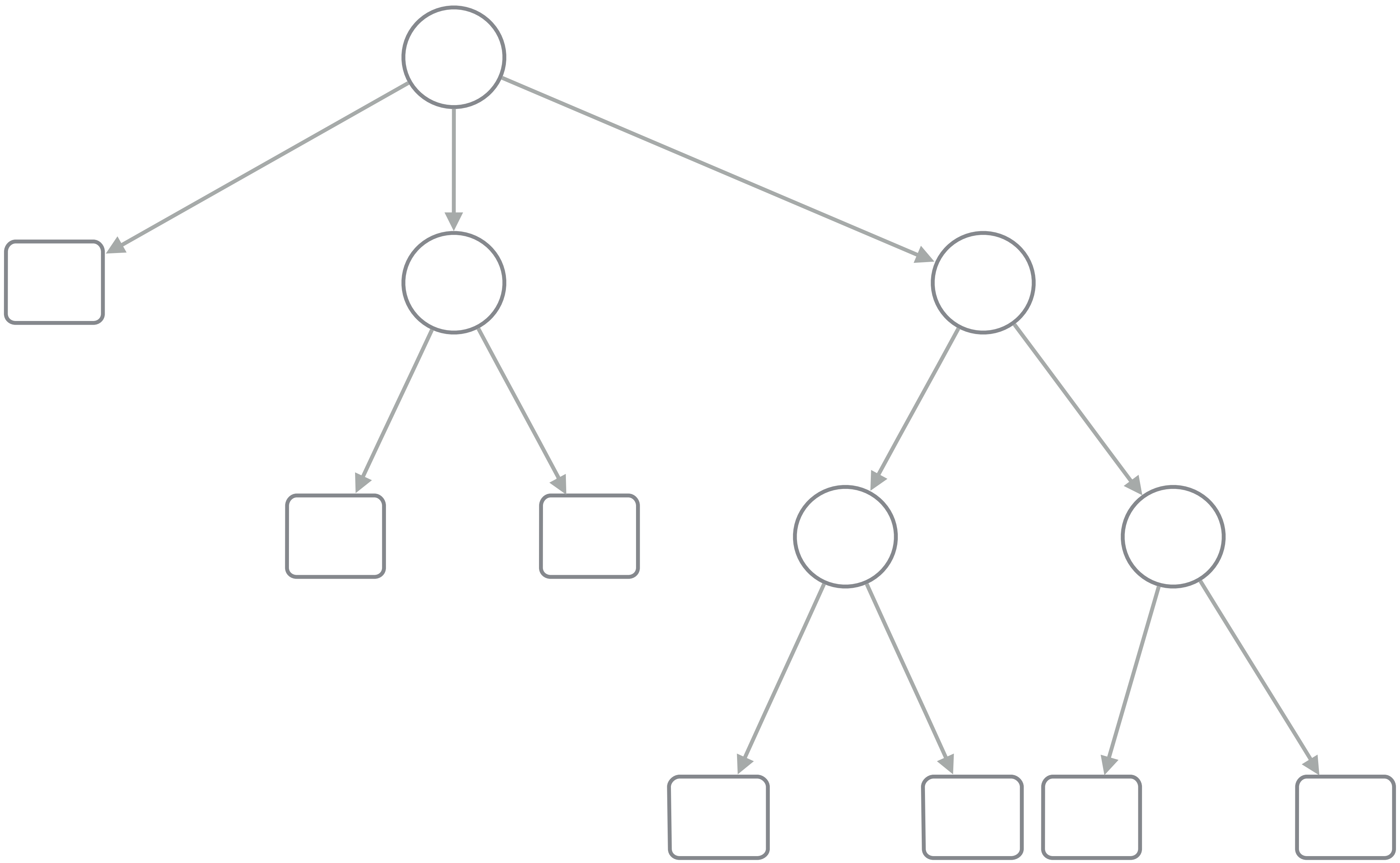
[LOUDS - Level-order unary degree sequence]



Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

Trivial: O(n log n) bits

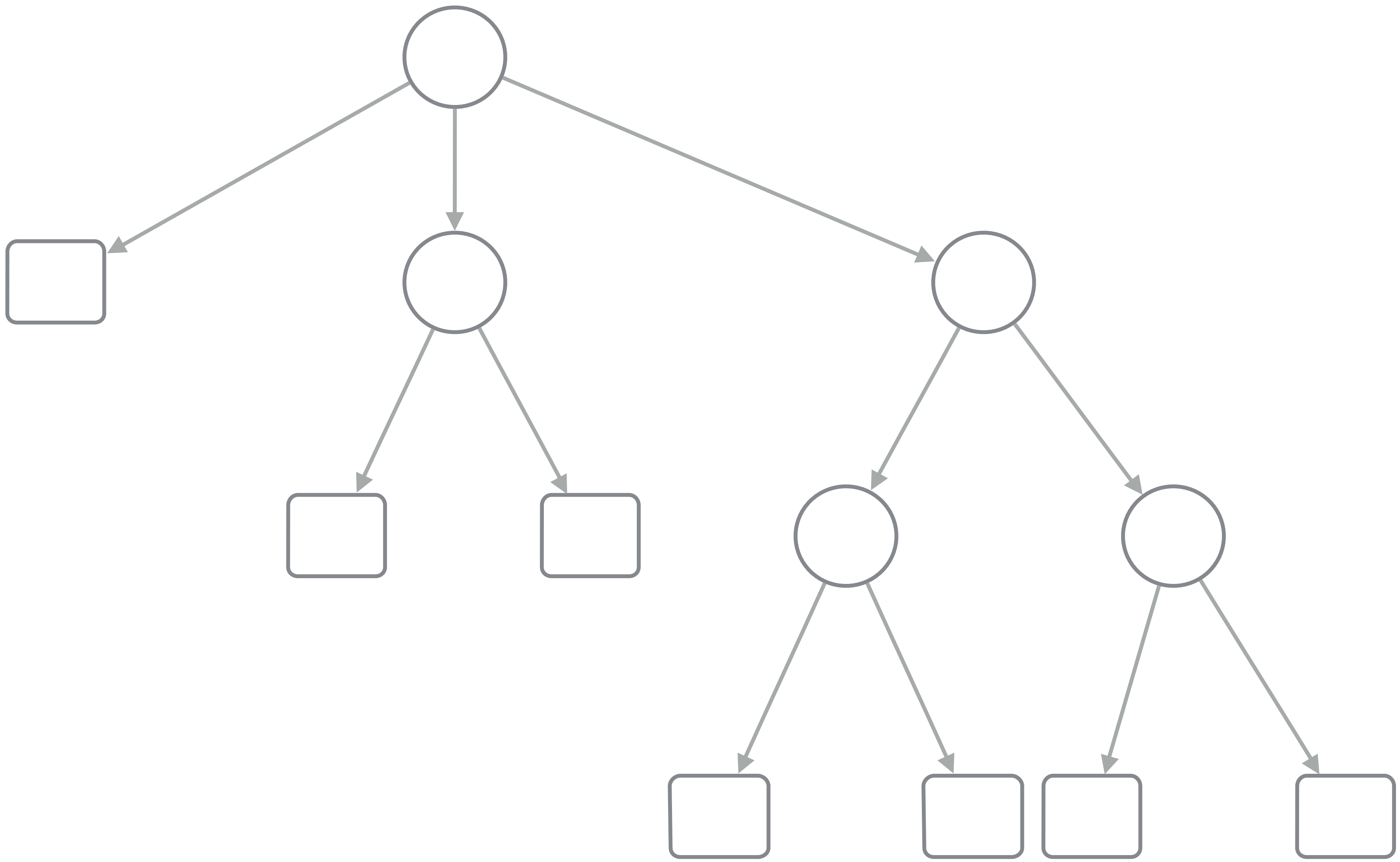


Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

Trivial: O(n log n) bits

Best: 2n bits



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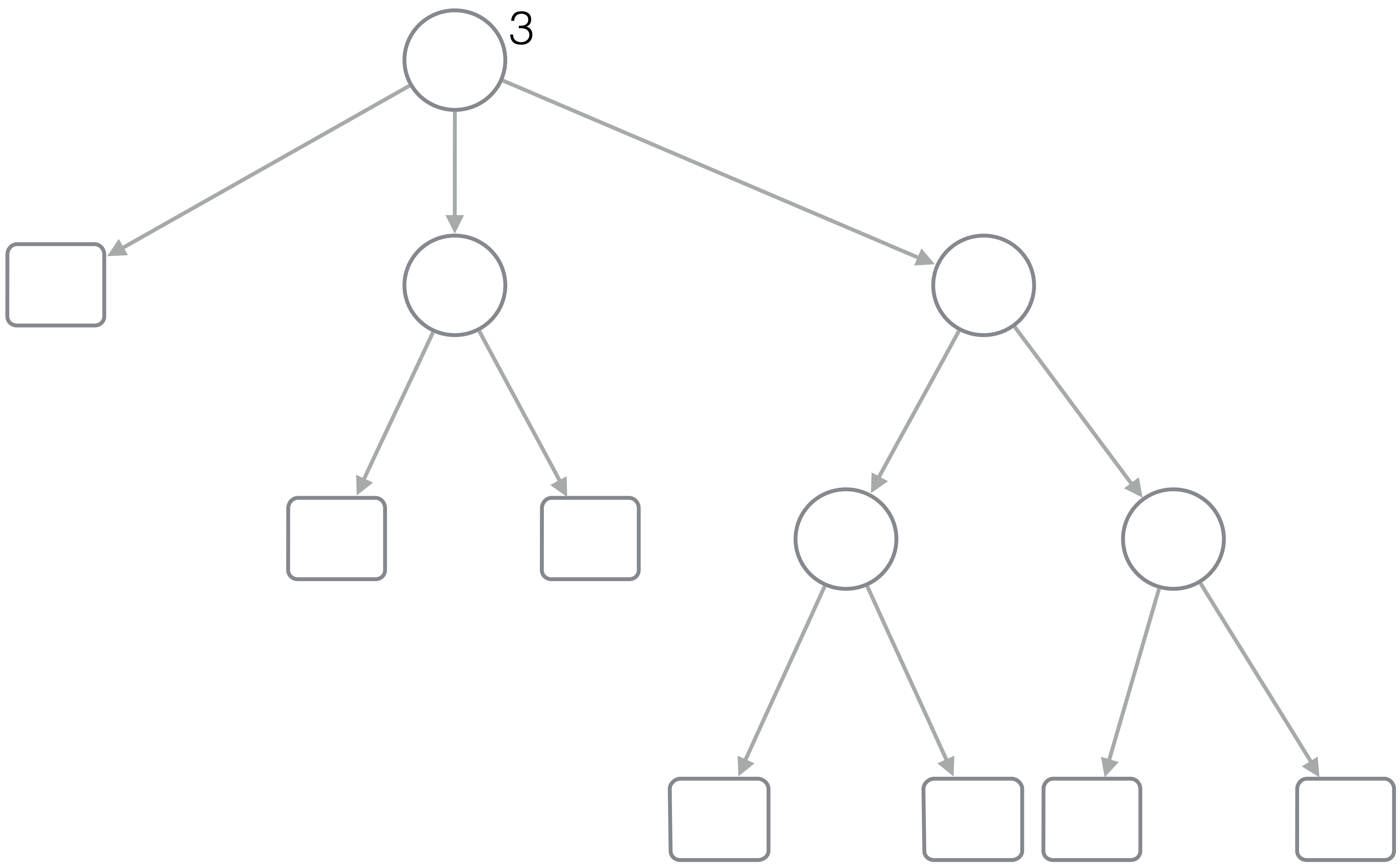

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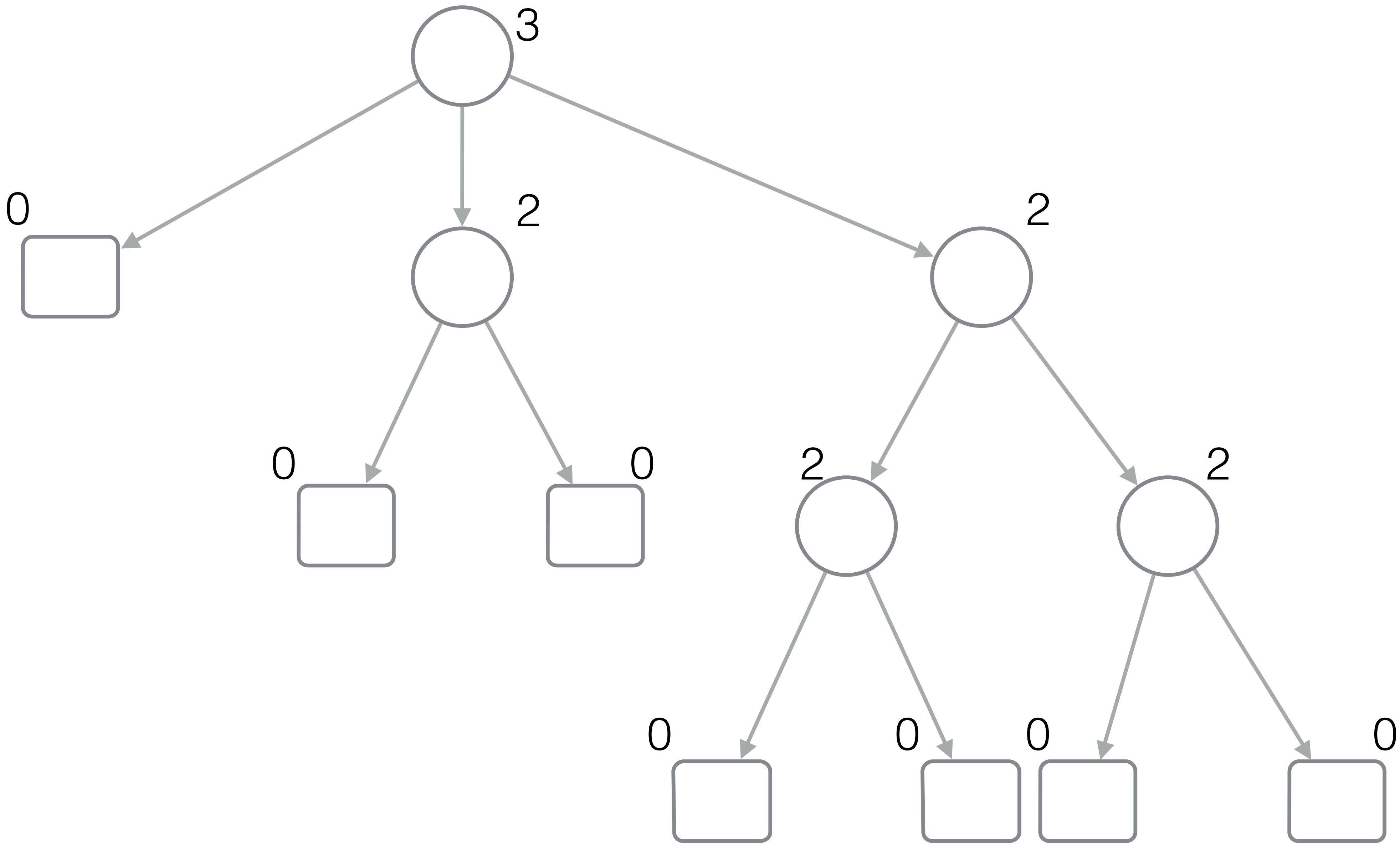

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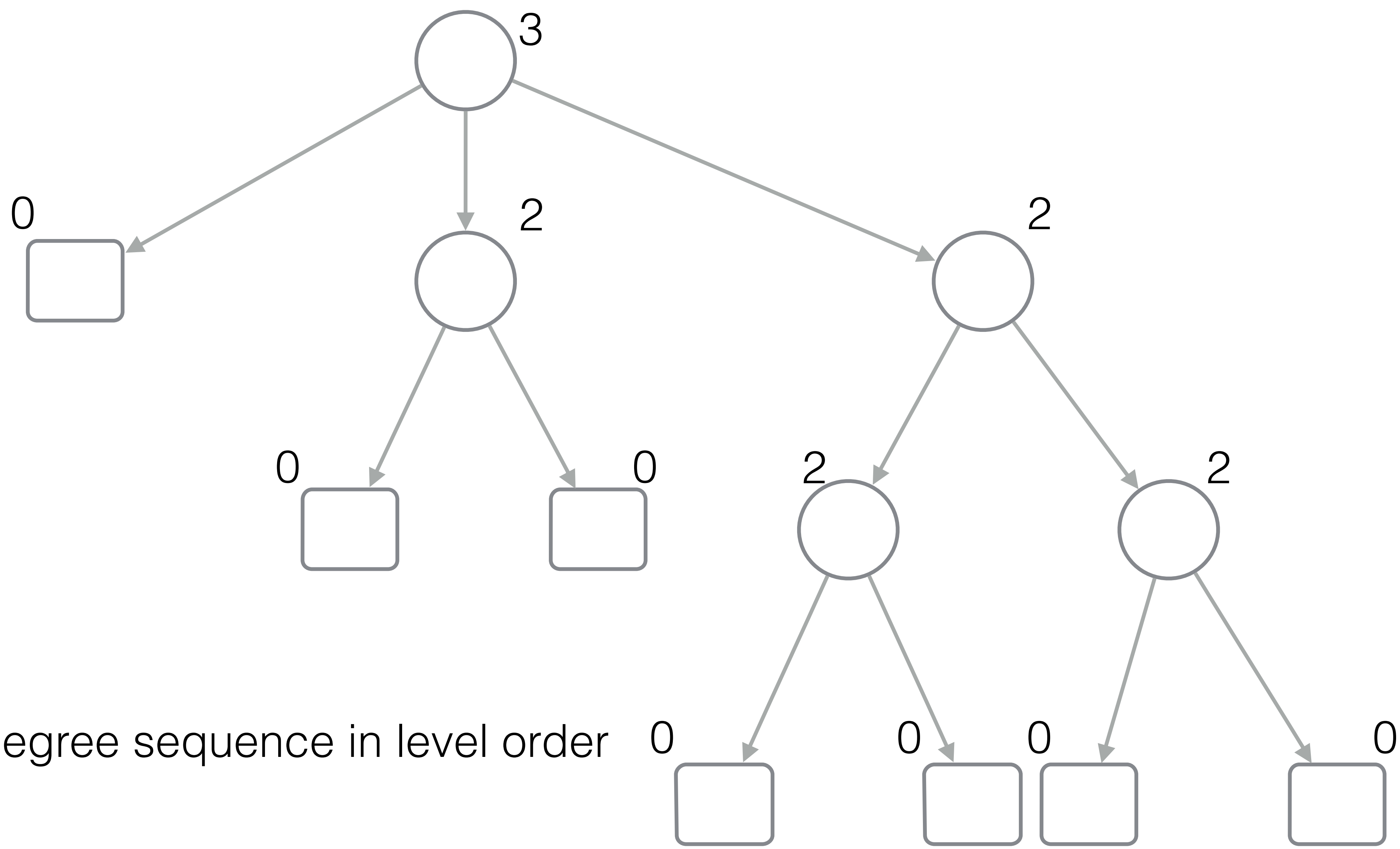

Write the degree sequence in level order

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

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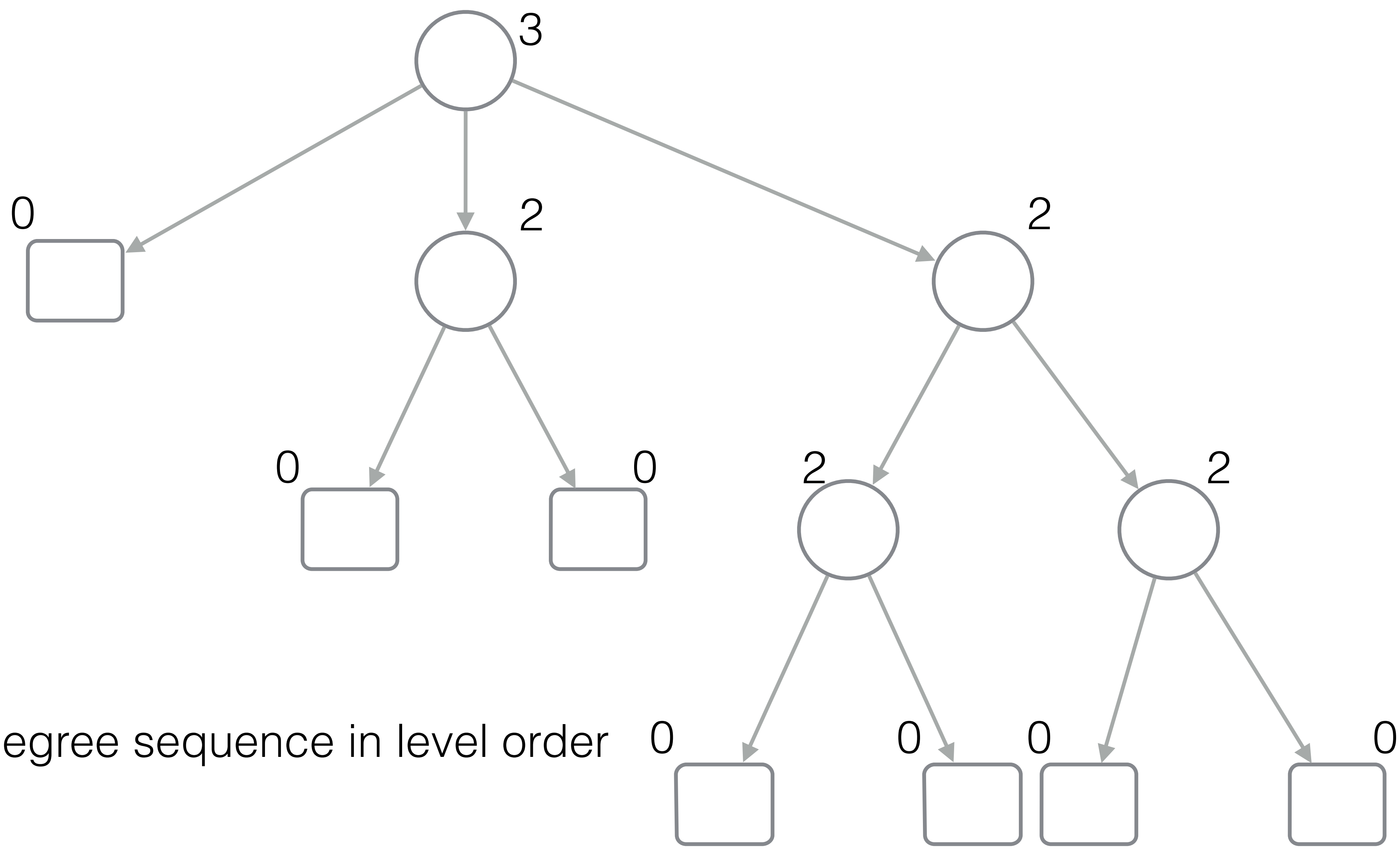


Write the degree sequence in level order

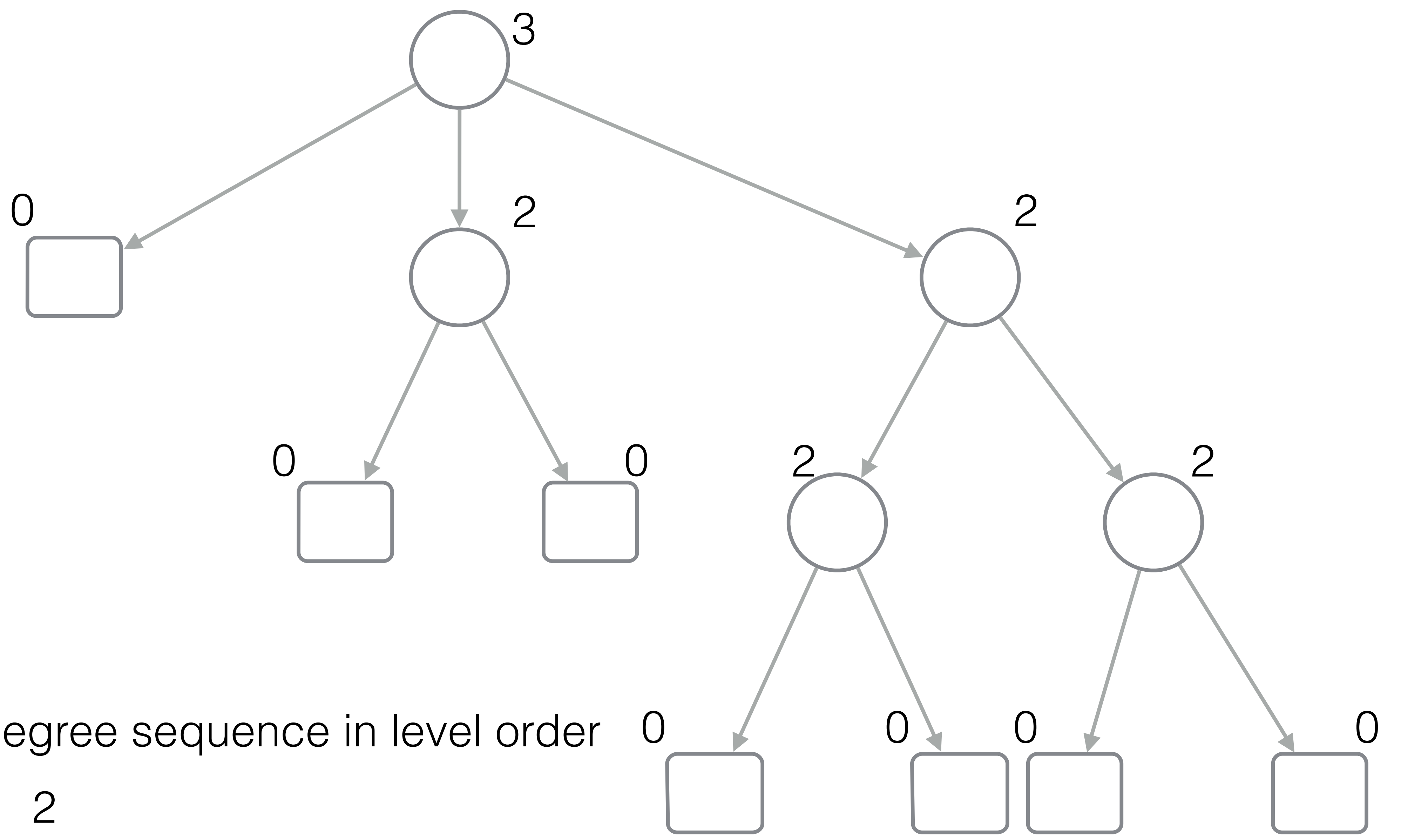
D 3

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

Trivial: O(n log n) bits

Best: 2n bits



Write the degree sequence in level order

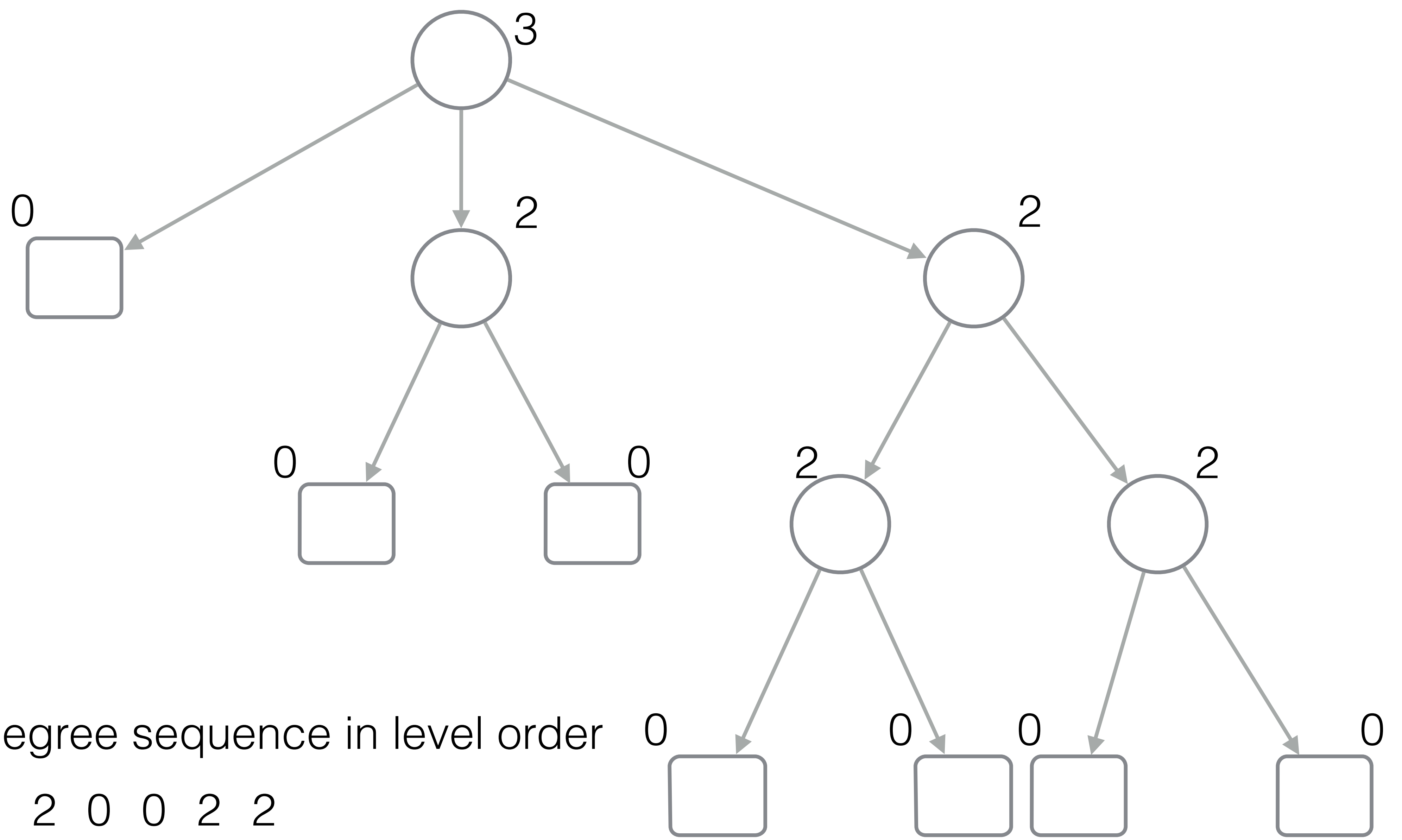
D 3 0 2 2

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

Trivial: O(n log n) bits

Best: 2n bits



Write the degree sequence in level order

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Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

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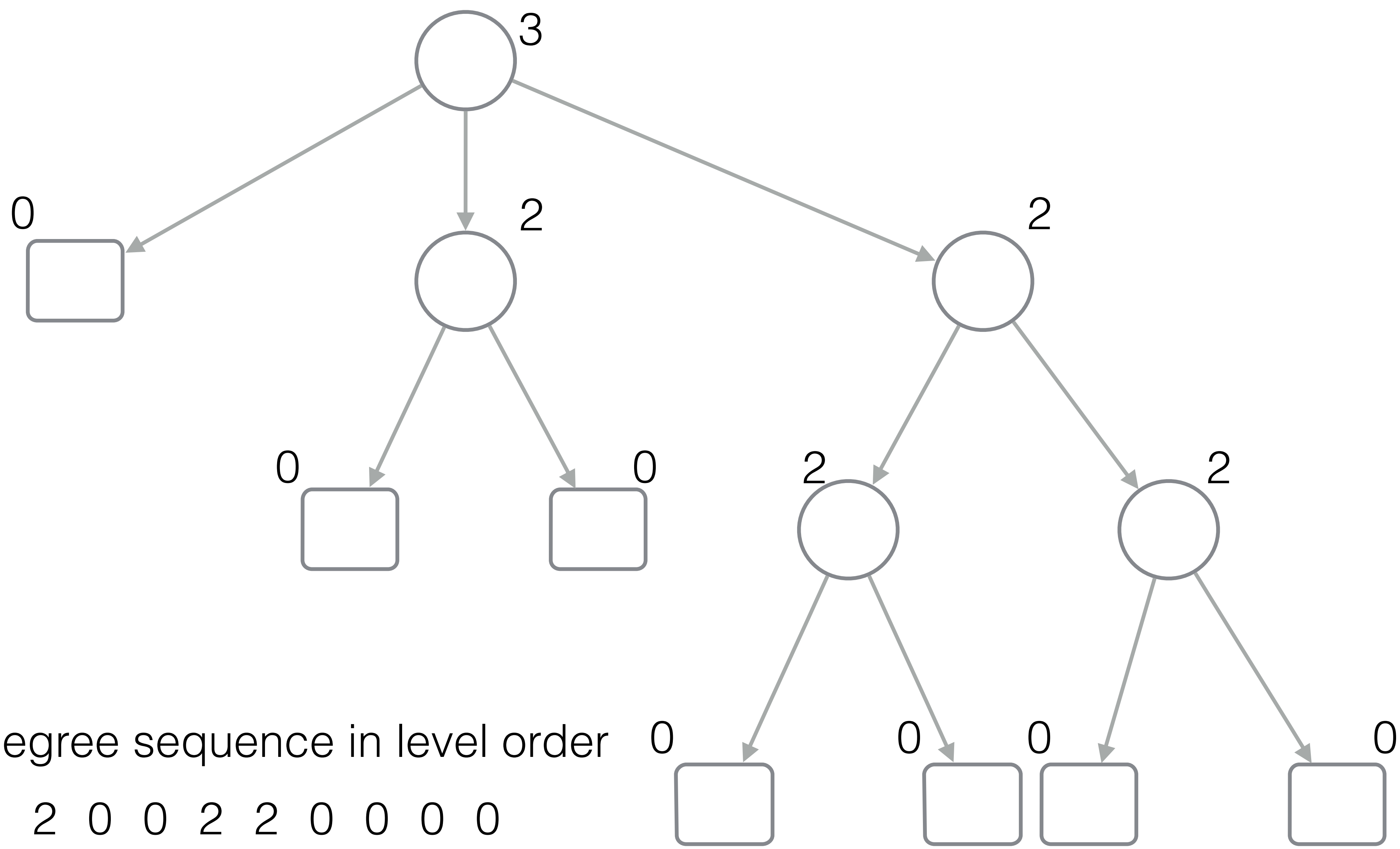

Write the degree sequence in level order

D 3 0 2 2 0 0 2 2 0 0 0 0

Succinct representation of trees (1)

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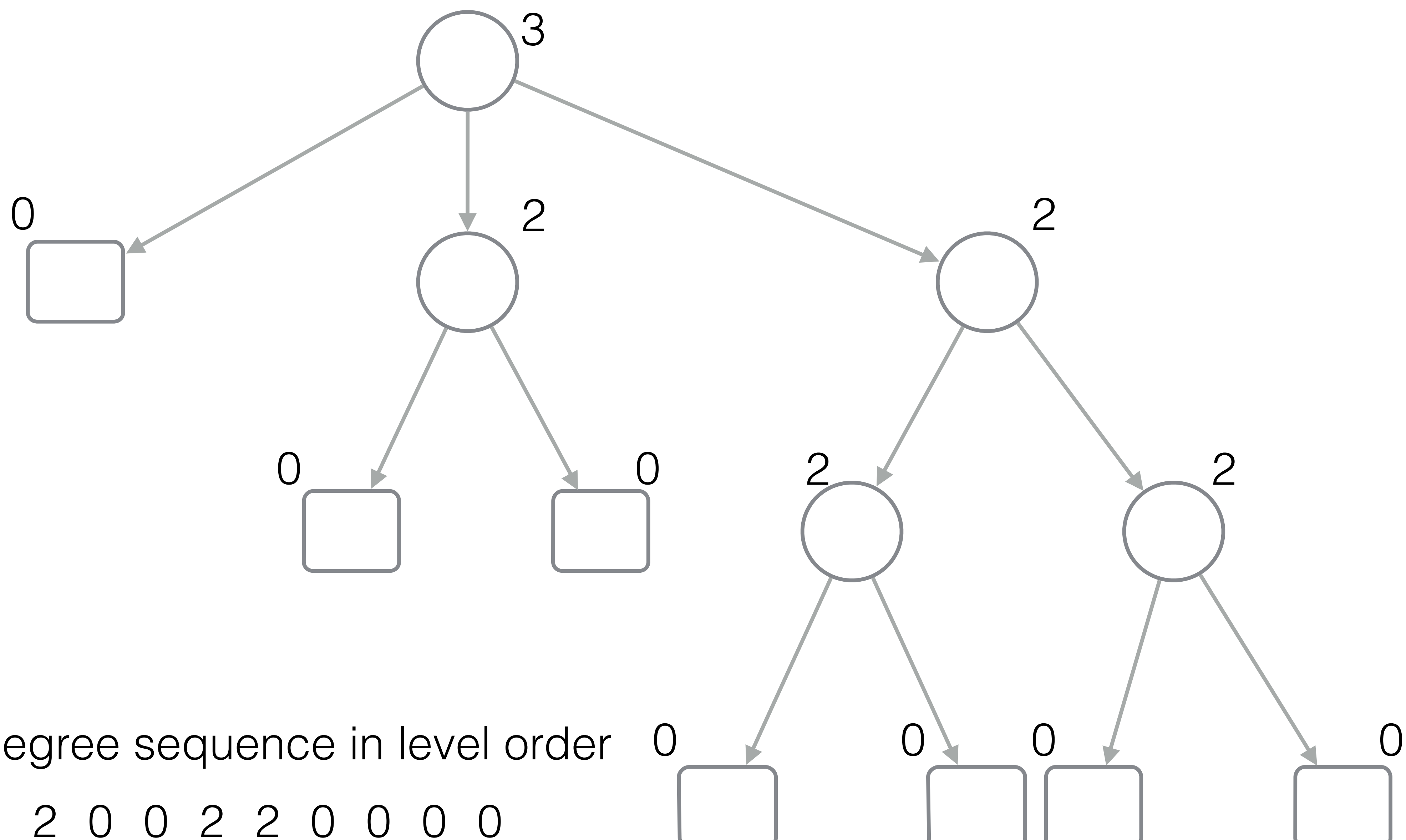
Best: 2n bits



Write the degree sequence in level order

D 3 0 2 2 0 0 2 2 0 0 0 0

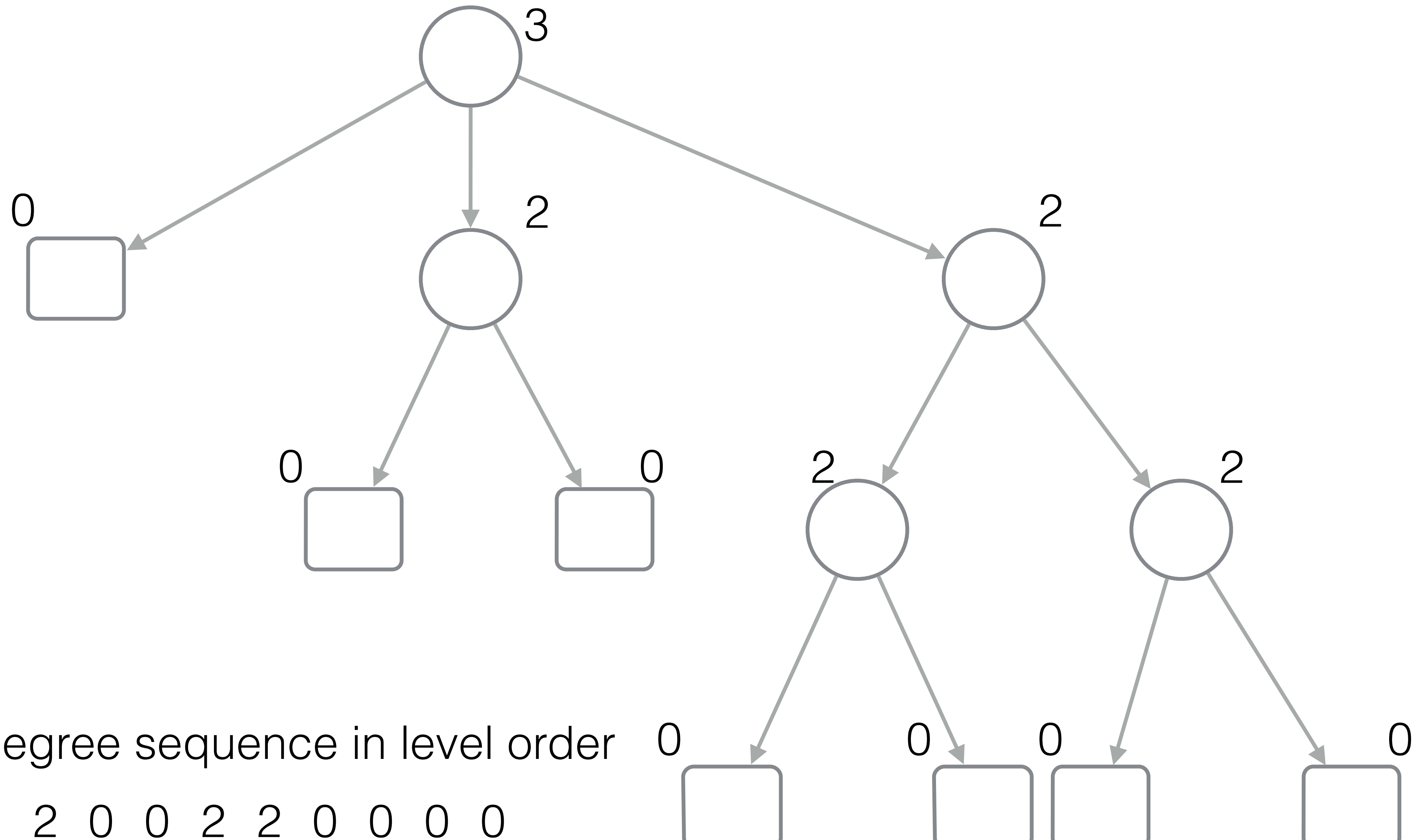
A tree is uniquely determined by the degree sequence

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

Trivial: O(n log n) bits

Best: 2n bits



Write the degree sequence in level order

D 3 0 2 2 0 0 2 2 0 0 0 0

A tree is uniquely determined by the degree sequence

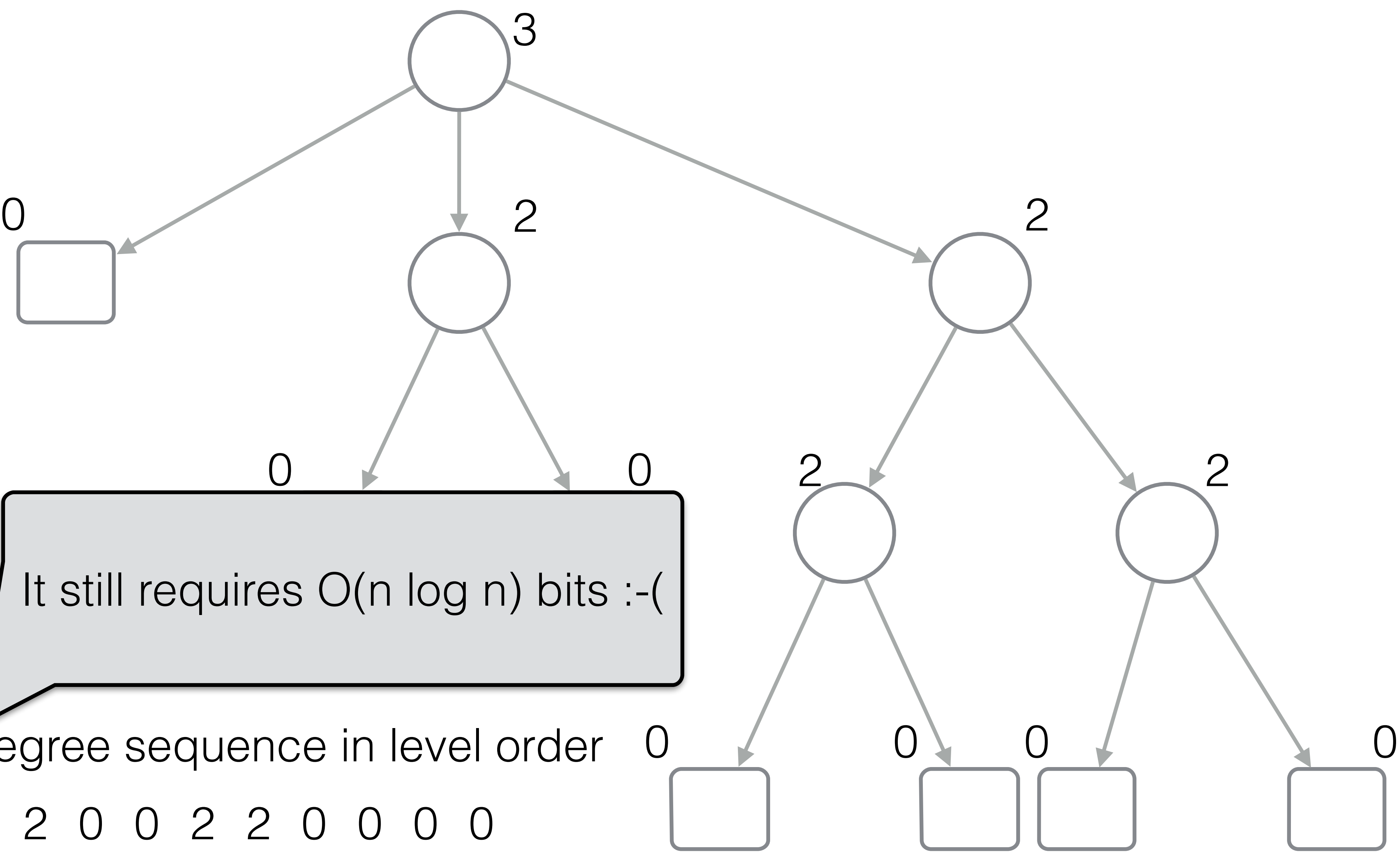
How reconstruct the tree?

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

Trivial: O(n log n) bits

Best: 2n bits



It still requires O(n log n) bits :-(

Write the degree sequence in level order

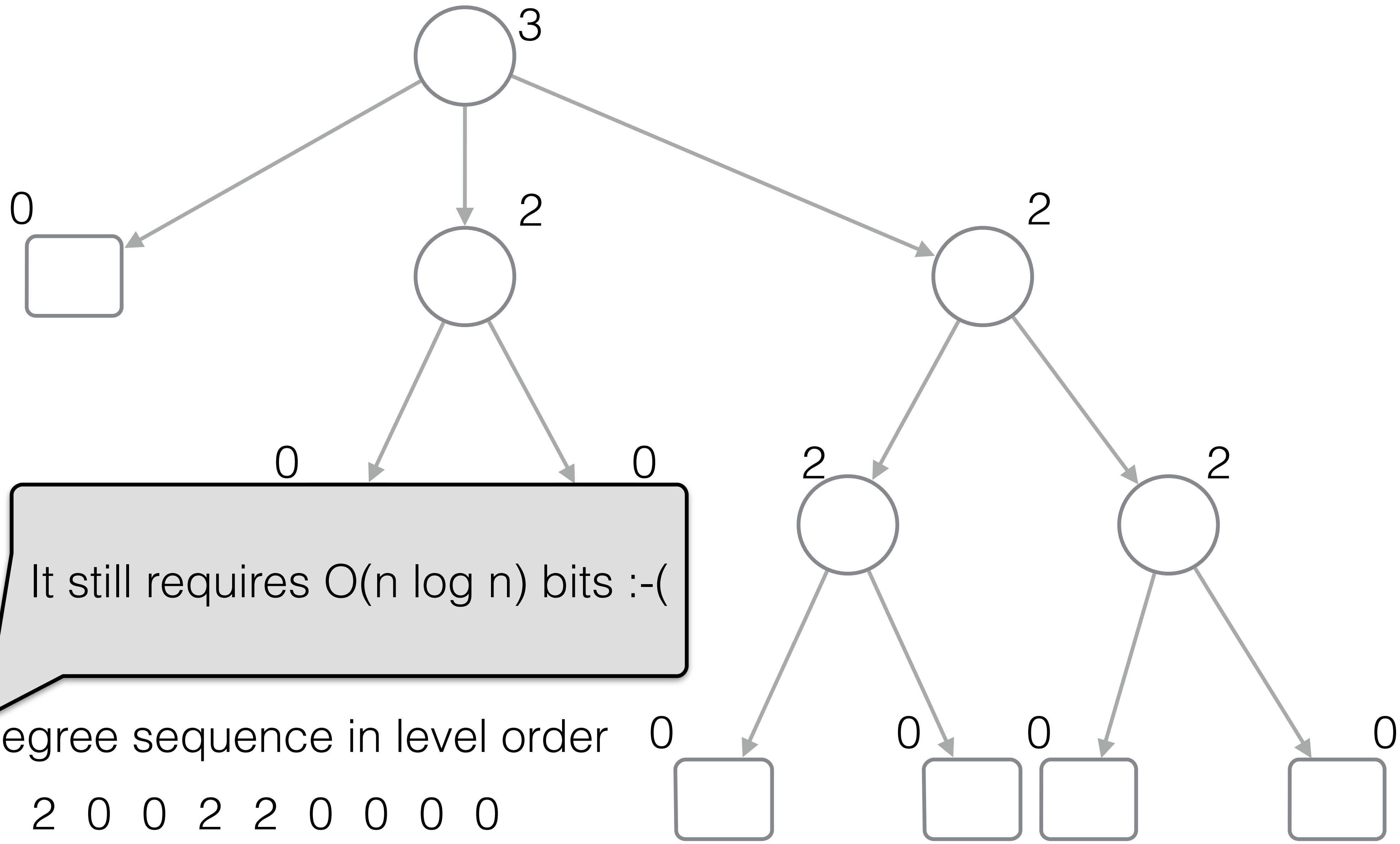
D 3 0 2 2 0 0 2 2 0 0 0 0

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

Trivial: O(n log n) bits

Best: 2n bits



It still requires O(n log n) bits :-(

Write the degree sequence in level order

D 3 0 2 2 0 0 2 2 0 0 0 0

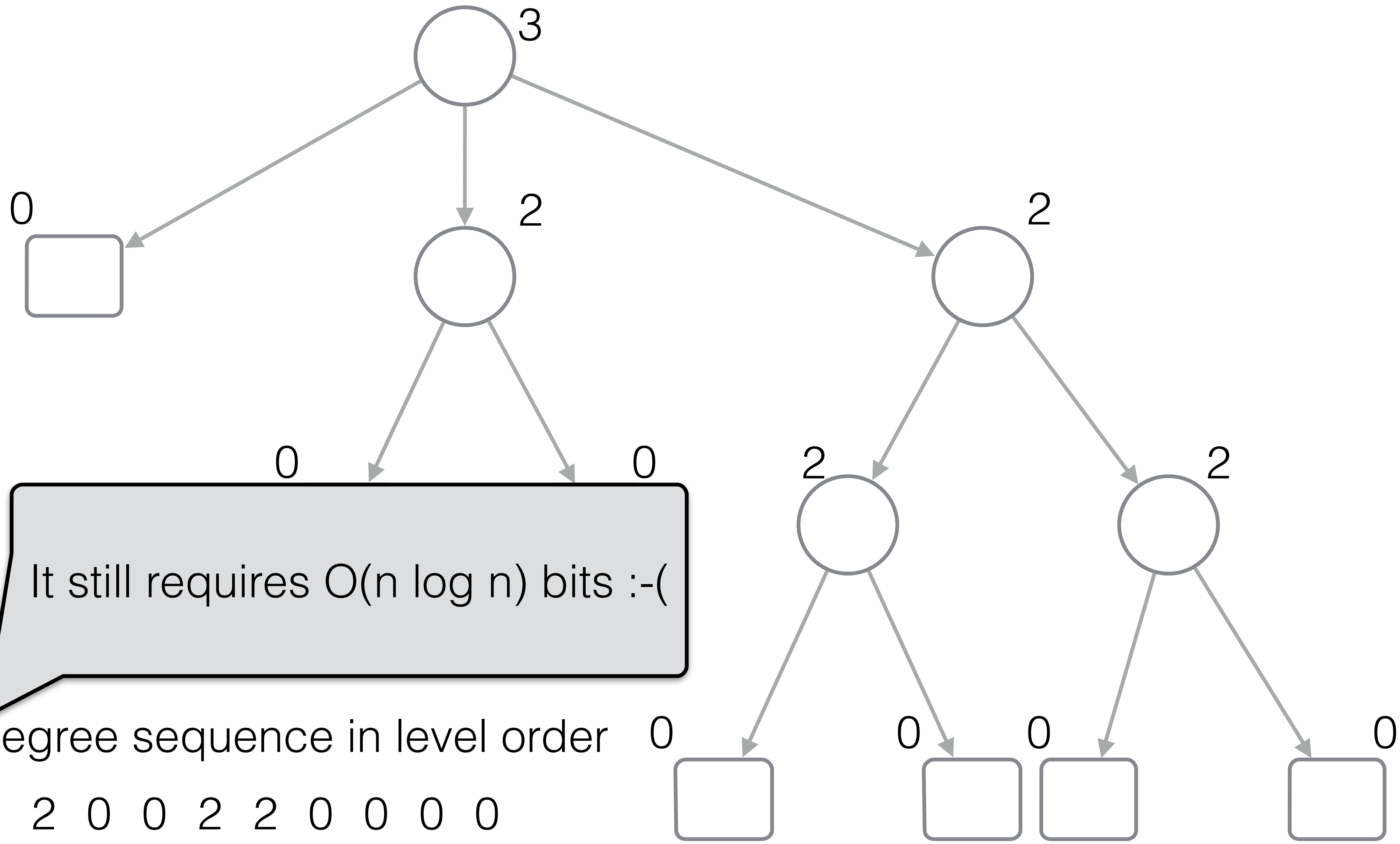
Solution: write them in unary

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

Trivial: O(n log n) bits

Best: 2n bits



It still requires O(n log n) bits :-(

Write the degree sequence in level order

D 3 0 2 2 0 0 2 2 0 0 0 0

Solution: write them in unary

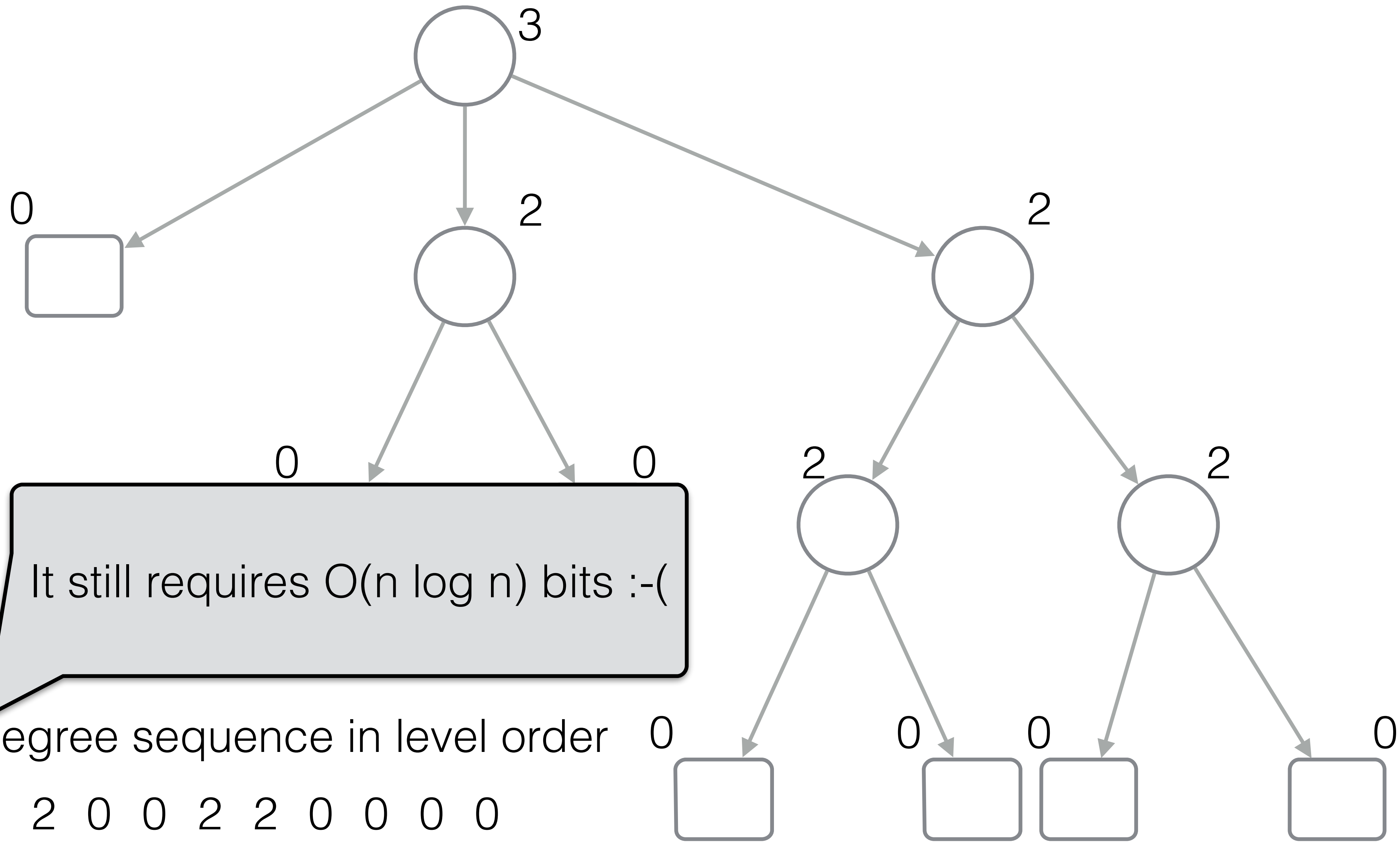
B

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

Trivial: O(n log n) bits

Best: 2n bits



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Write the degree sequence in level order

