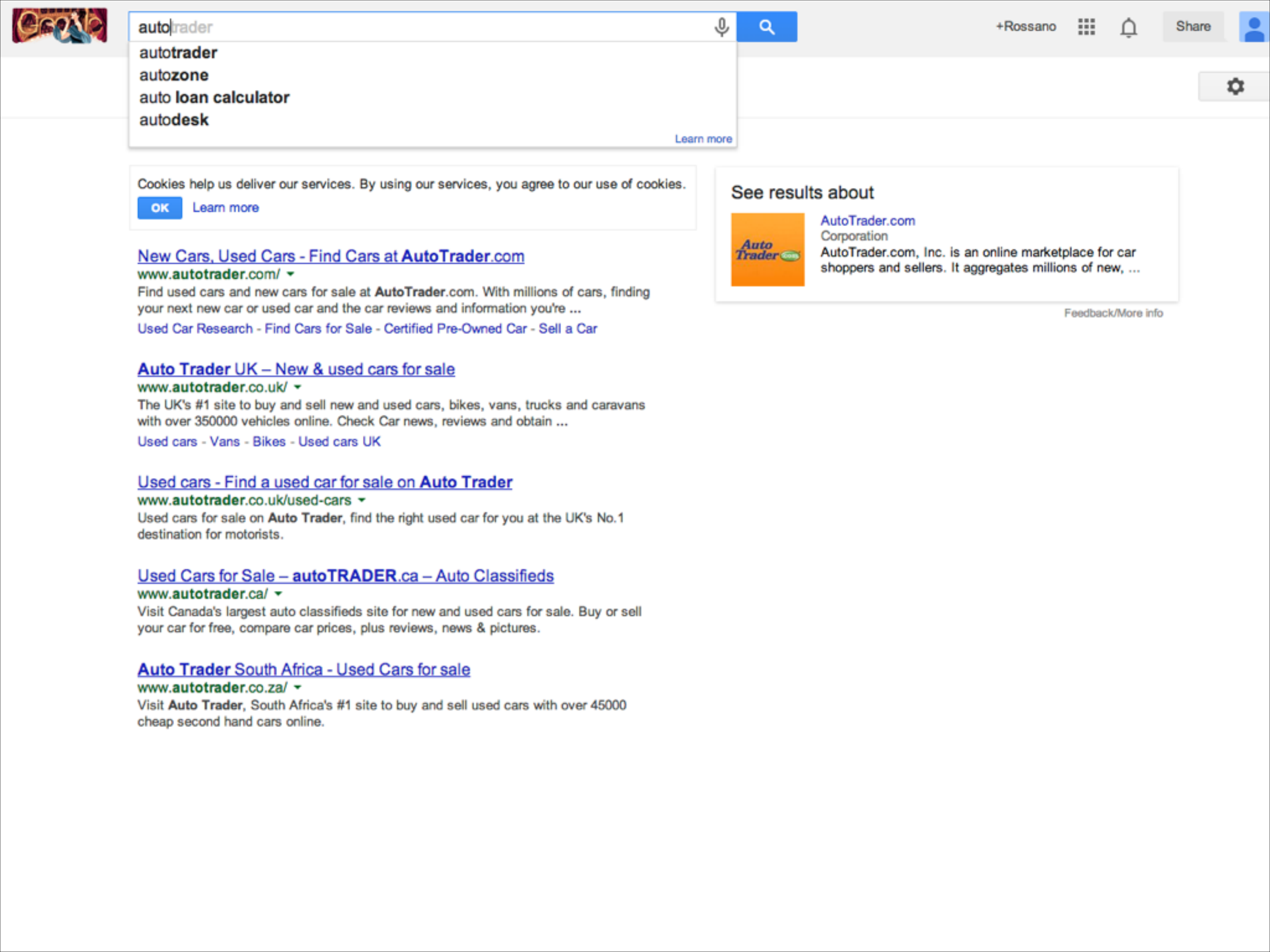
Succinct Data Structures

Auto-completion as our target application

Rossano Venturini



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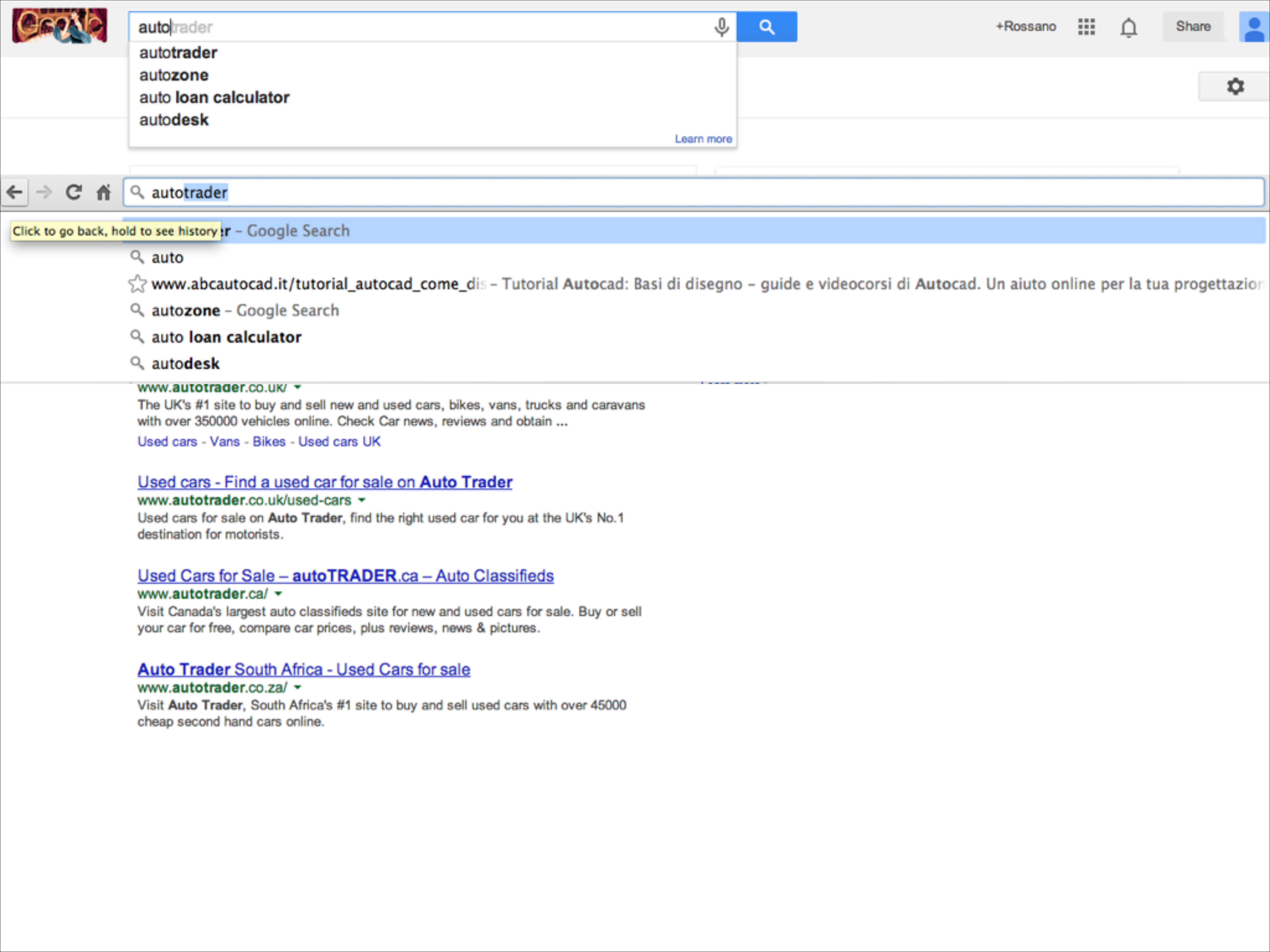