D 3 0 2 2 0 0 2 2 0 0 0 0

Solution: write them in unary

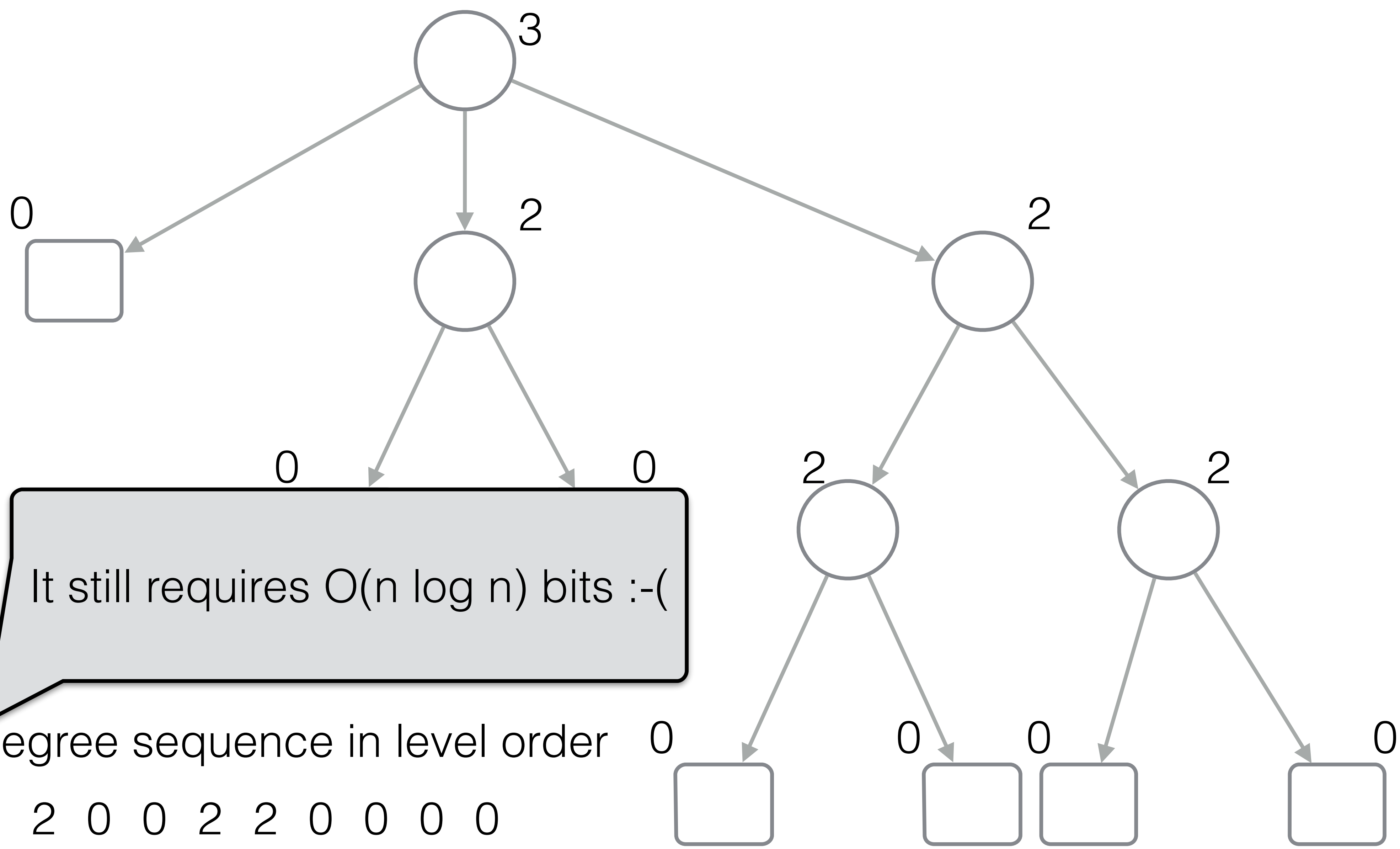
B 1110

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

Trivial: O(n log n) bits

Best: 2n bits



It still requires O(n log n) bits :-(

Write the degree sequence in level order

D 3 0 2 2 0 0 2 2 0 0 0 0

Solution: write them in unary

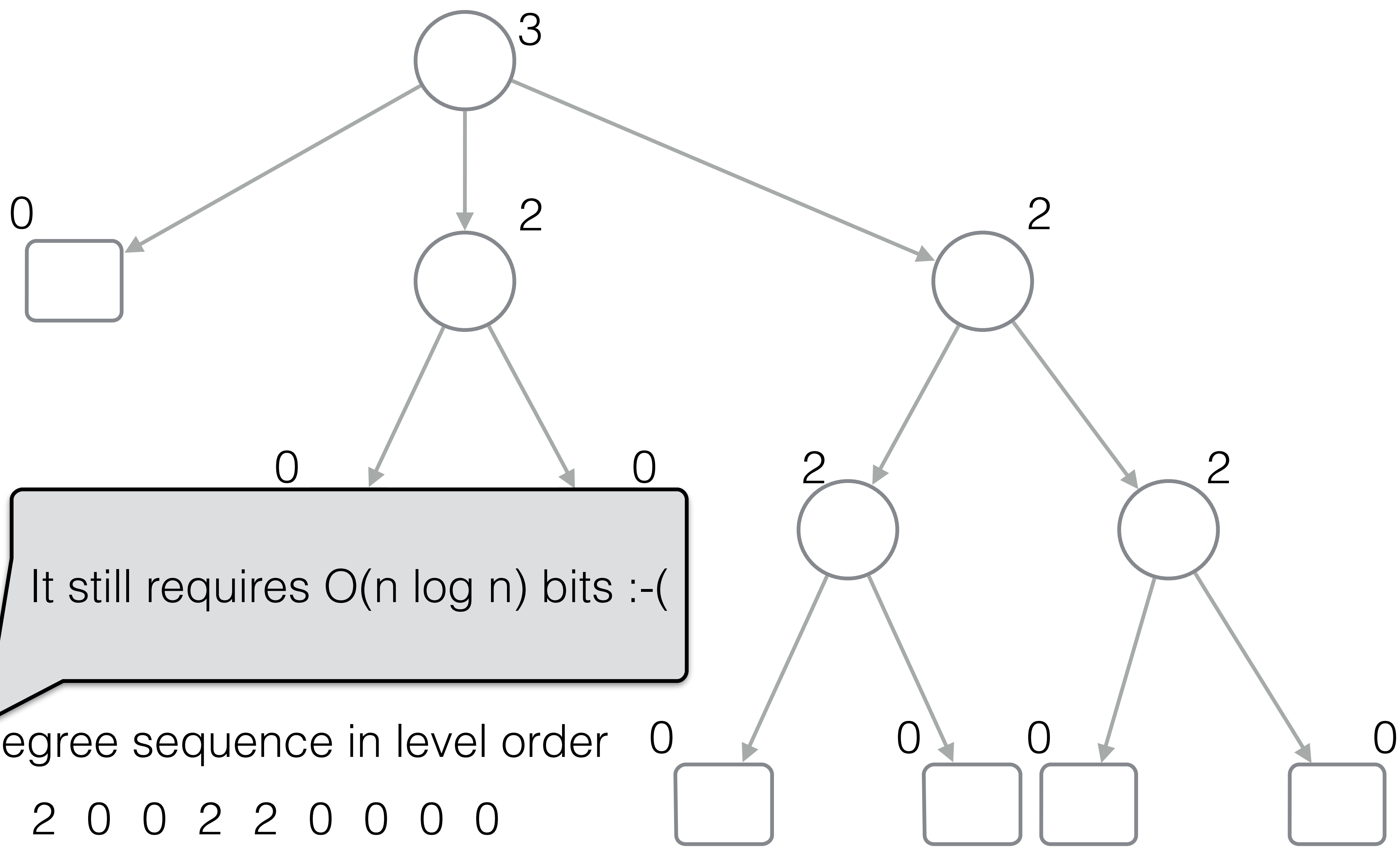
B 1110 0 110 110 0 0 110 110 0 0 0 0

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

Trivial: O(n log n) bits

Best: 2n bits



It still requires O(n log n) bits :-(

Write the degree sequence in level order

D 3 0 2 2 0 0 2 2 0 0 0 0

Solution: write them in unary

B 1110 0 110 110 0 0 110 110 0 0 0 0

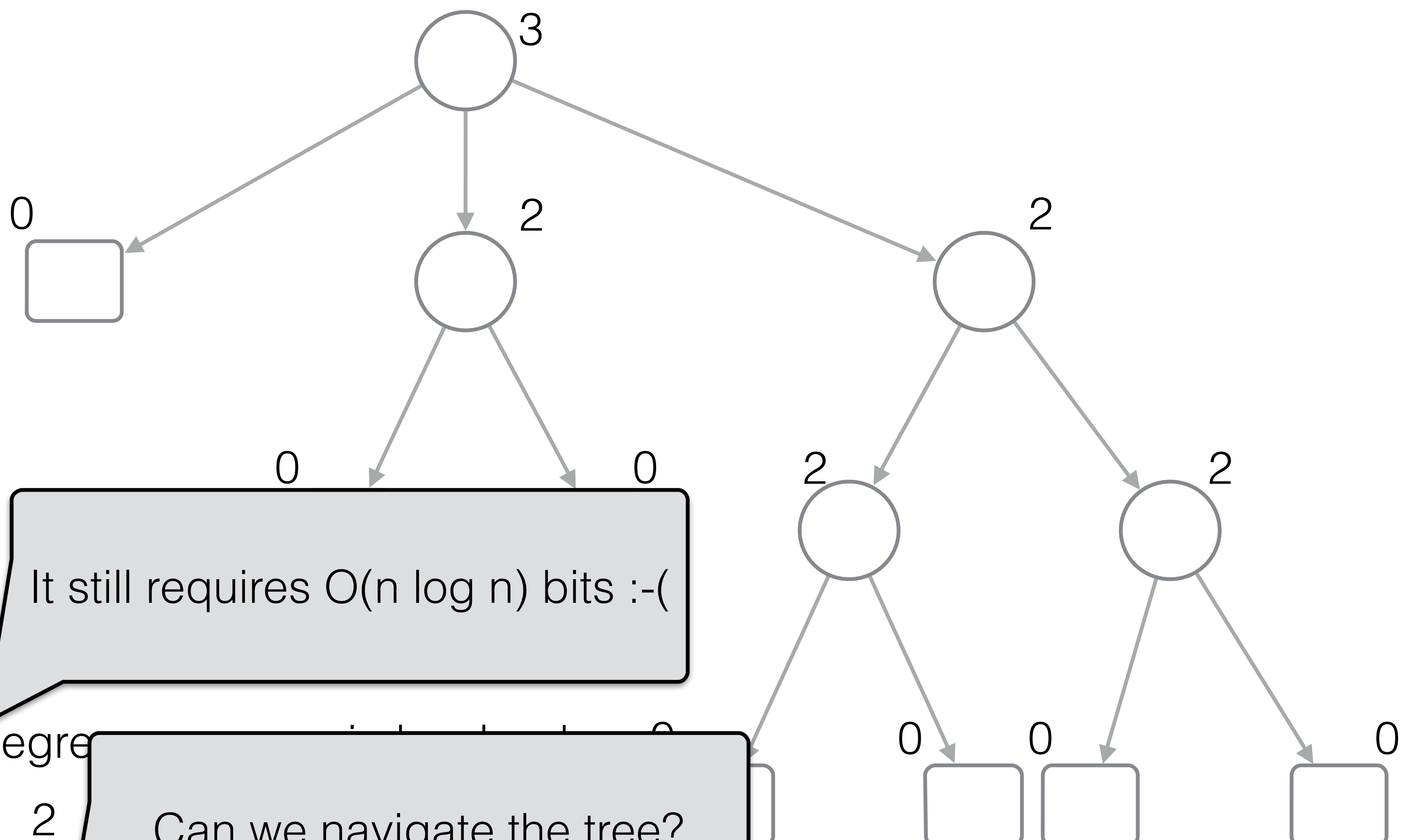
B takes 2n - 1 bits!
For each node we have a 0 and a 1
(but the root)

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

Trivial: O(n log n) bits

Best: 2n bits



It still requires O(n log n) bits :-(

Write the degre

D 3 0 2 2

Can we navigate the tree?

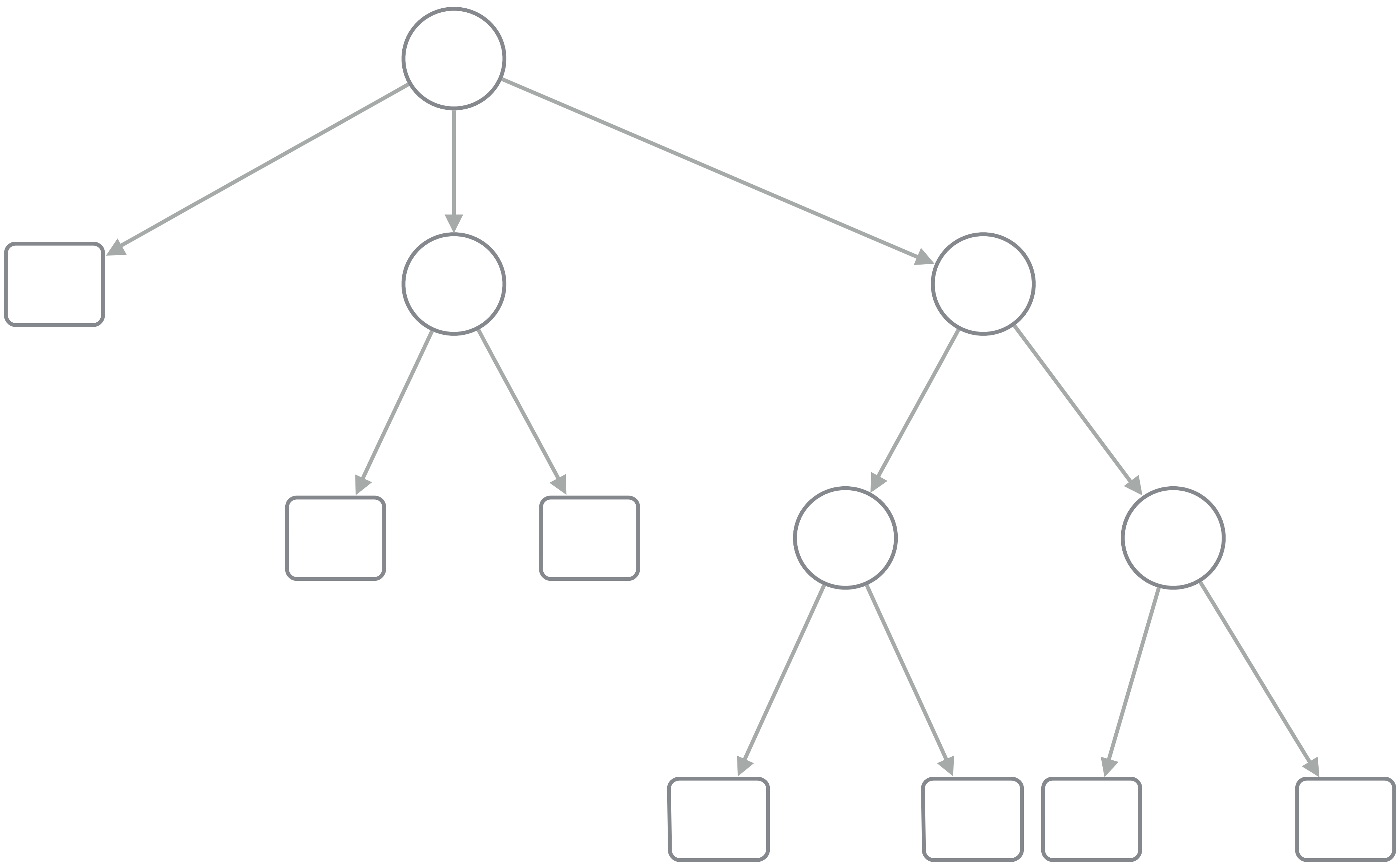
Solution: write ...em in unary

B 1110 0 110 110 0 0 110 110 0 0 0 0

B takes 2n - 1 bits!
For each node we have a 0 and a 1
(but the root)

Succinct representation of trees (1)

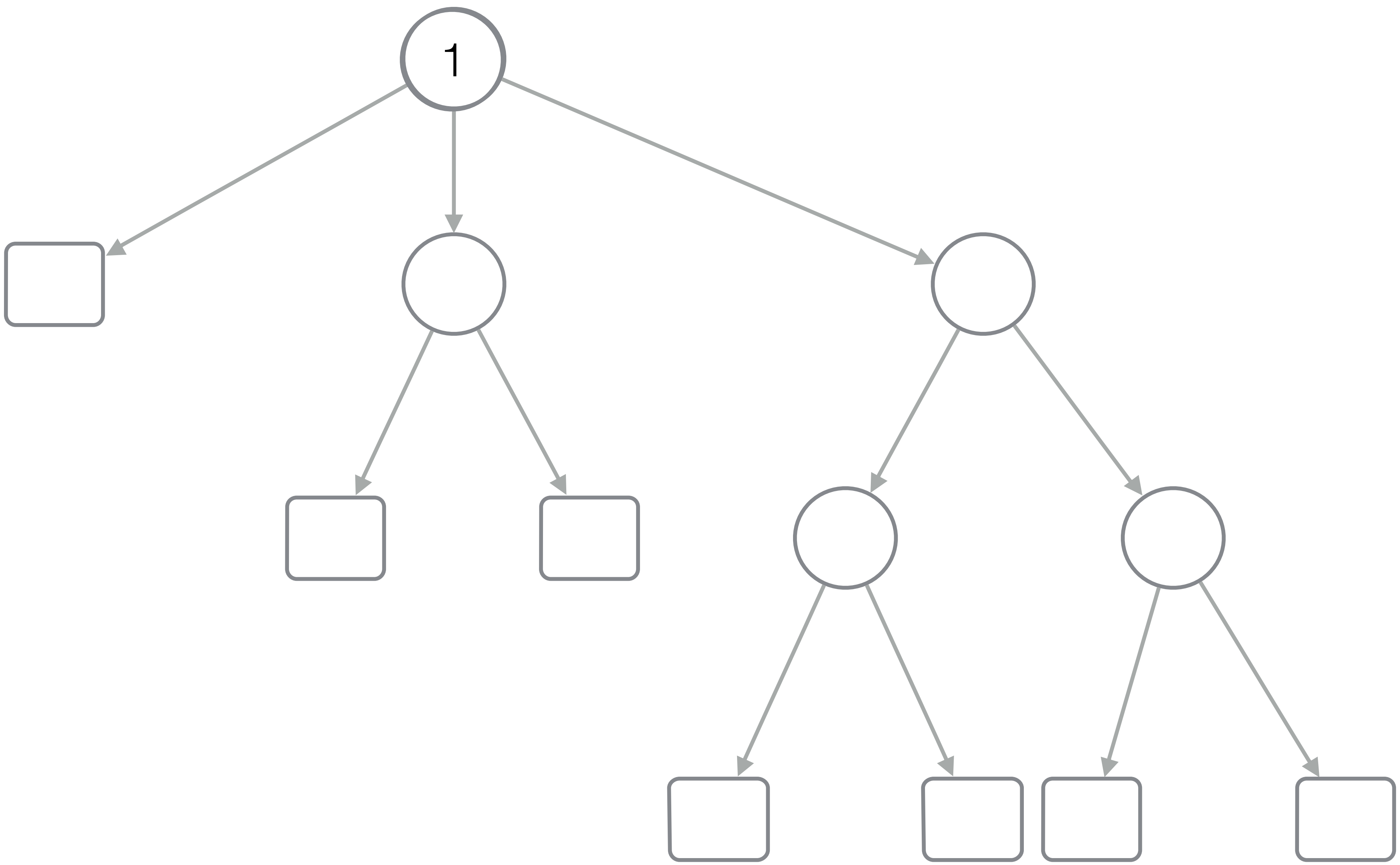
[LOUDS - Level-order unary degree sequence]



B 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

Succinct representation of trees (1)

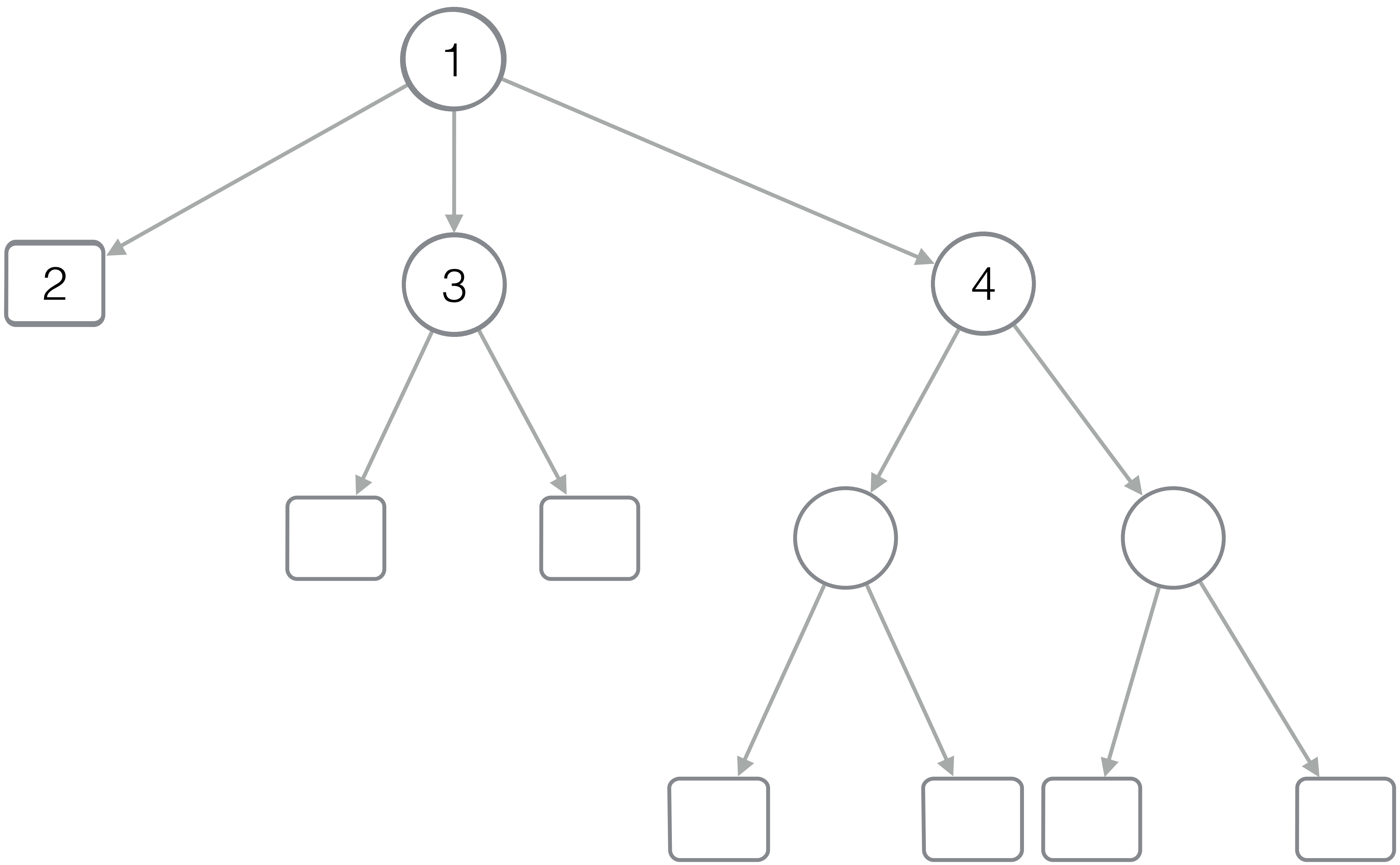
[LOUDS - Level-order unary degree sequence]



B 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

Succinct representation of trees (1)

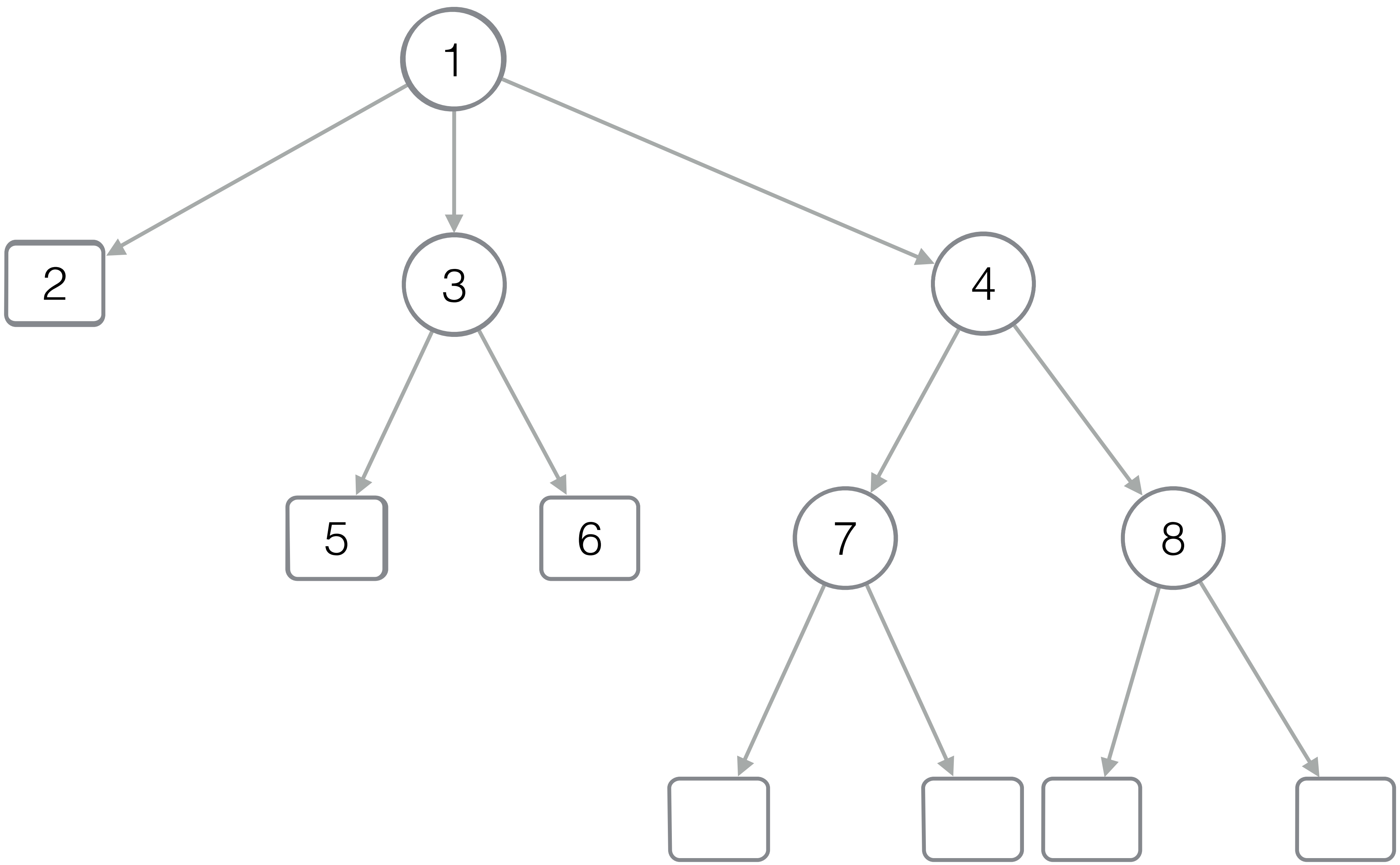
[LOUDS - Level-order unary degree sequence]



B 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

Succinct representation of trees (1)

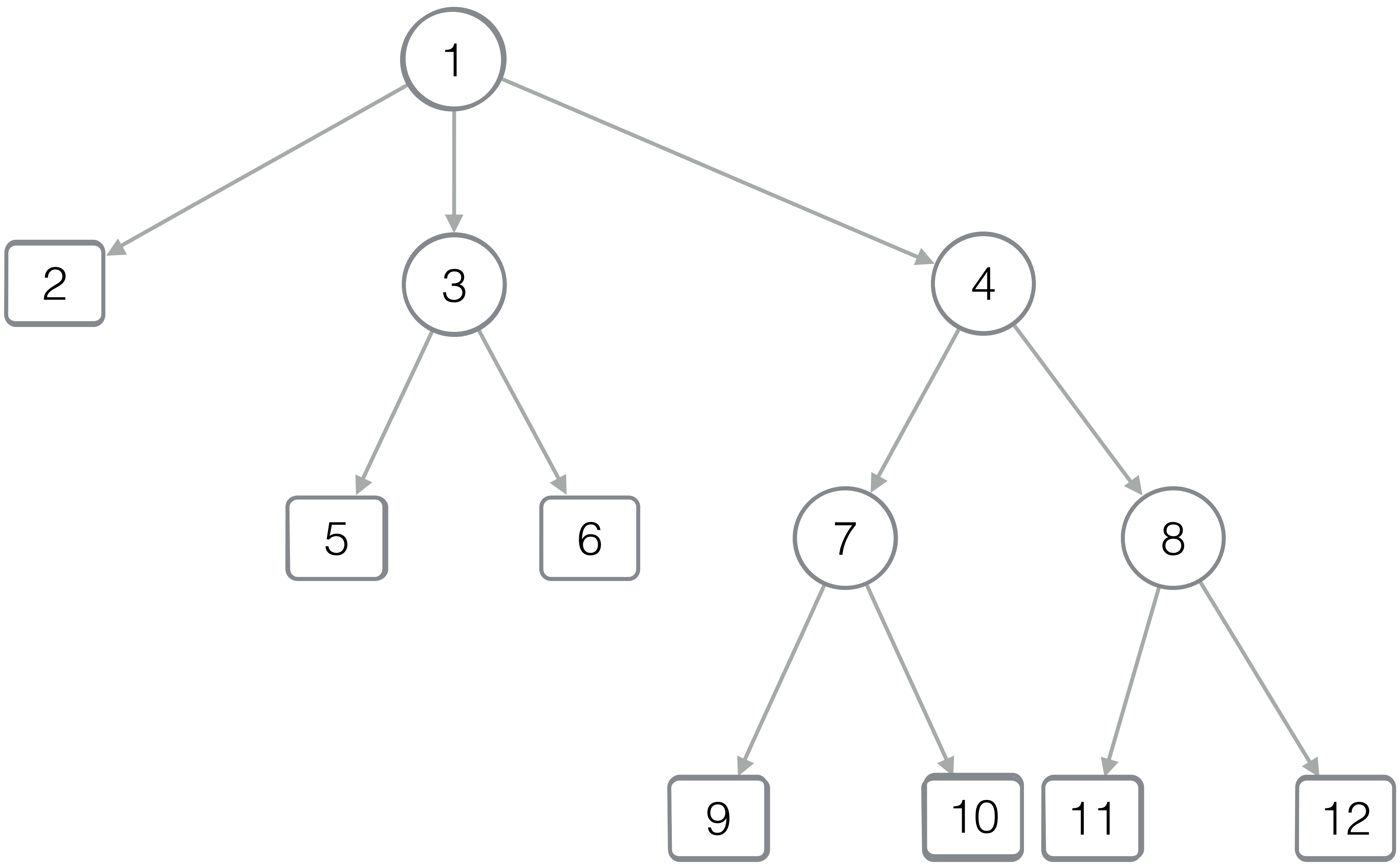
[LOUDS - Level-order unary degree sequence]



B 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

Succinct representation of trees (1)

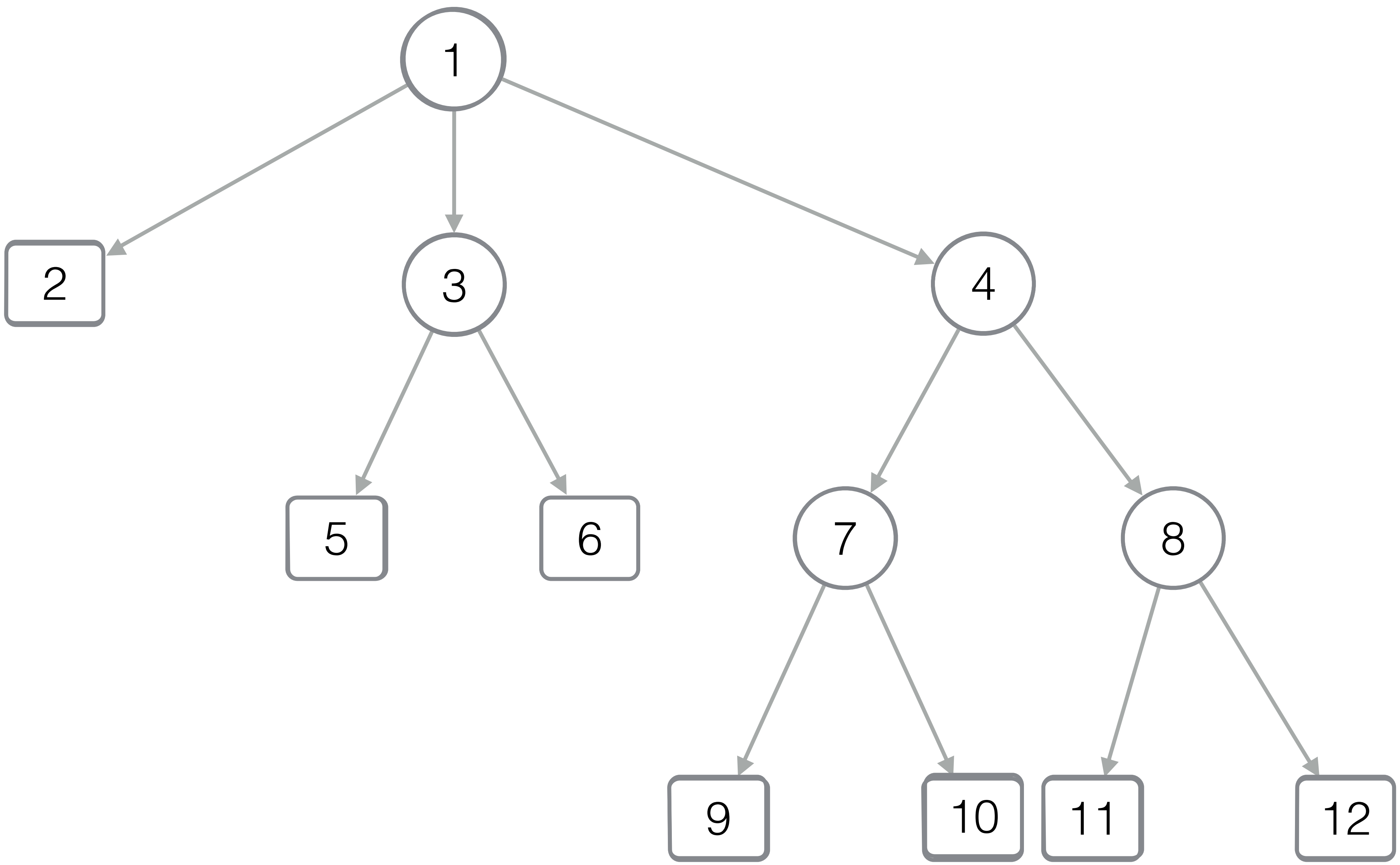
[LOUDS - Level-order unary degree sequence]



B 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

Succinct representation of trees (1)

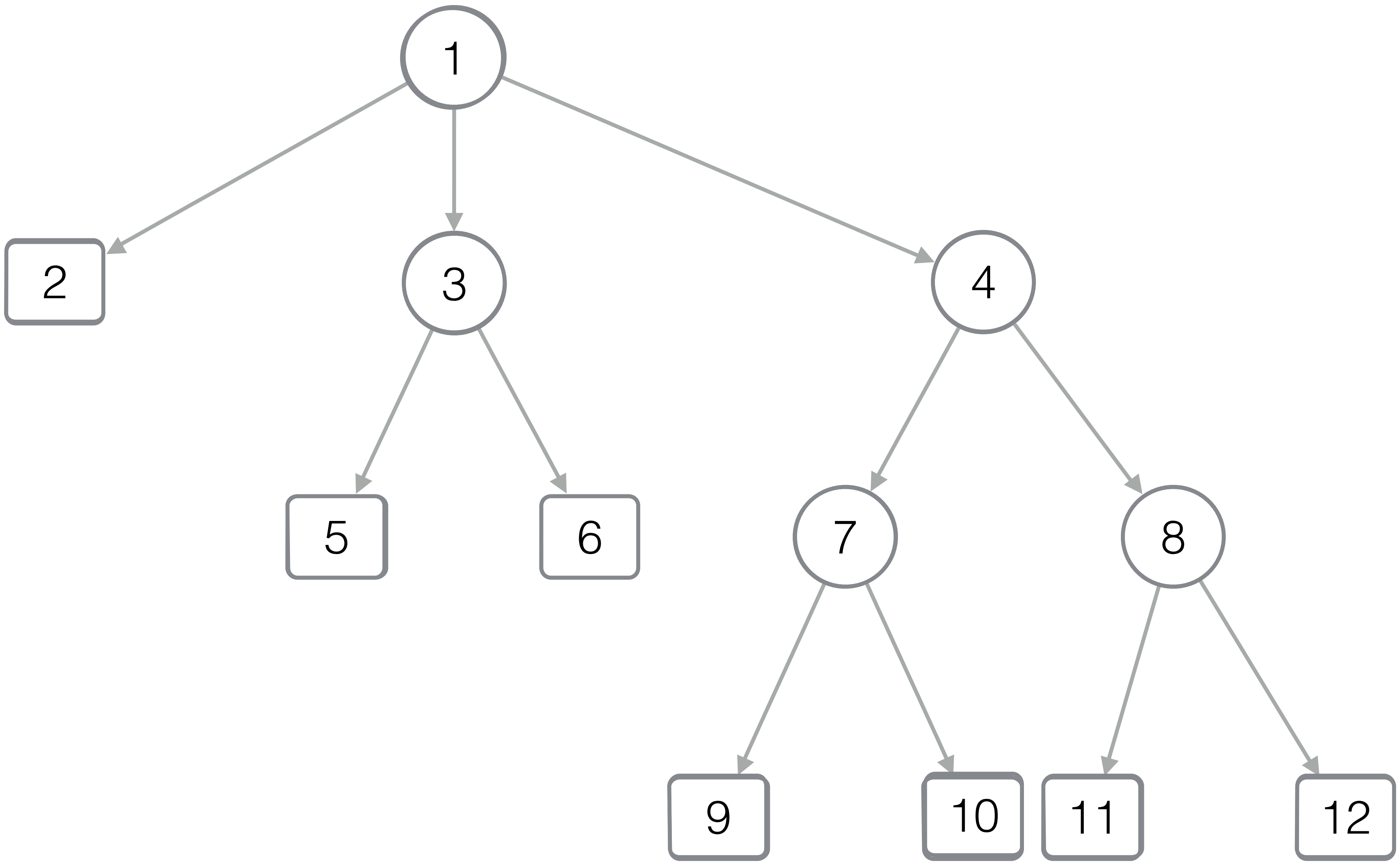
[LOUDS - Level-order unary degree sequence]



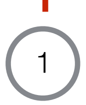
B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

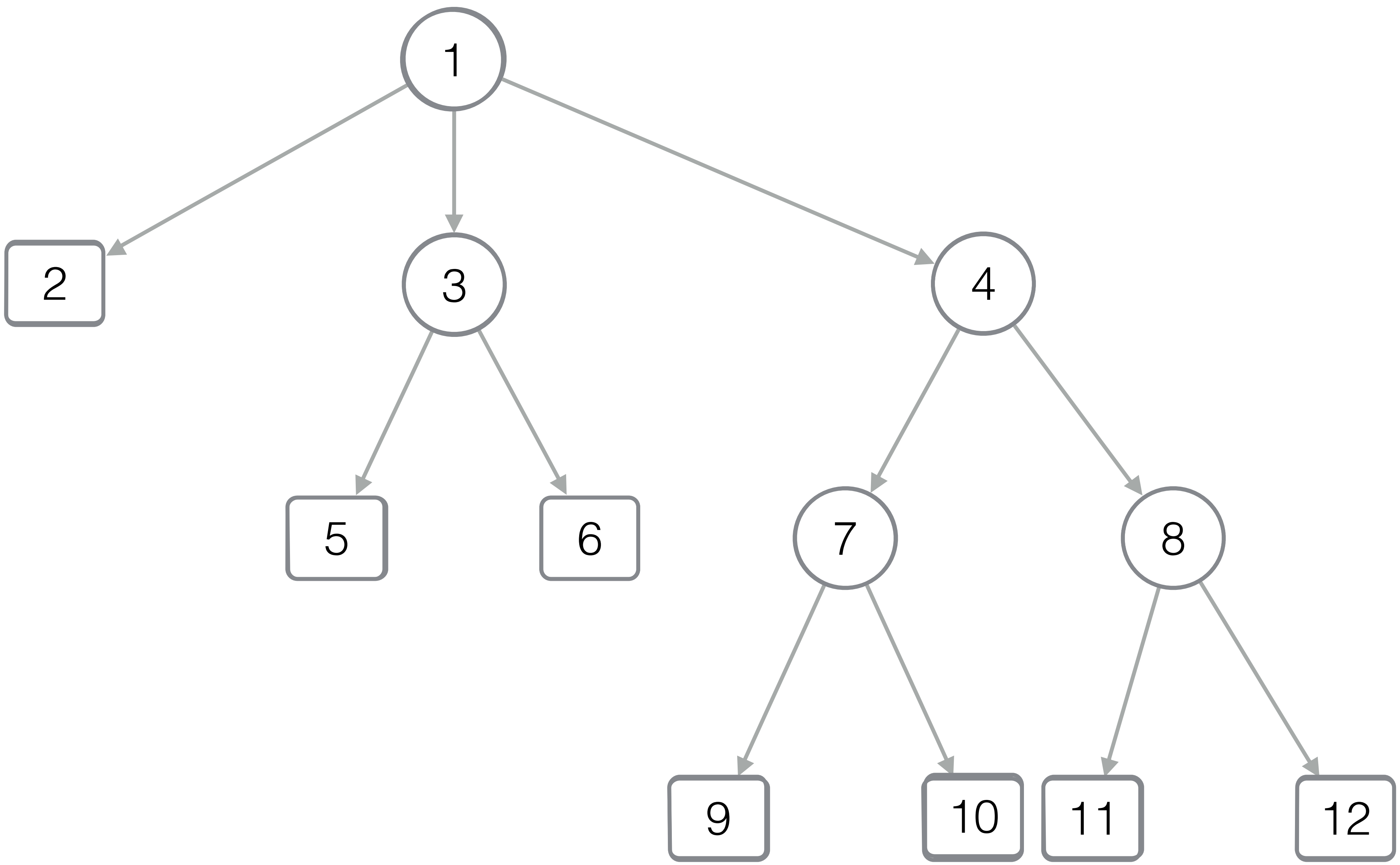


B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0



Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]



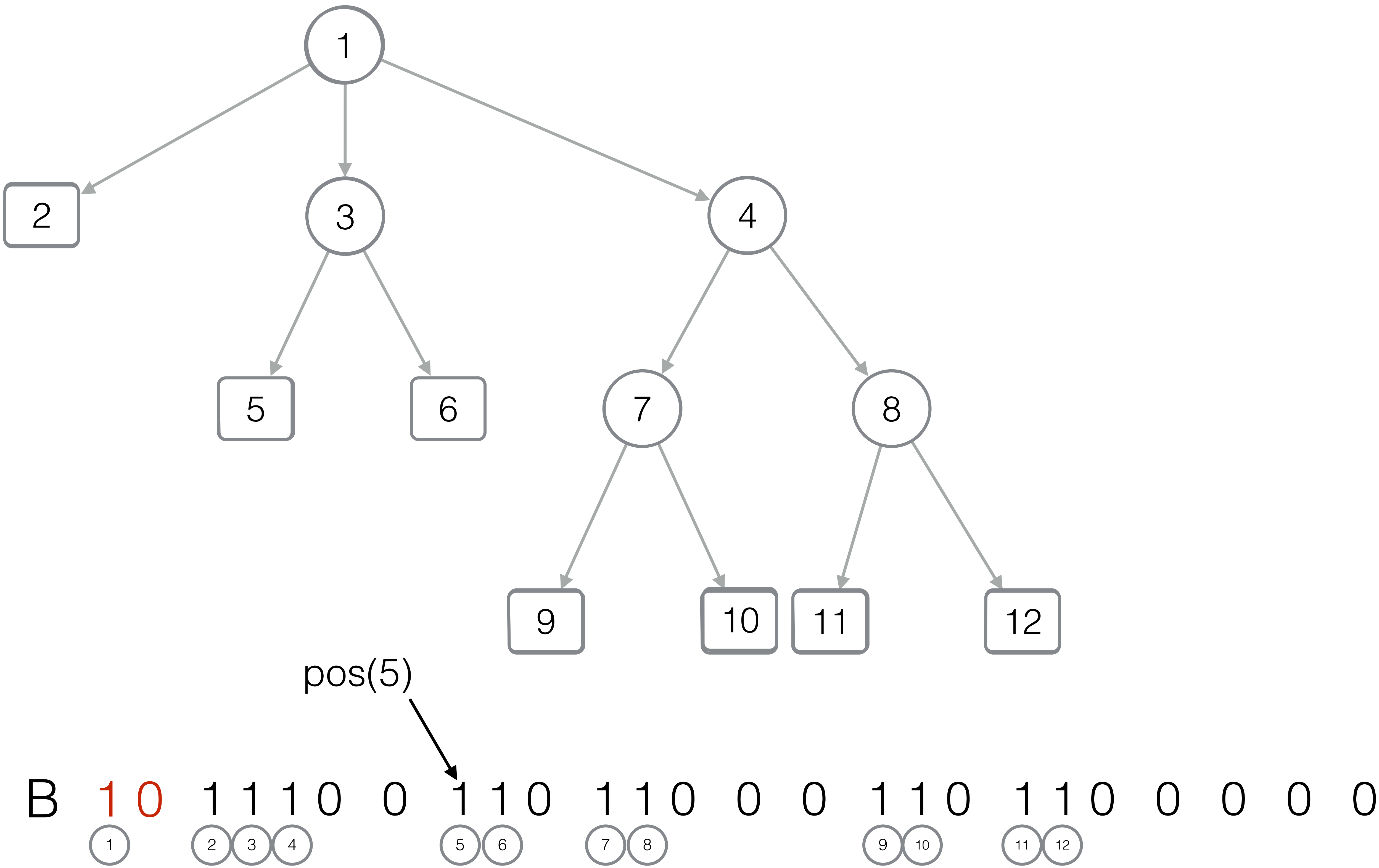
B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

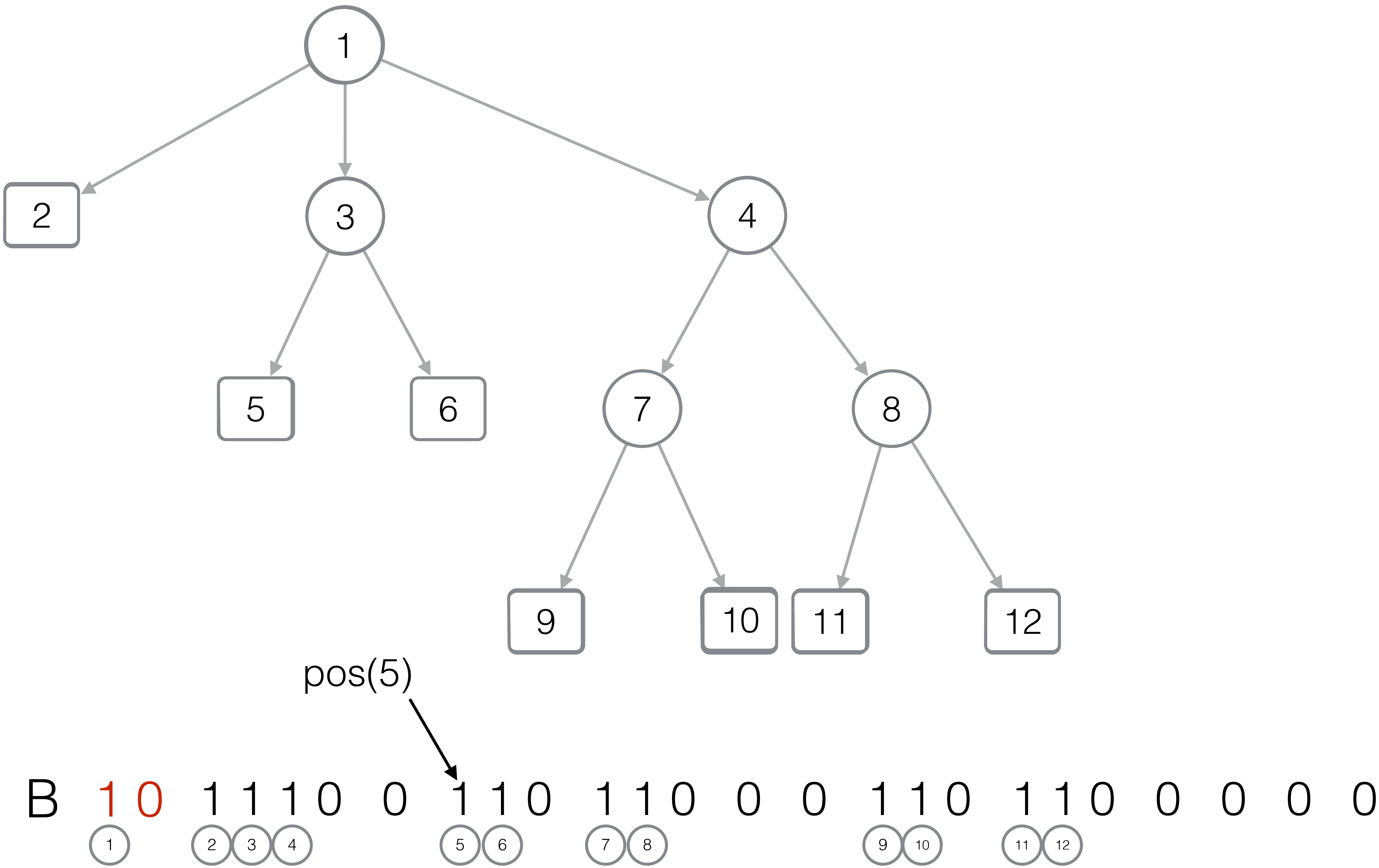
pos(x) =



Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

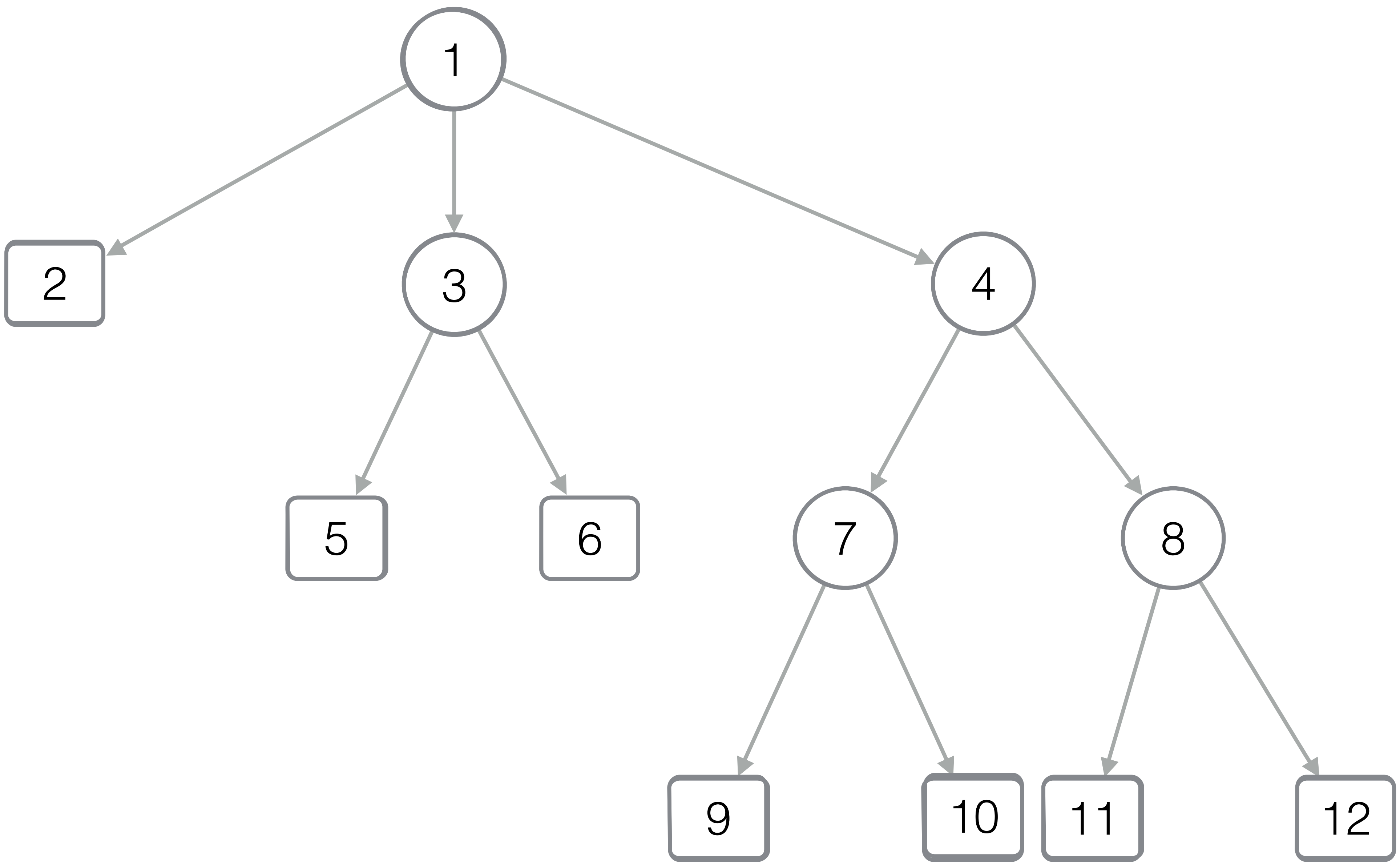


Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

$firstChild(x) = ?$



B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

(1) (2) (3) (4) (5) (6) (7) (8) (9) (10) (11) (12)