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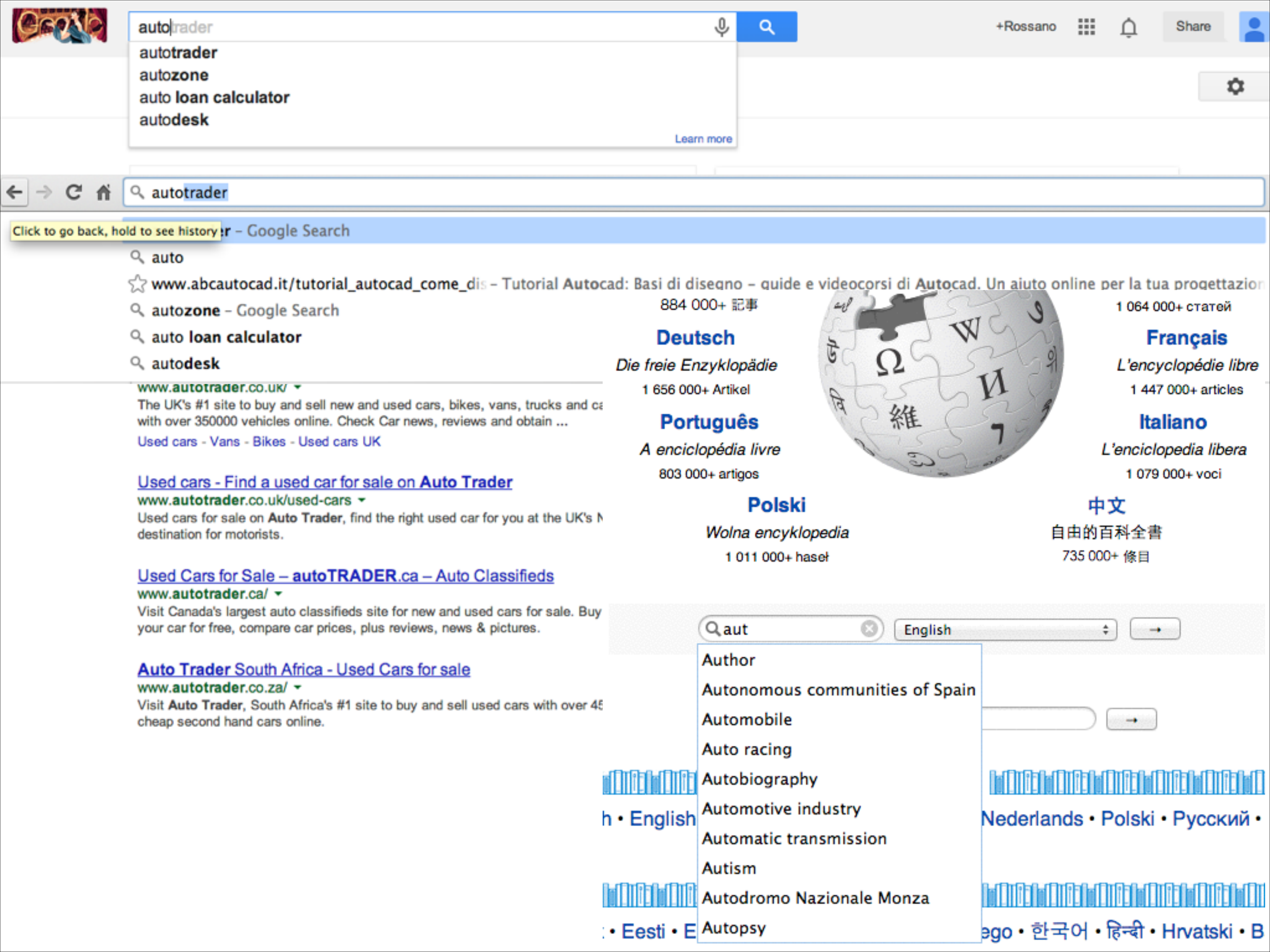

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