Succinct representation of trees (1)

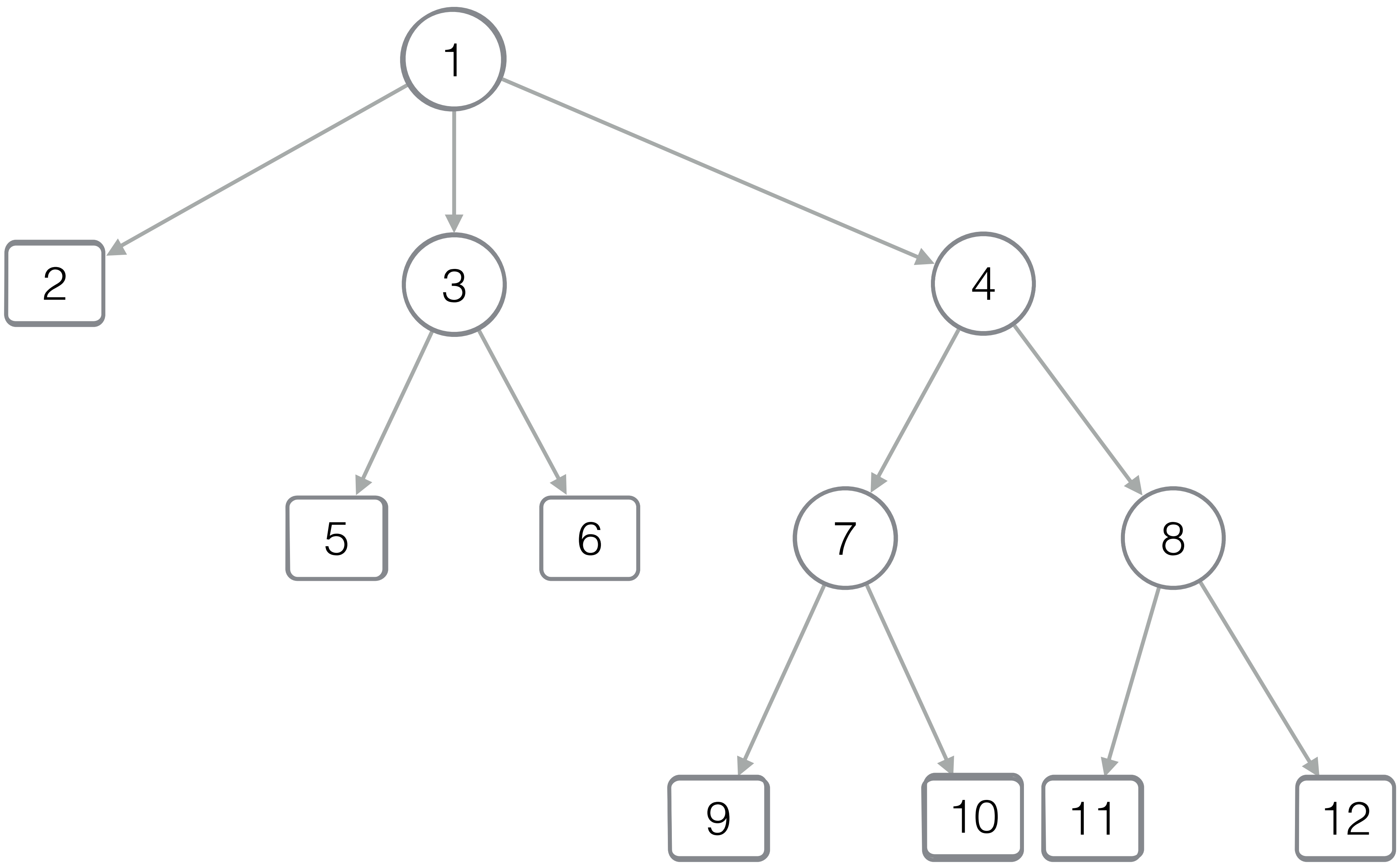
[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B



B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

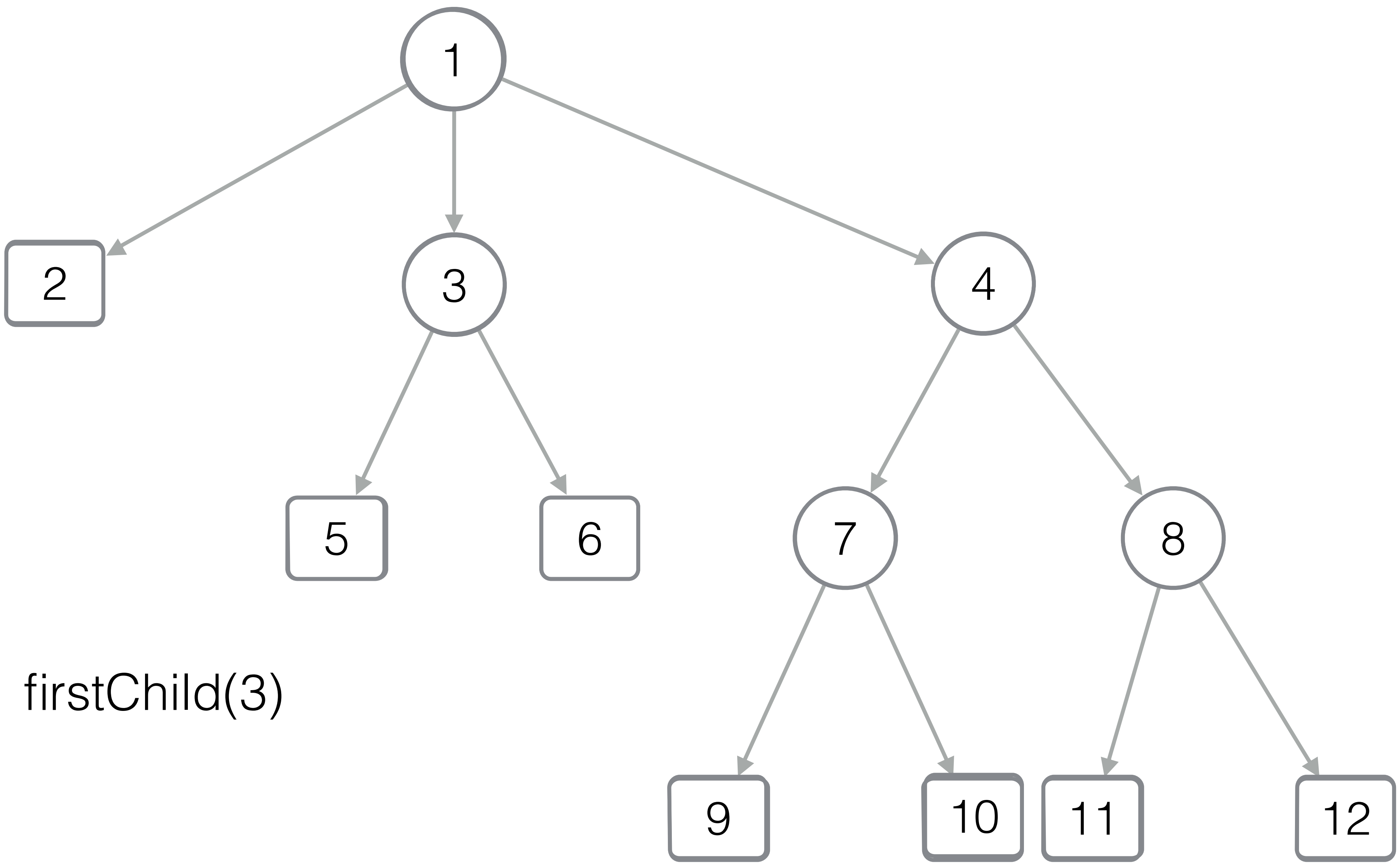
[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B



firstChild(3)

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

Succinct representation of trees (1)

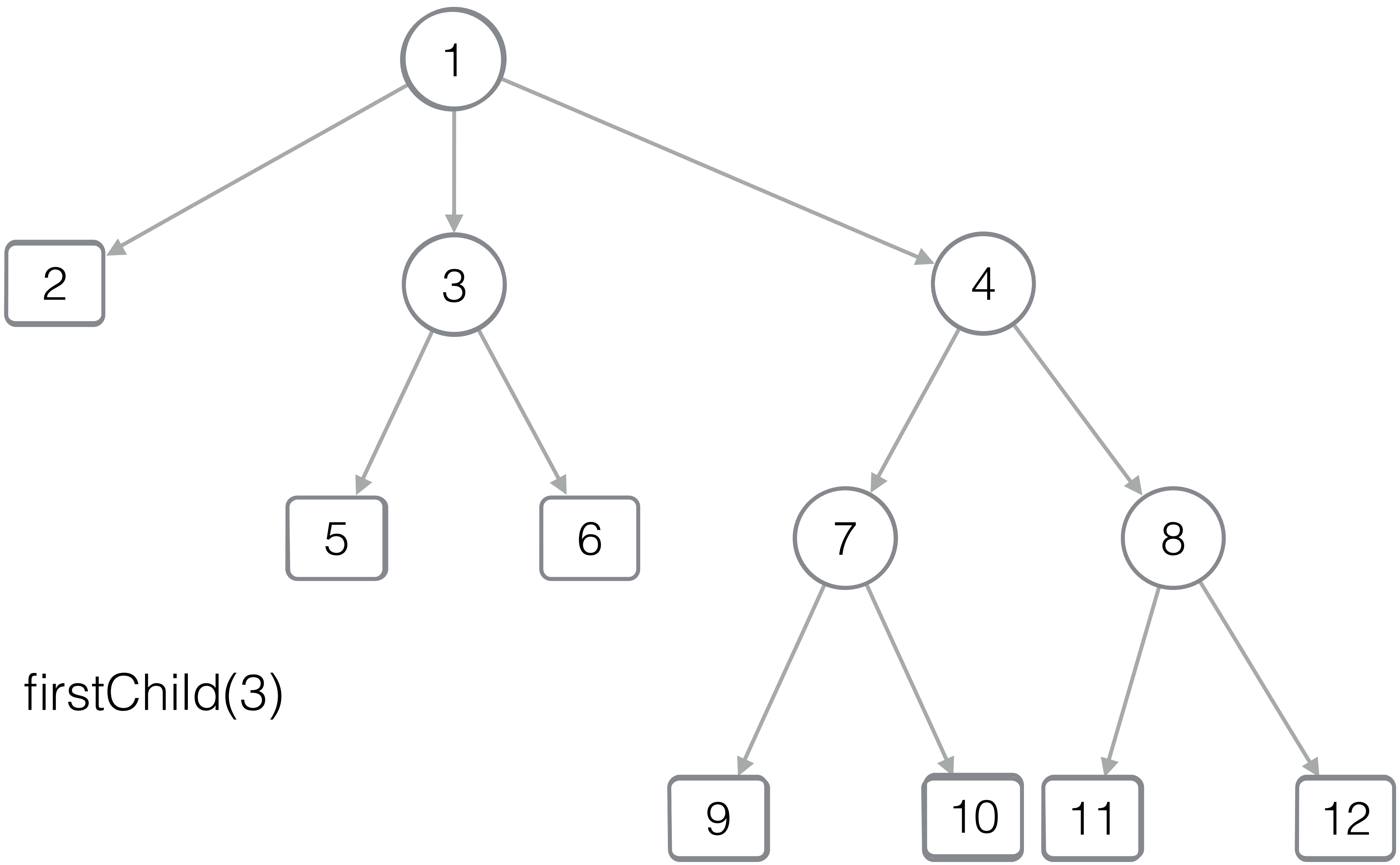
[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B



firstChild(3)

$y = Select_0(3)+1=8$

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

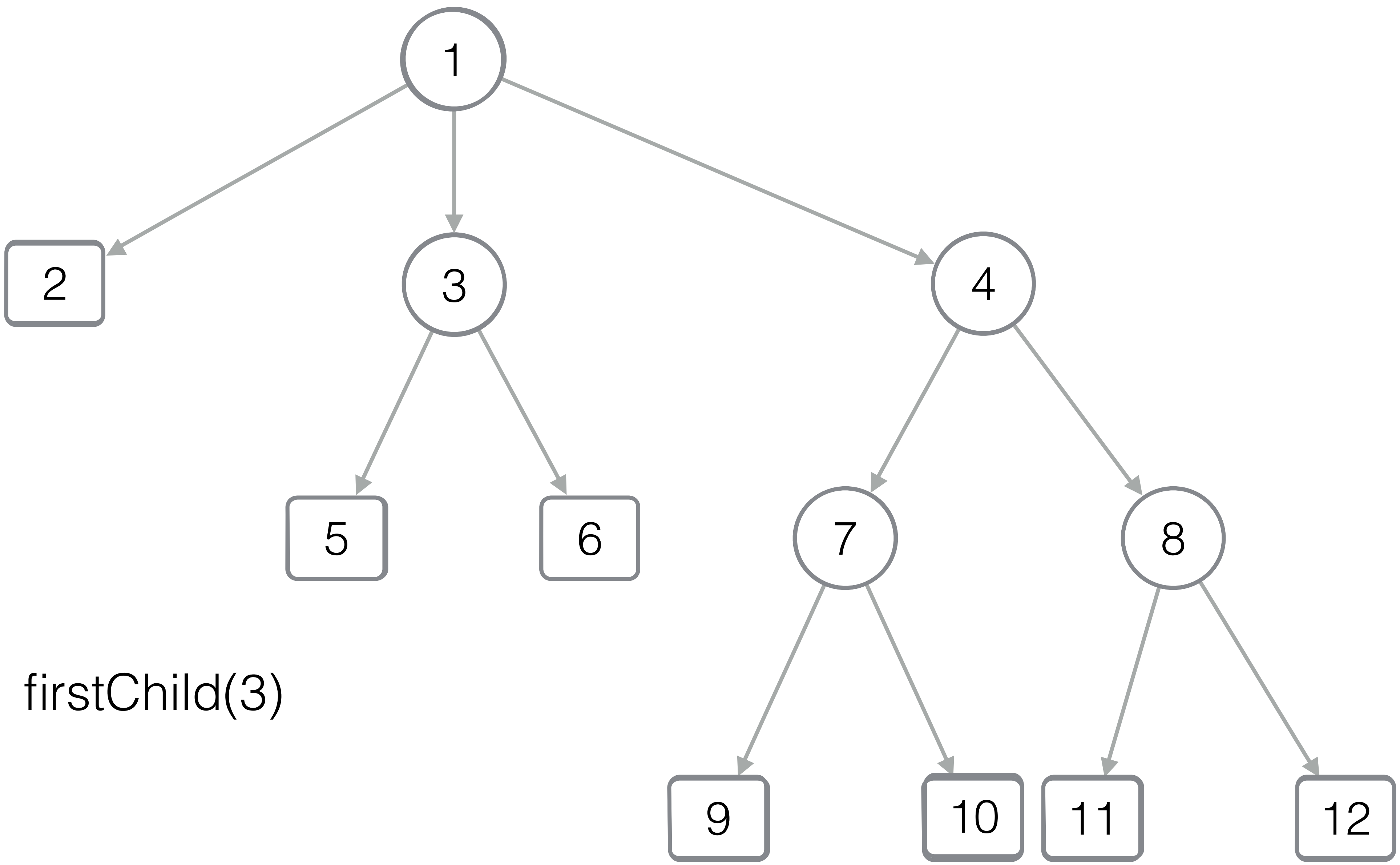
$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B

if $B[y] == 0$

return -1 // is a leaf



firstChild(3)

$y = Select_0(3)+1=8$

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

(1) (2) (3) (4) (5) (6) (7) (8) (9) (10) (11) (12)

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

firstChild(x) = ?

$y = Select_0(x)+1$

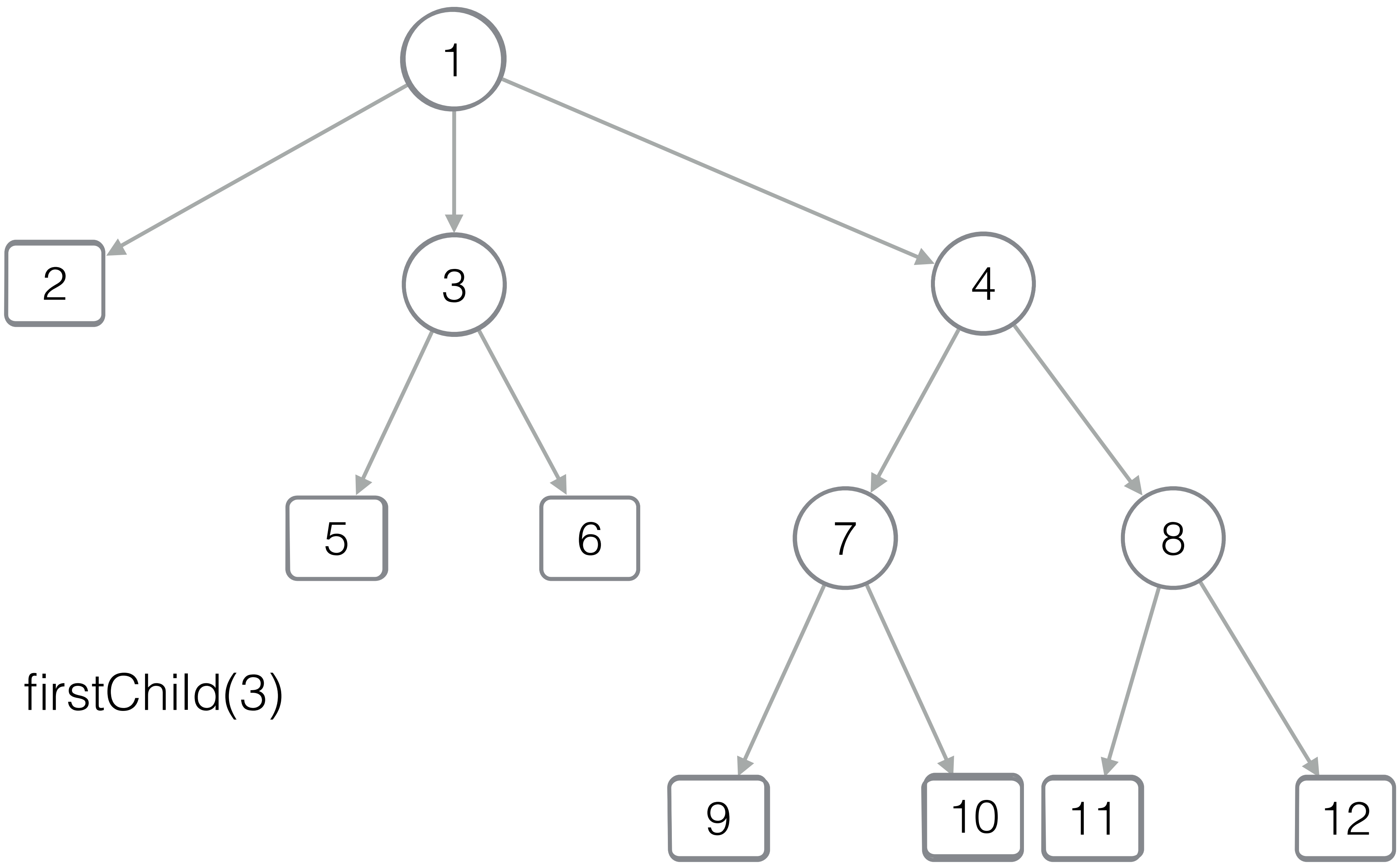
// start of x's children in B

if B[y] == 0

return -1 // is a leaf

else

return y-x // $Rank_1(y)$



firstChild(3)

$y = Select_0(3)+1=8$

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

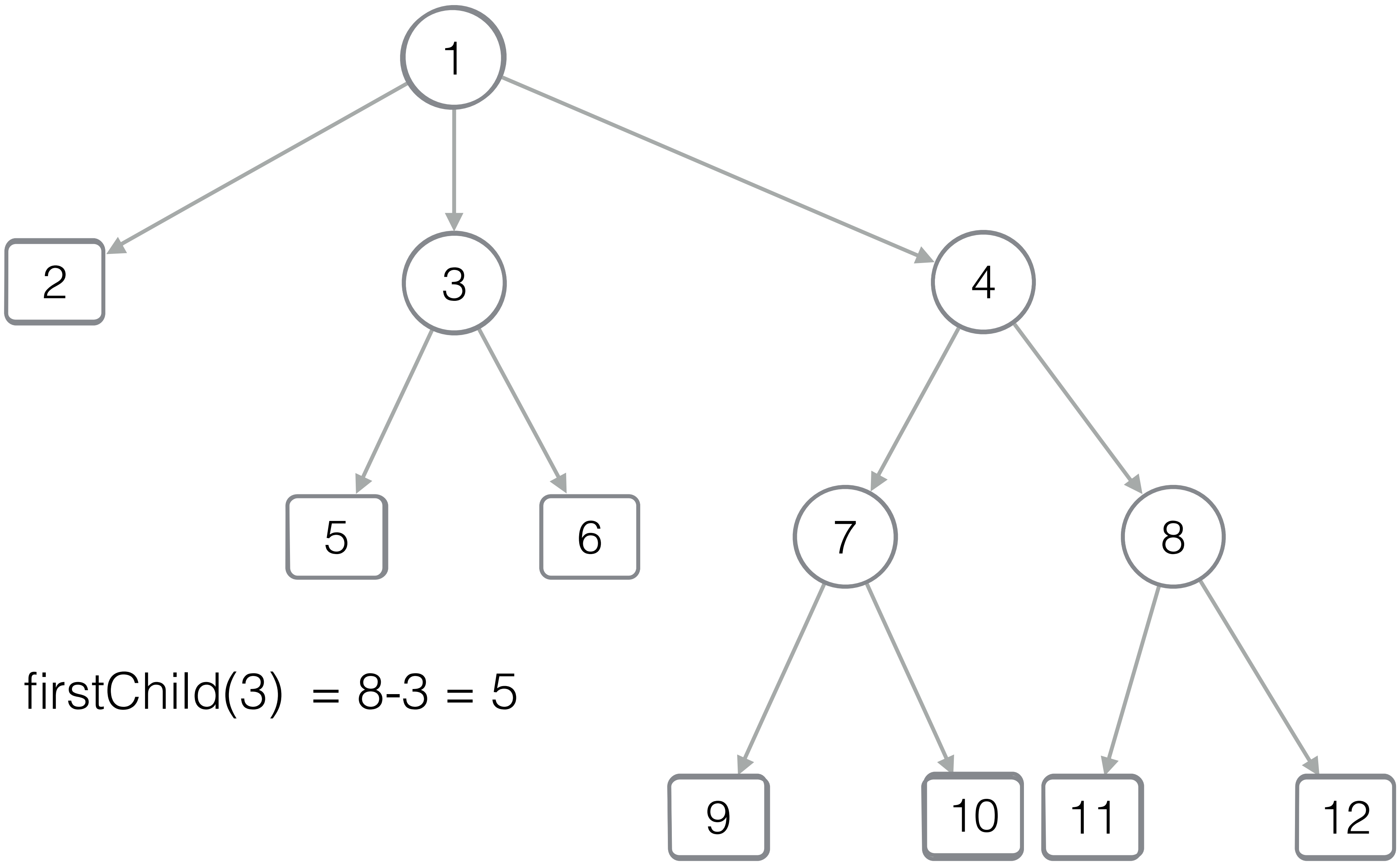
$pos(x) = Select_1(x)$

firstChild(x) = ?

$y = Select_0(x)+1$

// start of x's children in B

```
if B[y] == 0
    return -1   // is a leaf
else
    return y-x  // Rank1(y)
```



firstChild(3) = 8-3 = 5

$y = Select_0(3)+1=8$

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

(1) (2) (3) (4) (5) (6) (7) (8) (9) (10) (11) (12)

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

firstChild(x) = ?

$y = Select_0(x)+1$

// start of x's children in B

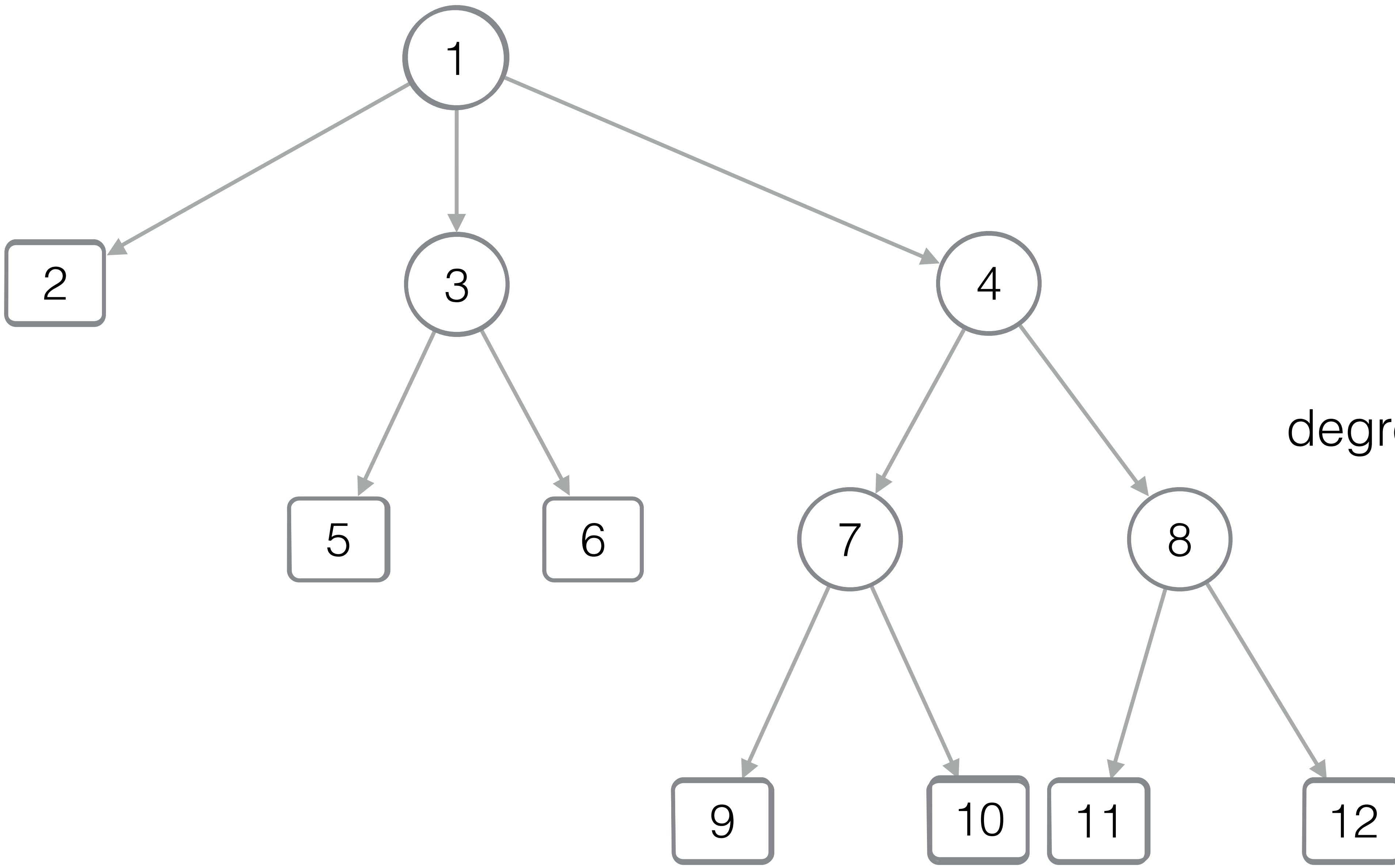
if B[y] == 0

return -1 // is a leaf

else

return y-x // $Rank_1(y)$

degree(x) = ?



$y= Select_0(3)+1=8$

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B

if $B[y] == 0$

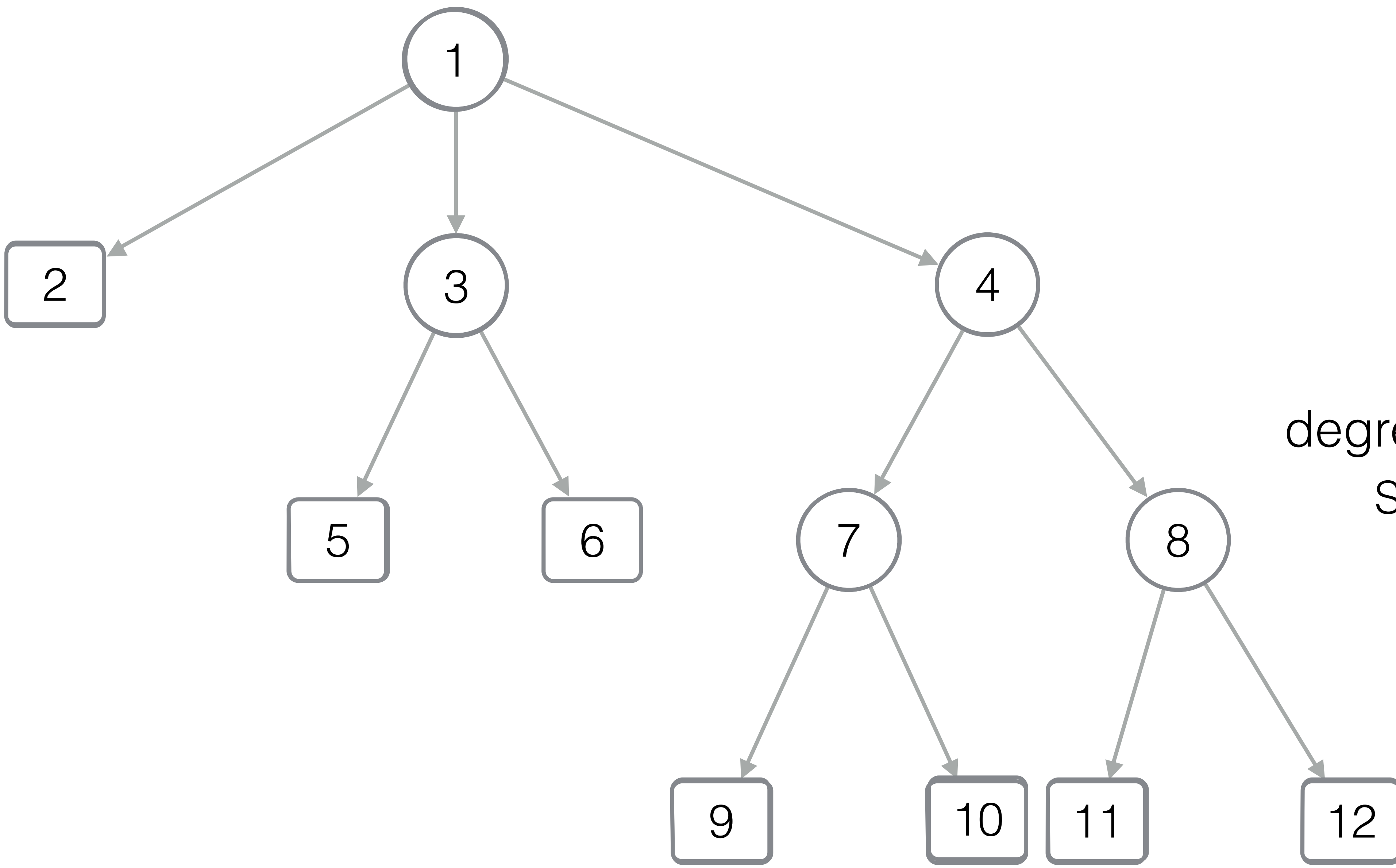
return -1 // is a leaf

else

return $y-x$ // $Rank_1(y)$

$degree(x) = ?$

$Select_0(x+1) - (Select_0(x) + 1)$



$y= Select_0(3)+1=8$

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

(1) (2) (3) (4) (5) (6) (7) (8) (9) (10) (11) (12)

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B

if $B[y] == 0$

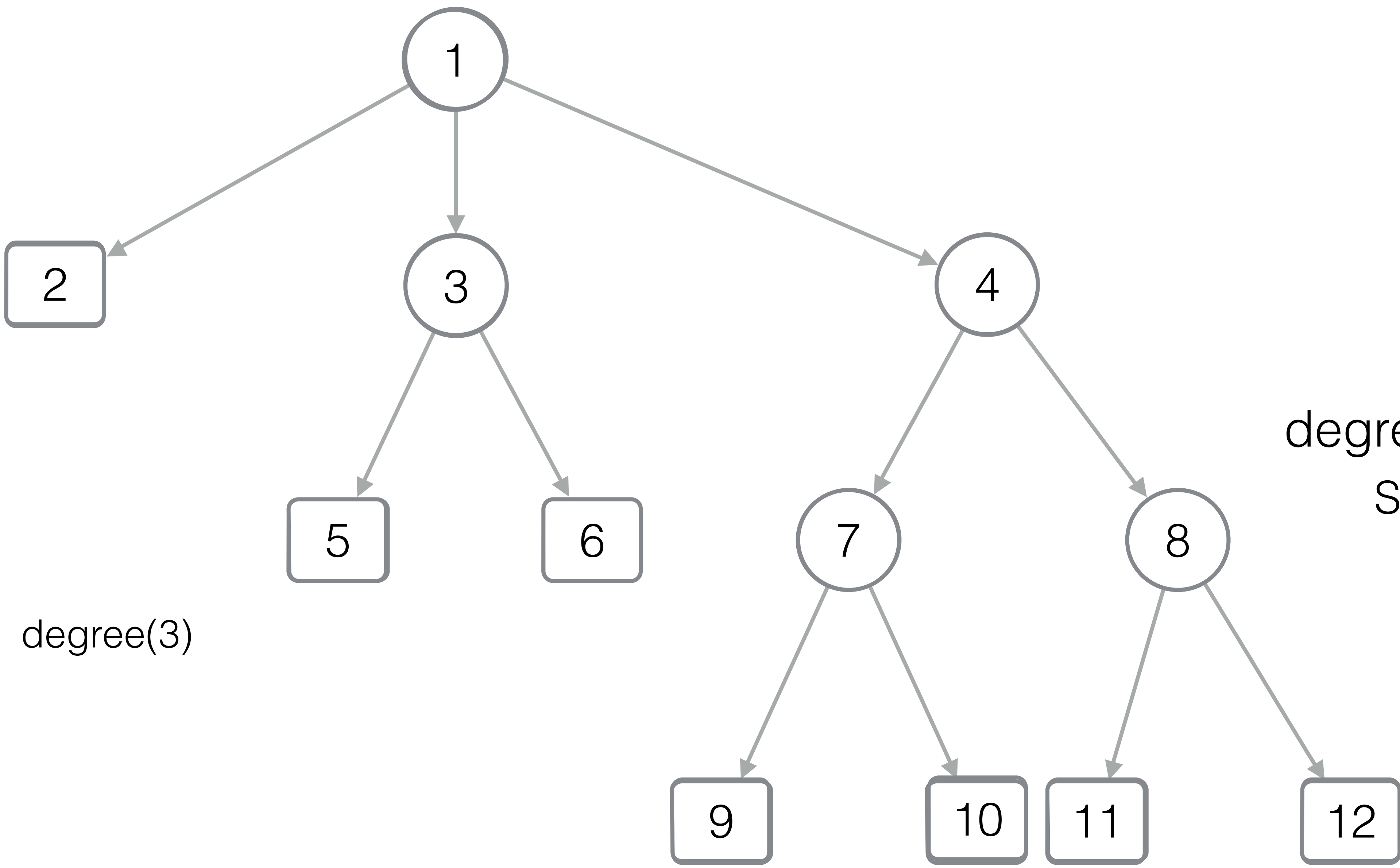
return -1 // is a leaf

else

return y-x // $Rank_1(y)$

$degree(x) = ?$

$Select_0(x+1) - (Select_0(x) + 1)$



degree(3)

$y = Select_0(3)+1=8$

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B

if $B[y] == 0$

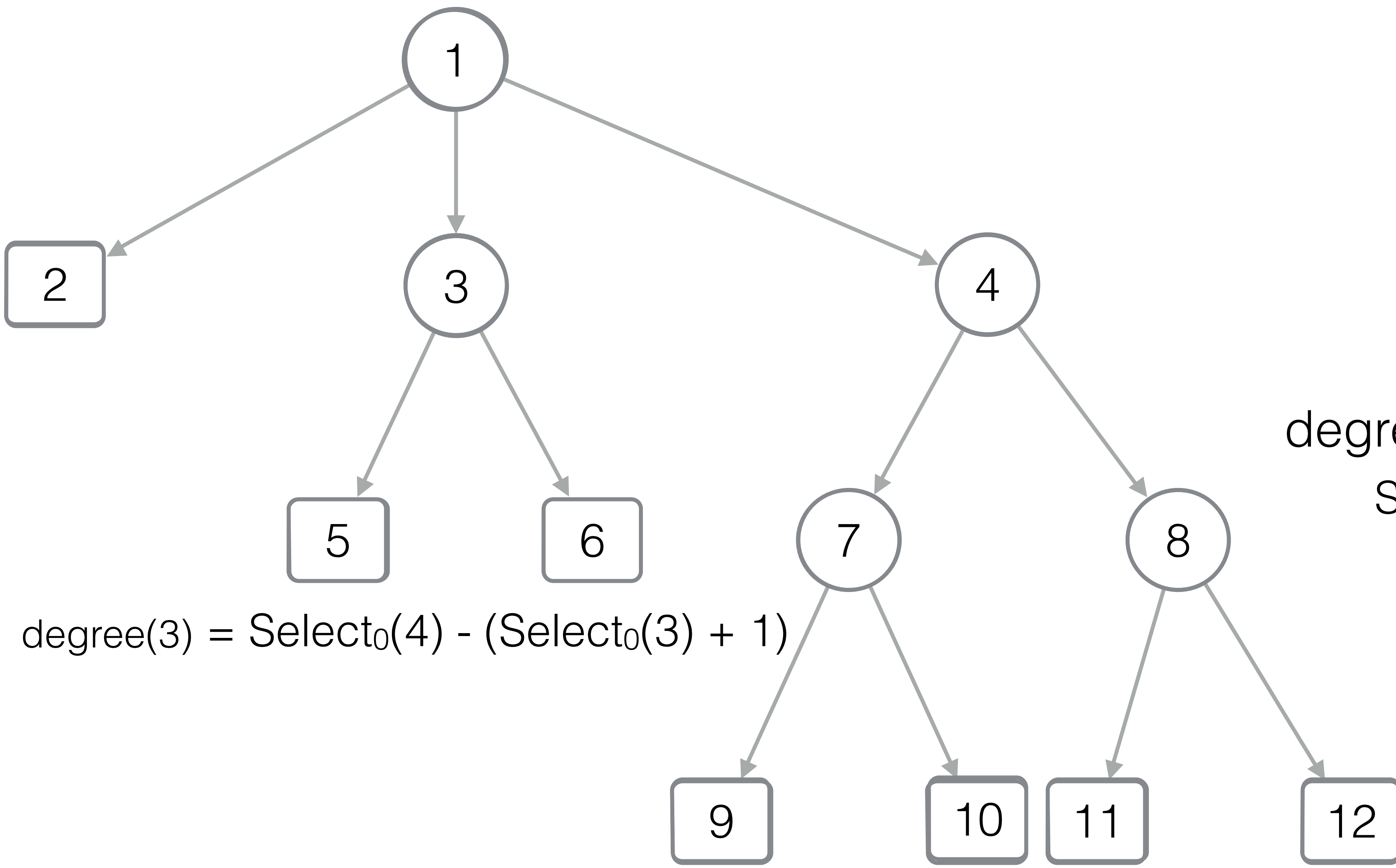
return -1 // is a leaf

else

return y-x // $Rank_1(y)$

$degree(x) = ?$

$Select_0(x+1) - (Select_0(x) + 1)$



$degree(3) = Select_0(4) - (Select_0(3) + 1)$

$y = Select_0(3)+1=8$

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

(1) (2) (3) (4) (5) (6) (7) (8) (9) (10) (11) (12)

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B

if $B[y] == 0$

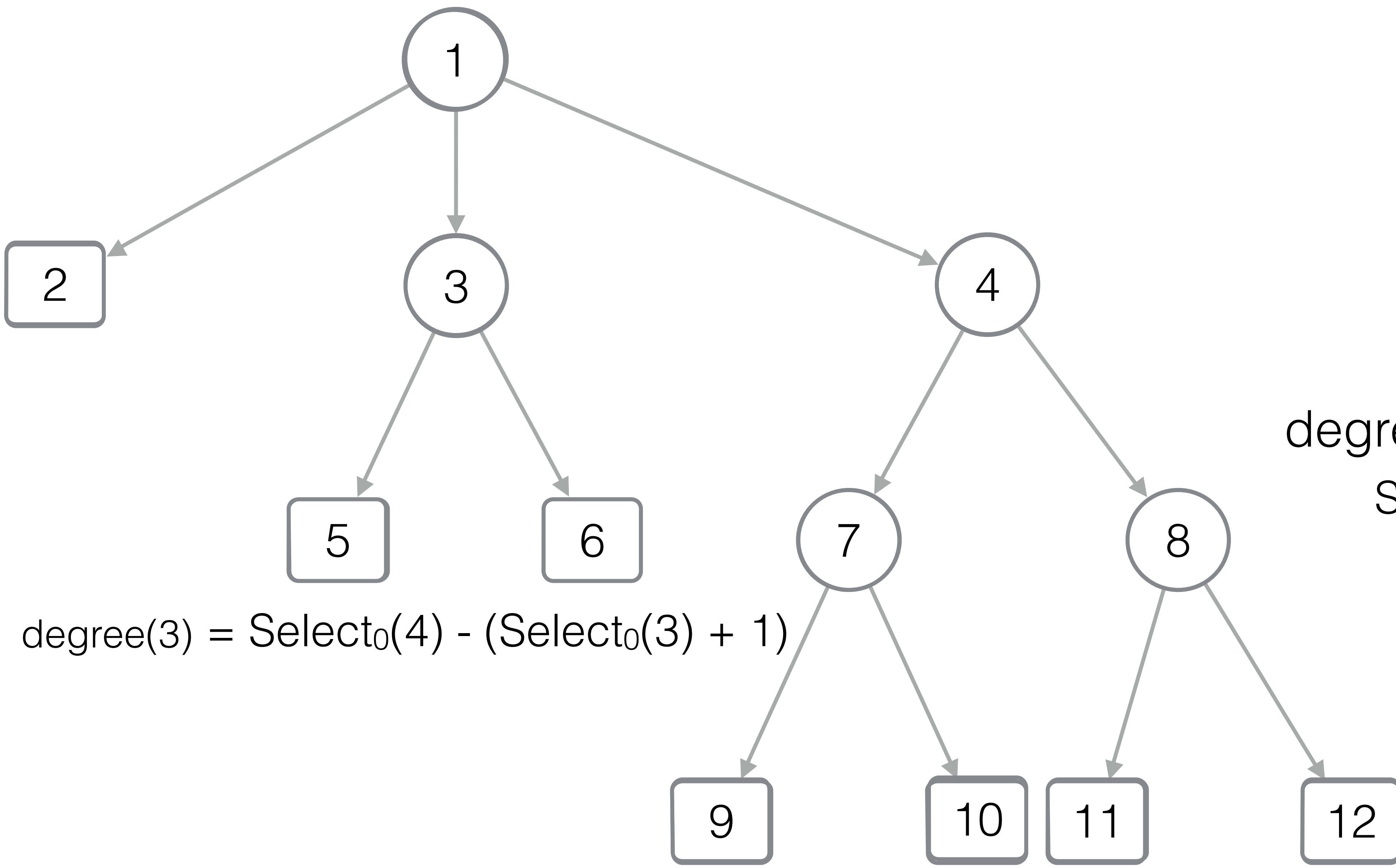
return -1 // is a leaf

else

return y-x // $Rank_1(y)$

$degree(x) = ?$

$Select_0(x+1) - (Select_0(x) + 1)$



$degree(3) = Select_0(4) - (Select_0(3) + 1)$

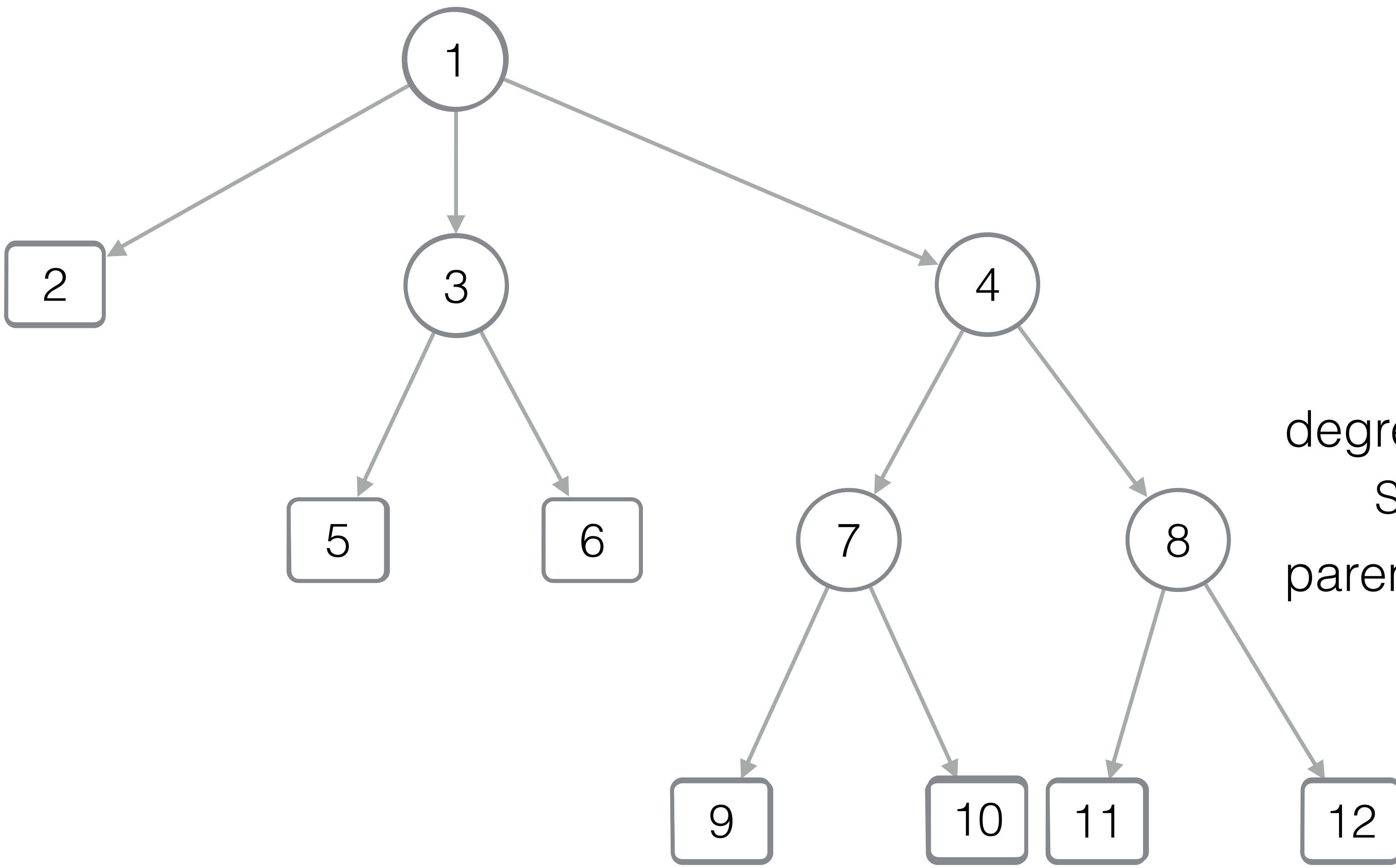
$y= Select_0(3)+1=8$ $Select_0(4)=10$

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]



$pos(x) = Select_1(x)$

$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B

if B[y] == 0

return -1 // is a leaf

else

return y-x // $Rank_1(y)$

$degree(x) = ?$

$Select_0(x+1) - (Select_0(x) + 1)$

parent(x) =

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B

if $B[y] == 0$

return -1 // is a leaf

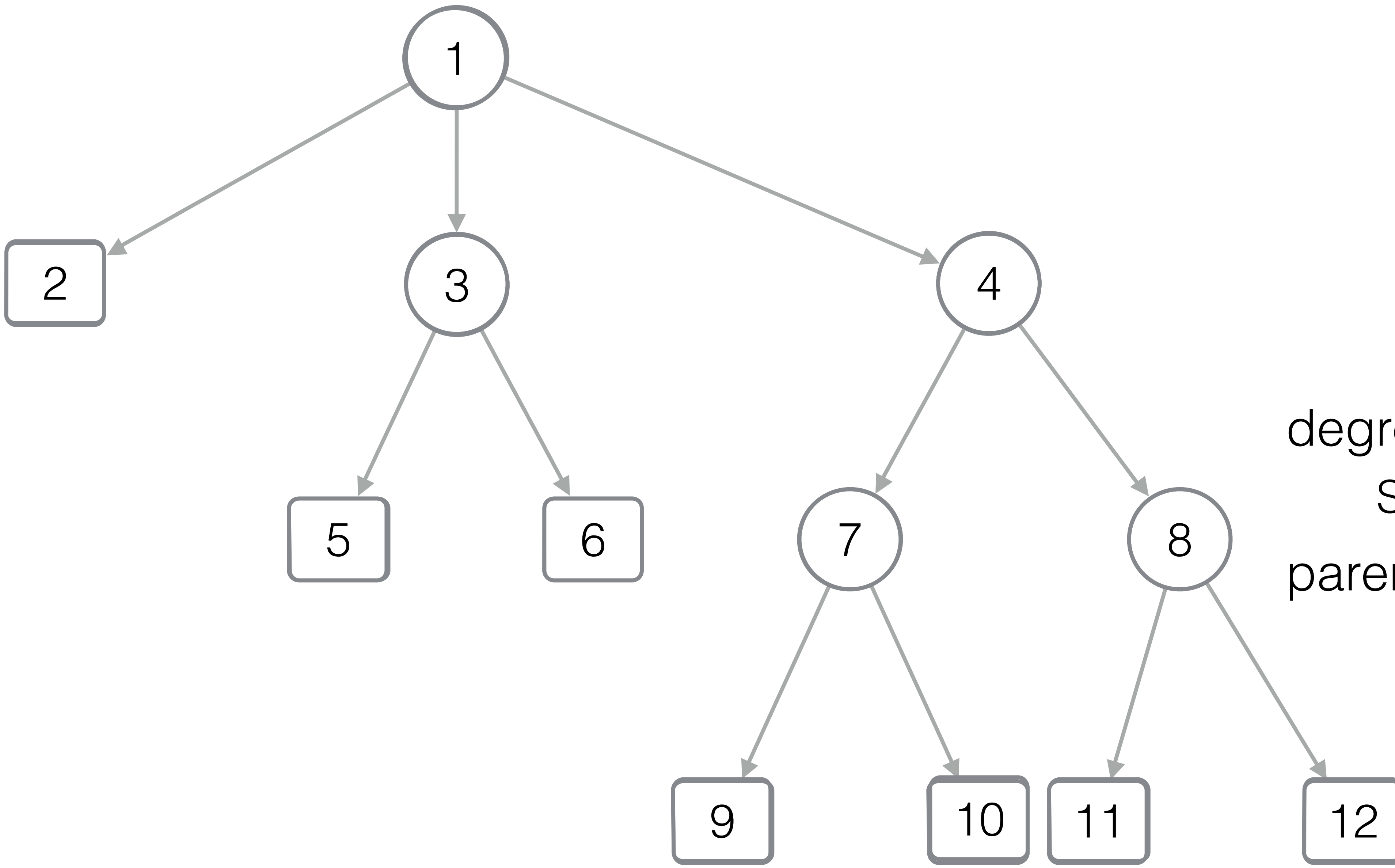
else

return $y-x$ // $Rank_1(y)$

$degree(x) = ?$

$Select_0(x+1) - (Select_0(x) + 1)$

$parent(x) = Rank_0(pos(x))$



B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B

if $B[y] == 0$

return -1 // is a leaf

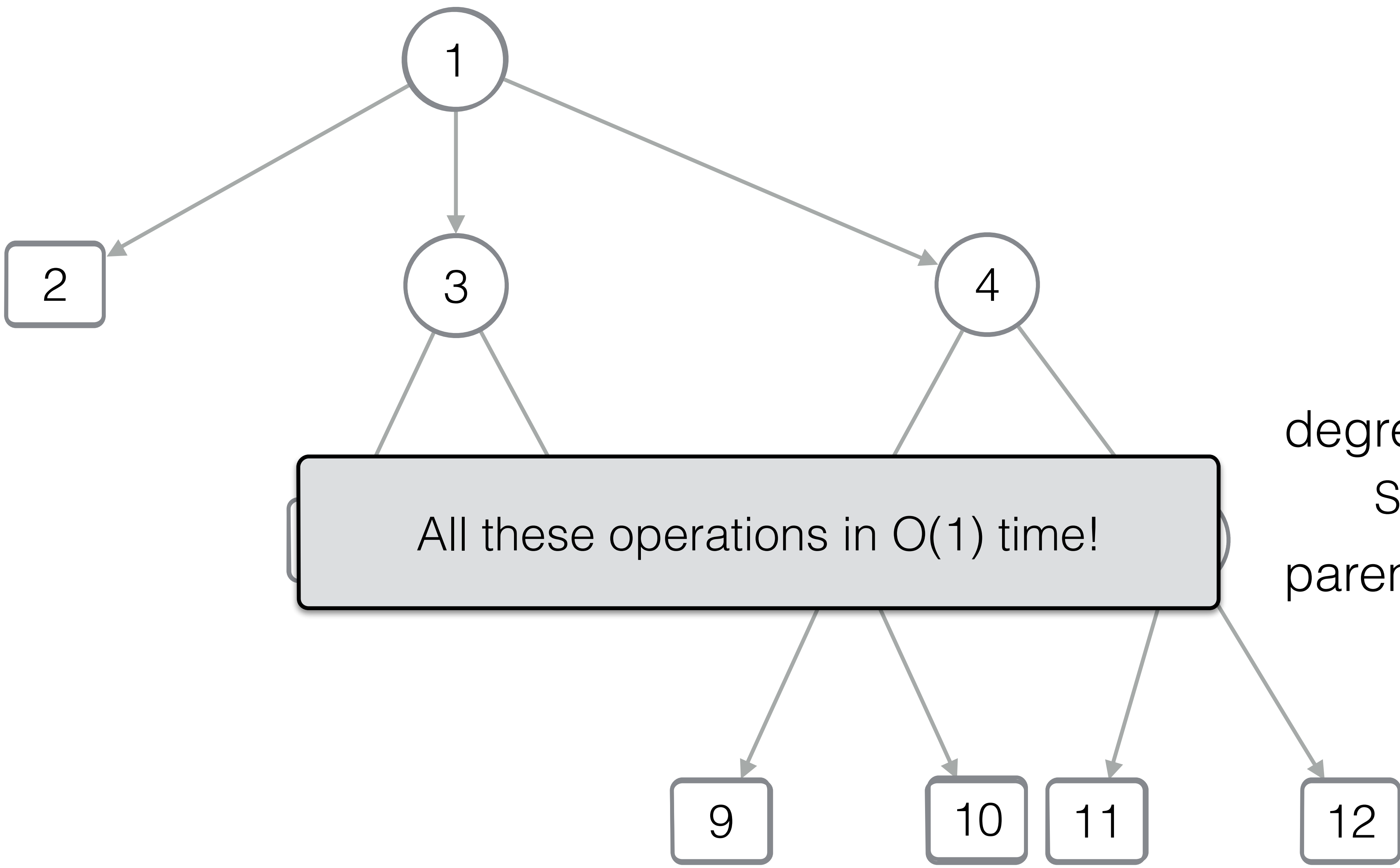
else

return y-x // $Rank_1(y)$

$degree(x) = ?$

$Select_0(x+1) - (Select_0(x) + 1)$

$parent(x) = Rank_0(pos(x))$

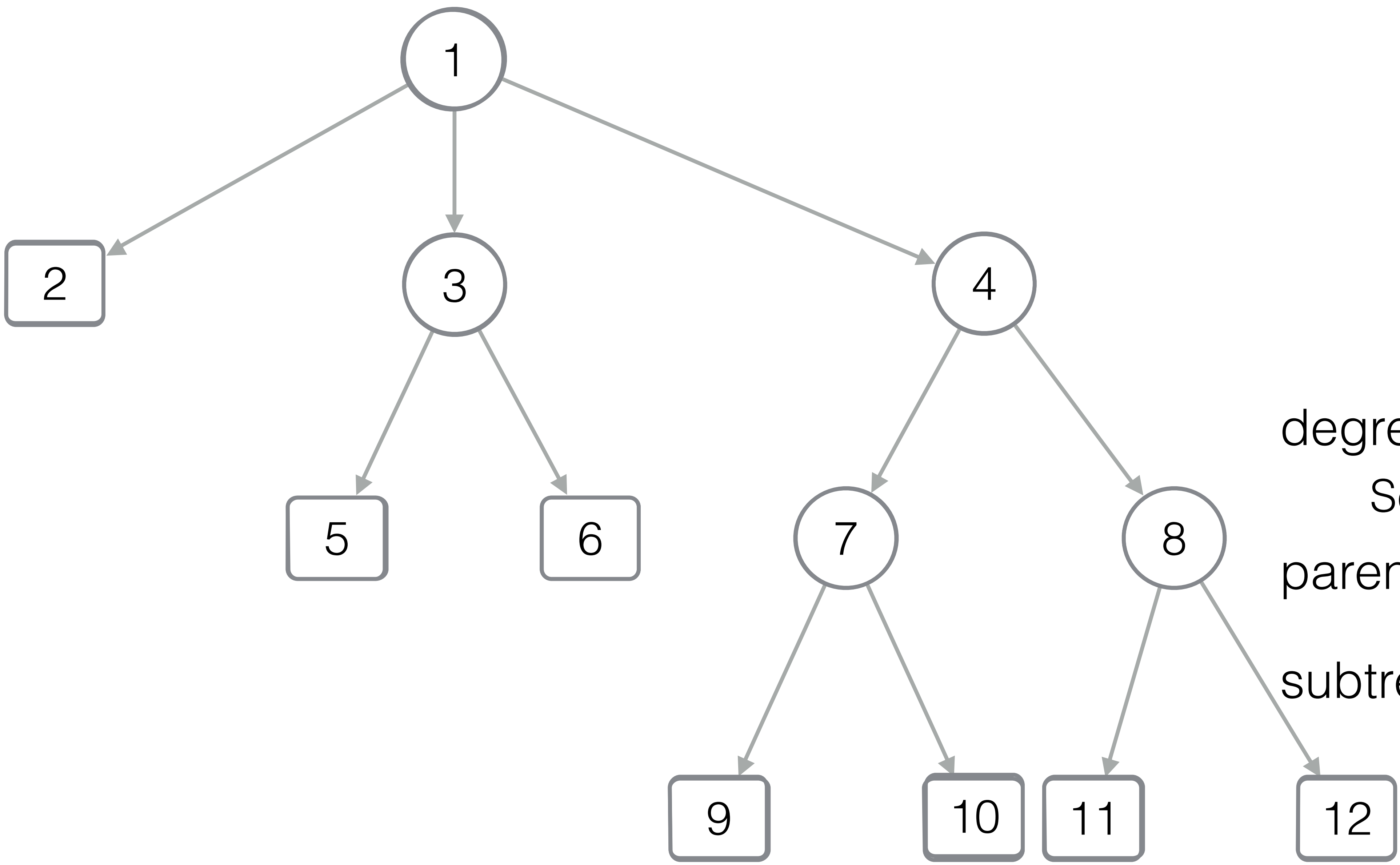


All these operations in O(1) time!