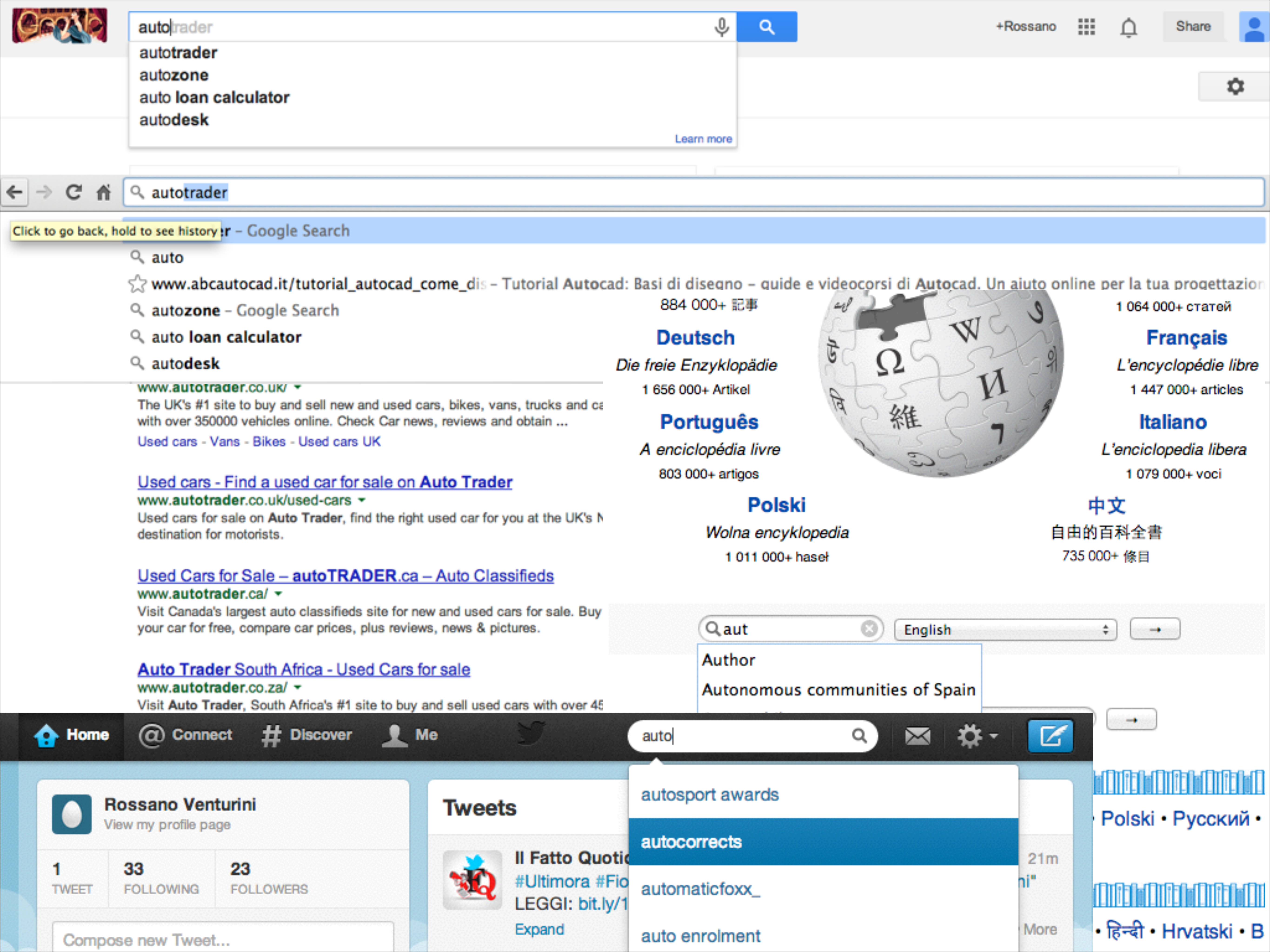

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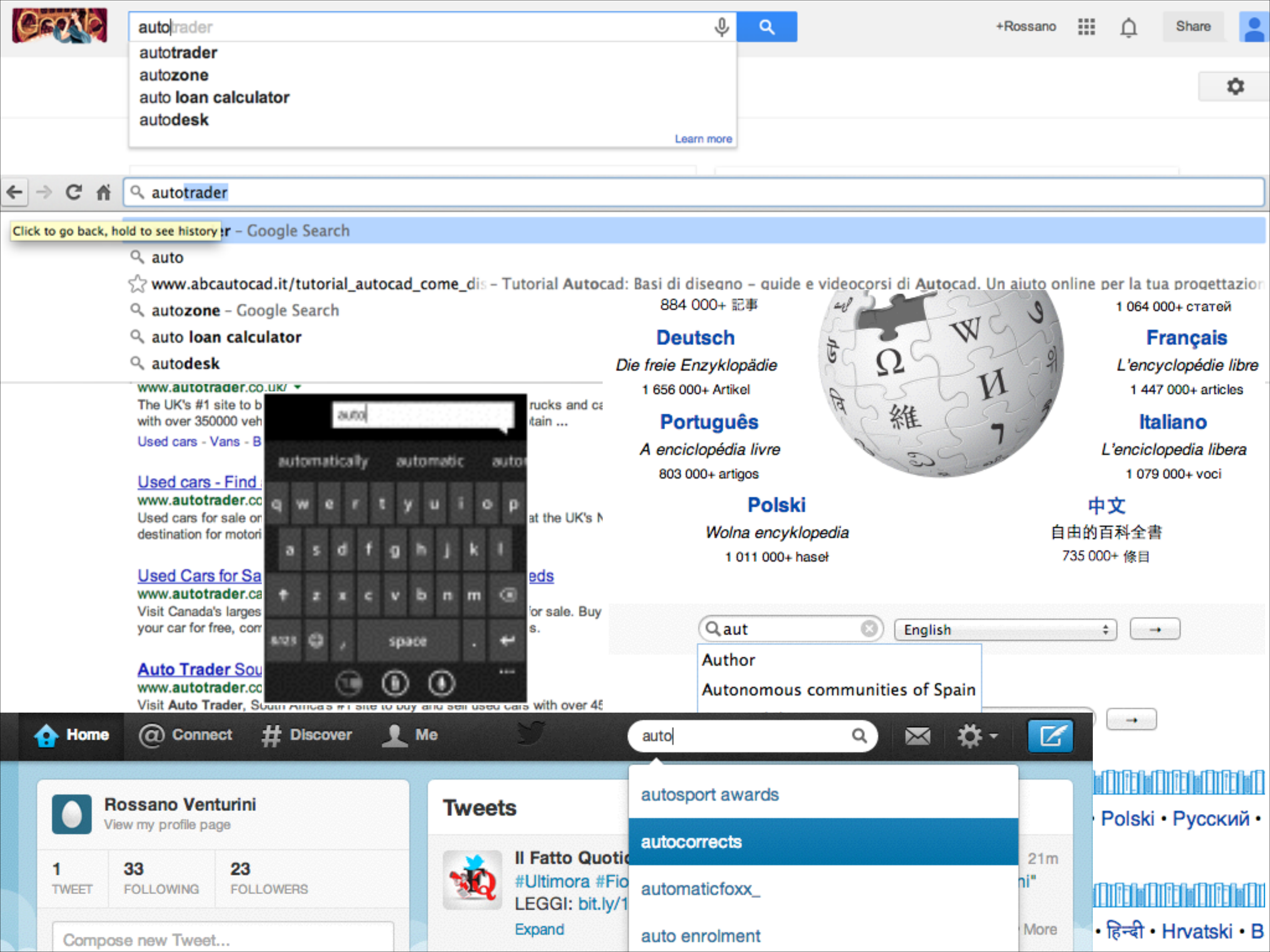

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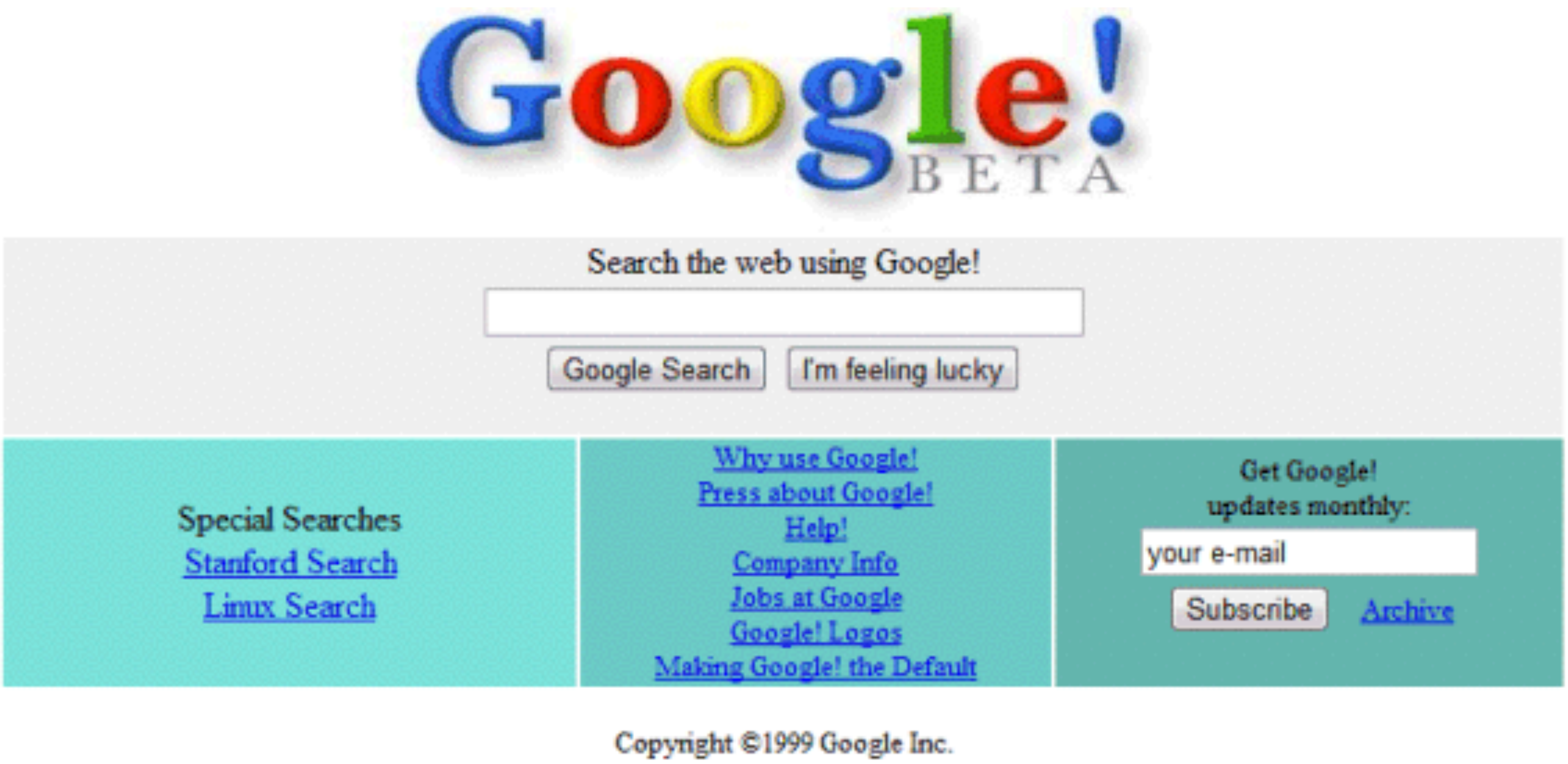