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)



[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B

if B[y] == 0

return -1 // is a leaf

else

return y-x // $Rank_1(y)$

$degree(x) = ?$

$Select_0(x+1) - (Select_0(x) + 1)$

$parent(x) = Rank_0(pos(x))$

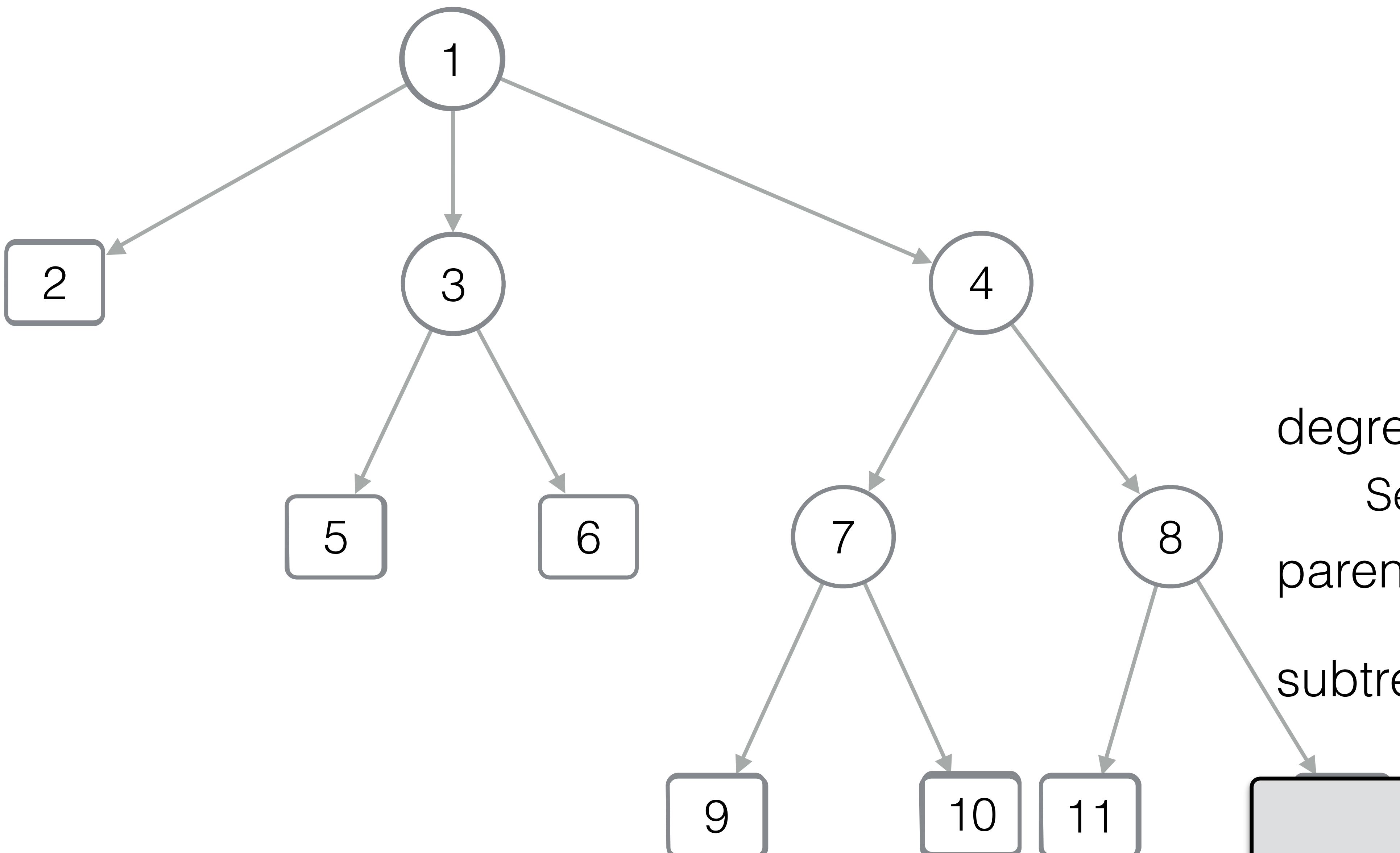
$subtreeSize(x) = ?$

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]



$pos(x) = Select_1(x)$

$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B

if $B[y] == 0$

return -1 // is a leaf

else

return y-x // $Rank_1(y)$

$degree(x) = ?$

$Select_0(x+1) - (Select_0(x) + 1)$

$parent(x) = Rank_0(pos(x))$

$subtreeSize(x) = ?$

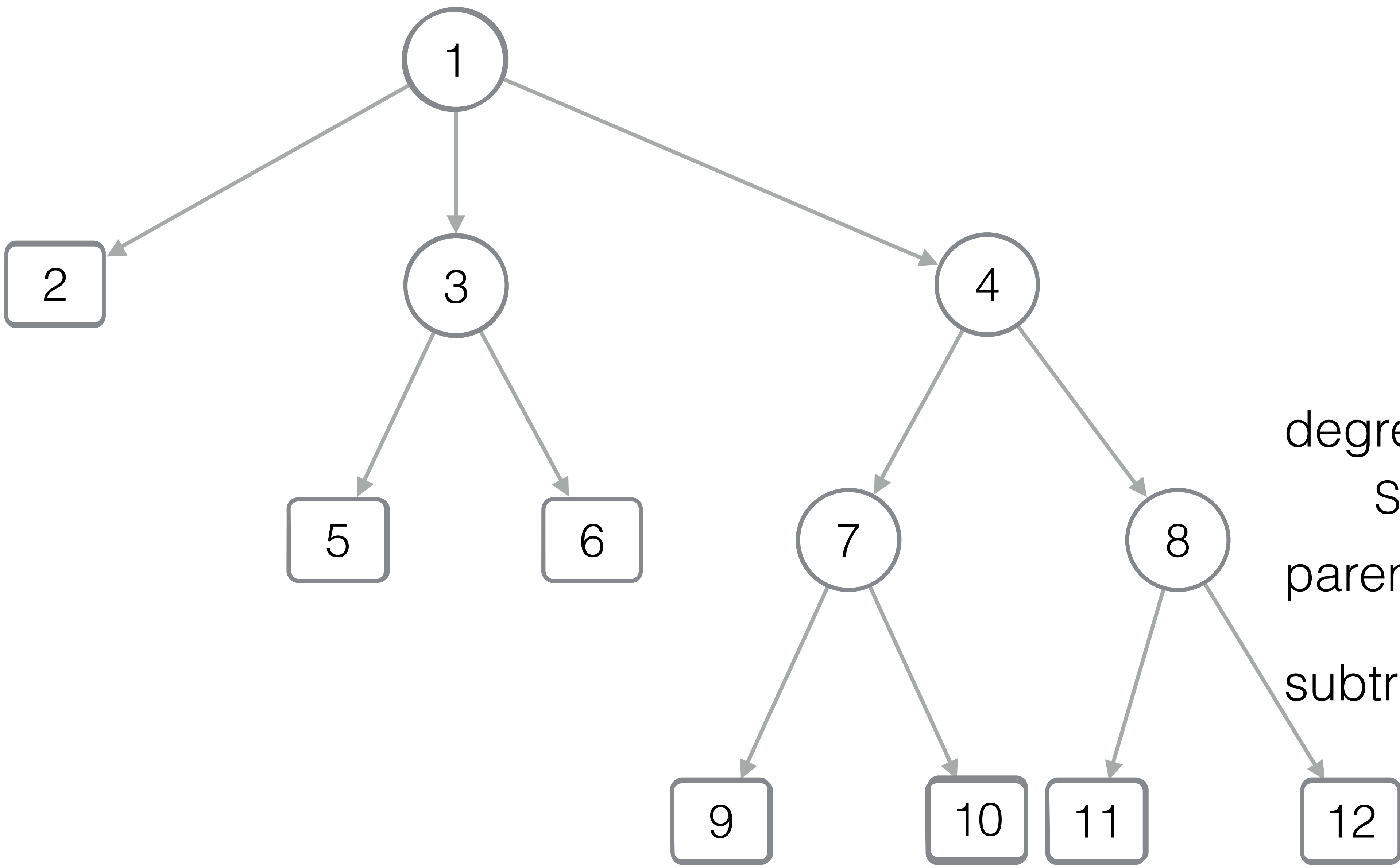
Not efficient!
Nodes of the subtree are spread in B

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

(1) (2)(3)(4) (5)(6) (7)(8) (9)(10) (11)(12)

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]



$pos(x) = Select_1(x)$

$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B

if $B[y] == 0$

return -1 // is a leaf

else

return $y-x$ // $Rank_1(y)$

$degree(x) = ?$

$Select_0(x+1) - (Select_0(x) + 1)$

$parent(x) = Rank_0(pos(x))$

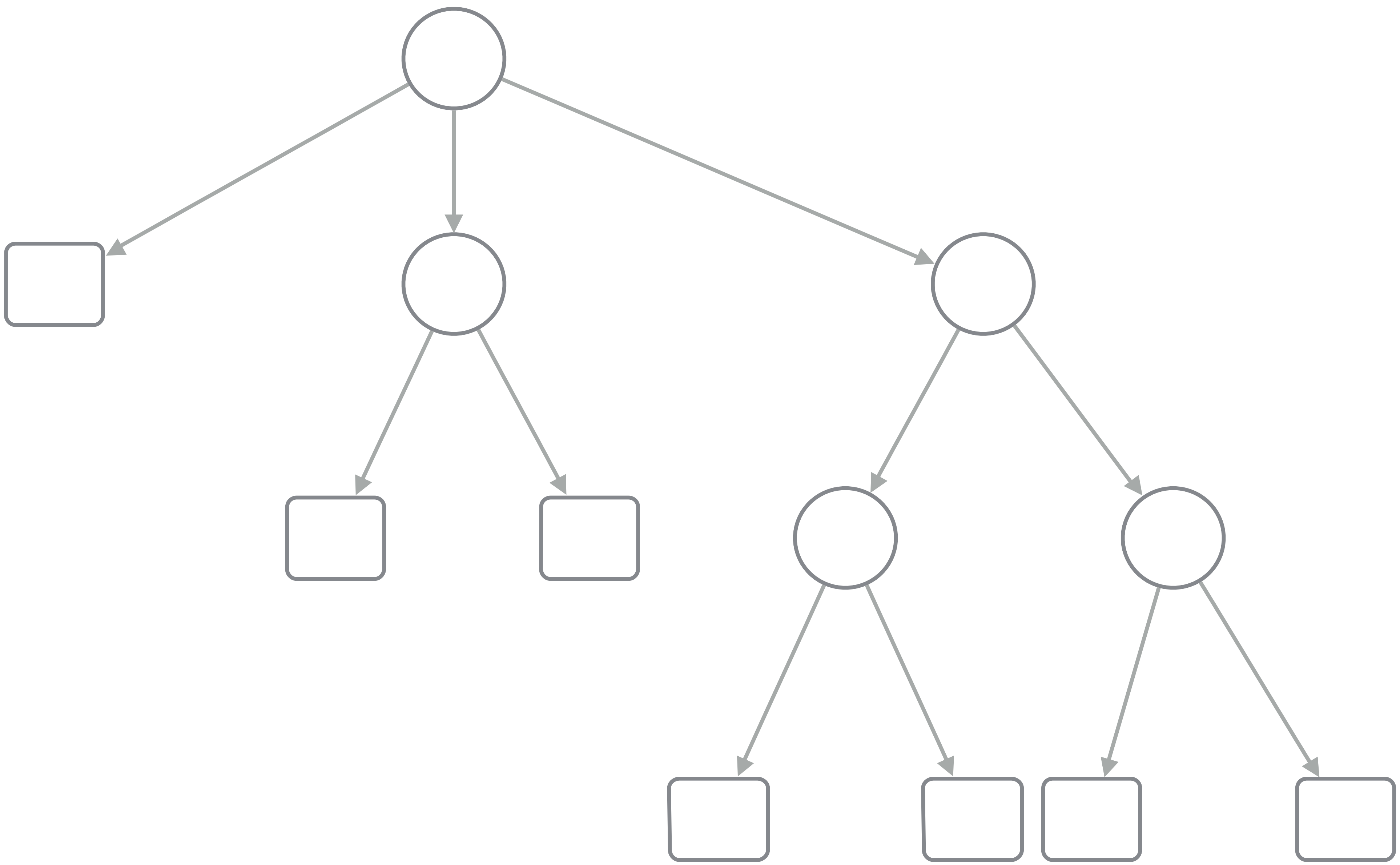
$subtreeSize(x) = ?$

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

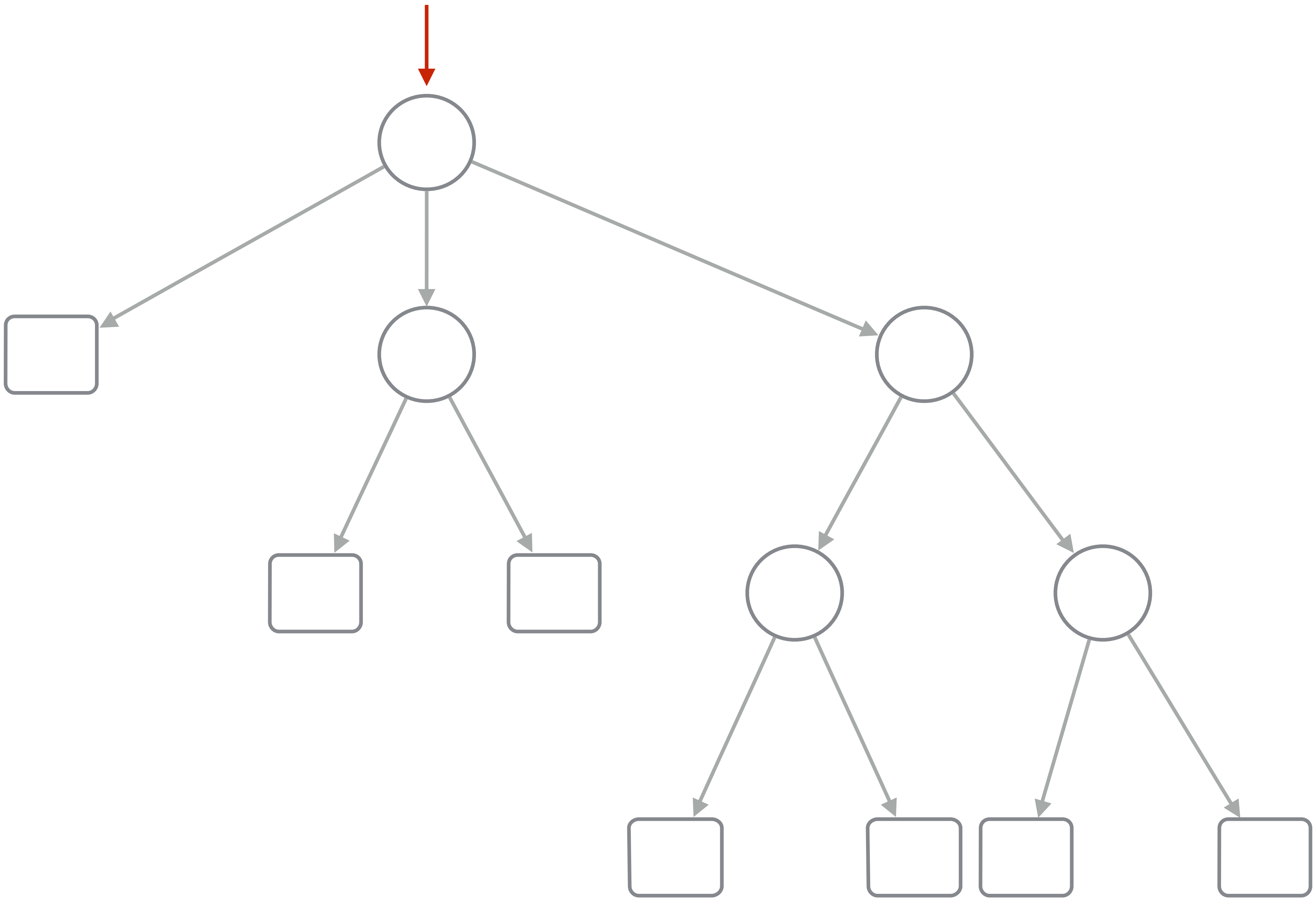
Succinct representation of trees (2)

[BP - Balanced parenthesis]



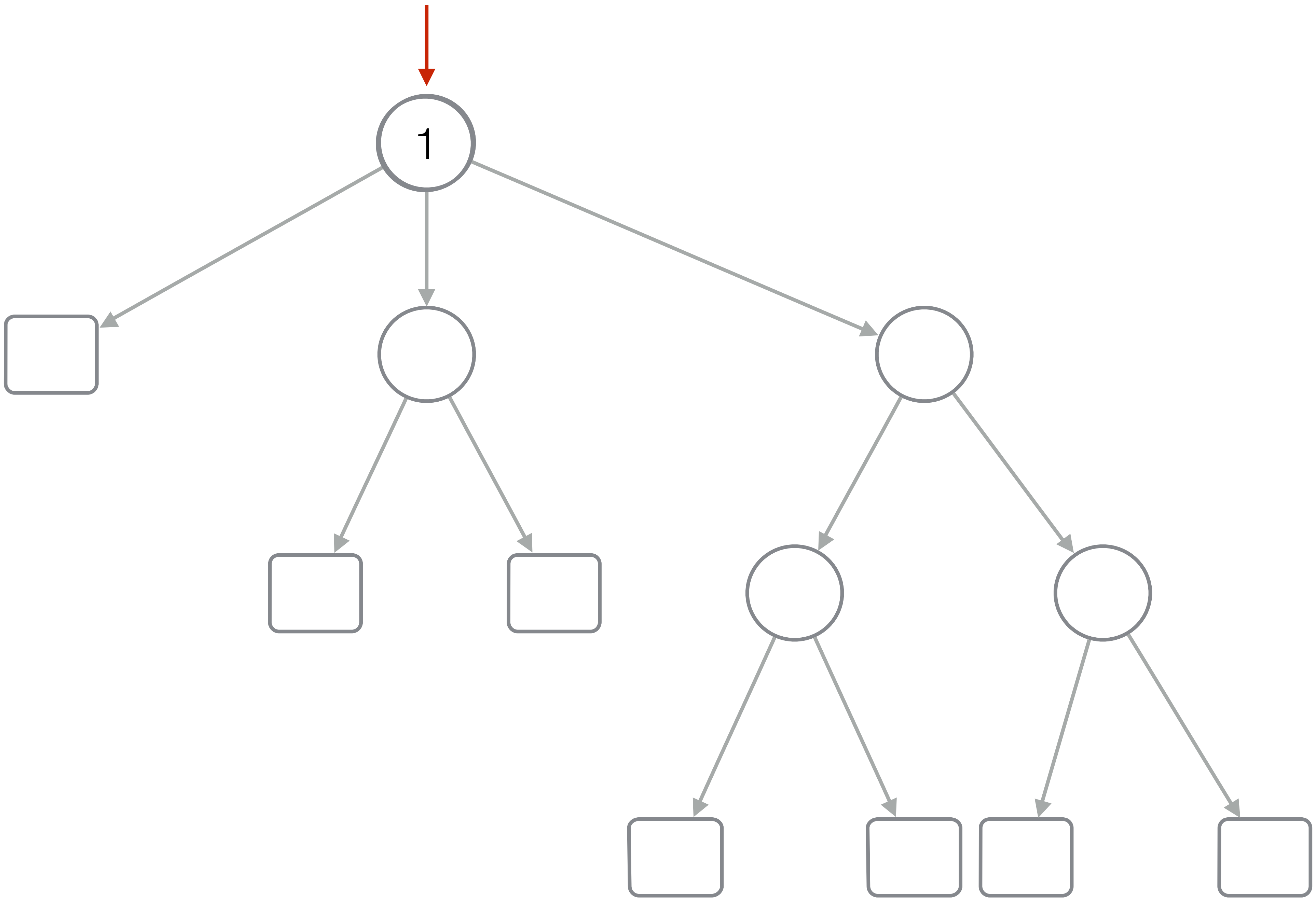
Succinct representation of trees (2)

[BP - Balanced parenthesis]



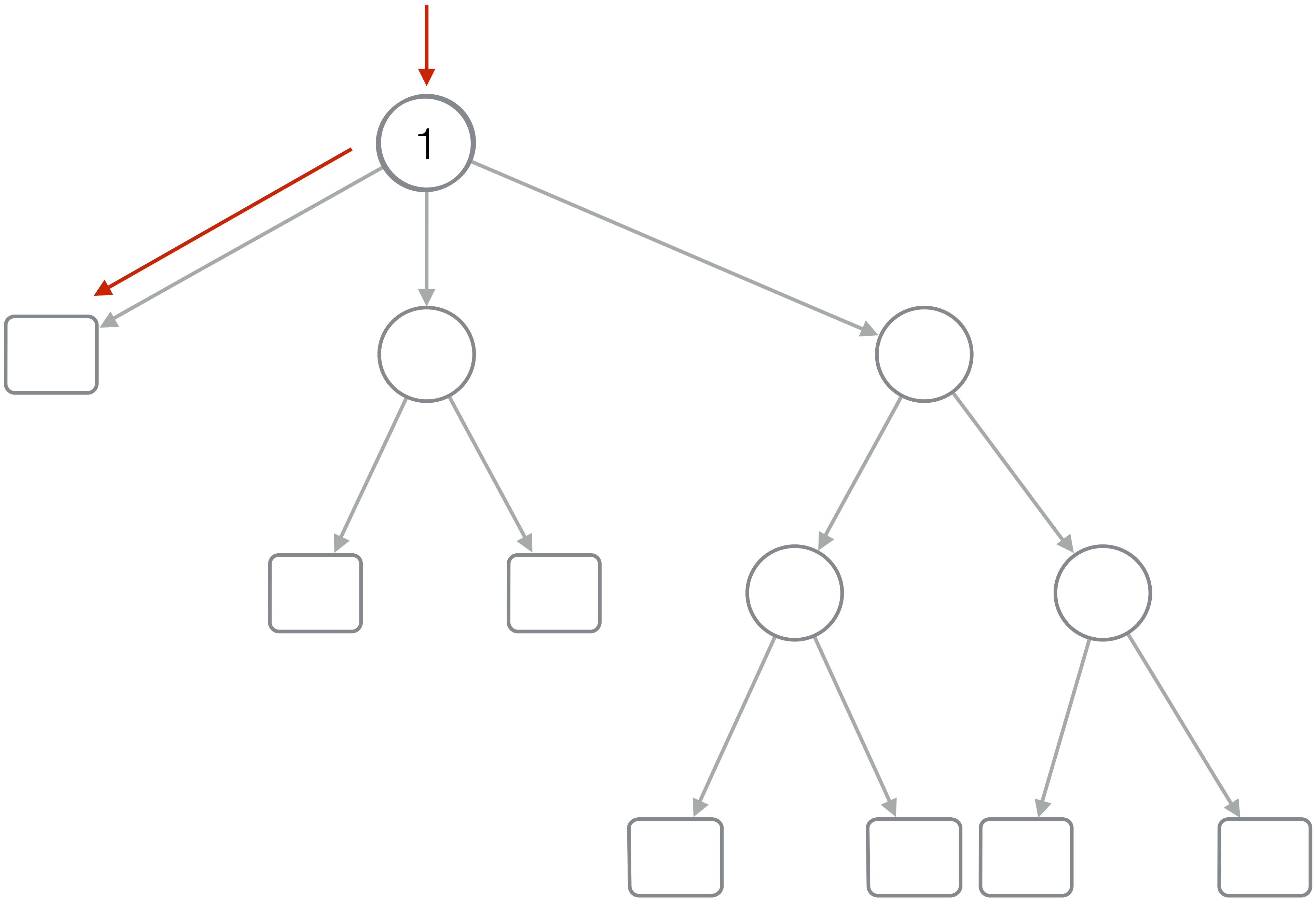
Succinct representation of trees (2)

[BP - Balanced parenthesis]



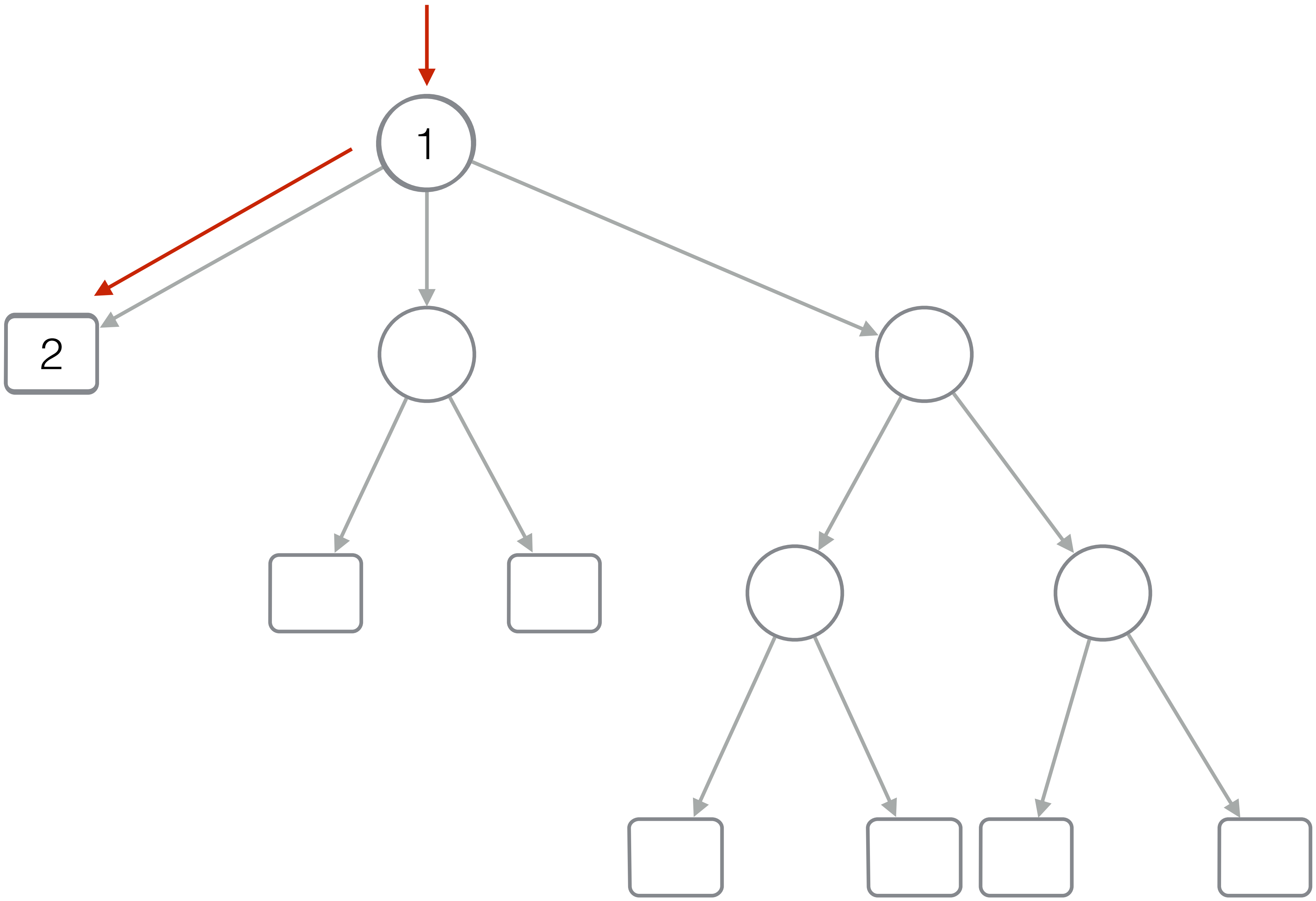
Succinct representation of trees (2)

[BP - Balanced parenthesis]



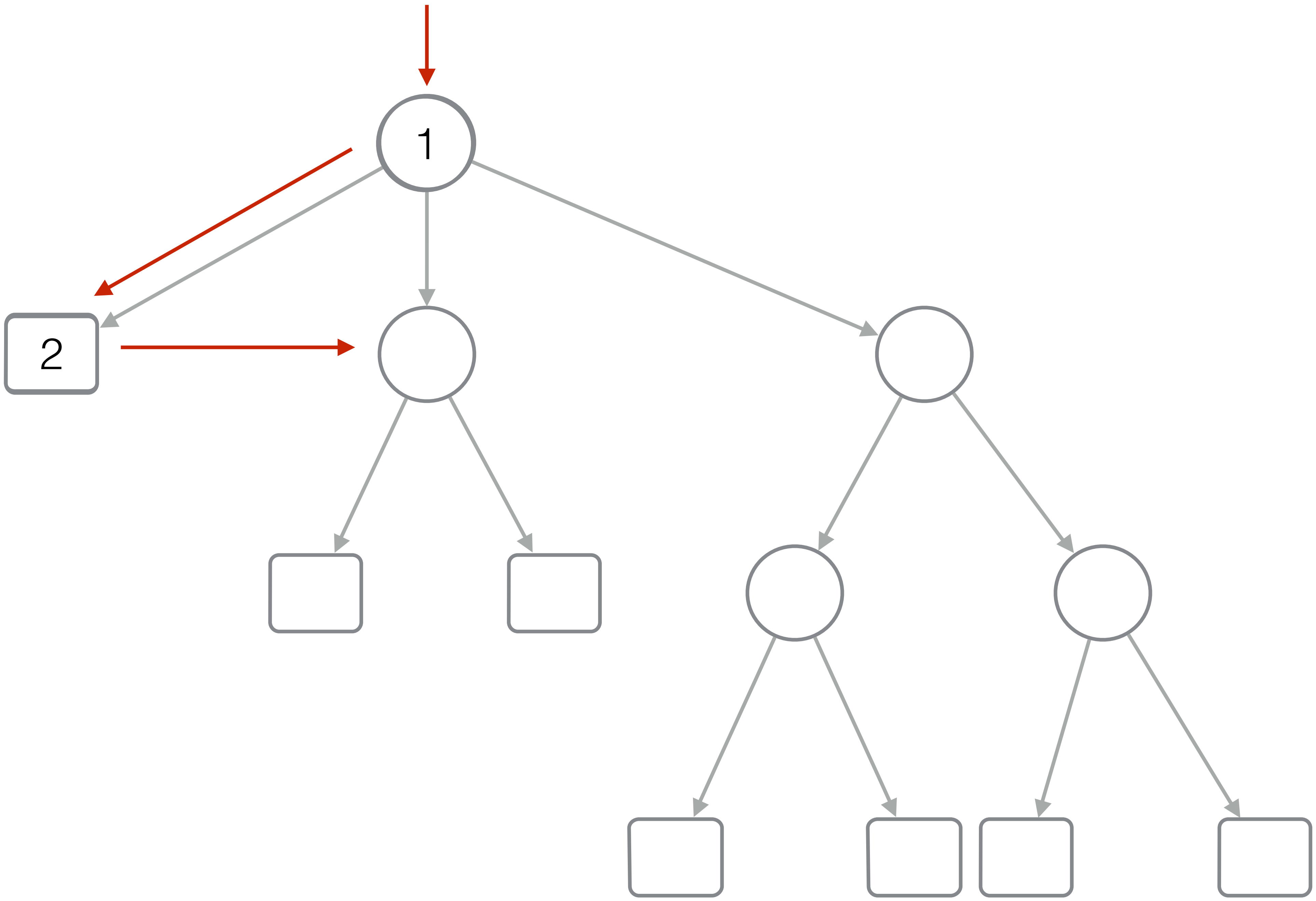
Succinct representation of trees (2)

[BP - Balanced parenthesis]



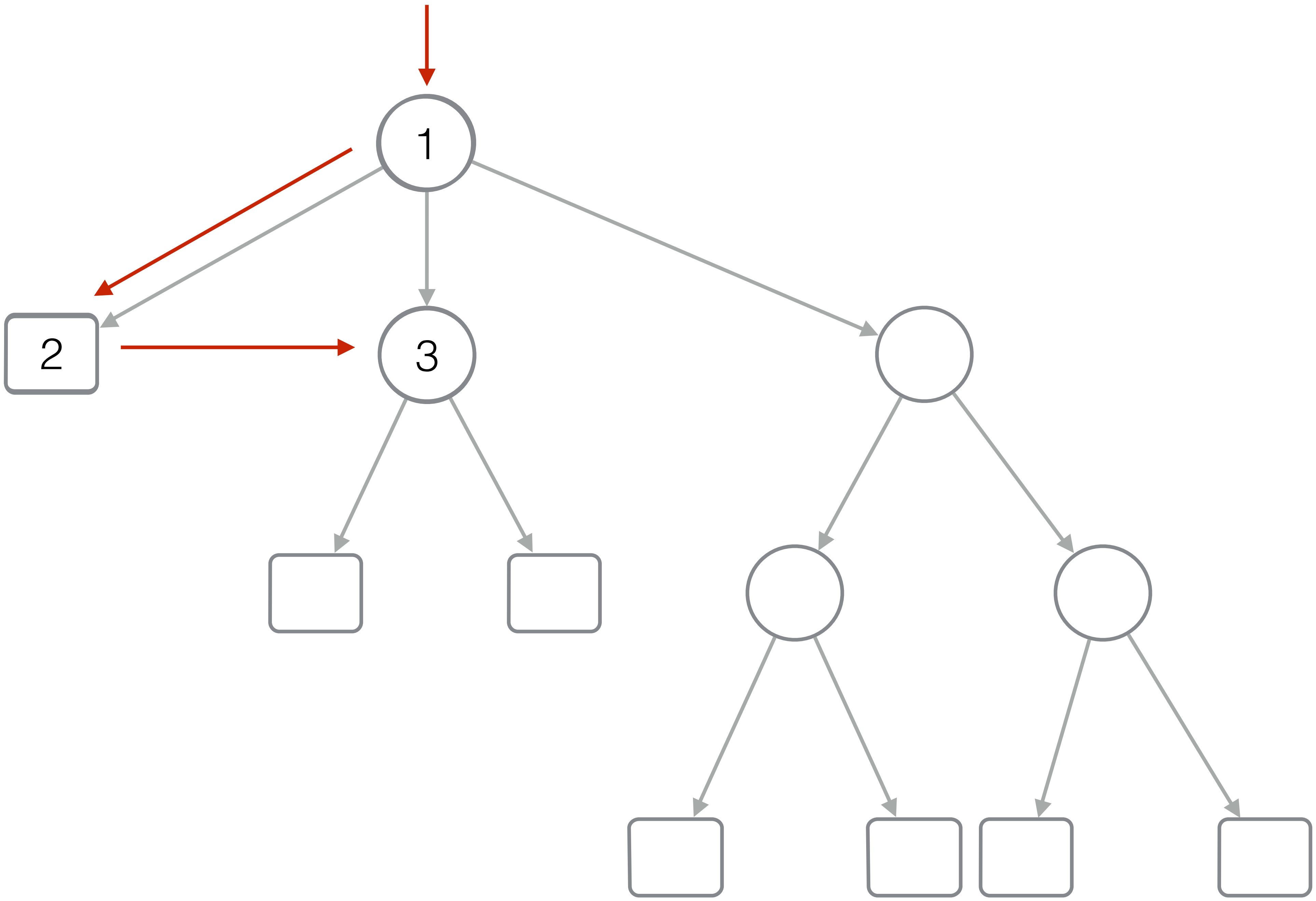
Succinct representation of trees (2)

[BP - Balanced parenthesis]



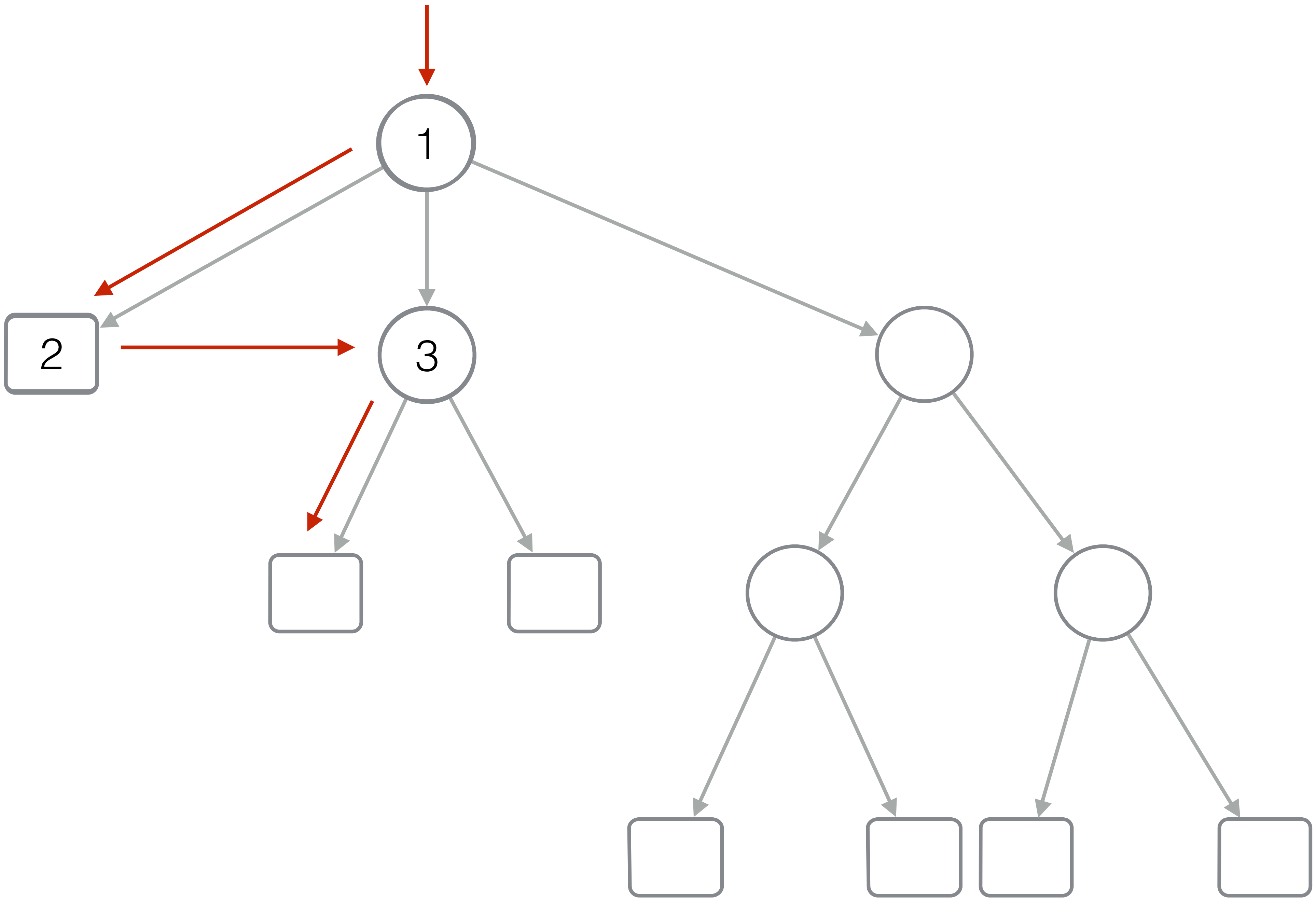
Succinct representation of trees (2)

[BP - Balanced parenthesis]



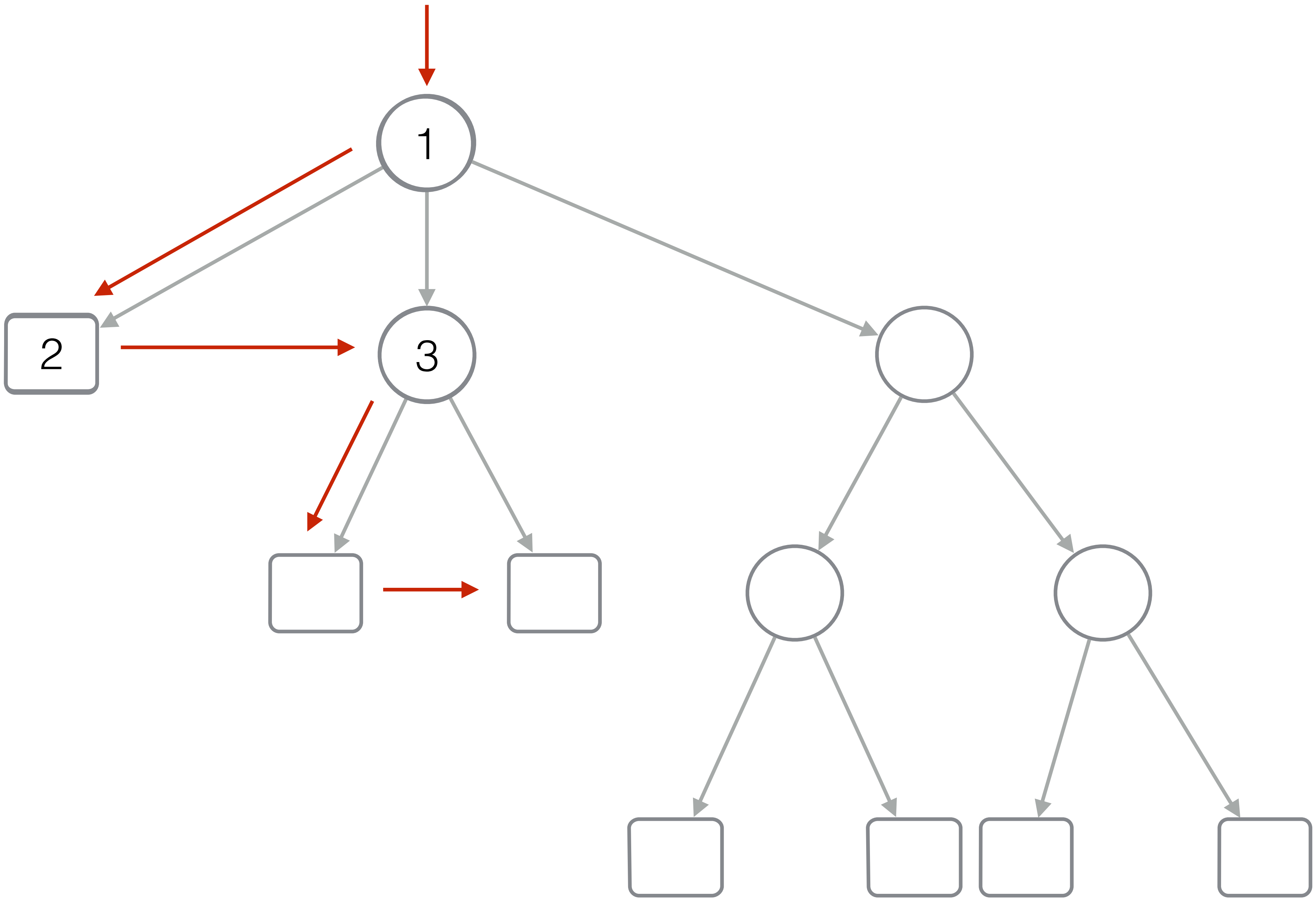
Succinct representation of trees (2)

[BP - Balanced parenthesis]



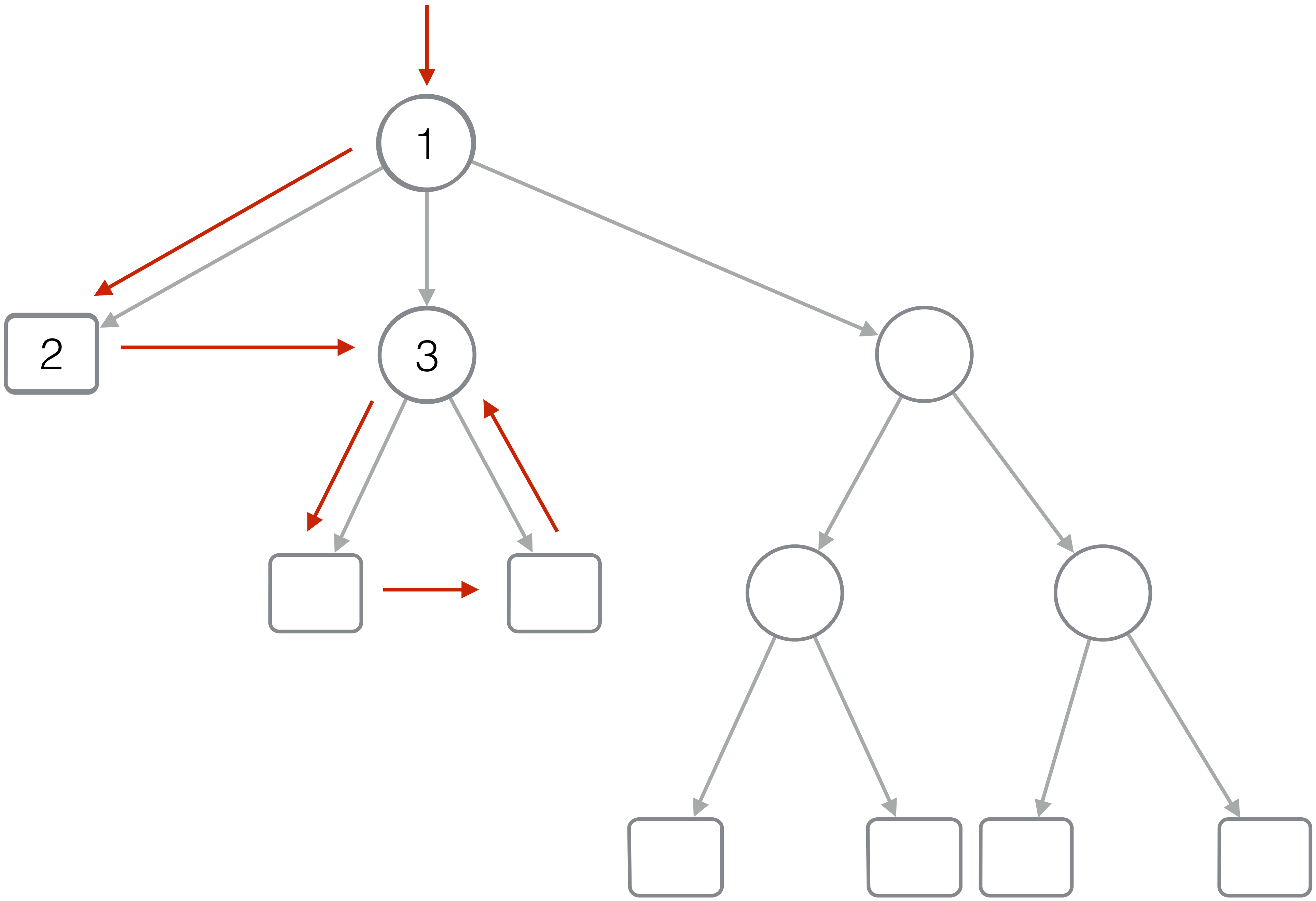
Succinct representation of trees (2)

[BP - Balanced parenthesis]



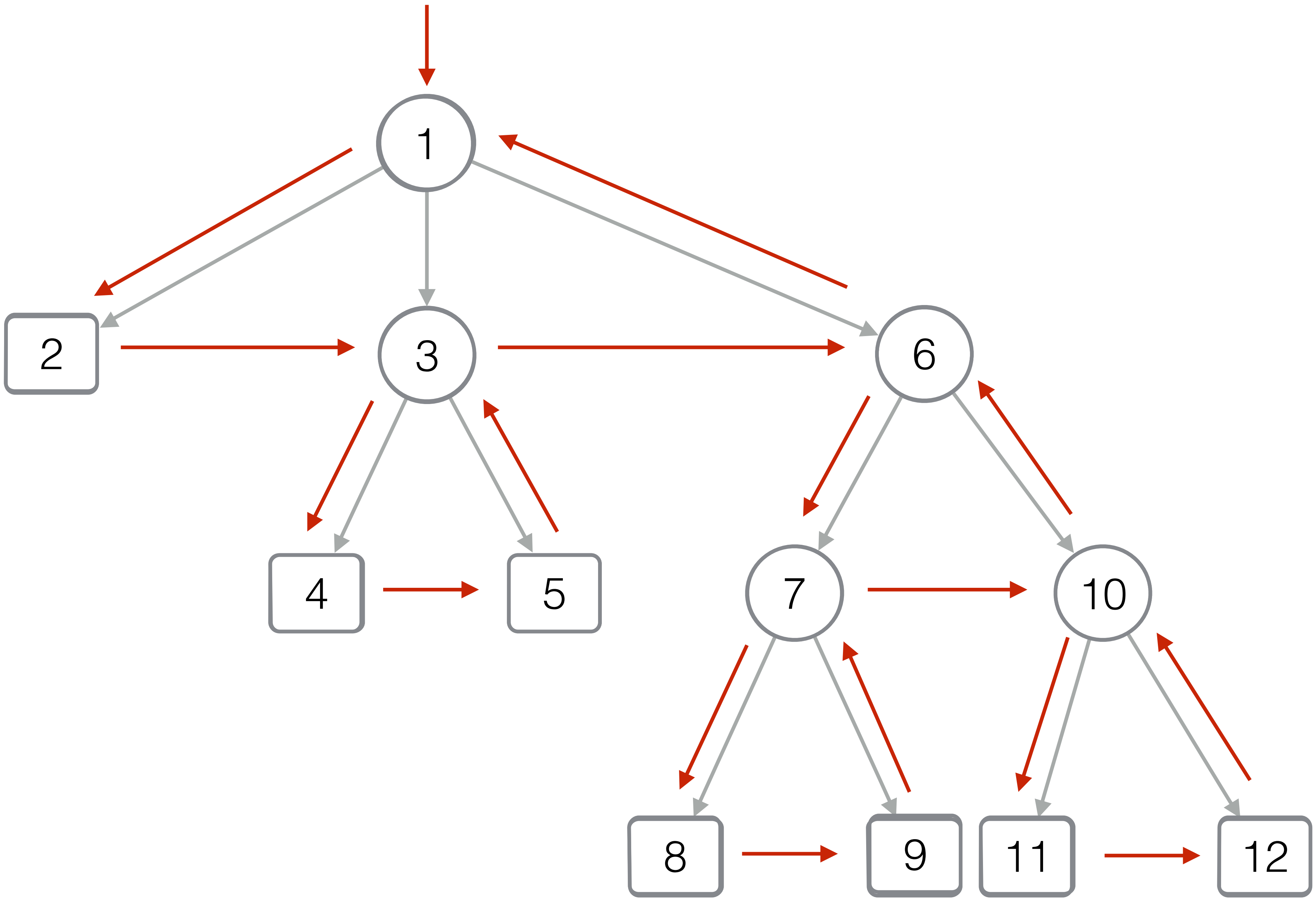
Succinct representation of trees (2)

[BP - Balanced parenthesis]



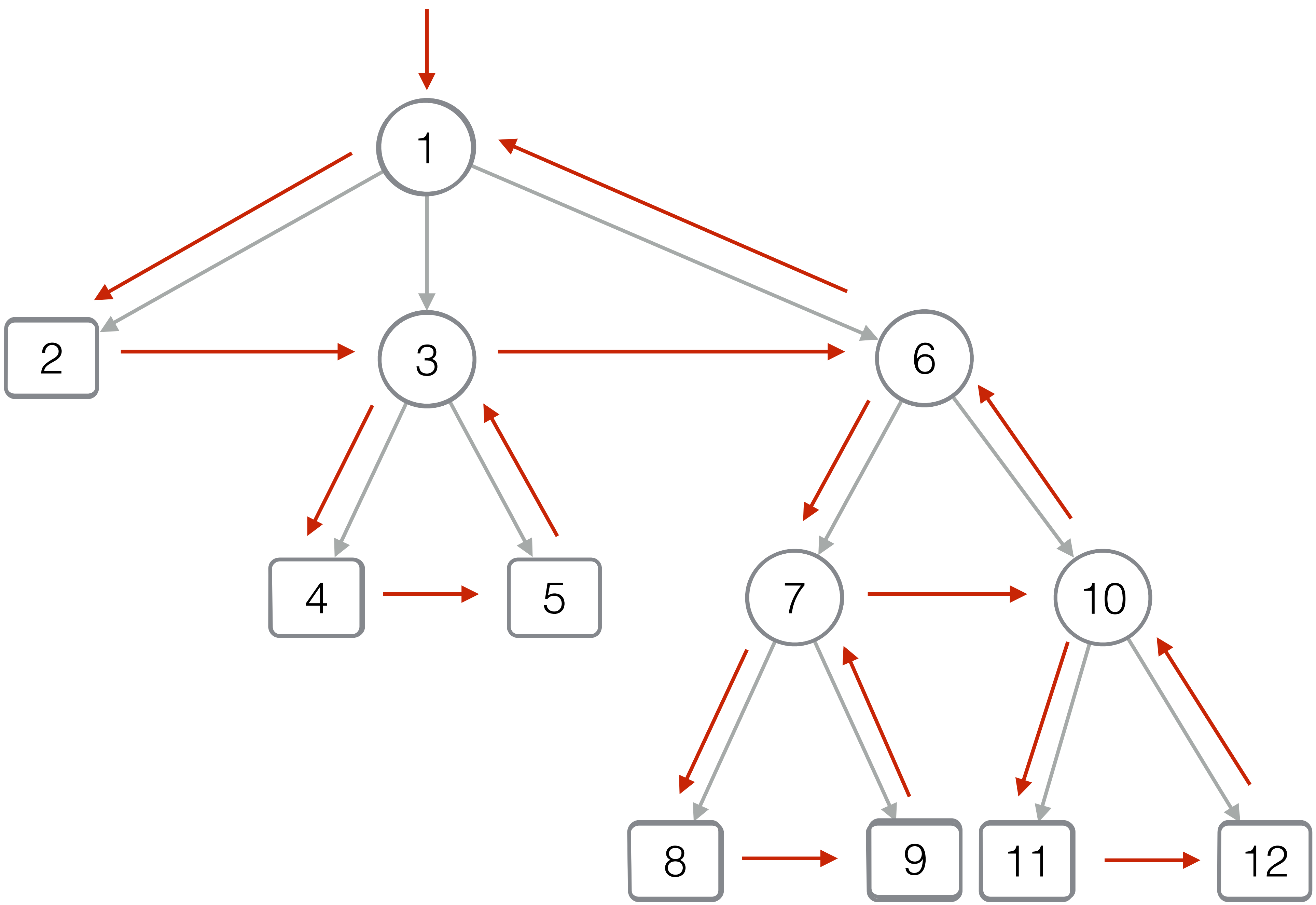
Succinct representation of trees (2)

[BP - Balanced parenthesis]



Succinct representation of trees (2)

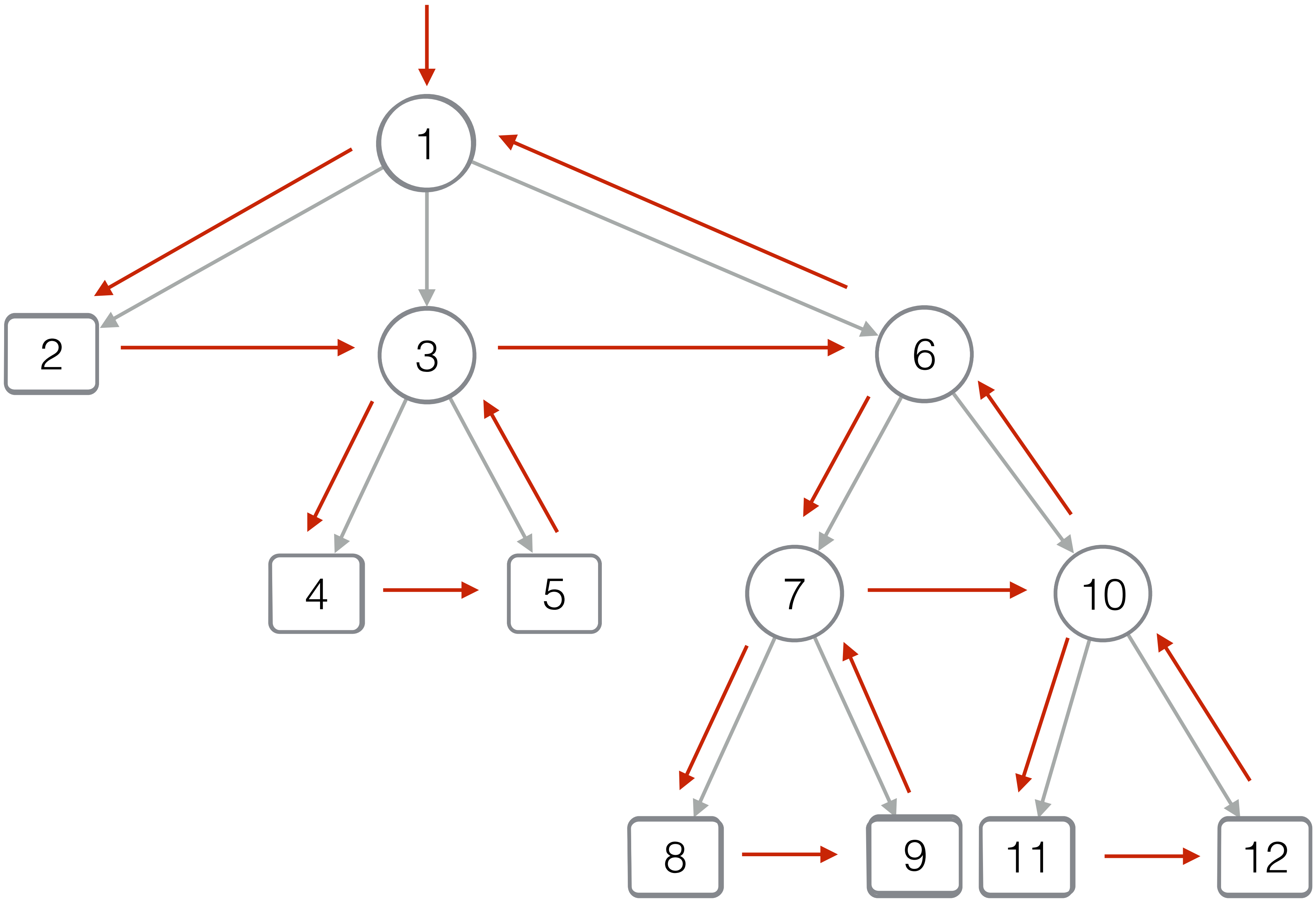
[BP - Balanced parenthesis]



B (( ) (( ) ( )) ((( ) ( )) (( ) ( ))))

Succinct representation of trees (2)

[BP - Balanced parenthesis]



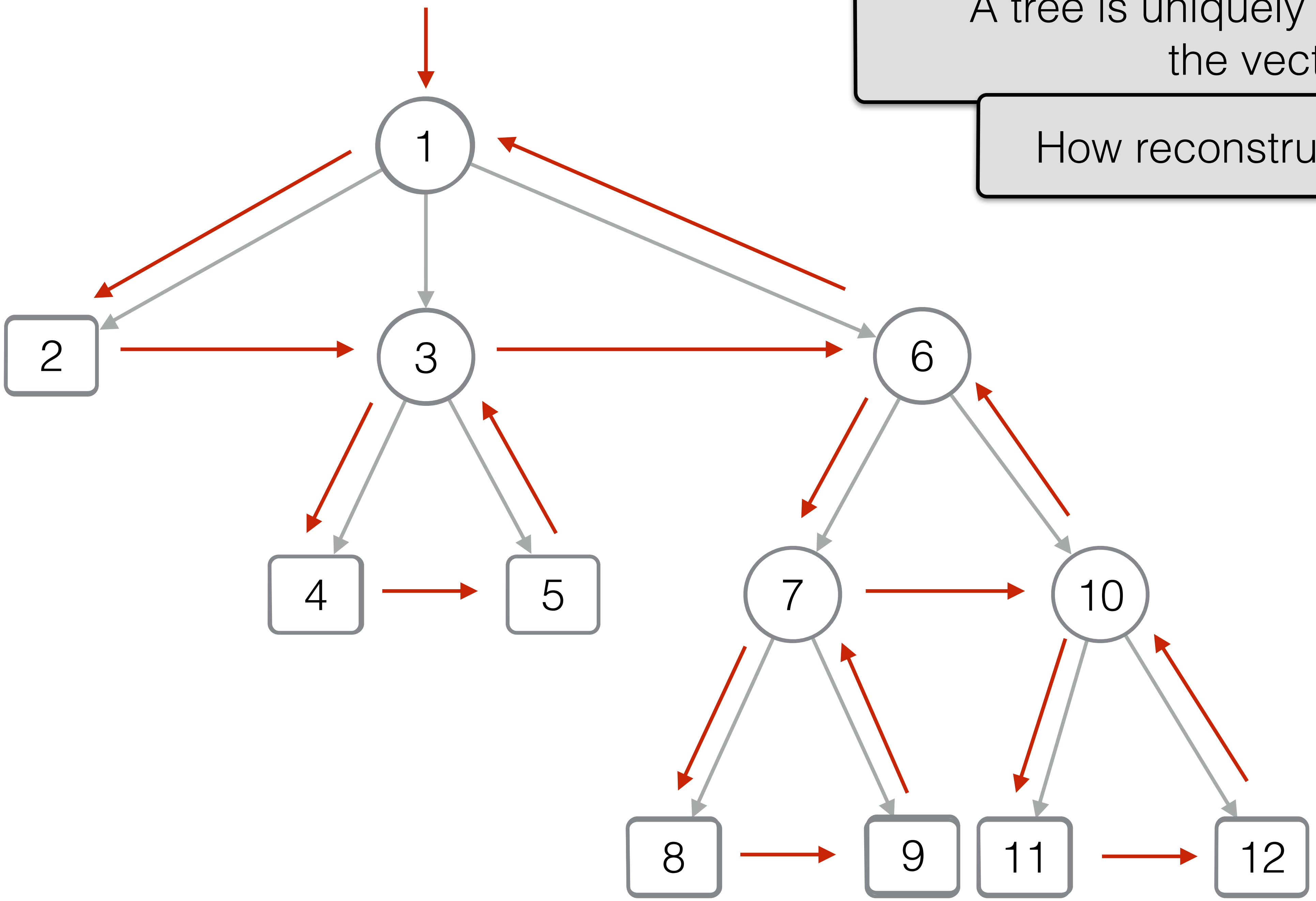
B	(	(	)	(	(	)	(	)	)	(	(	(	)	(	)	)	(	(	)	(	)	)	)	)
	1	2	2	3	4	4	5	5	3	6	7	8	8	9	9	7	10	11	11	12	12	10	6	1

Succinct representation of trees (2)

[BP - Balanced parenthesis]

A tree is uniquely determined by the vector B

How reconstruct the tree?

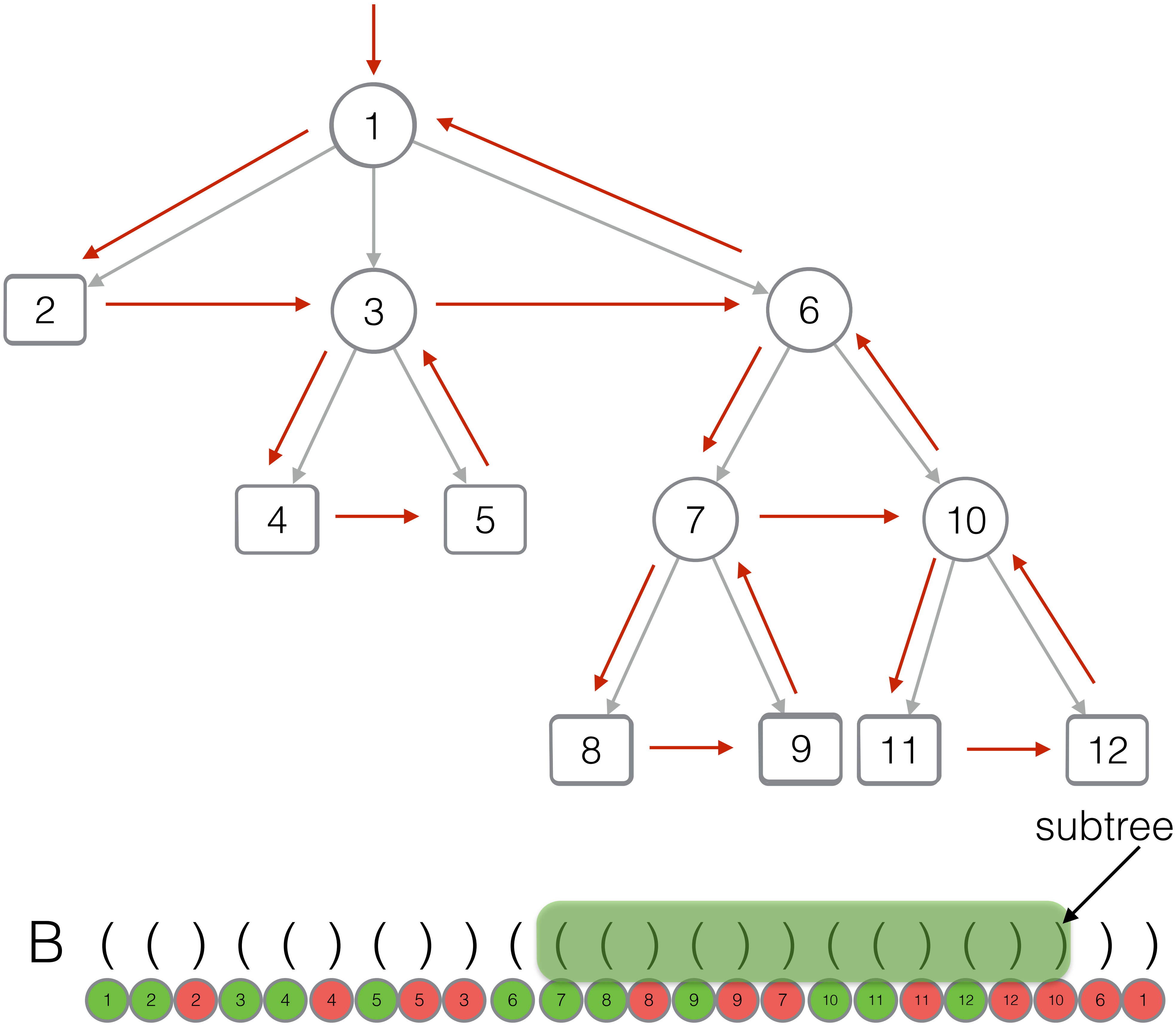


B (( ) (( ) ( )) ((( ) ( )) (( ) ( ))))

1 2 2 3 4 4 5 5 3 6 7 8 8 9 9 7 10 11 11 12 12 10 6 1

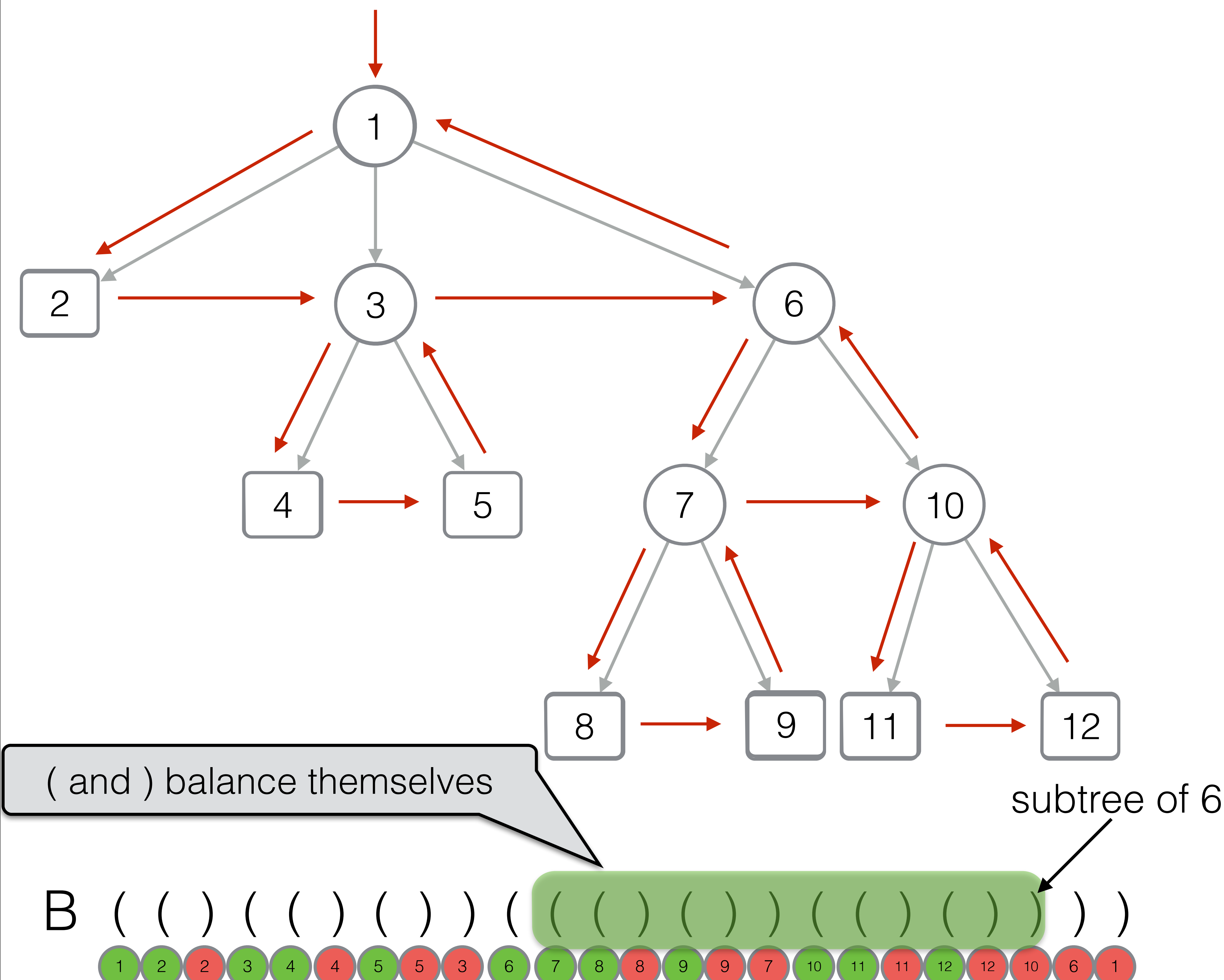
Succinct representation of trees (2)

[BP - Balanced parenthesis]



Succinct representation of trees (2)

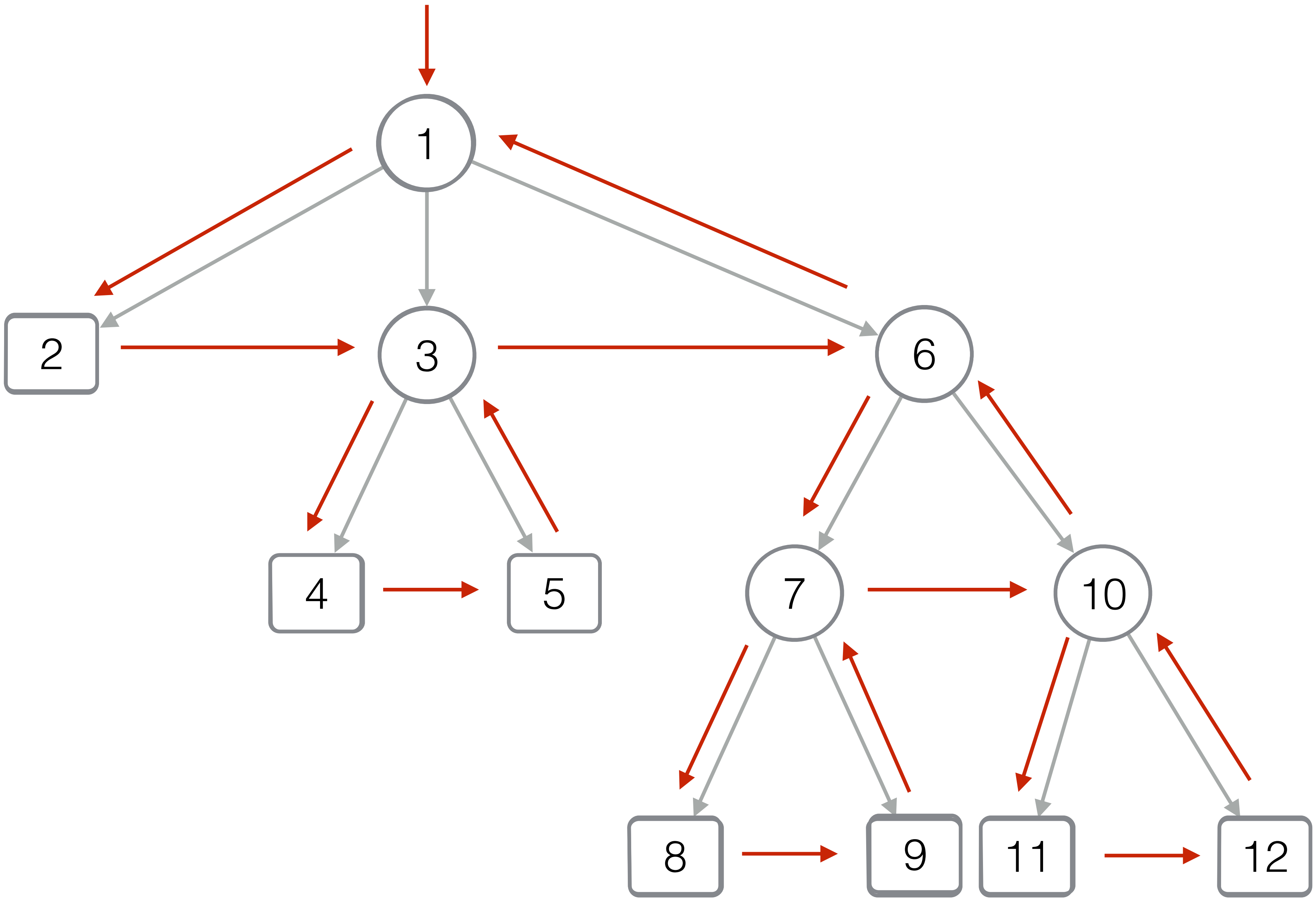
[BP - Balanced parenthesis]



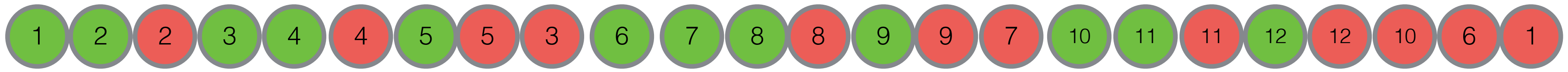
Succinct representation of trees (2)

[BP - Balanced parenthesis]

pos(x) =



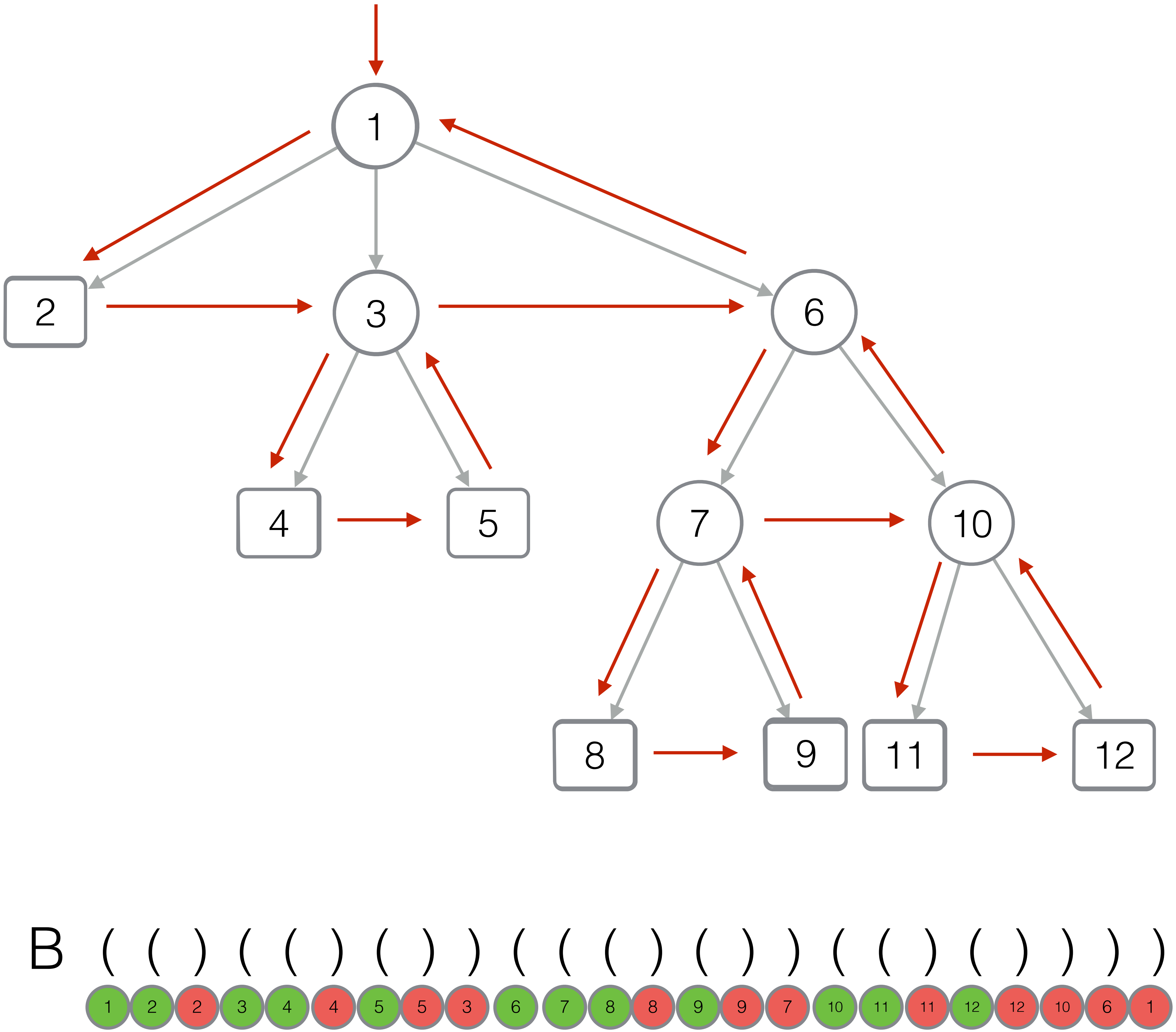
B (( ) (( ) ( )) ((( ) ( )) (( ) ( ))))



Succinct representation of trees (2)

[BP - Balanced parenthesis]

pos(x) = Select ((x)

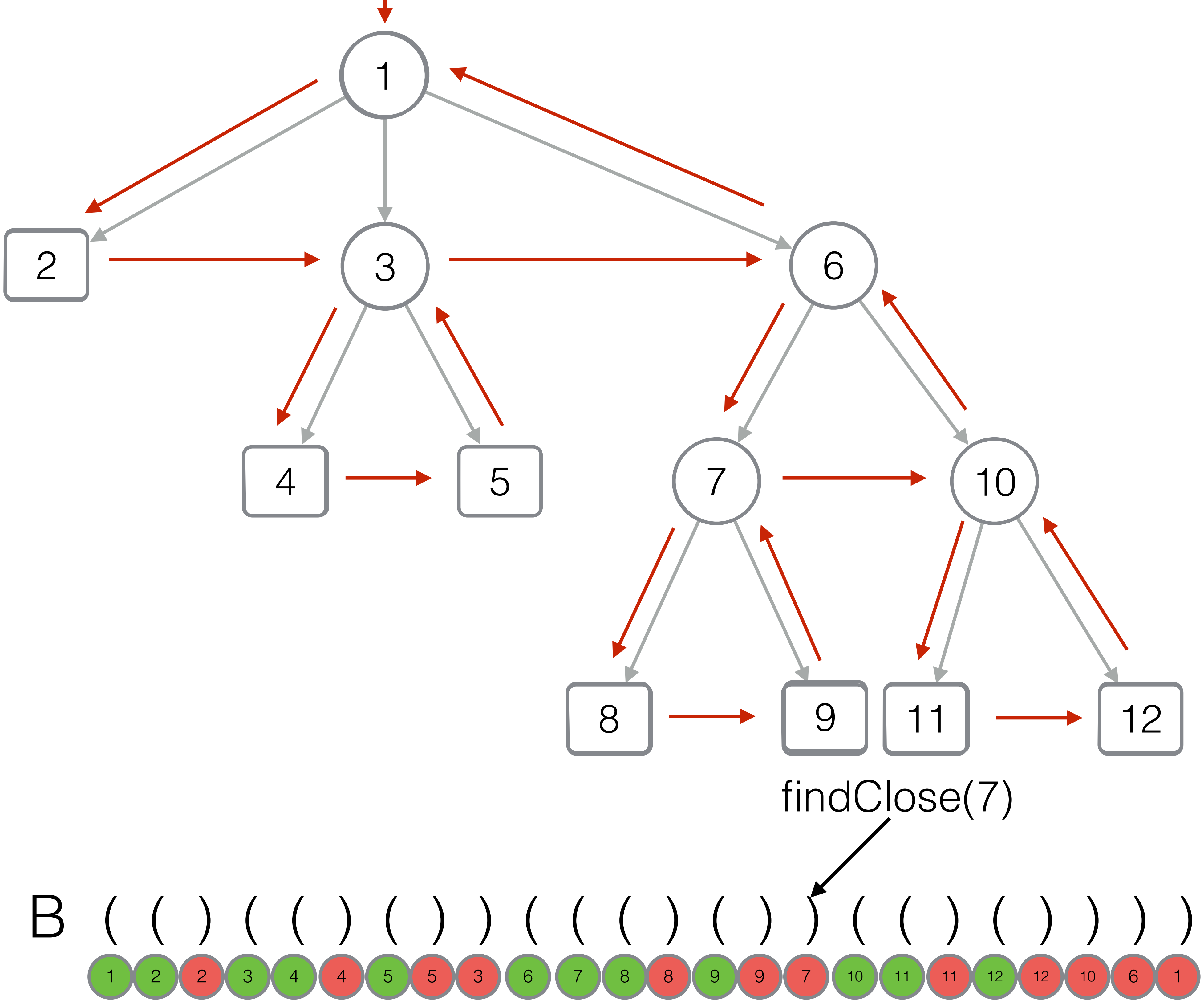


Succinct representation of trees (2)

[BP - Balanced parenthesis]

pos(x) = Select $_($ (x)

findClose(x) = returns the position of) matching x-th (



Succinct representation of trees (2)

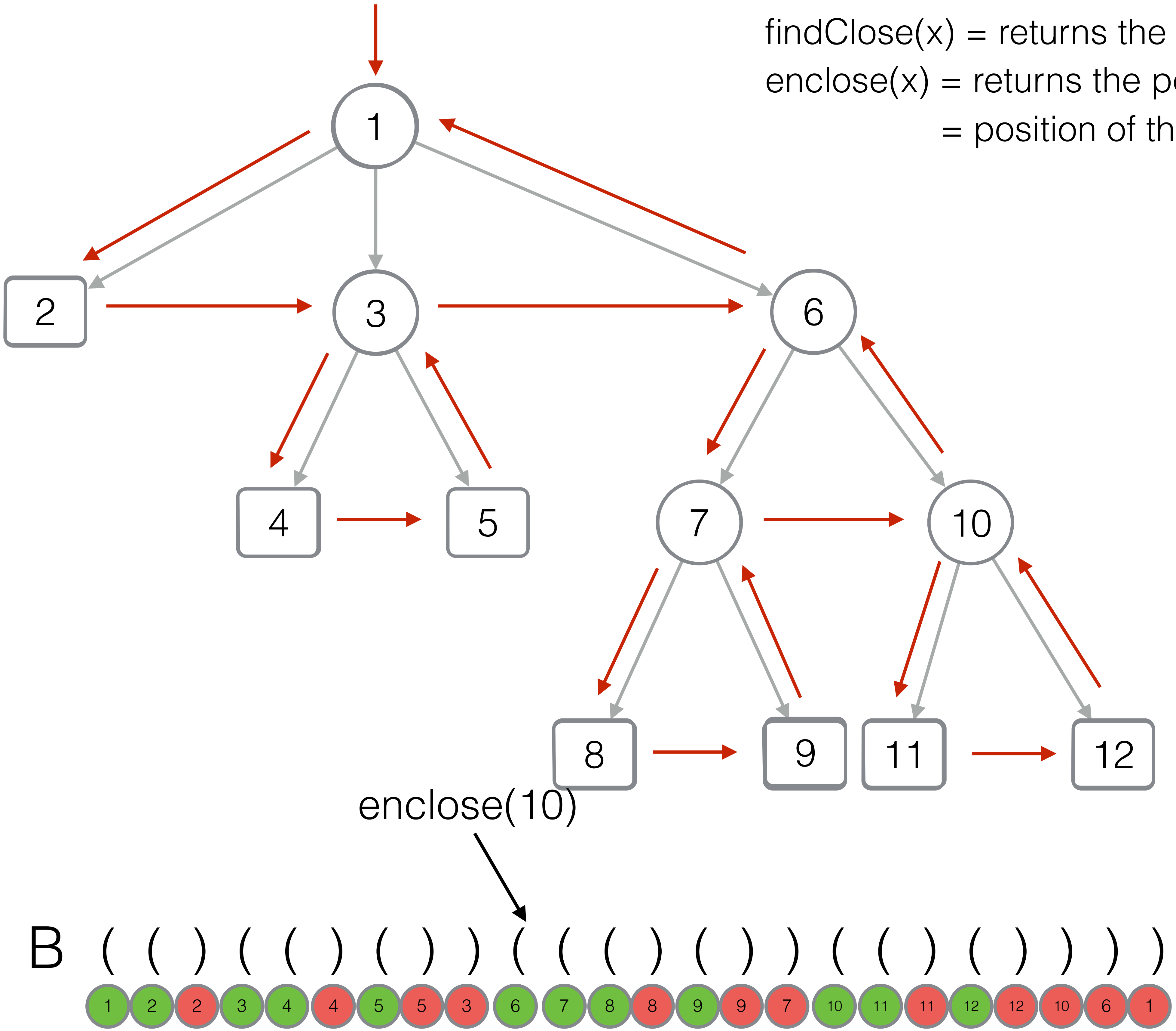
[BP - Balanced parenthesis]

pos(x) = Select $_($ (x)

findClose(x) = returns the position of) matching x-th (

enclose(x) = returns the position of (enclosing x-th (

= position of the parent of x in B



Succinct representation of trees (2)

[BP - Balanced parenthesis]

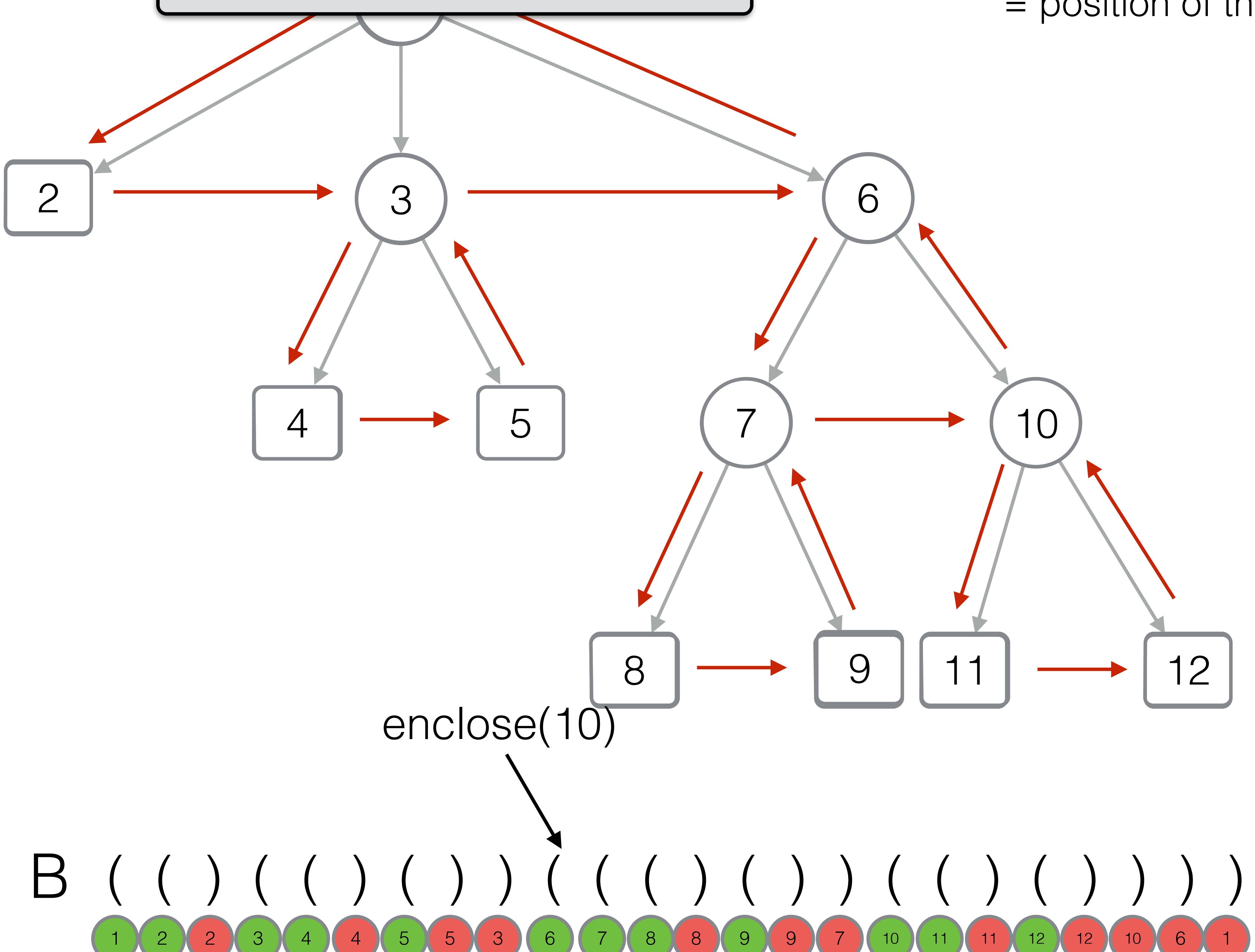
They can be implemented in O(1) time.

pos(x) = $\text{Select}_{(}(x)$

findClose(x) = returns the position of) matching x-th (

enclose(x) = returns the position of (enclosing x-th (

= position of the parent of x in B



Succinct representation of trees (2)

[BP - Balanced parenthesis]

They can be implemented in O(1) time.

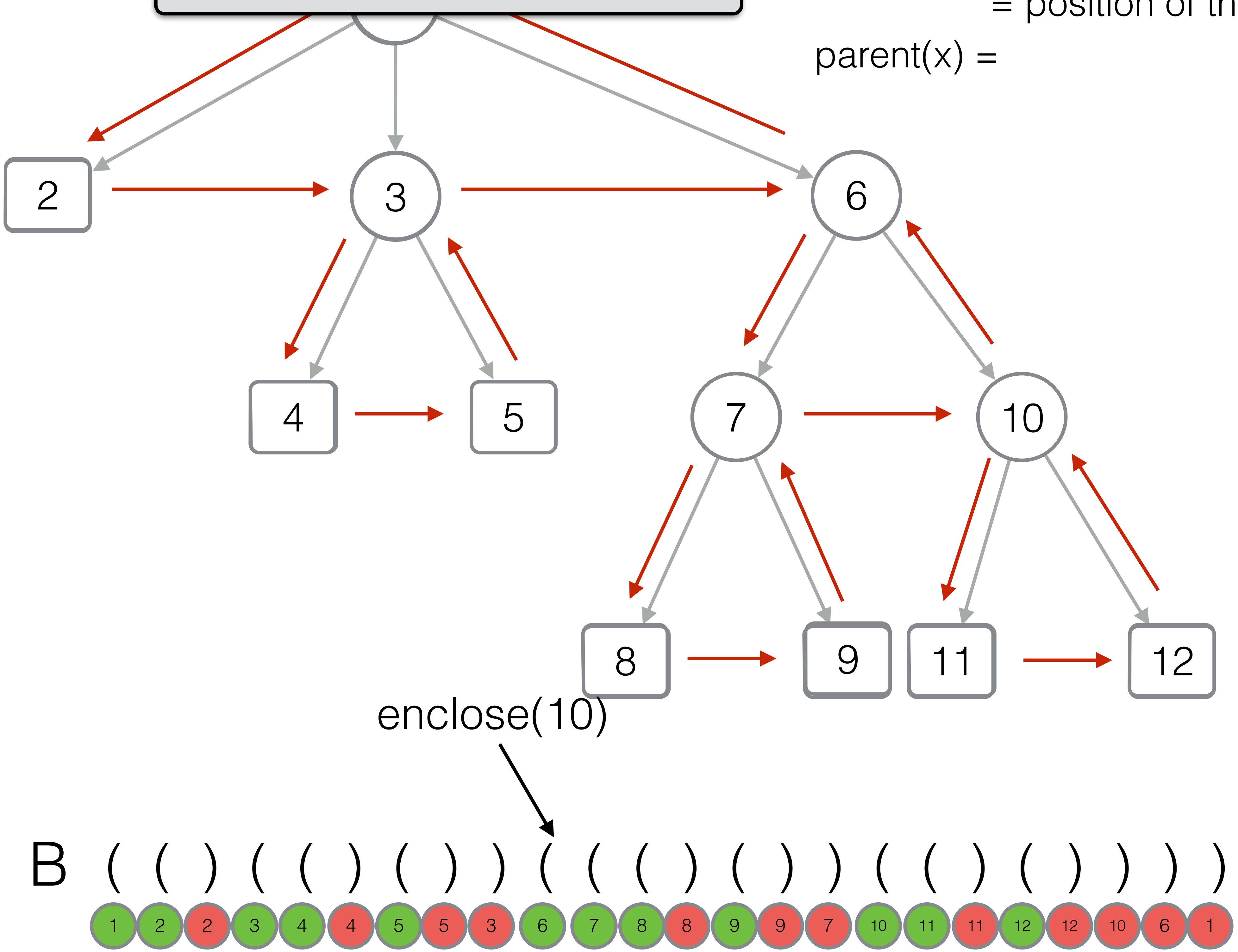
pos(x) = Select $_($ (x)

findClose(x) = returns the position of) matching x-th (

enclose(x) = returns the position of (enclosing x-th (

= position of the parent of x in B

parent(x) =



Succinct representation of trees (2)

[BP - Balanced parenthesis]

They can be implemented in O(1) time.

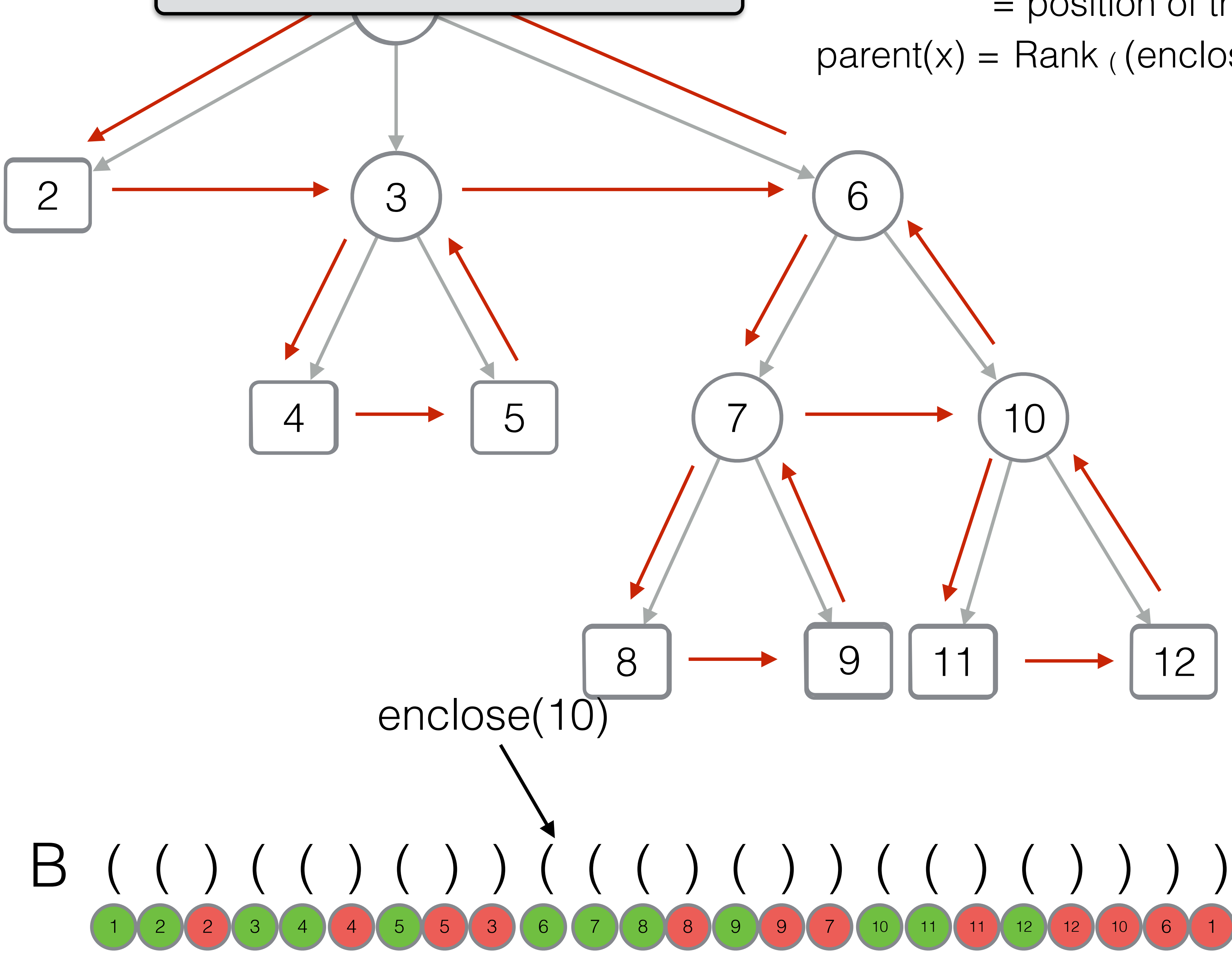
pos(x) = Select $_($ (x)

findClose(x) = returns the position of) matching x-th (

enclose(x) = returns the position of (enclosing x-th (

= position of the parent of x in B

parent(x) = Rank $_($ (enclose(x))



Succinct representation of trees (2)

[BP - Balanced parenthesis]

They can be implemented in O(1) time.

pos(x) = Select ((x)

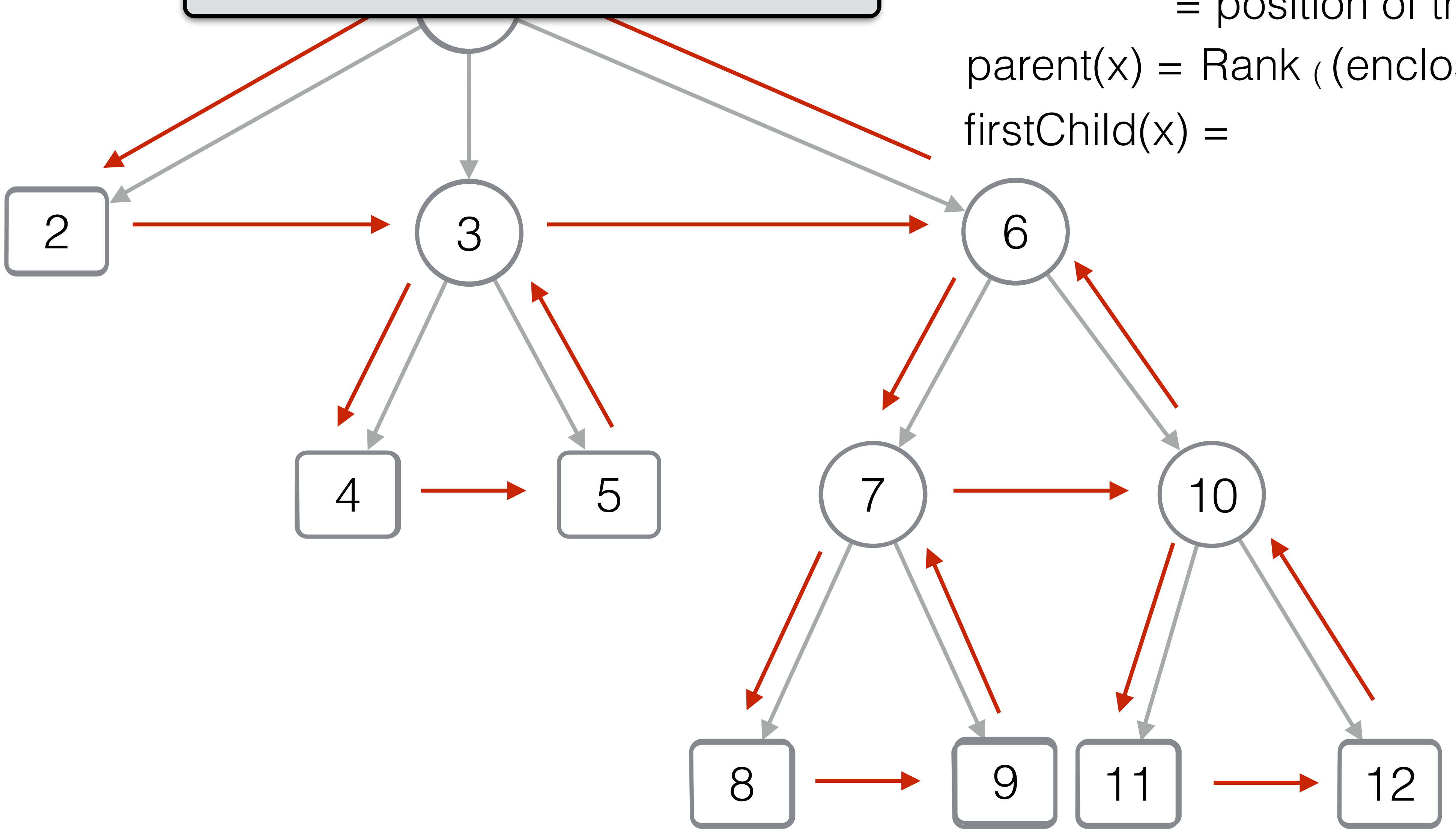
findClose(x) = returns the position of) matching x-th (

enclose(x) = returns the position of (enclosing x-th (

= position of the parent of x in B

parent(x) = Rank ((enclose(x))

firstChild(x) =



B (( ) (( ) ( )) ((( ) ( )) (( ) ( ))))

1 2 2 3 4 4 5 5 3 6 7 8 8 9 9 7 10 11 11 12 12 10 6 1

Succinct representation of trees (2)

[BP - Balanced parenthesis]

They can be implemented in O(1) time.

pos(x) = Select ((x)

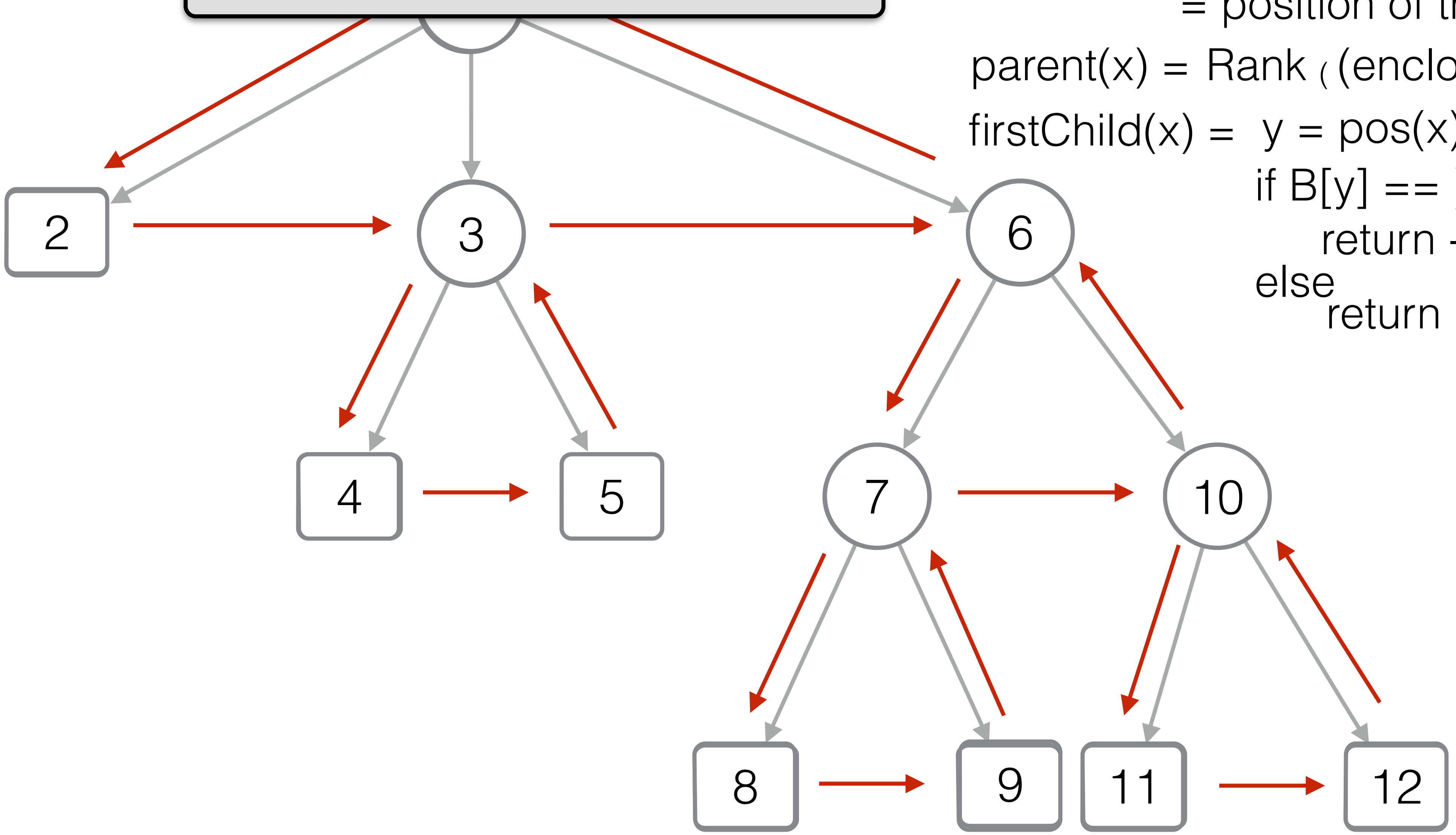
findClose(x) = returns the position of) matching x-th (

enclose(x) = returns the position of (enclosing x-th (

= position of the parent of x in B

parent(x) = Rank ((enclose(x))

```
firstChild(x) =  y = pos(x)+1
                 if B[y] == )
                      return -1      // is a leaf
                 else
                      return Rank ( (y)
```