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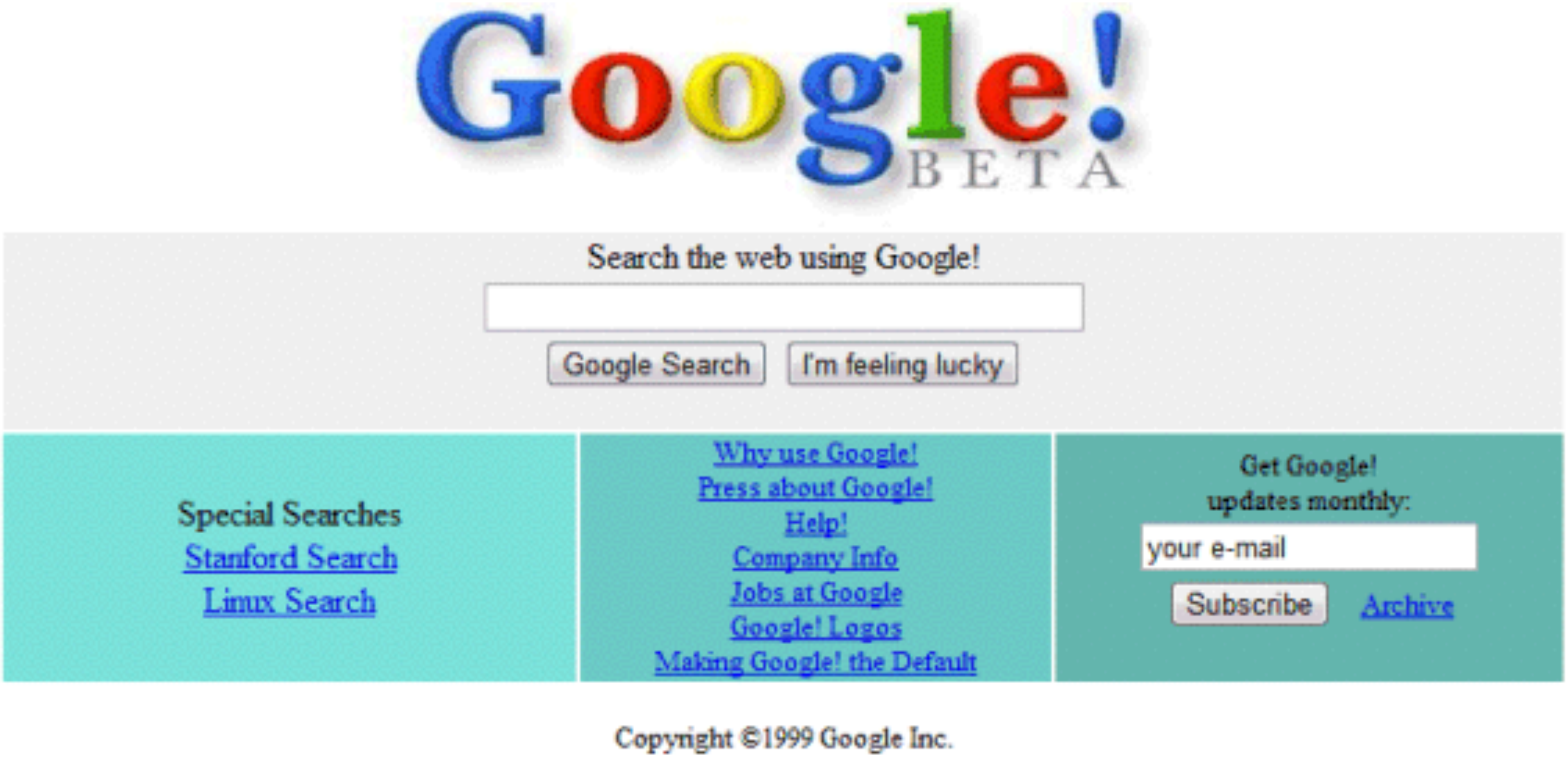
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