B (( ) (( ) ( )) ((( ) ( )) (( ) ( ))))

1 2 2 3 4 4 5 5 3 6 7 8 8 9 9 7 10 11 11 12 12 10 6 1

Succinct representation of trees (2)

[BP - Balanced parenthesis]

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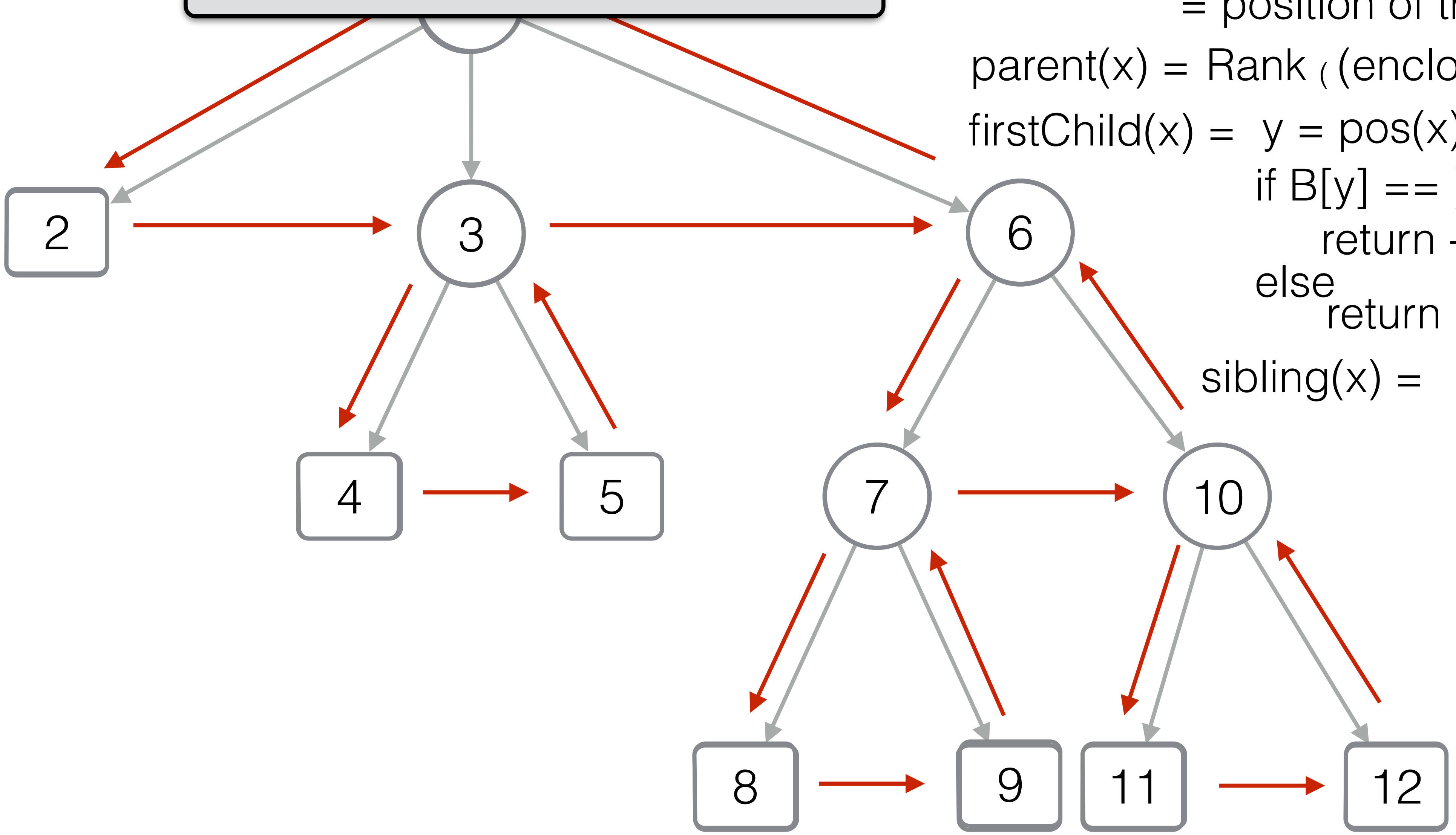
parent(x) = Rank ((enclose(x))

```
firstChild(x) = y = pos(x)+1
                if B[y] == )
                    return -1     // is a leaf
                else
                    return Rank ( (y)
```

sibling(x) =



B	(	(	)	(	(	)	(	)	)	(	(	(	)	(	)	)	(	(	)	(	)	)	)	)
	1	2	2	3	4	4	5	5	3	6	7	8	8	9	9	7	10	11	11	12	12	10	6	1

Succinct representation of trees (2)

[BP - Balanced parenthesis]

They can be implemented in O(1) time.

pos(x) = Select$_($(x)

findClose(x) = returns the position of) matching x-th (

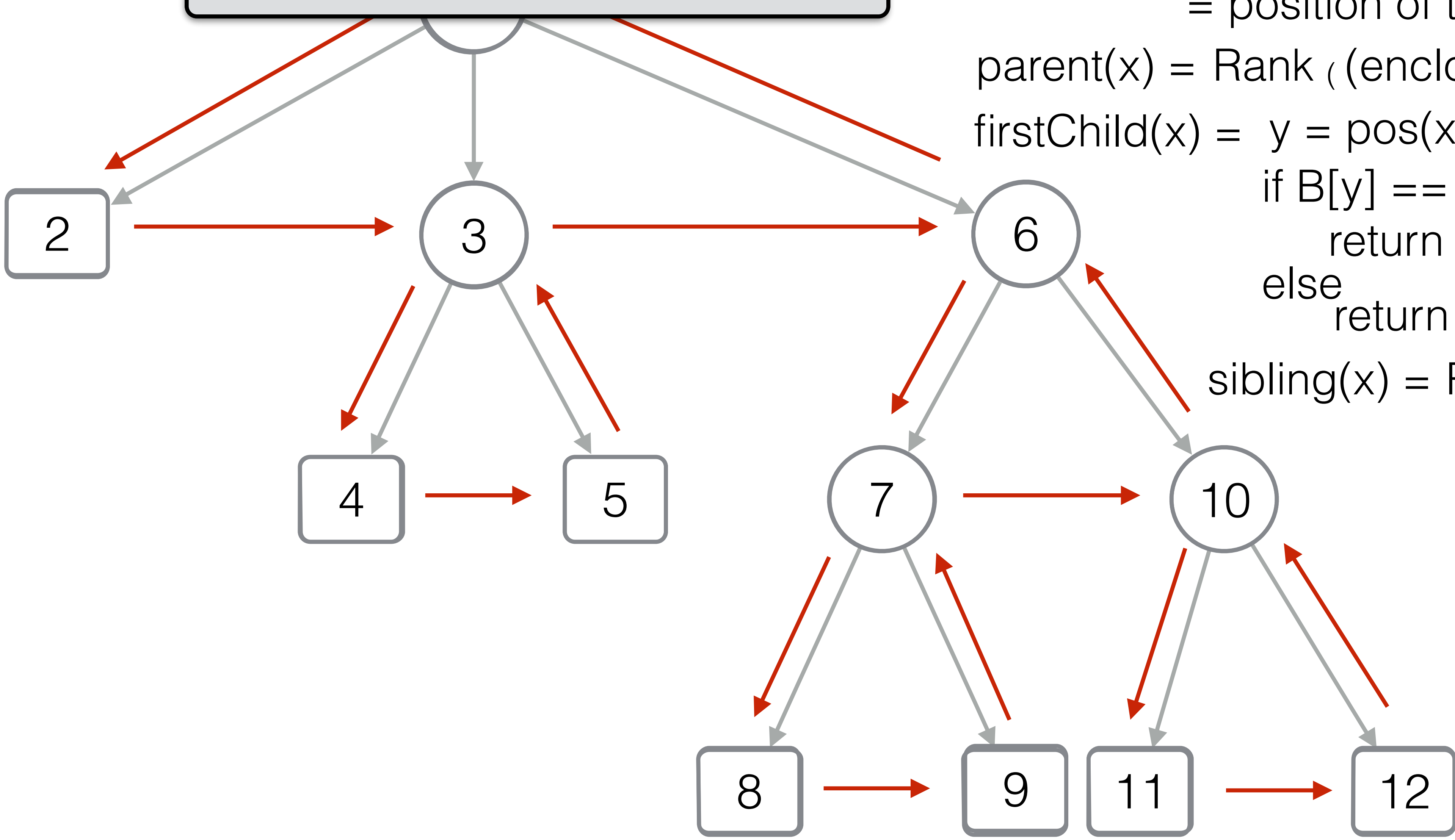
enclose(x) = returns the position of (enclosing x-th (

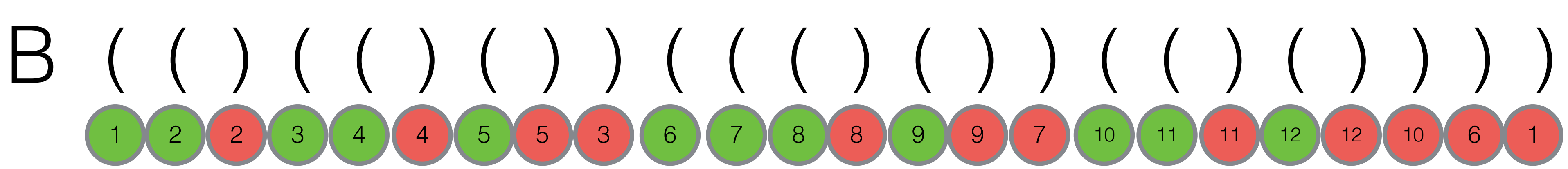
= position of the parent of x in B

parent(x) = Rank$_($(enclose(x))

firstChild(x) = y = pos(x)+1
if B[y] ==)
return -1 // is a leaf
else
return Rank$_($(y)

sibling(x) = Rank$_($(findClose(x)+1) (if any)





Succinct representation of trees (2)

[BP - Balanced parenthesis]

They can be implemented in O(1) time.

pos(x) = Select $_($ (x)

findClose(x) = returns the position of) matching x-th (

enclose(x) = returns the position of (enclosing x-th (

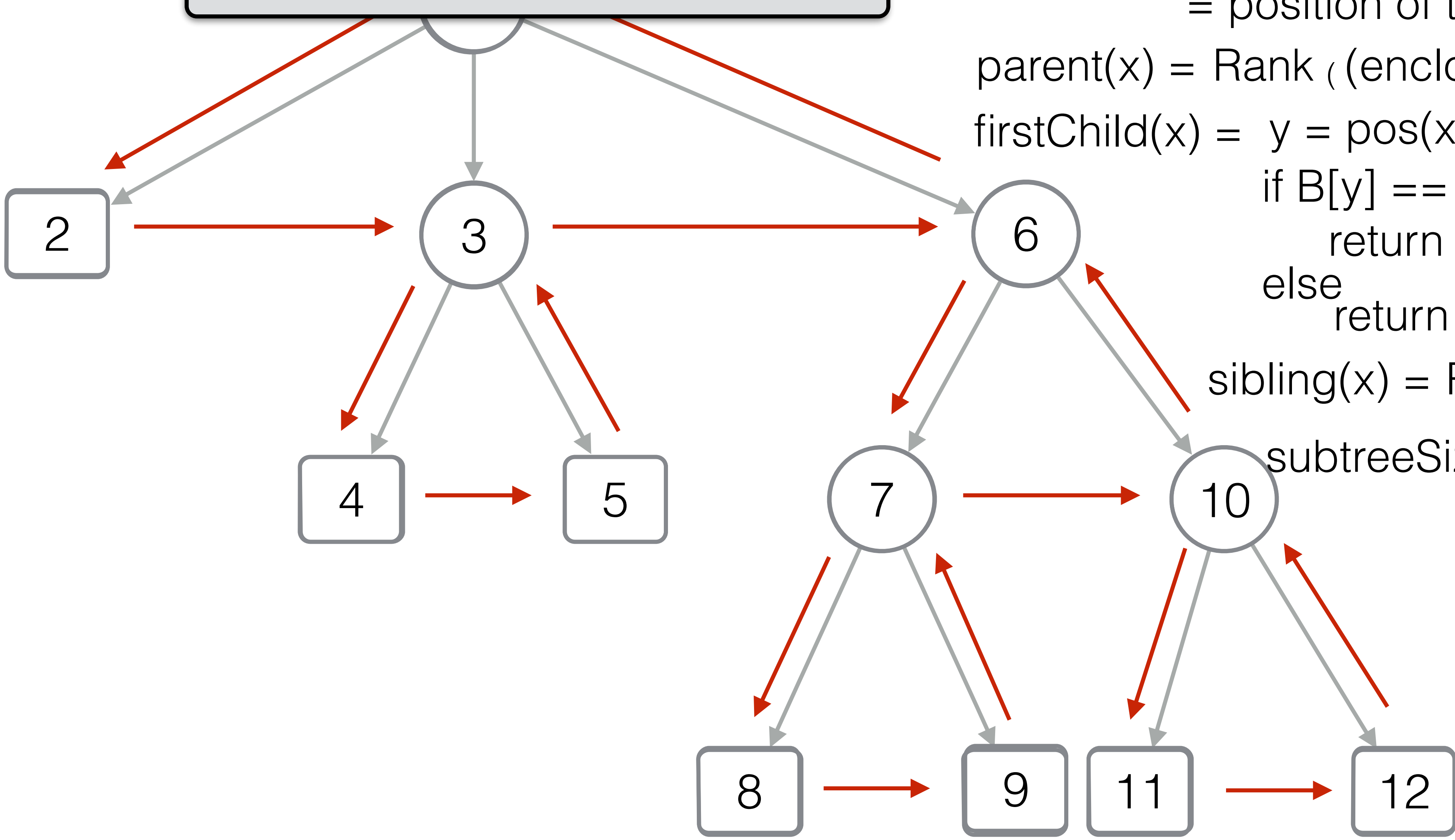
= position of the parent of x in B

parent(x) = Rank $_($ (enclose(x))

firstChild(x) = y = pos(x)+1
if B[y] ==)
return -1 // is a leaf
else
return Rank $_($ (y)

sibling(x) = Rank $_($ (findClose(x)+1) (if any)

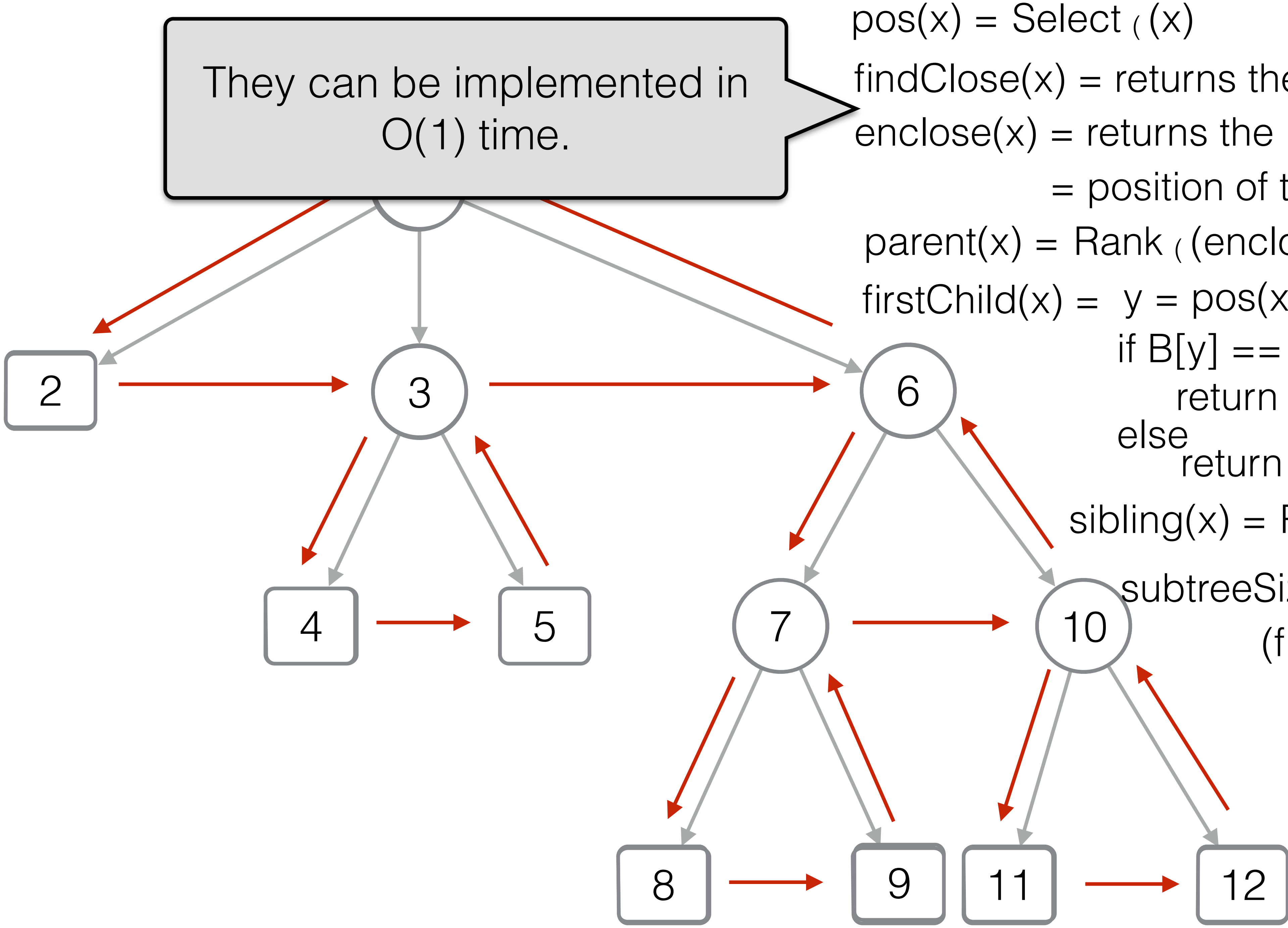
subtreeSize(x) =



B	(	(	)	(	(	)	(	)	)	(	(	(	)	(	)	)	(	(	)	(	)	)	)	)
	1	2	2	3	4	4	5	5	3	6	7	8	8	9	9	7	10	11	11	12	12	10	6	1

Succinct representation of trees (2)

[BP - Balanced parenthesis]



pos(x) = Select $_($ (x)

findClose(x) = returns the position of) matching x-th (

enclose(x) = returns the position of (enclosing x-th (

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parent(x) = Rank $_($ (enclose(x))

firstChild(x) = y = pos(x)+1

if B[y] ==)

return -1 // is a leaf

else

return Rank $_($ (y)

sibling(x) = Rank $_($ (findClose(x)+1) (if any)

subtreeSize(x) =

(findClose(x) - pos(x) + 1) / 2

B (( ) (( ) ( )) ((( ) ( )) (( ) ( ))))

1 2 2 3 4 4 5 5 3 6 7 8 8 9 9 7 10 11 11 12 12 10 6 1

Succinct representation of trees (2)

[BP - Balanced parenthesis]

> They can be implemented in O(1) time.

pos(x) = Select ((x)

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enclose(x) = returns the position of (enclosing x-th (

= position of the parent of x in B

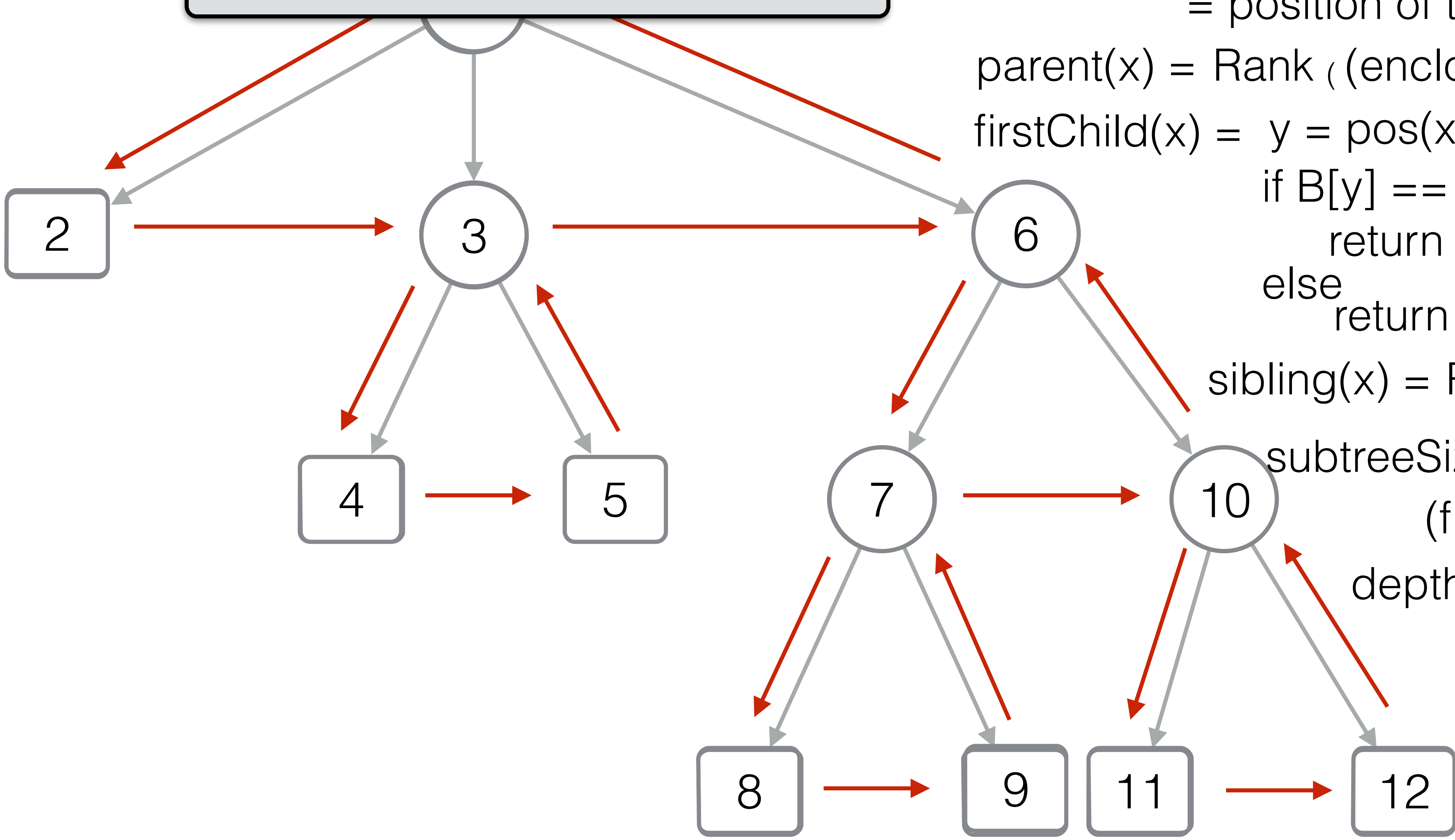
parent(x) = Rank ((enclose(x))

firstChild(x) = y = pos(x)+1
if B[y] ==)
return -1 // is a leaf
else
return Rank ((y)

sibling(x) = Rank ((findClose(x)+1) (if any)

subtreeSize(x) =
(findClose(x) - pos(x) + 1) / 2

depth(x) =



B (( ) (( ) ( )) ((( ) ( )) (( ) ( ))))

1 2 2 3 4 4 5 5 3 6 7 8 8 9 9 7 10 11 11 12 12 10 6 1

Succinct representation of trees (2)

[BP - Balanced parenthesis]

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findClose(x) = returns the position of) matching x-th (

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else

return Rank$_($ (y)

sibling(x) = Rank$_($ (findClose(x)+1) (if any)

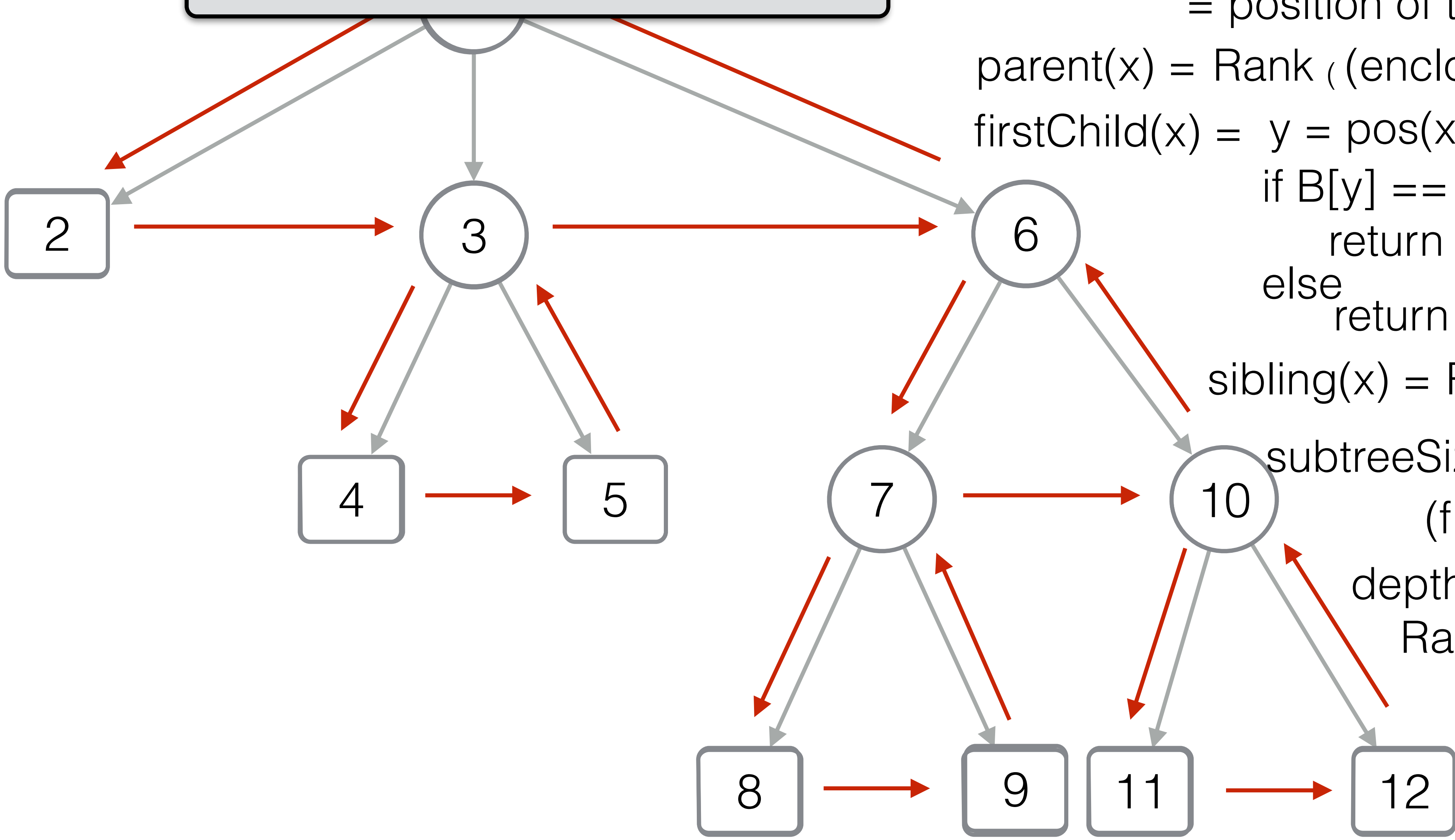
subtreeSize(x) =

(findClose(x) - pos(x) + 1) / 2

depth(x) =

Rank$_($ (pos(x)) - Rank$_)$ (pos(x))

They can be implemented in O(1) time.



B (( ) (( ) ( )) ((( ) ( )) (( ) ( ))))

1 2 2 3 4 4 5 5 3 6 7 8 8 9 9 7 10 11 11 12 12 10 6 1

Succinct representation of trees (2)

[BP - Balanced parenthesis]

They can be implemented in O(1) time.

$pos(x) = Select_{(}(x)$

findClose(x) = returns the position of) matching x-th (

enclose(x) = returns the position of (enclosing x-th (

= position of the parent of x in B

$parent(x) = Rank_{(}(enclose(x))$

firstChild(x) = y = pos(x)+1

if B[y] ==)

return -1 // is a leaf

else

$return\ Rank_{(}(y)$

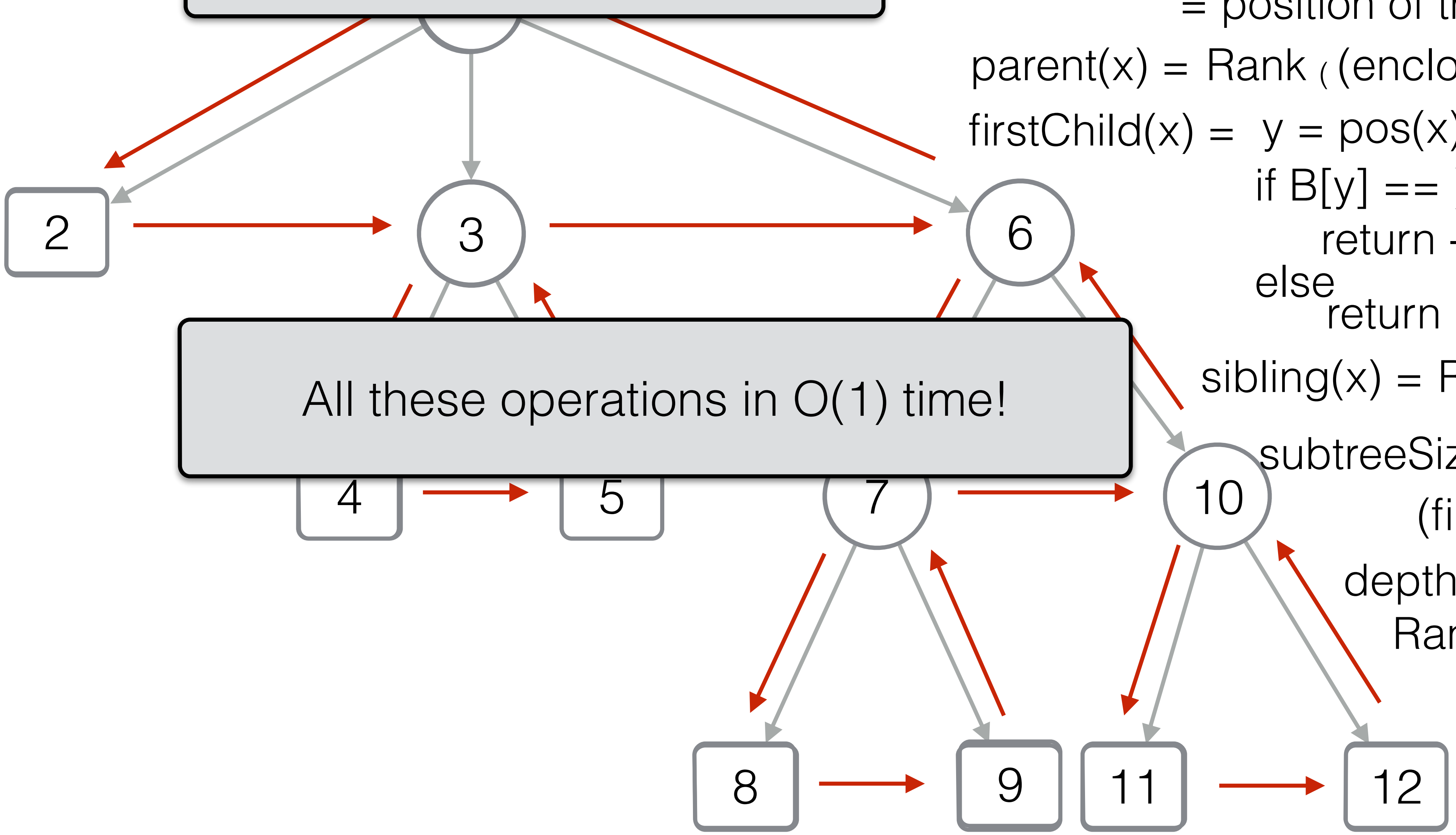
$sibling(x) = Rank_{(}(findClose(x)+1)$ (if any)

subtreeSize(x) =

(findClose(x) - pos(x) + 1) / 2

depth(x) =

$Rank_{(}(pos(x)) - Rank_{)}(pos(x))$



All these operations in O(1) time!

B (( ) (( ) ( )) ((( ) ( )) (( ) ( ))))

1 2 2 3 4 4 5 5 3 6 7 8 8 9 9 7 10 11 11 12 12 10 6 1

Succinct representation of trees (2)

[BP - Balanced parenthesis]

They can be implemented in O(1) time.

pos(x) = Select$_{(}$(x)

findClose(x) = returns the position of) matching x-th (

enclose(x) = returns the position of (enclosing x-th (

= position of the parent of x in B

parent(x) = Rank$_{(}$(enclose(x))

firstChild(x) = y = pos(x)+1

if B[y] ==)

return -1 // is a leaf

else

return Rank$_{(}$(y)

sibling(x) = Rank$_{(}$(findClose(x)+1) (if any)

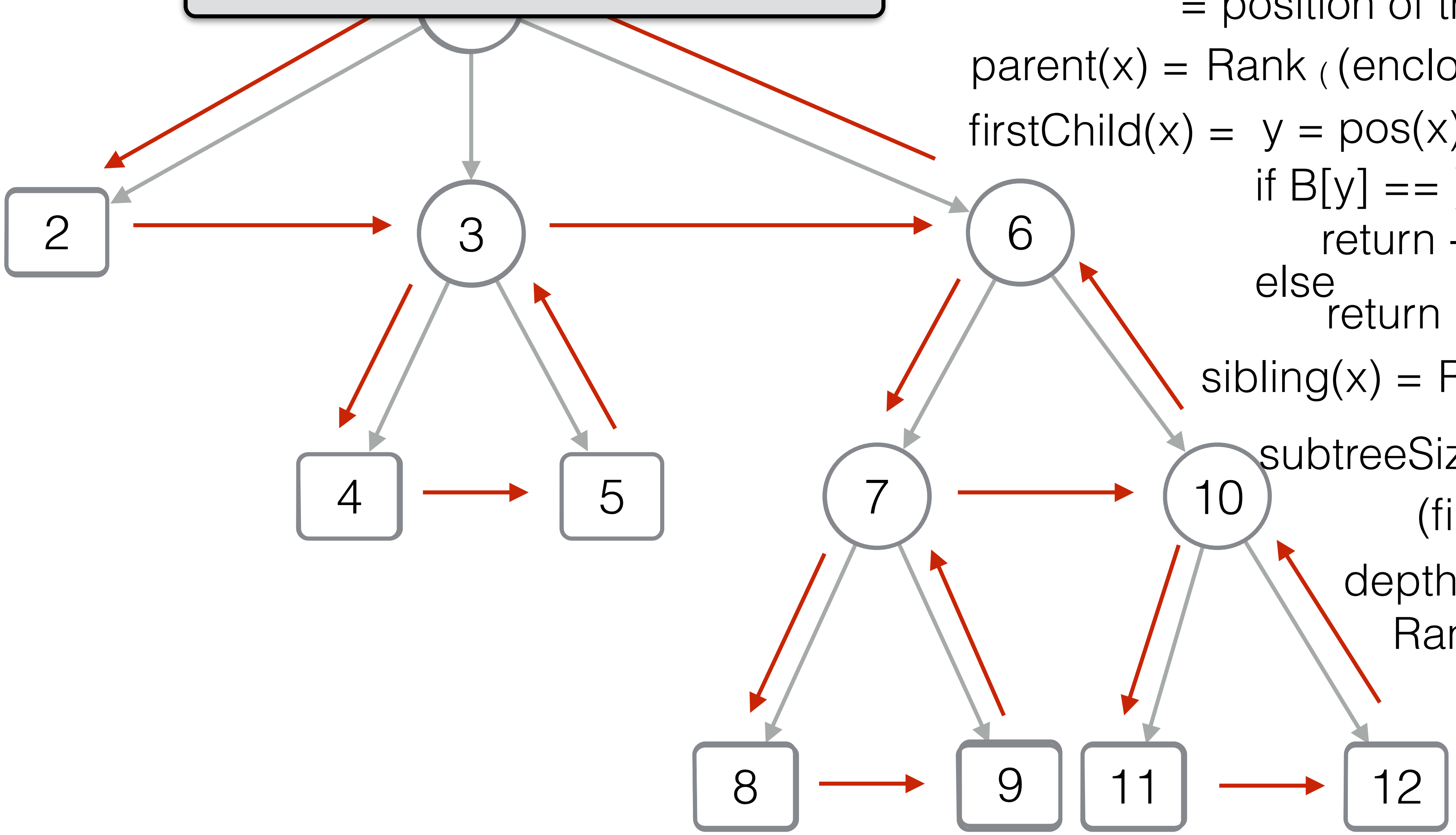
subtreeSize(x) =

(findClose(x) - pos(x) + 1) / 2

depth(x) =

Rank$_{(}$(pos(x)) - Rank$_{)}$(pos(x))

degree(x) = ?



B	(	(	)	(	(	)	(	)	)	(	(	(	)	(	)	)	(	(	)	(	)	)	)	)
	1	2	2	3	4	4	5	5	3	6	7	8	8	9	9	7	10	11	11	12	12	10	6	1

Succinct representation of trees (2)

[BP - Balanced parenthesis]

They can be implemented in O(1) time.

pos(x) = Select$_{(}$ (x)

findClose(x) = returns the position of) matching x-th (

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= position of the parent of x in B

parent(x) = Rank$_{(}$ (enclose(x))

firstChild(x) = y = pos(x)+1

if B[y] ==)

return -1 // is a leaf

else

return Rank$_{(}$ (y)

sibling(x) = Rank$_{(}$ (findClose(x)+1) (if any)

subtreeSize(x) =

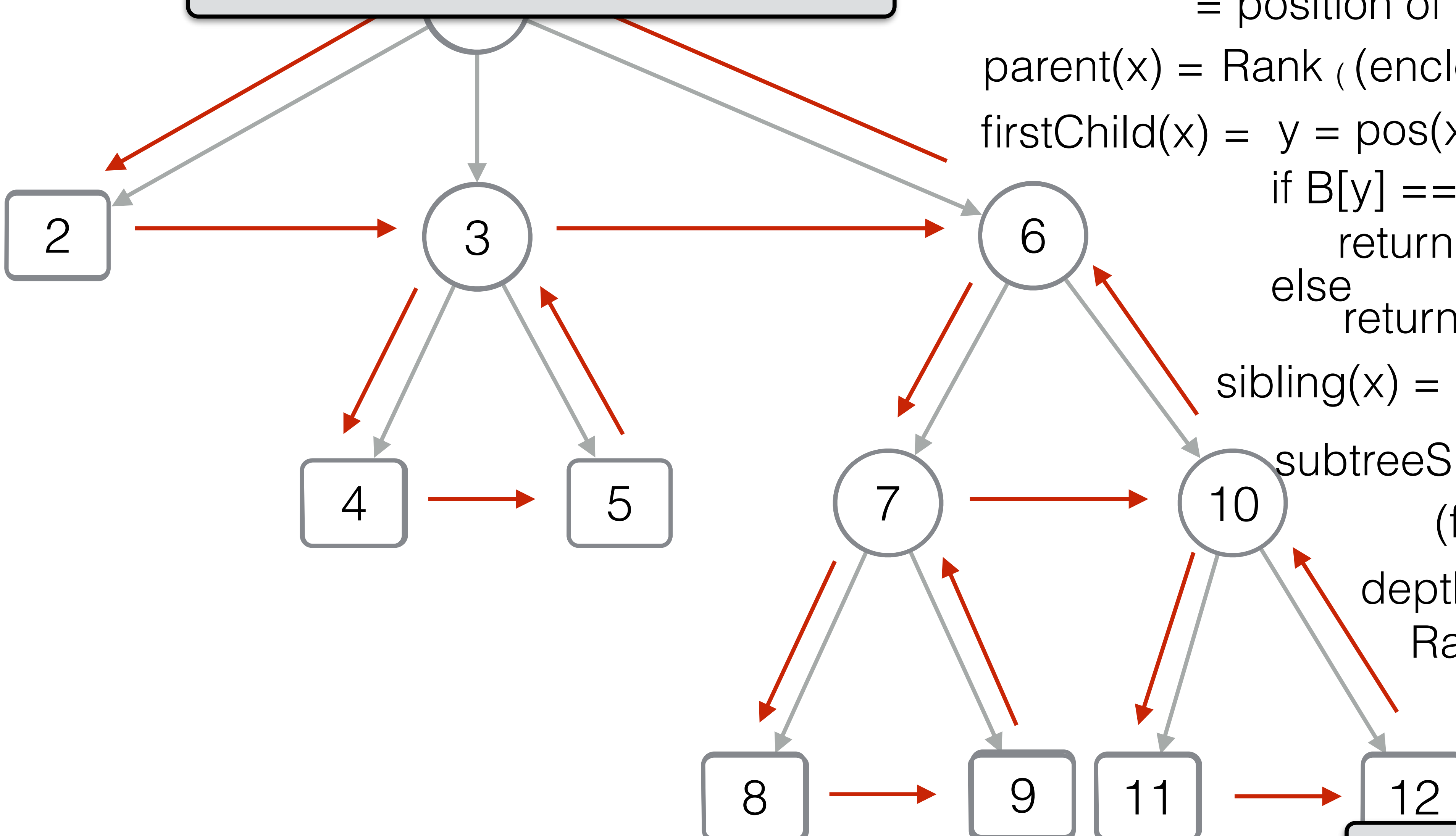
(findClose(x) - pos(x) + 1) / 2

depth(x) =

Rank$_{(}$ (pos(x)) - Rank$_{)}$ (pos(x))

degree(x) = ?

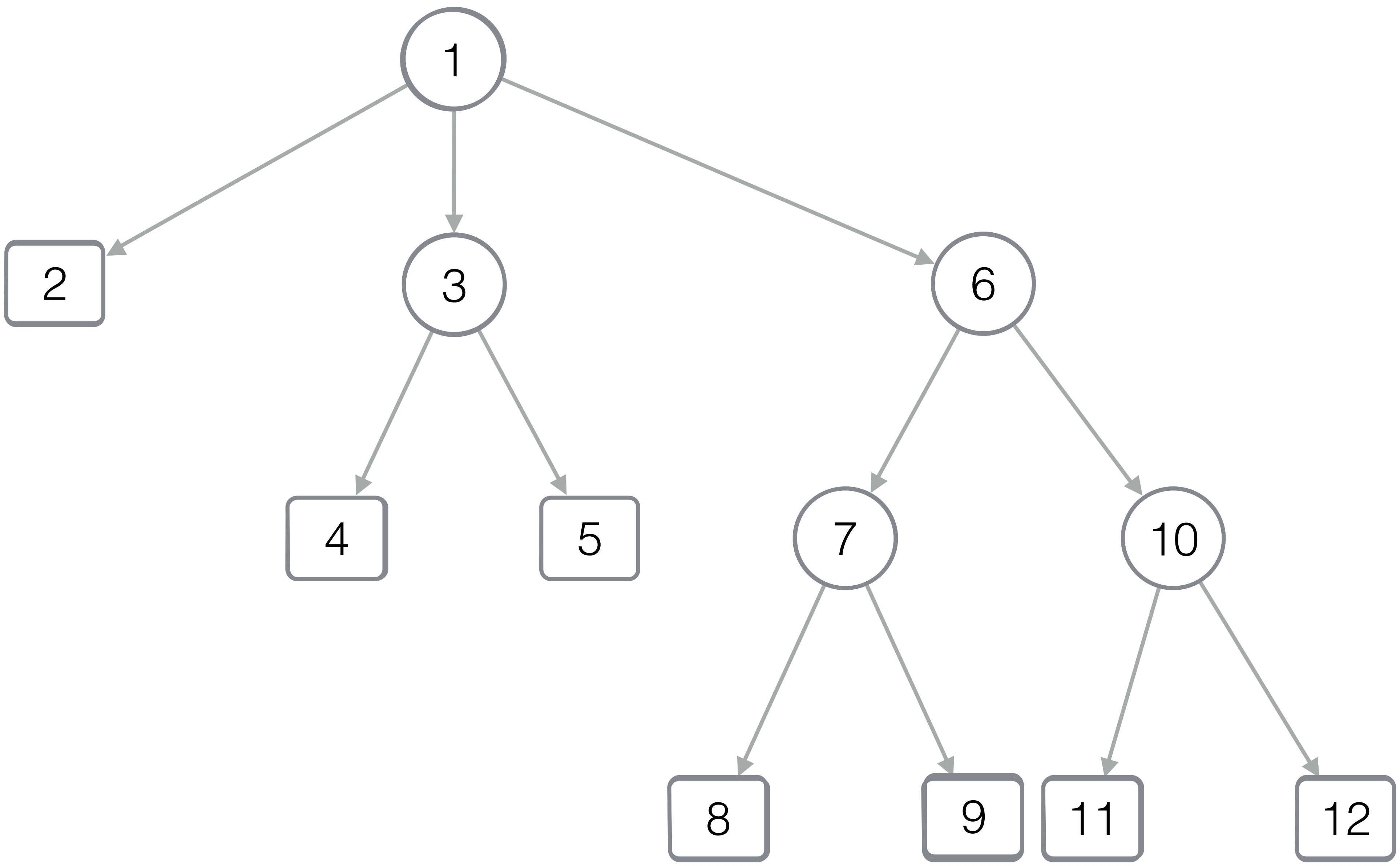
Quite inefficient!
Solved by repeatedly calling sibling to scan x's children.



B	(	(	)	(	(	)	(	)	)	(	(	(	)	(	)	)	(	(	)	(	)	)	)	)
	1	2	2	3	4	4	5	5	3	6	7	8	8	9	9	7	10	11	11	12	12	10	6	1

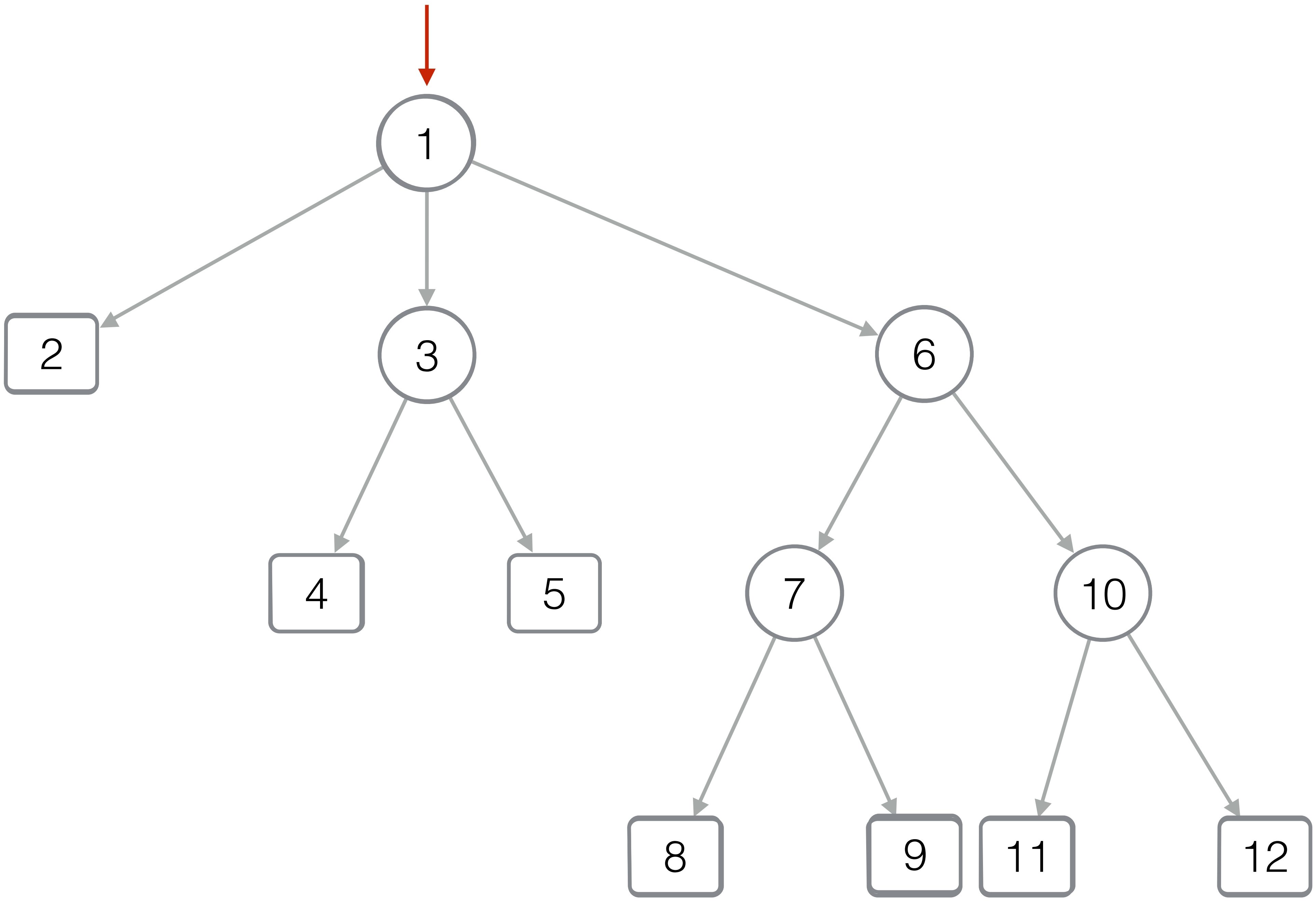
Succinct representation of trees (3)

[DFUDS - Depth First Unary Degree Sequence



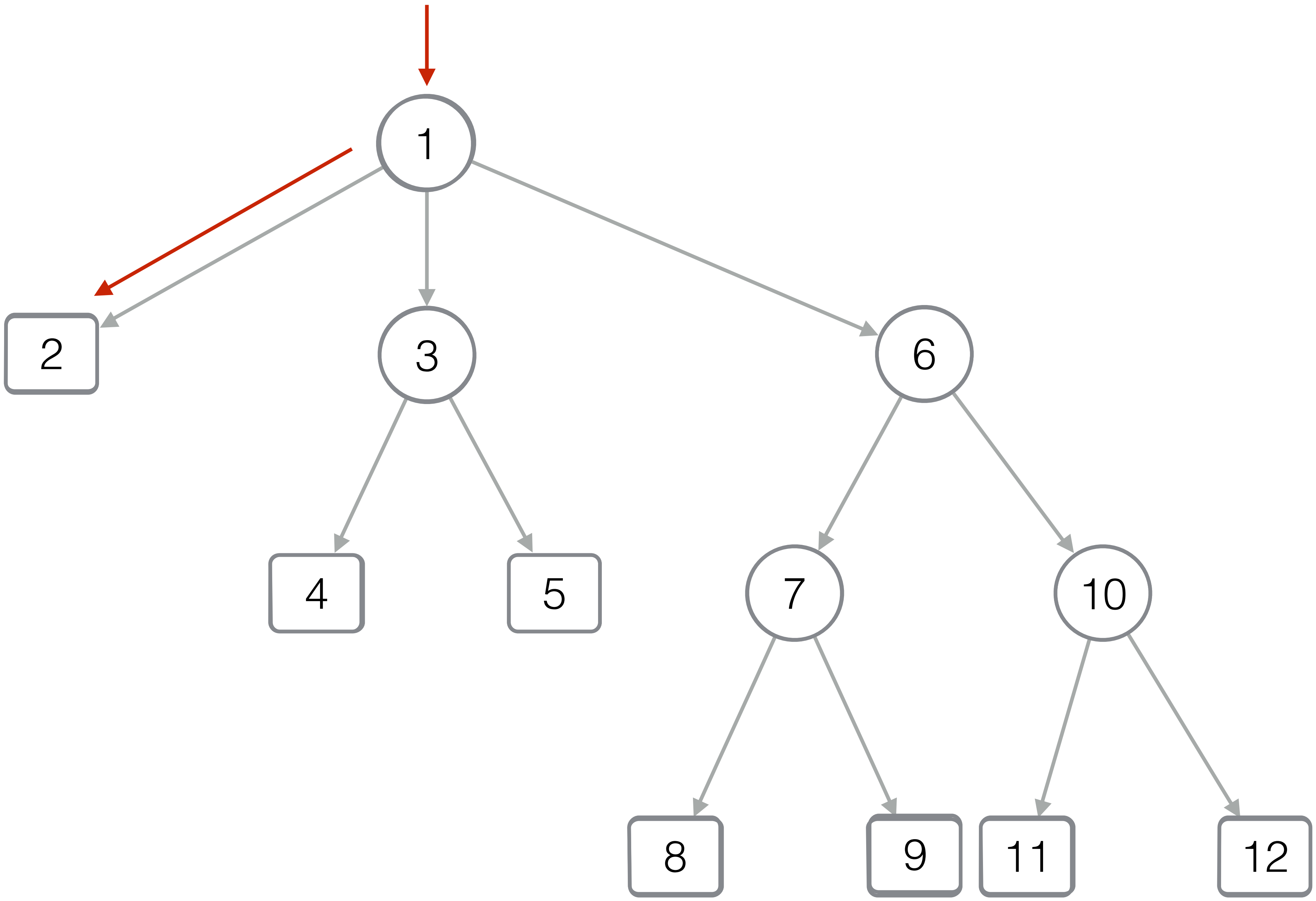
Succinct representation of trees (3)

[DFUDS - Depth First Unary Degree Sequence



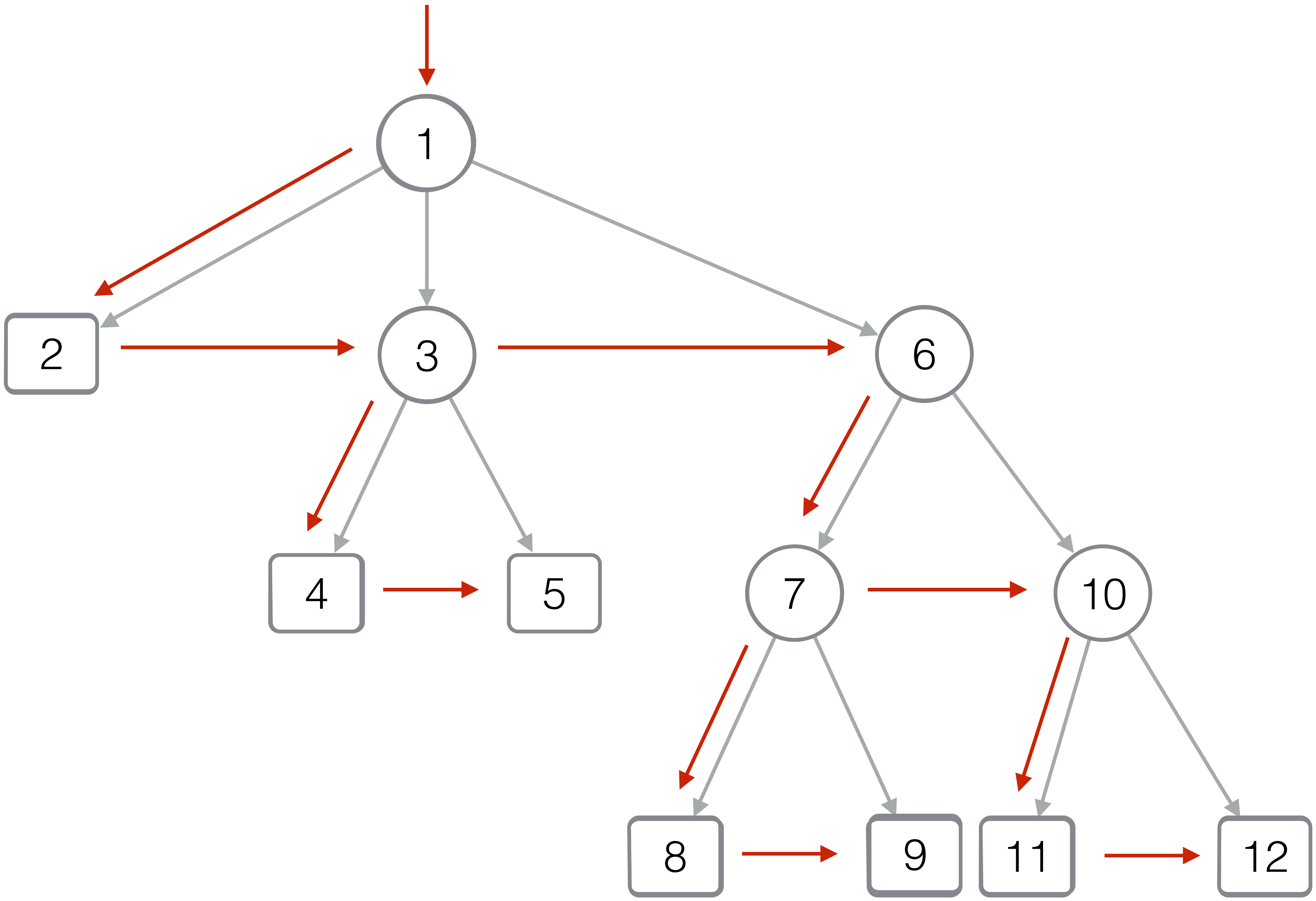
Succinct representation of trees (3)

[DFUDS - Depth First Unary Degree Sequence



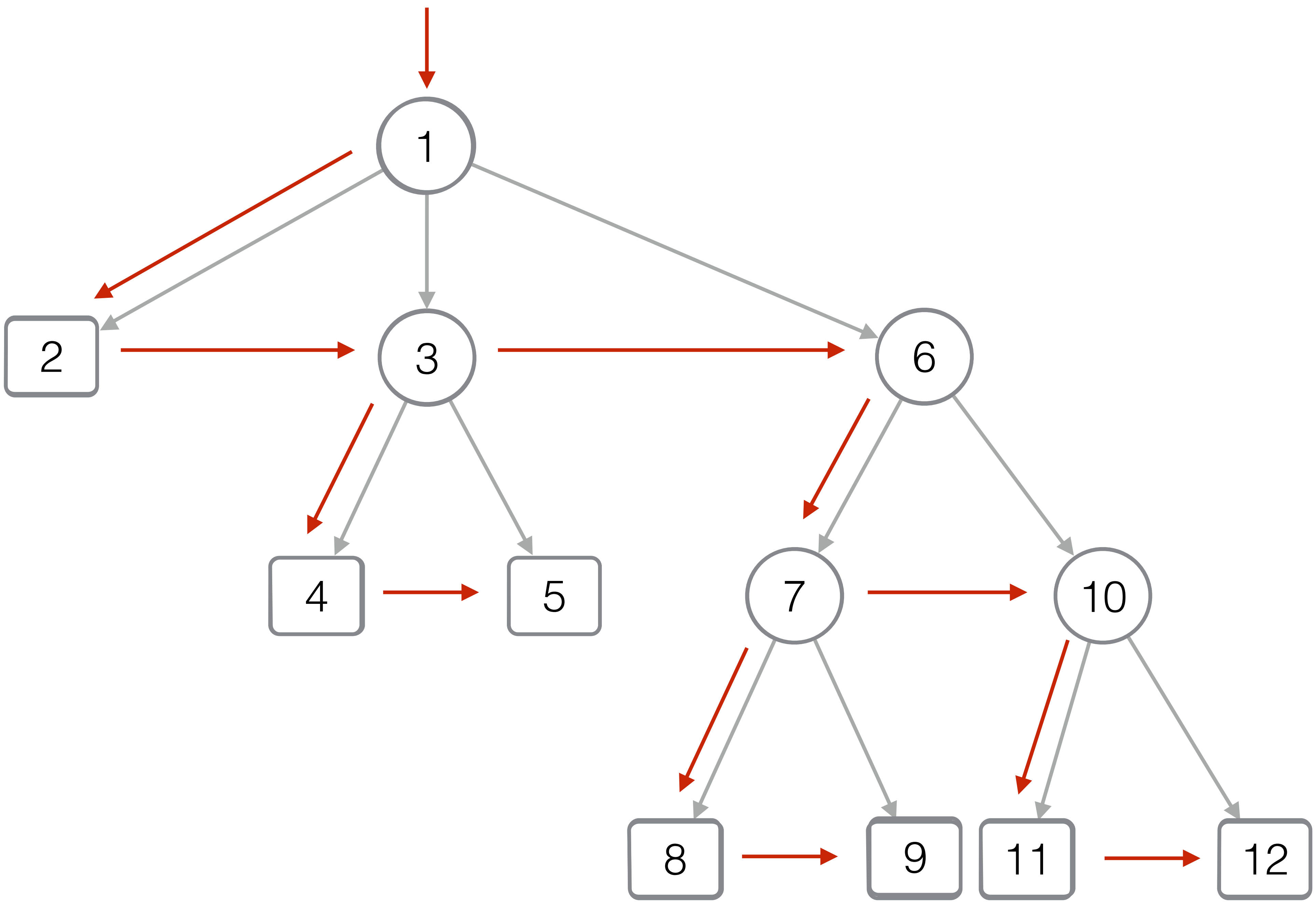
Succinct representation of trees (3)

[DFUDS - Depth First Unary Degree Sequence



Succinct representation of trees (3)

[DFUDS - Depth First Unary Degree Sequence

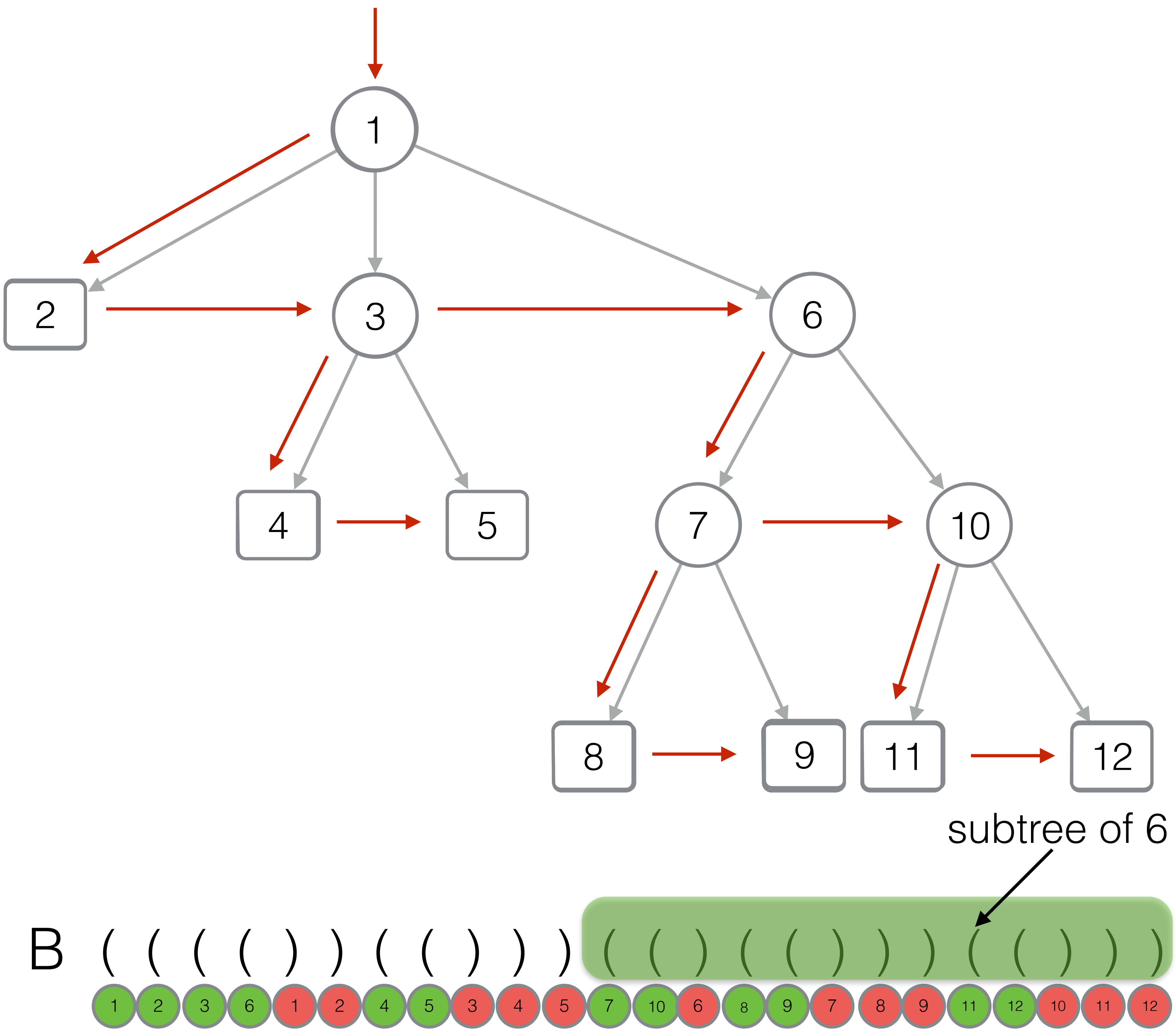


B (((( )) (( ))) (( ) (( ))) (( )))

1 2 3 6 1 2 4 5 3 4 5 7 10 6 8 9 7 8 9 11 12 10 11 12

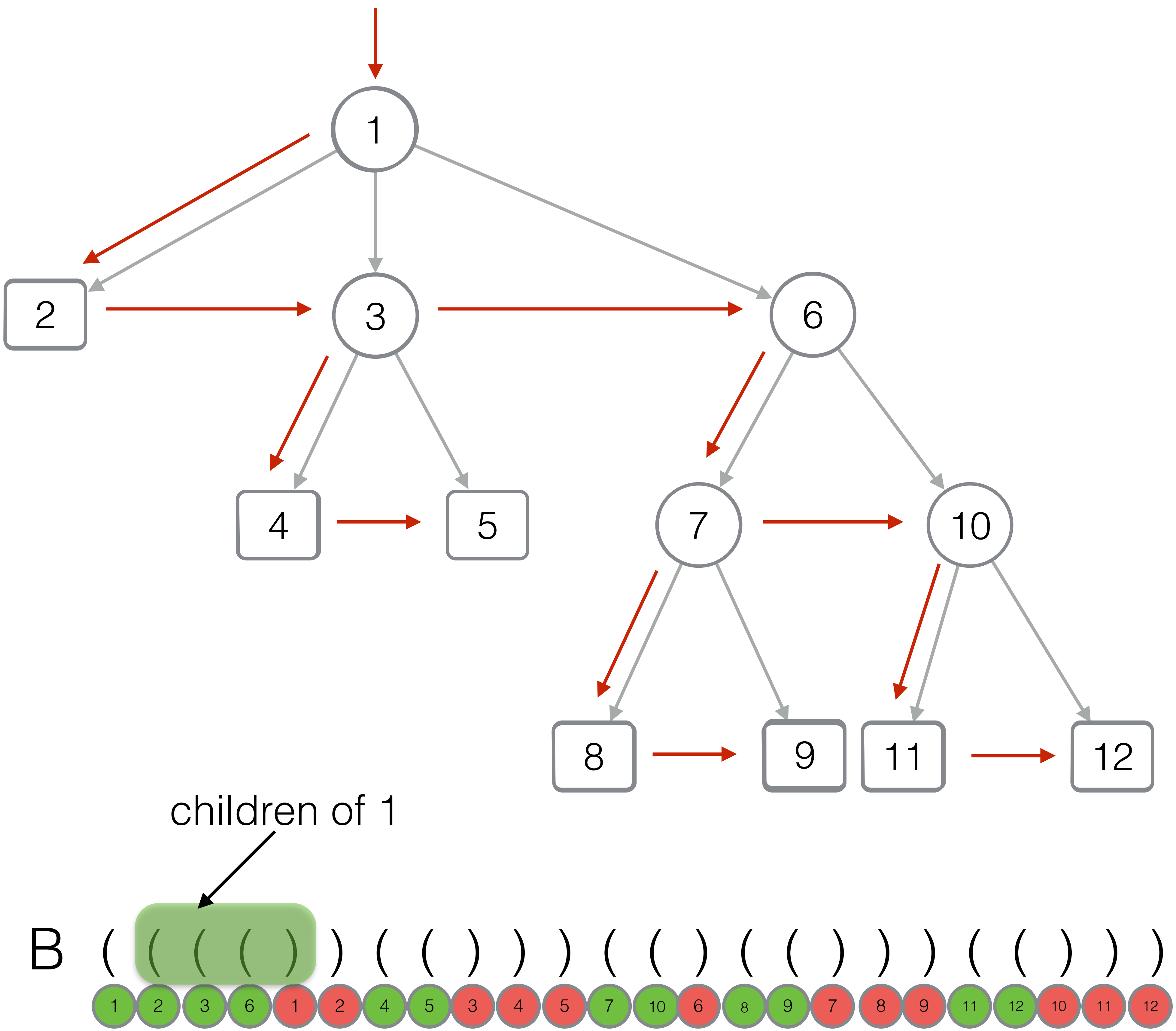
Succinct representation of trees (3)

[DFUDS - Depth First Unary Degree Sequence



Succinct representation of trees (3)

[DFUDS - Depth First Unary Degree Sequence



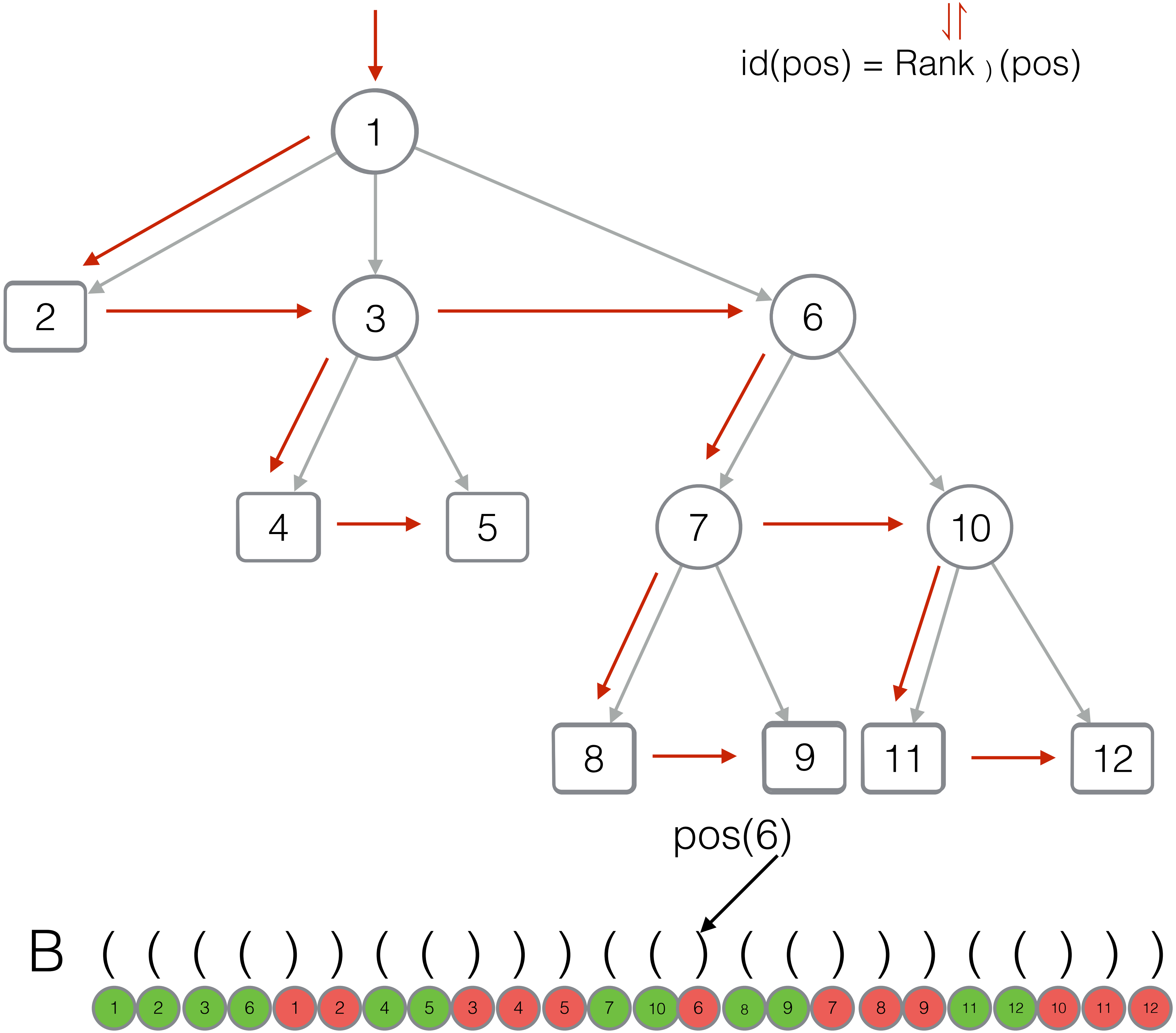
Succinct representation of trees (3)

[DFUDS - Depth First Unary Degree Sequence

$pos(x) = Select_{)}(x)$ // closing)

⇃↾

$id(pos) = Rank_{)}(pos)$



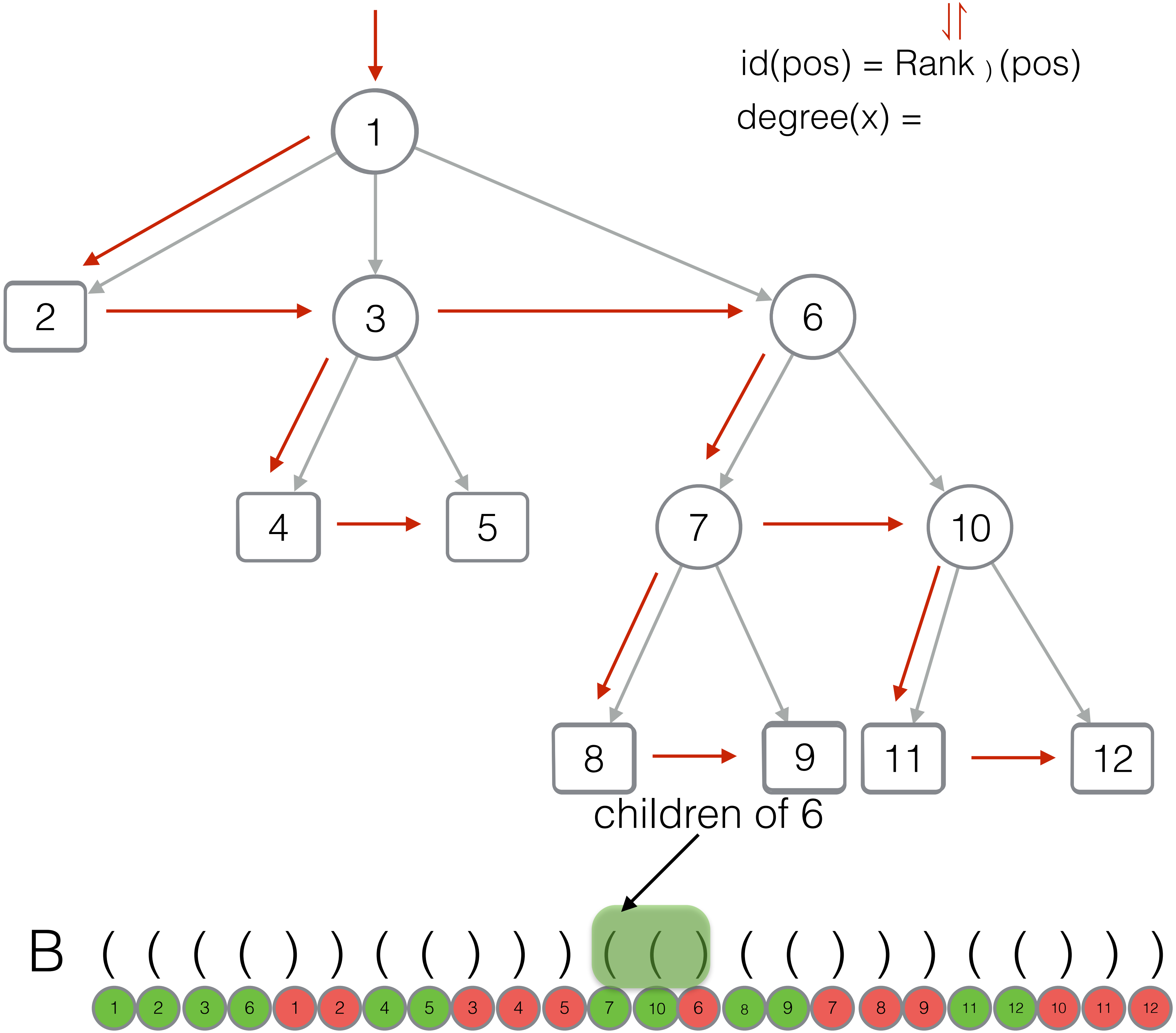
Succinct representation of trees (3)

[DFUDS - Depth First Unary Degree Sequence

pos(x) = $Select_{)}(x)$ // closing)

id(pos) = $Rank_{)}(pos)$

degree(x) =



Succinct representation of trees (3)

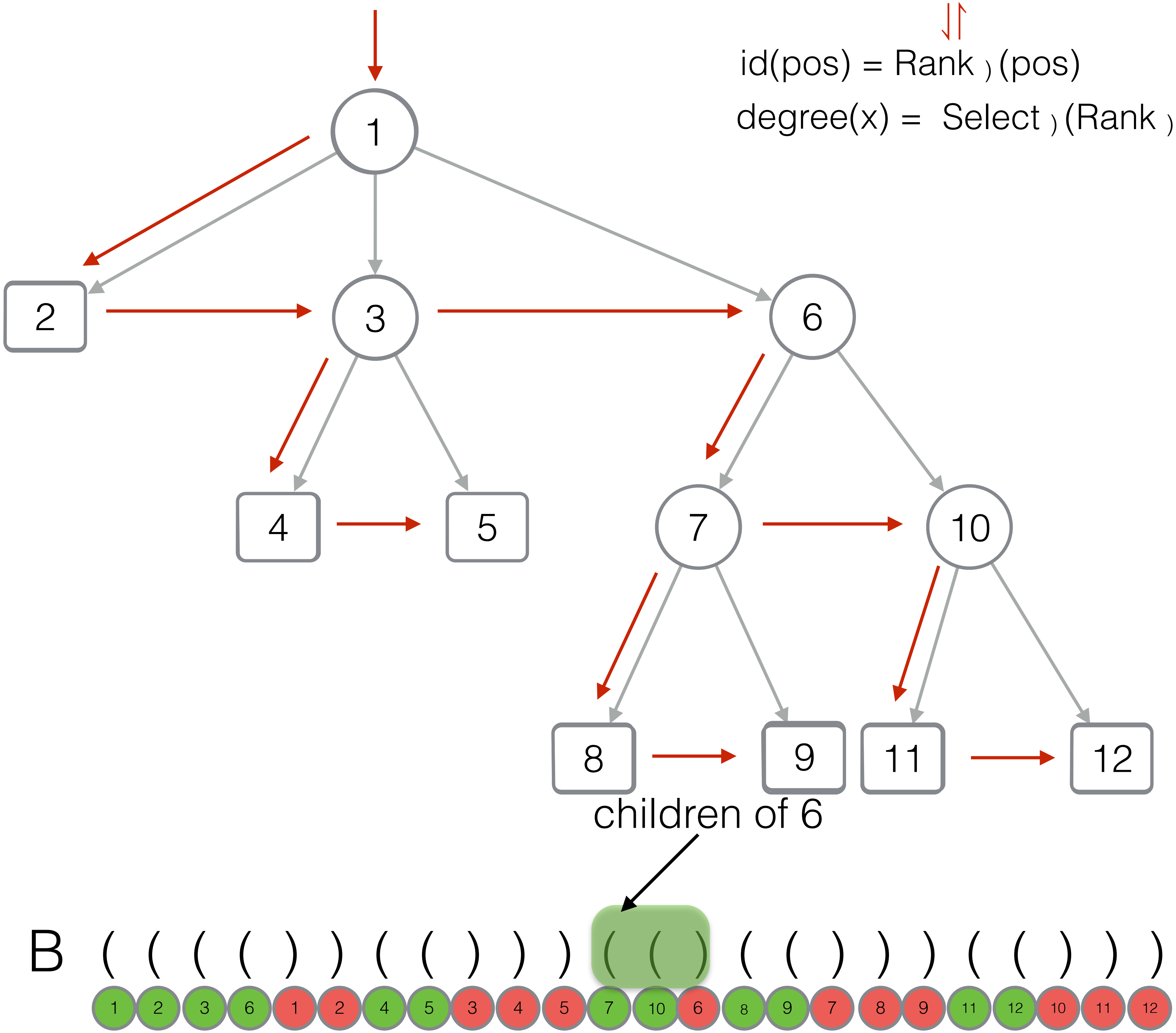
[DFUDS - Depth First Unary Degree Sequence

pos(x) = $Select_{)}(x)$ // closing)

⇃↾

id(pos) = $Rank_{)}(pos)$

degree(x) = $Select_{)}(Rank_{)}(pos(x))) - x$



Succinct representation of trees (3)

[DFUDS - Depth First Unary Degree Sequence

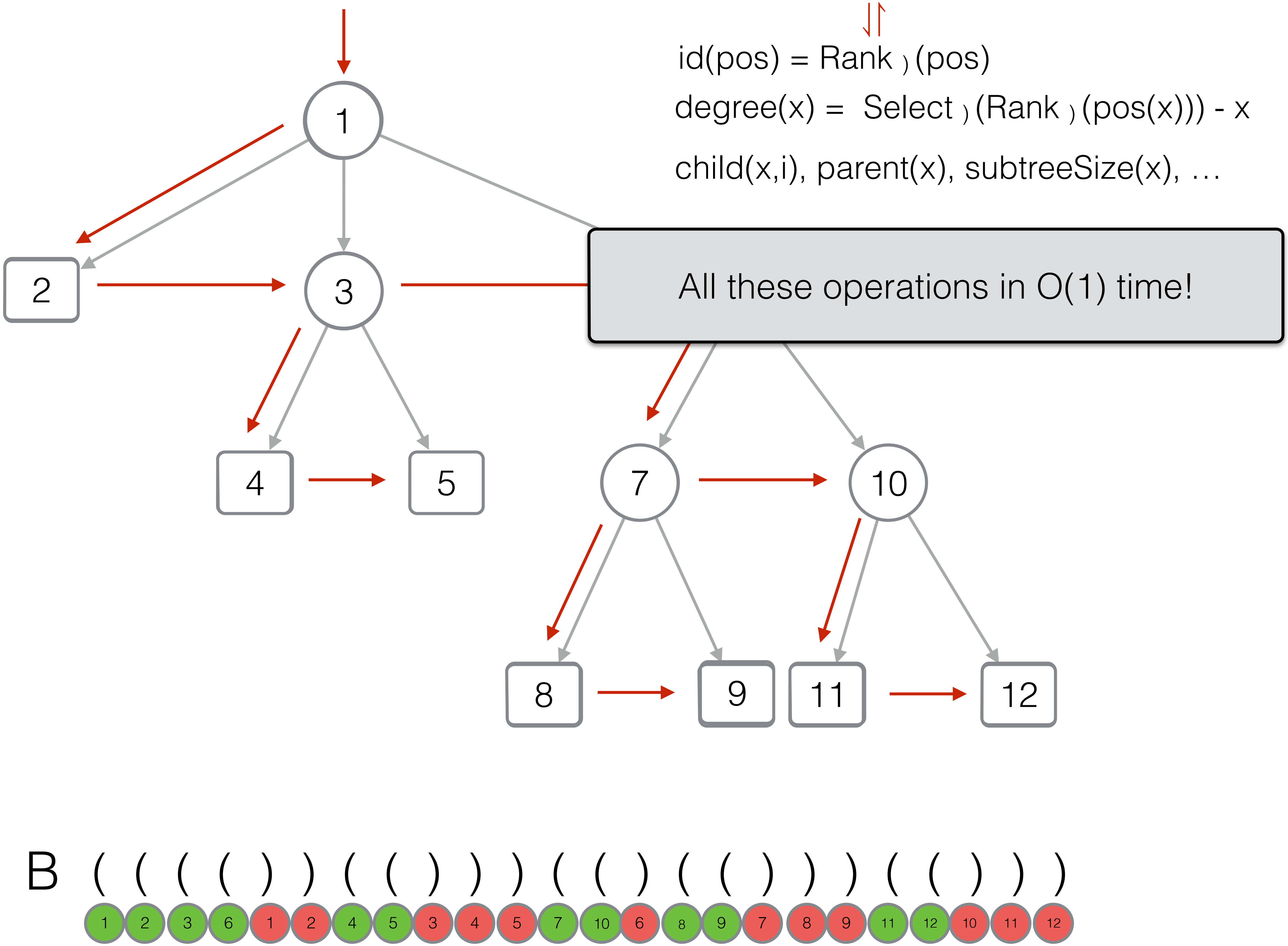
$pos(x) = \mathrm{Select}_{)}(x)$ // closing)

⇃↾

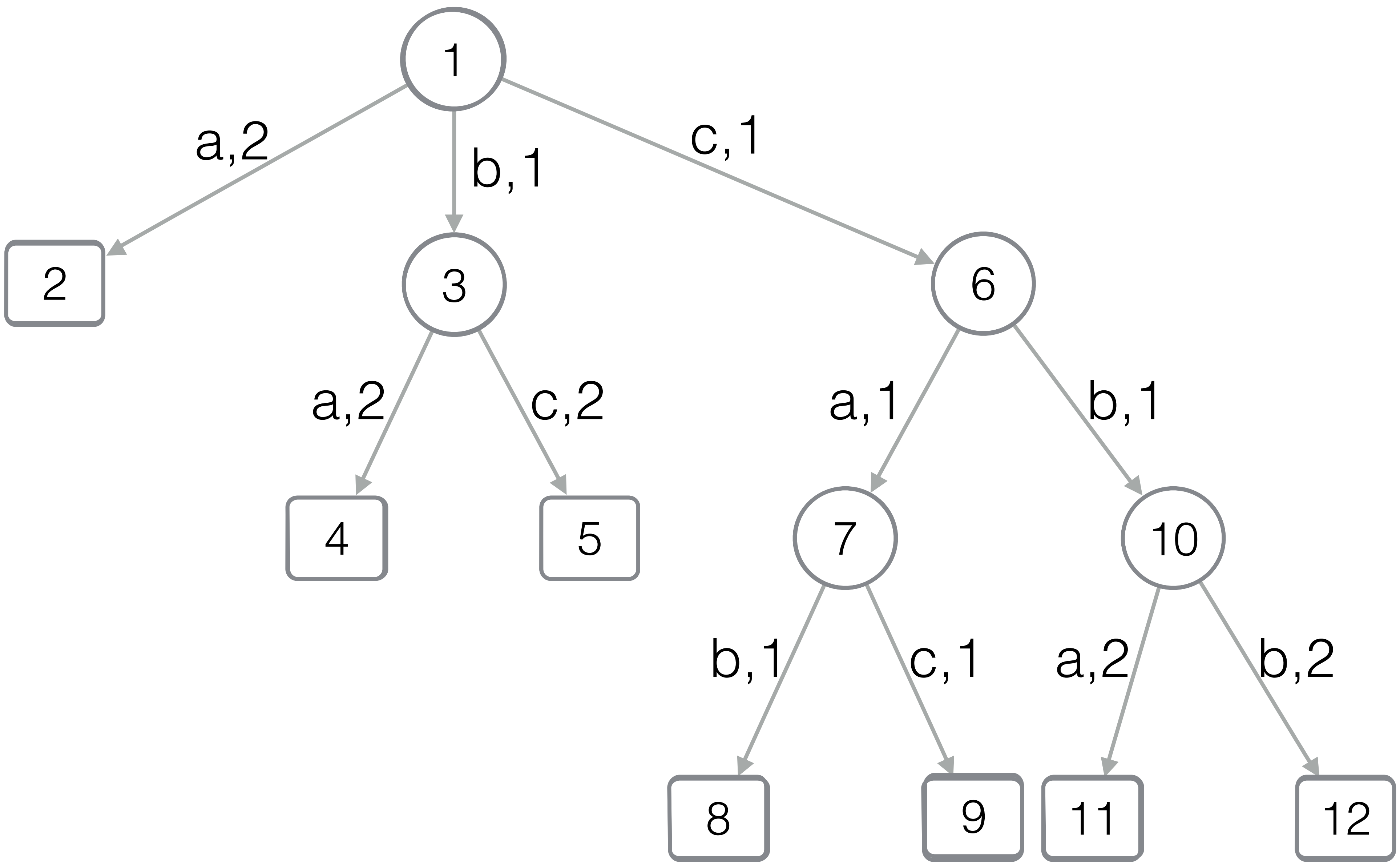
$id(pos) = \mathrm{Rank}_{)}(pos)$

$degree(x) = \mathrm{Select}_{)}(\mathrm{Rank}_{)}(pos(x))) - x$

child(x,i), parent(x), subtreeSize(x), …



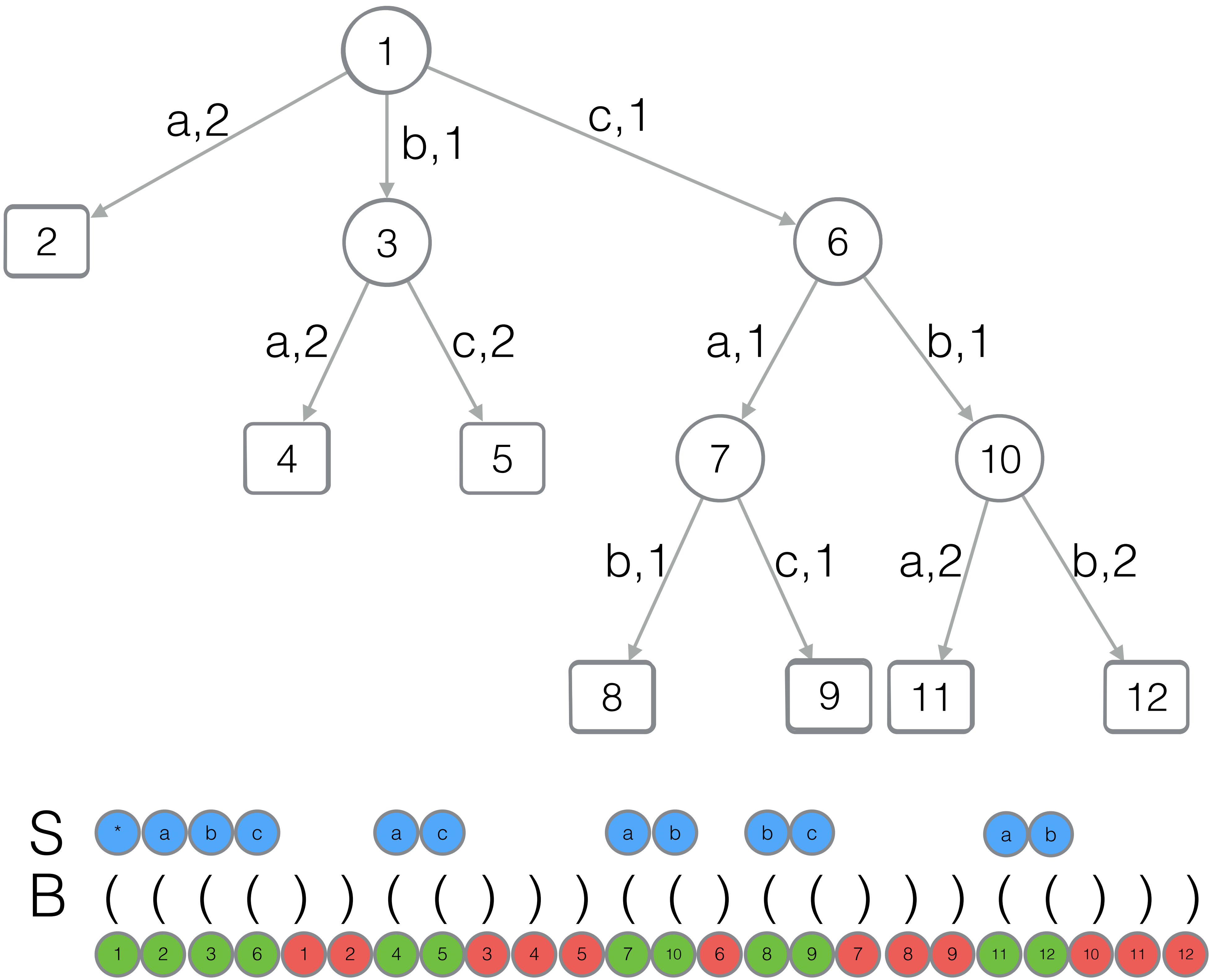
Patricia trie with DFUDS



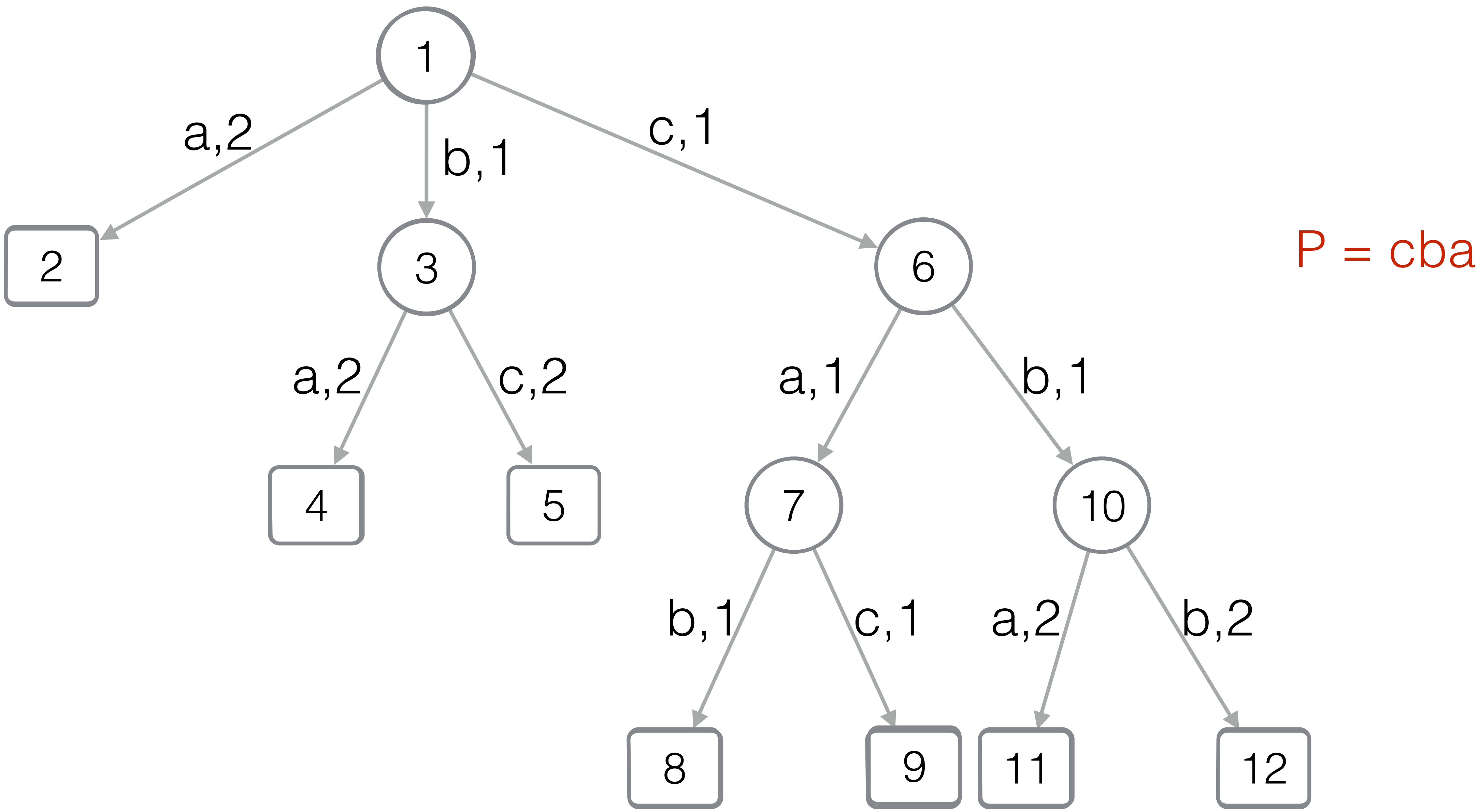
B (((( )) (( ))) (( ) (( ))) (( )))

1 2 3 6 1 2 4 5 3 4 5 7 10 6 8 9 7 8 9 11 12 10 11 12

Patricia trie with DFUDS



Patricia trie with DFUDS

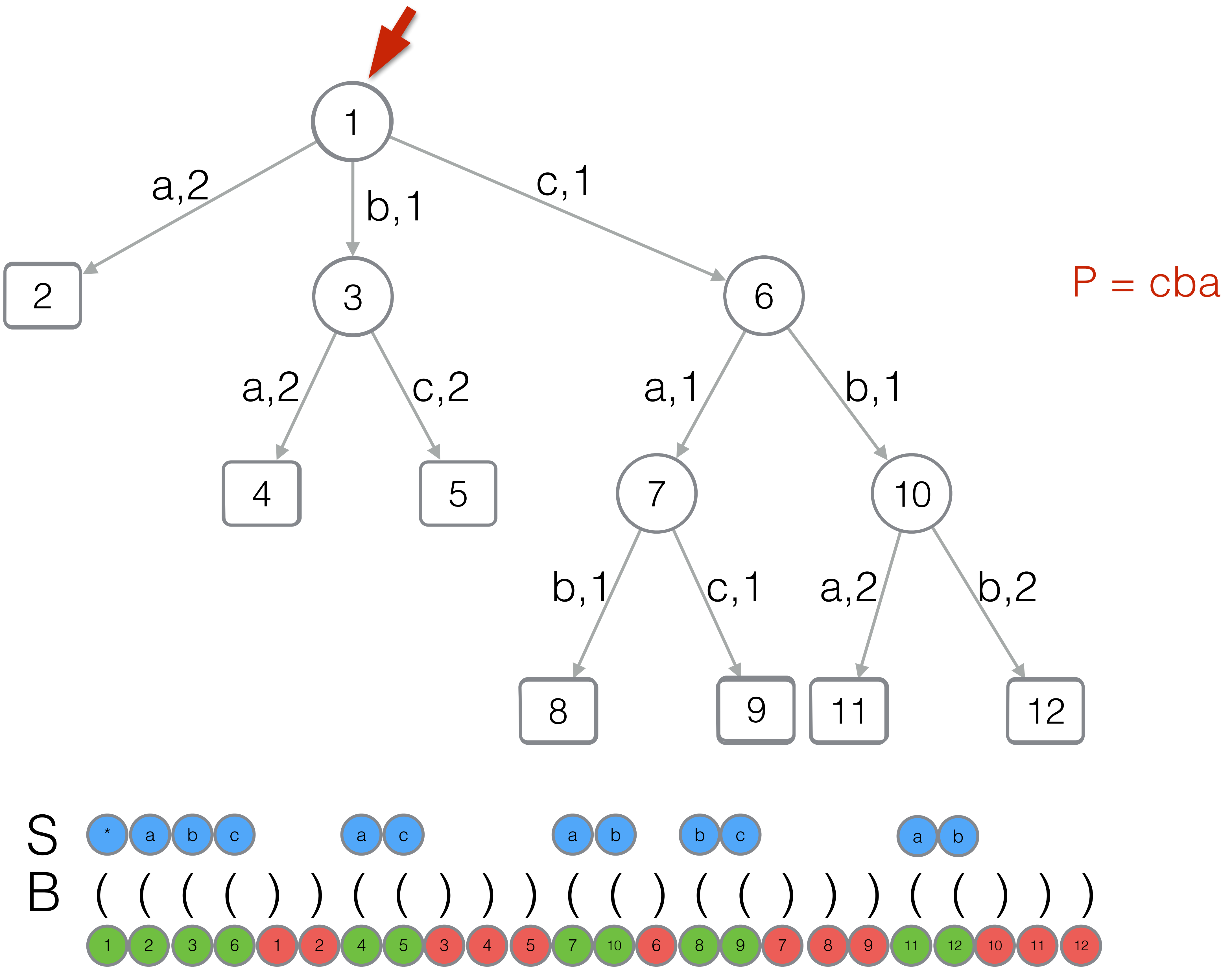


S: * a b c   a c   a b   b c   a b

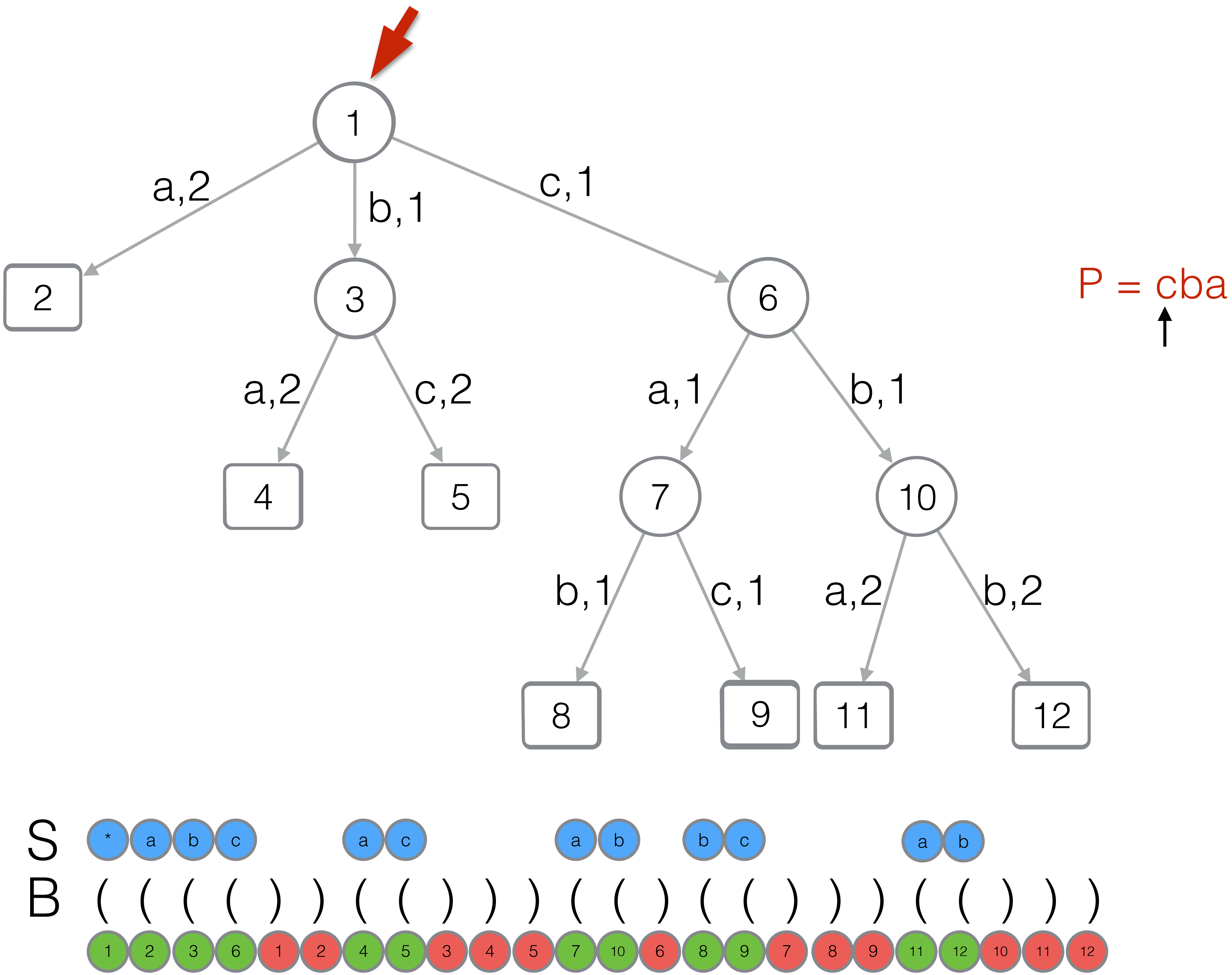
B: (((( )) (( ))) (( ) (( ))) (( )))

1 2 3 6 1 2 4 5 3 4 5 7 10 6 8 9 7 8 9 11 12 10 11 12

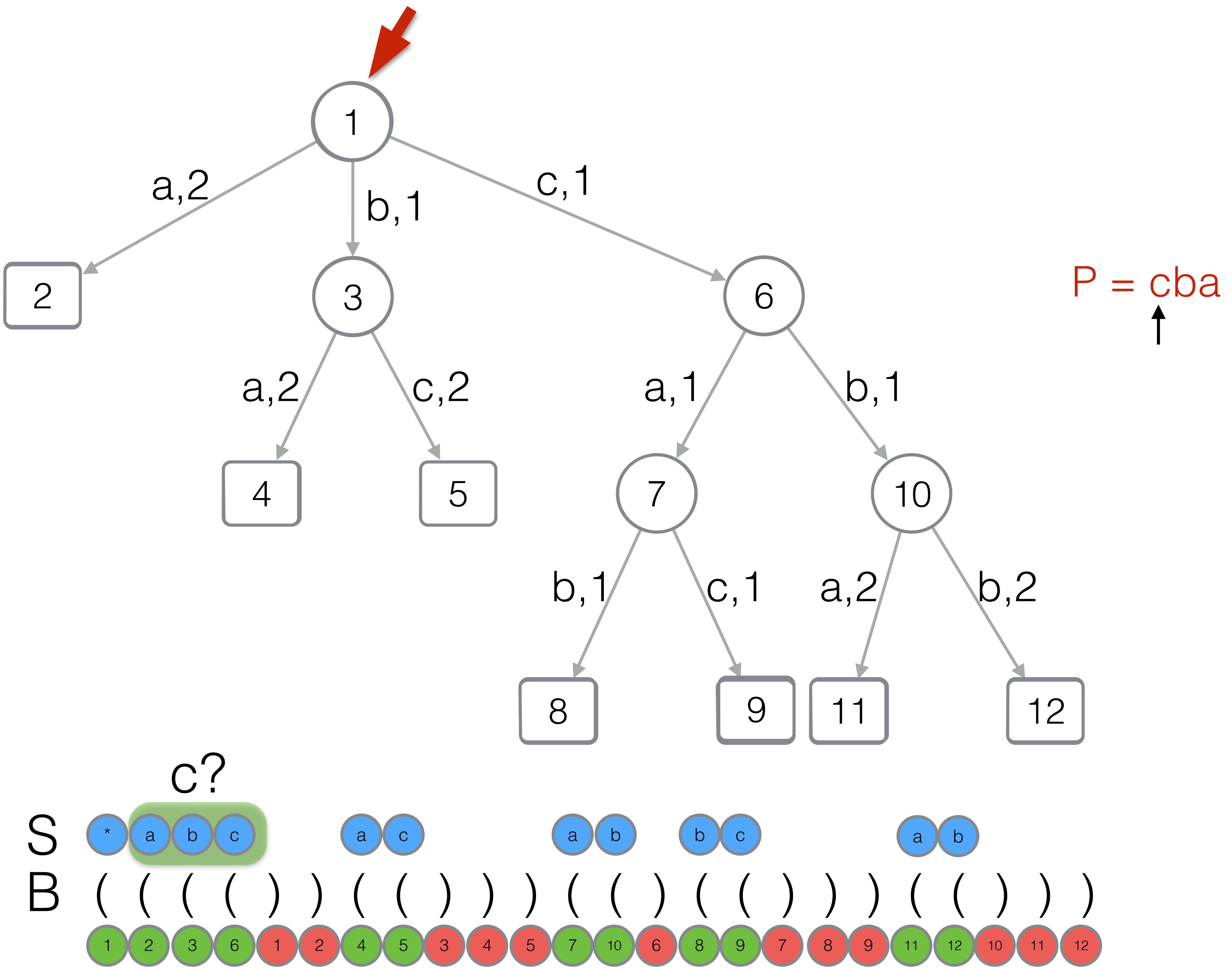
Patricia trie with DFUDS



Patricia trie with DFUDS



Patricia trie with DFUDS



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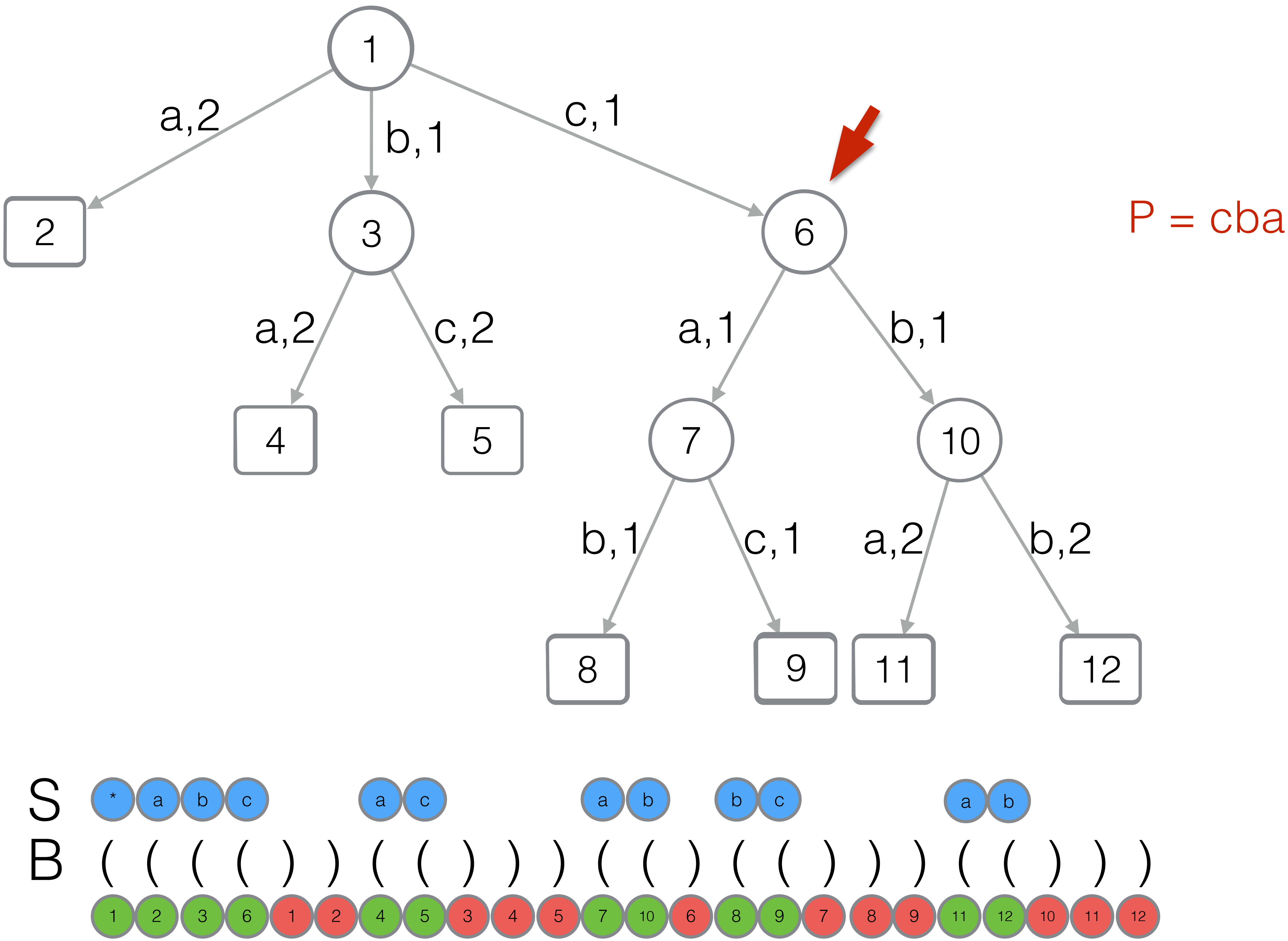