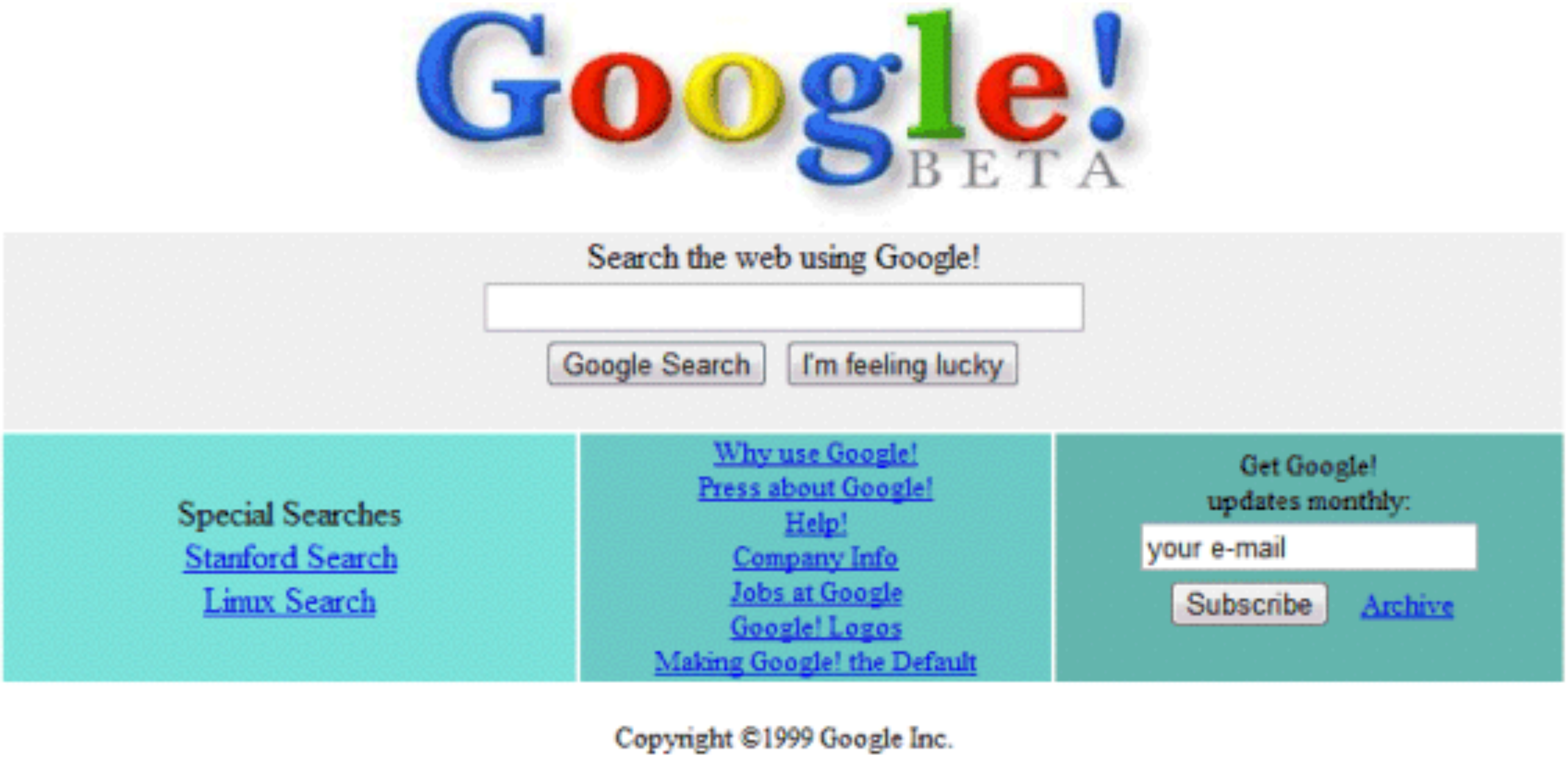

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