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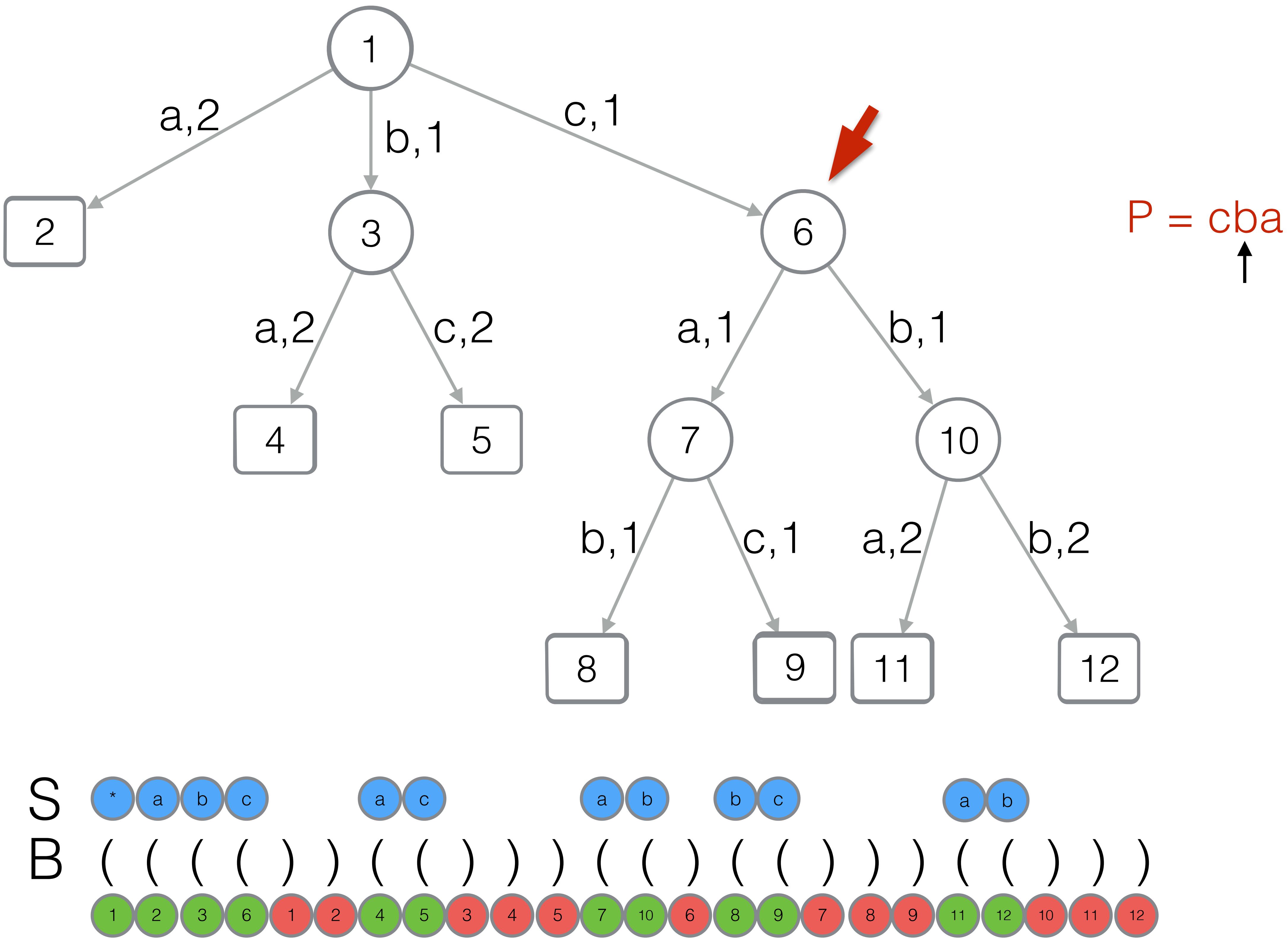

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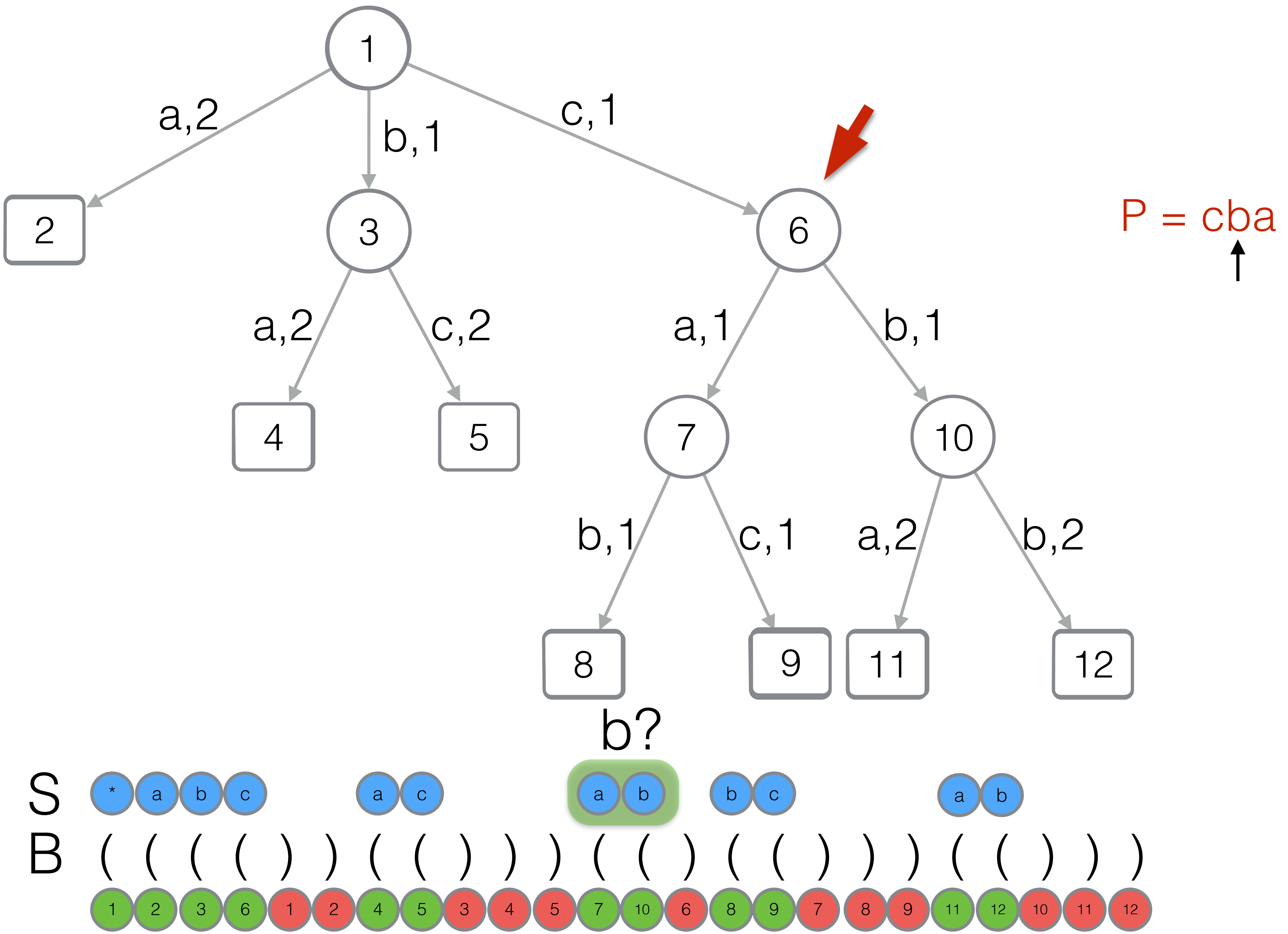

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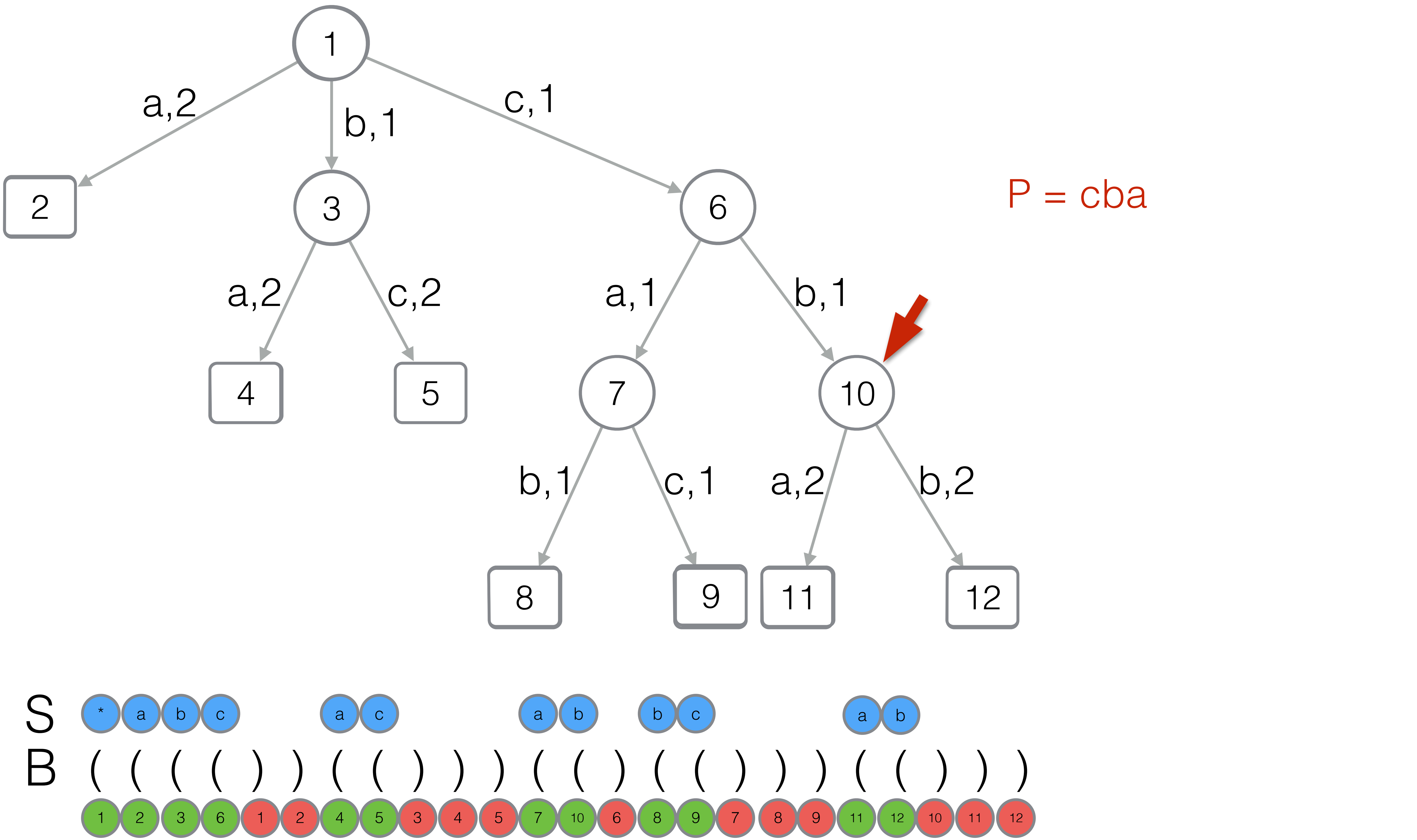

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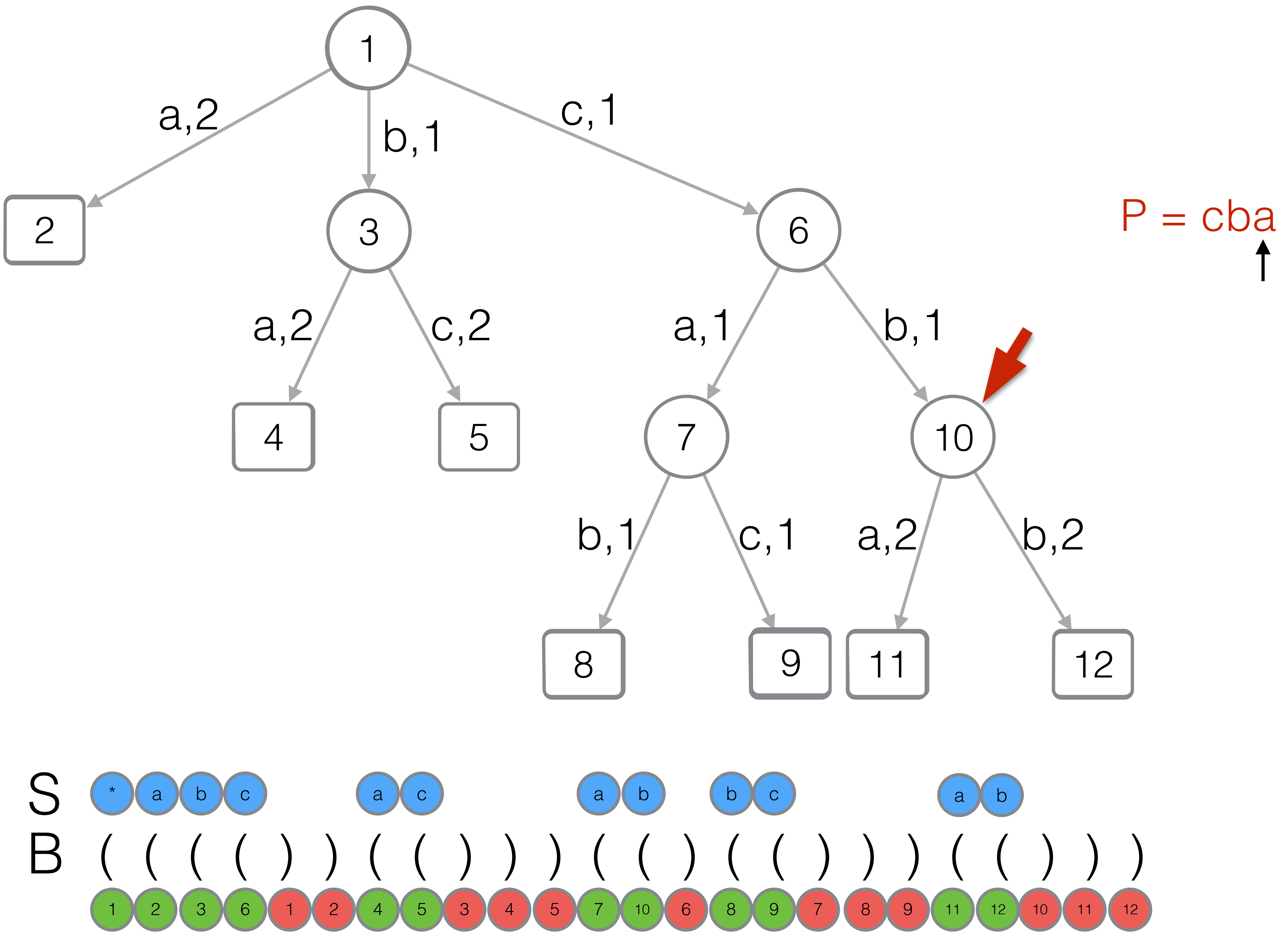

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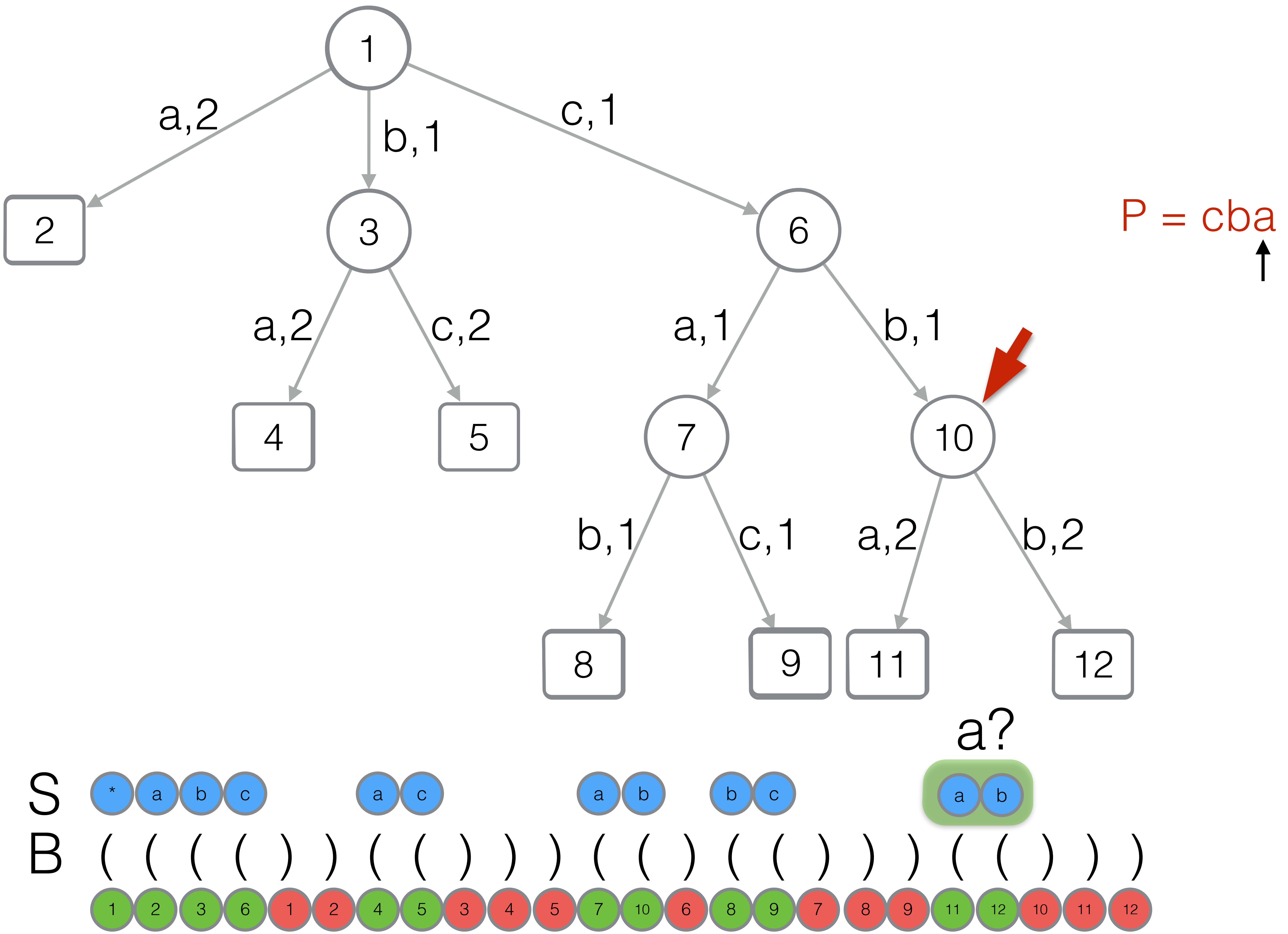

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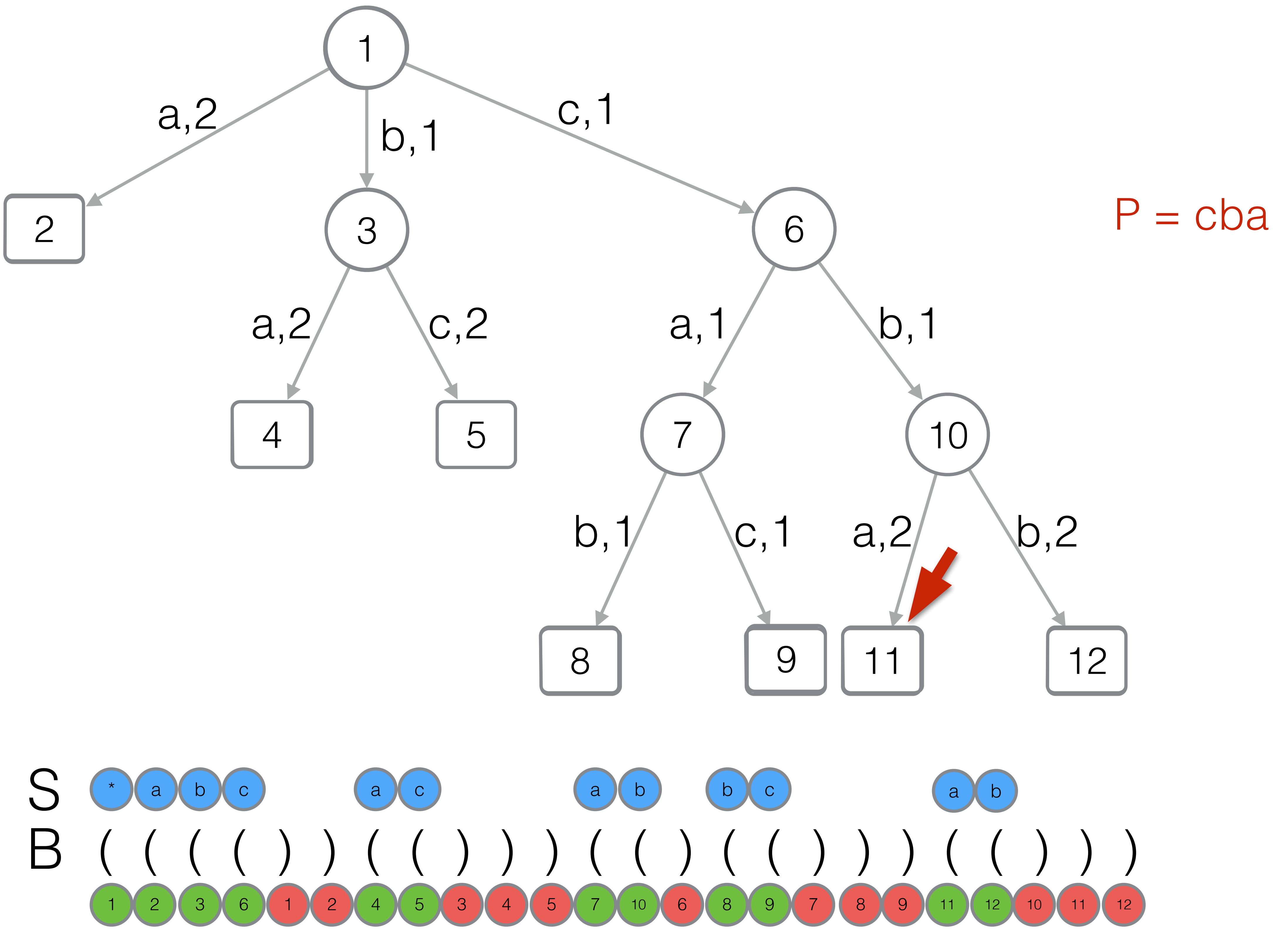

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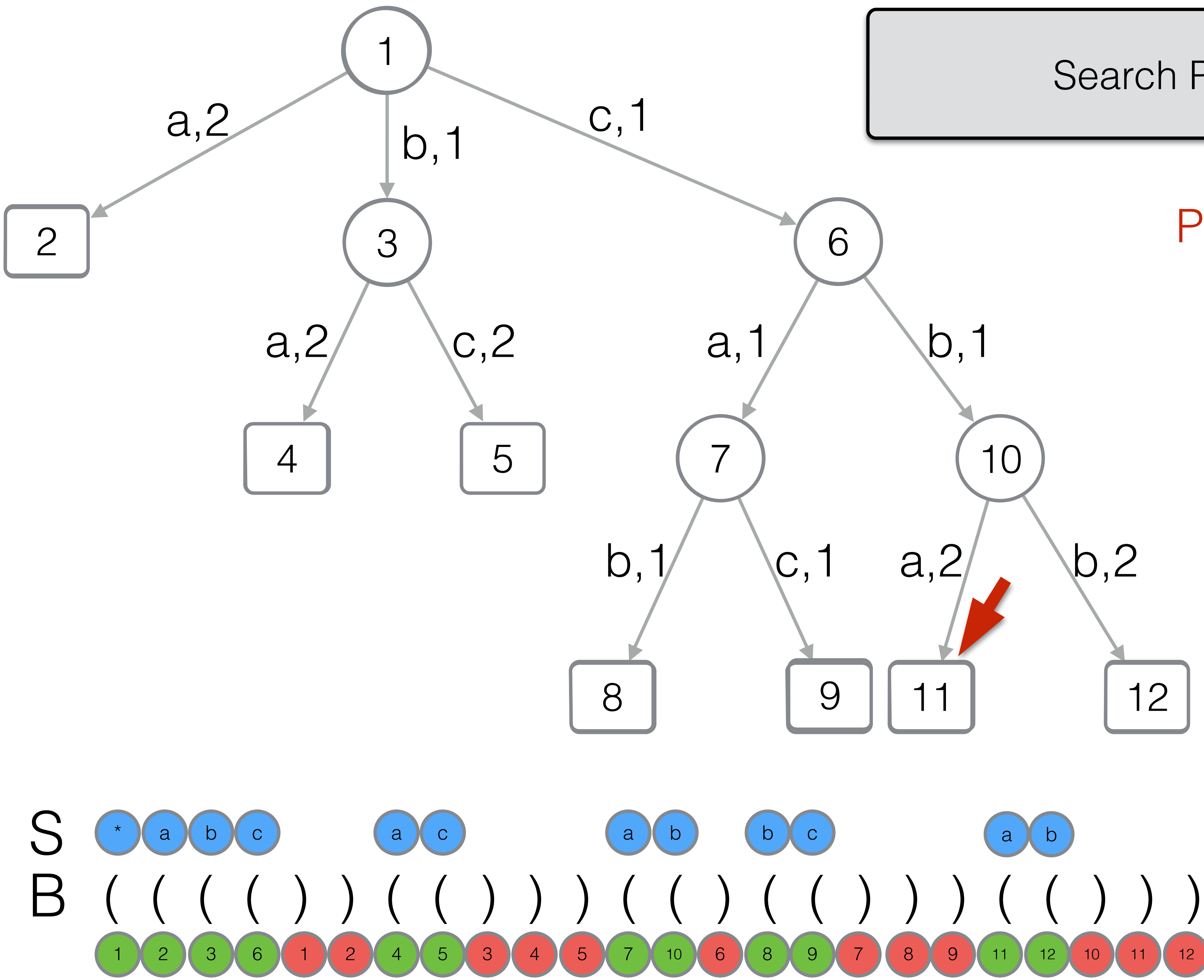
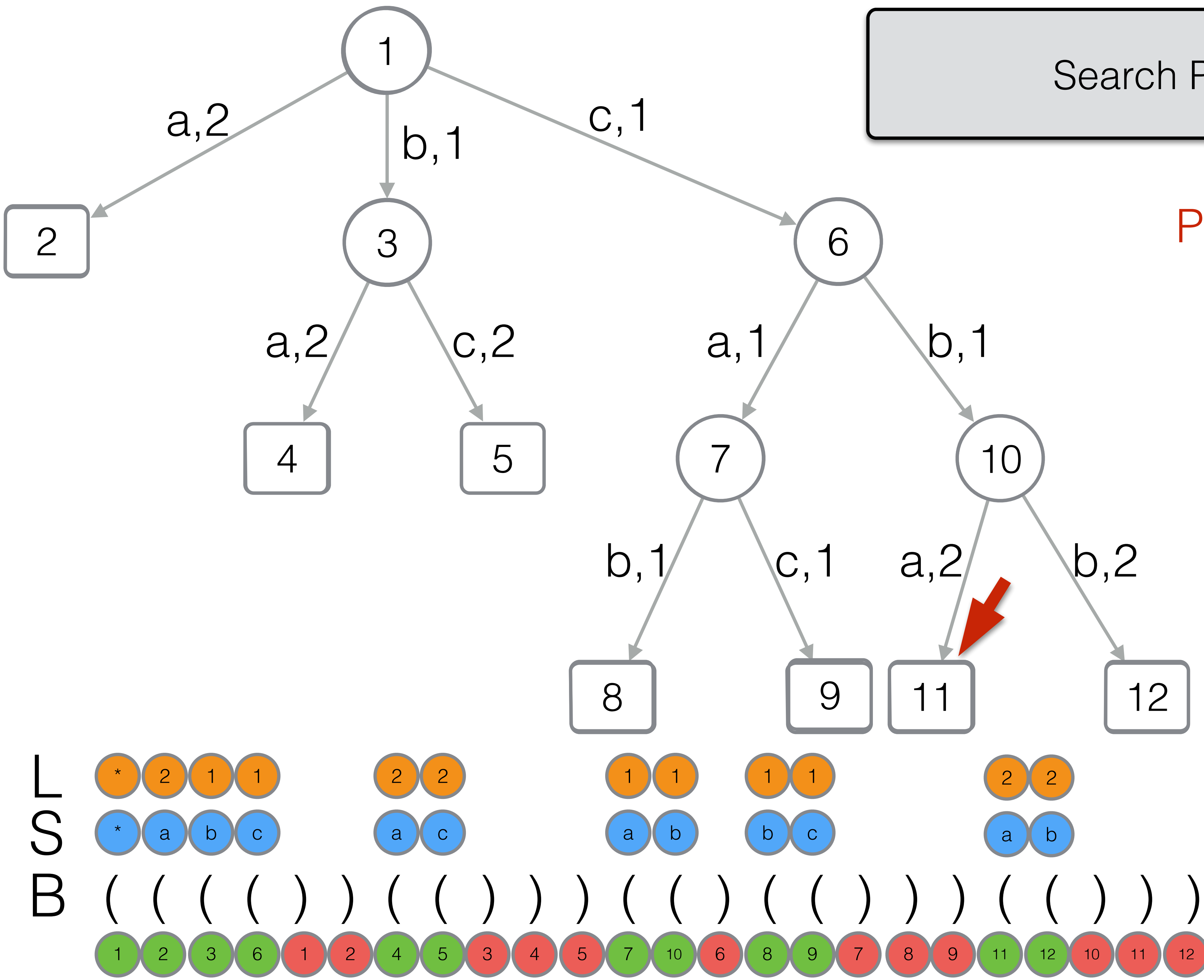


Search P in O(|P|) time!

P = cba

Patricia trie with DFUDS



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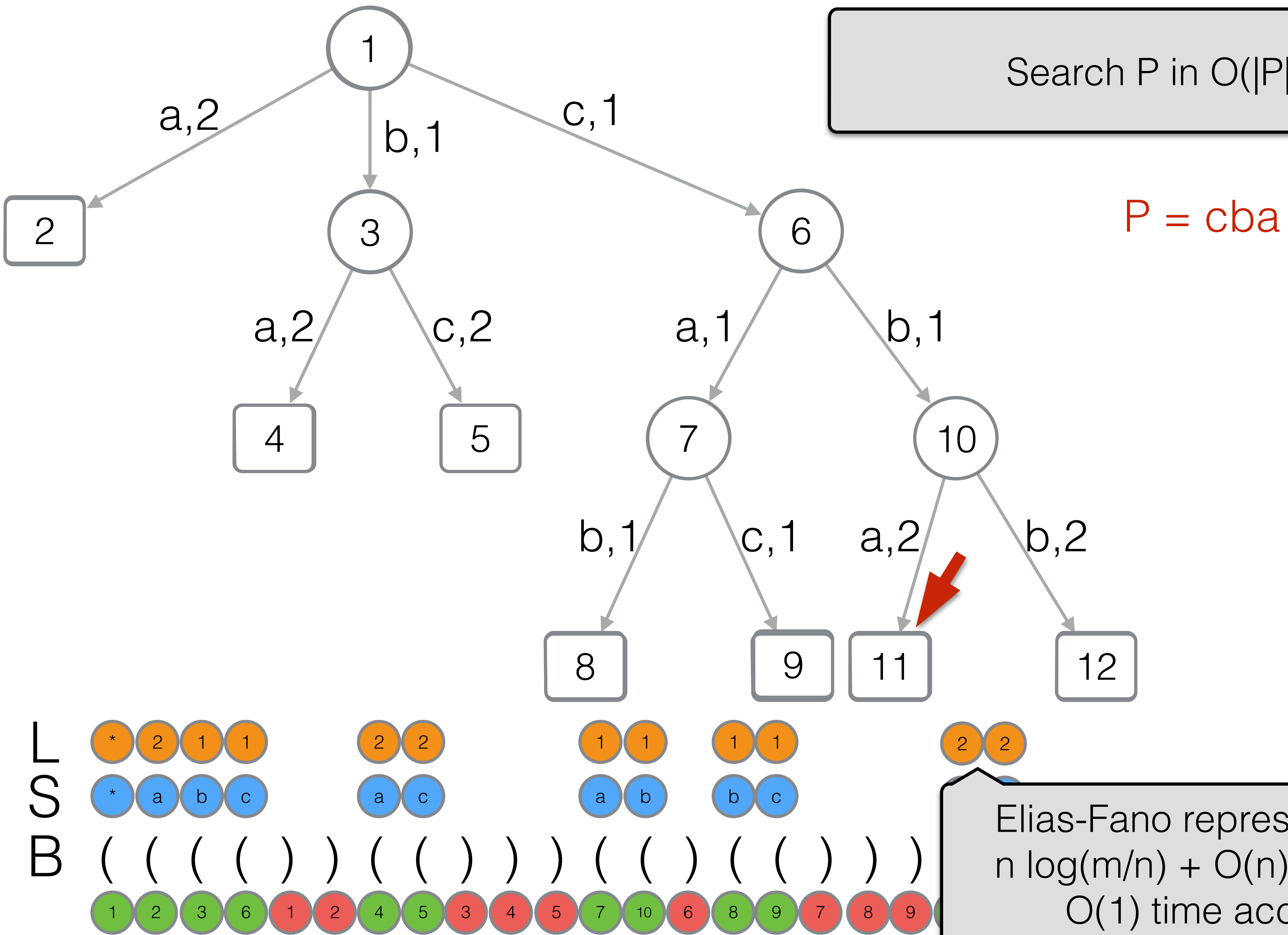


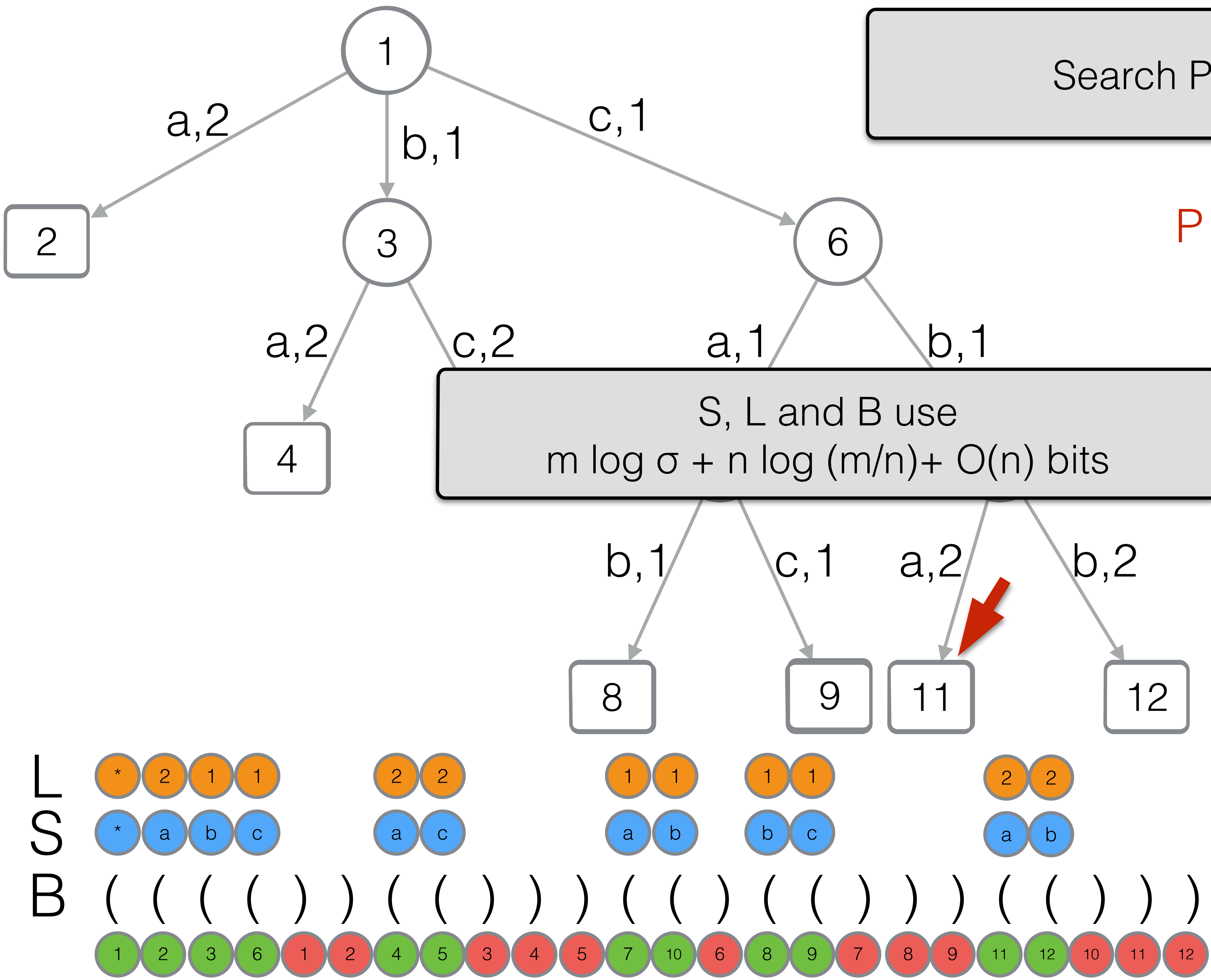
Elias-Fano representation: n log(m/n) + O(n) bits and O(1) time access.

Patricia trie with DFUDS

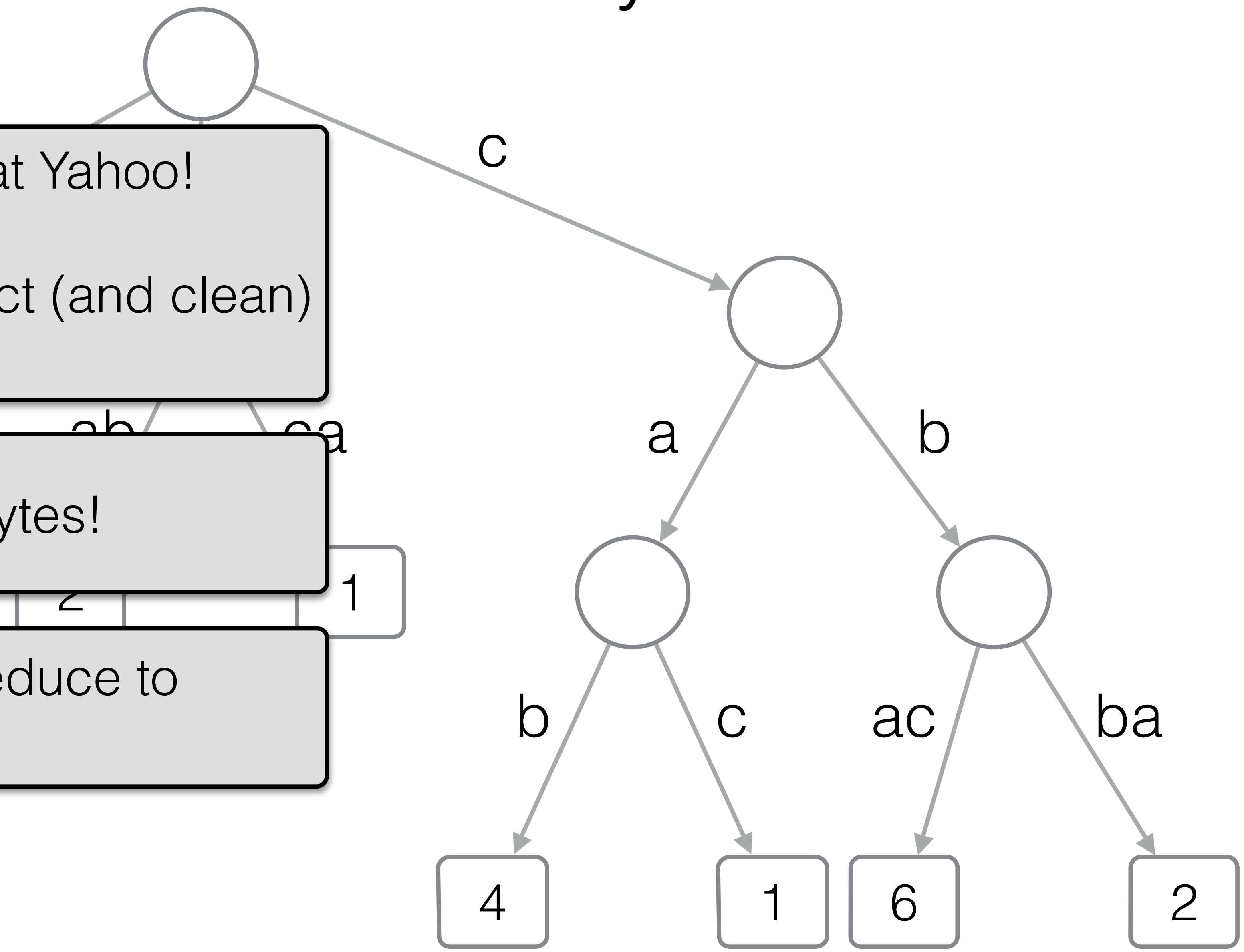
Search P in O(|P|) time!

P = cba

S, L and B use
$m \log \sigma + n \log (m/n) + O(n)$ bits



Summary



3 months query log at Yahoo!

≈600 million of distinct (and clean) queries

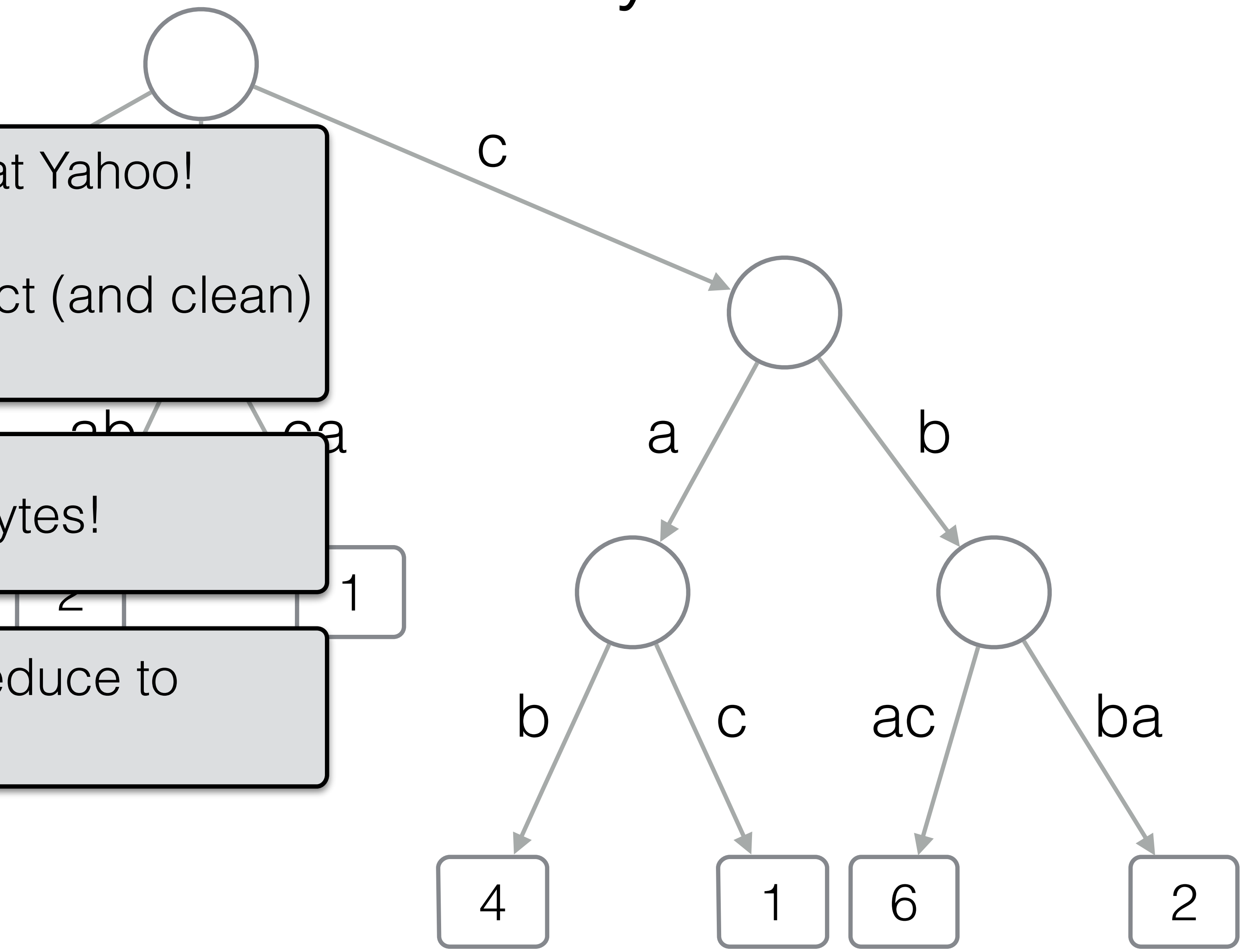
Trie requires ≈50 Gbytes!

We will see how to reduce to ≈5 Gbytes!

Find the node "prefixed" by P	O(|P|) time	
Compute the top-k strings	O(k log k) time	O(n) bits

n = |D|, m total length of strings in D

Summary



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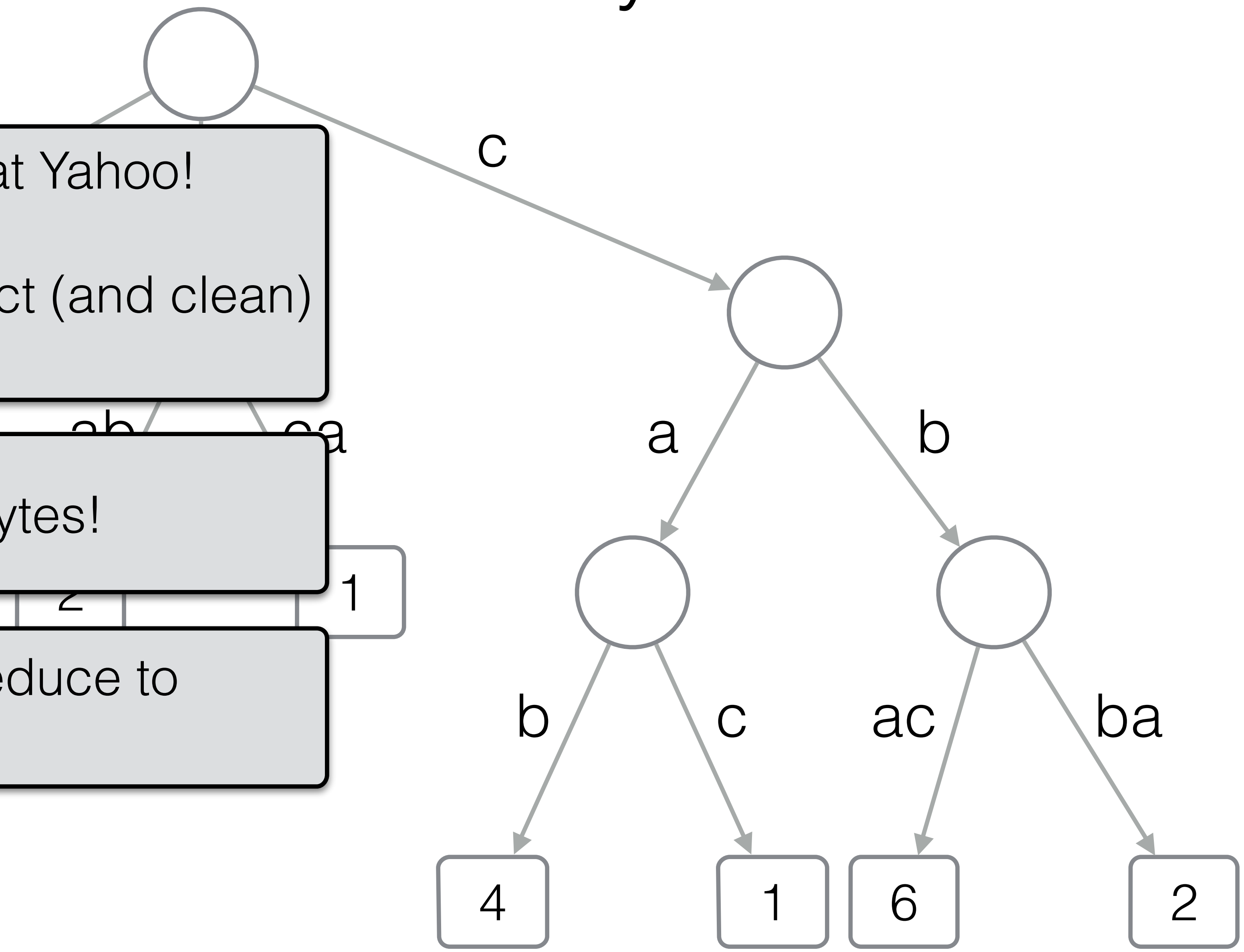
Summary



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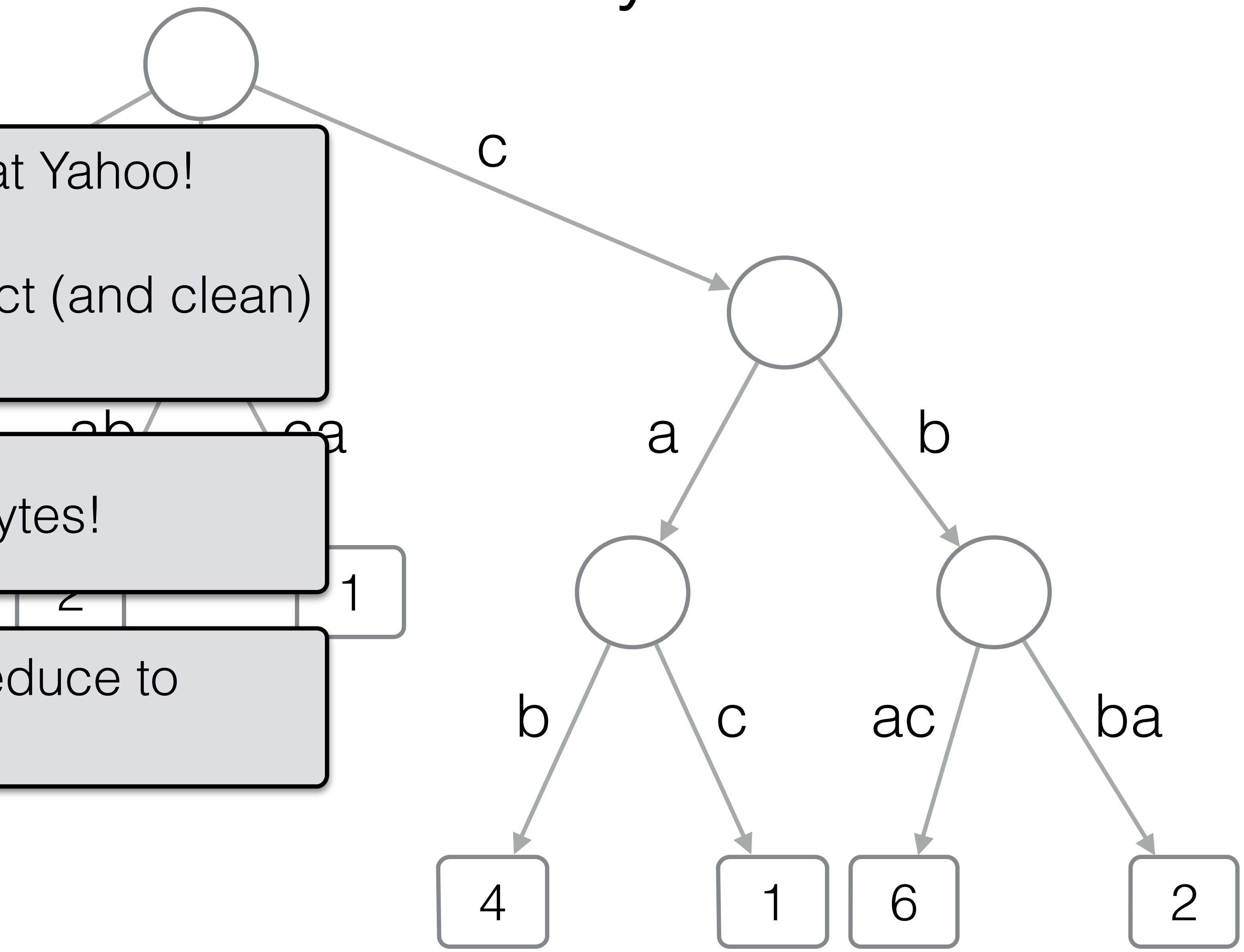
Find the node "prefixed" by P	O(|P|) time	$m \log \sigma + n \log (m/n) + O(n)$ bits
Compute the top-k strings	O(k log k) time	to be compared with $O(m \log \sigma + n \log m)$ bits

n = |D|, m total length of strings in D

Summary



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Find the node “prefix

Compute the top-k st

(n=) 1 billion of strings of average length 64

$m=64*10^9$ symbols and $m/n = 64$

$m \log \sigma + n \log (m/n) + O(n)$ bits

to be compared with
$O(m \log \sigma + n \log m)$ bits

$n = |D|$, m total length of strings in D