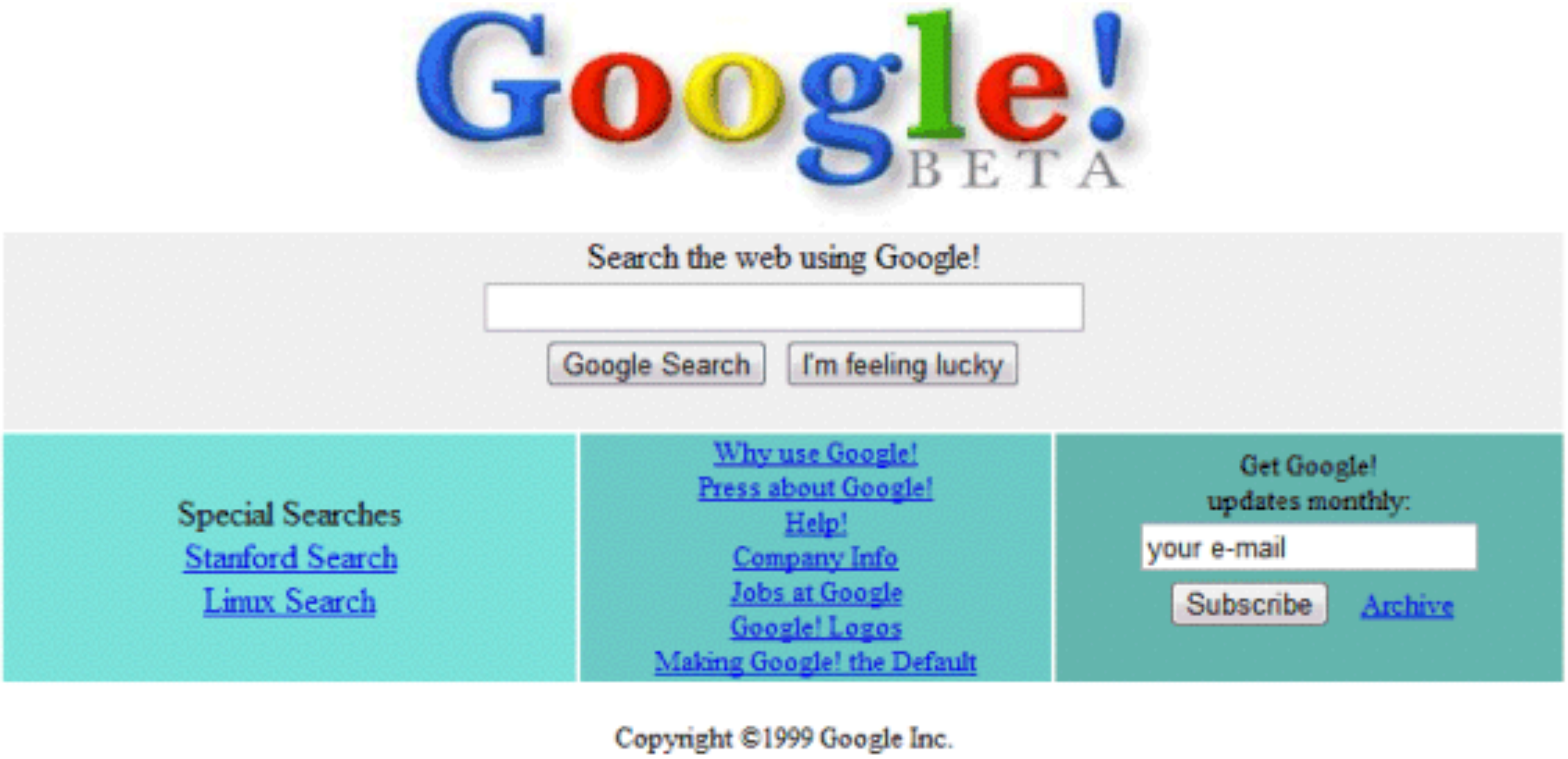

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How to find top-k efficiently?

Trie

Trie

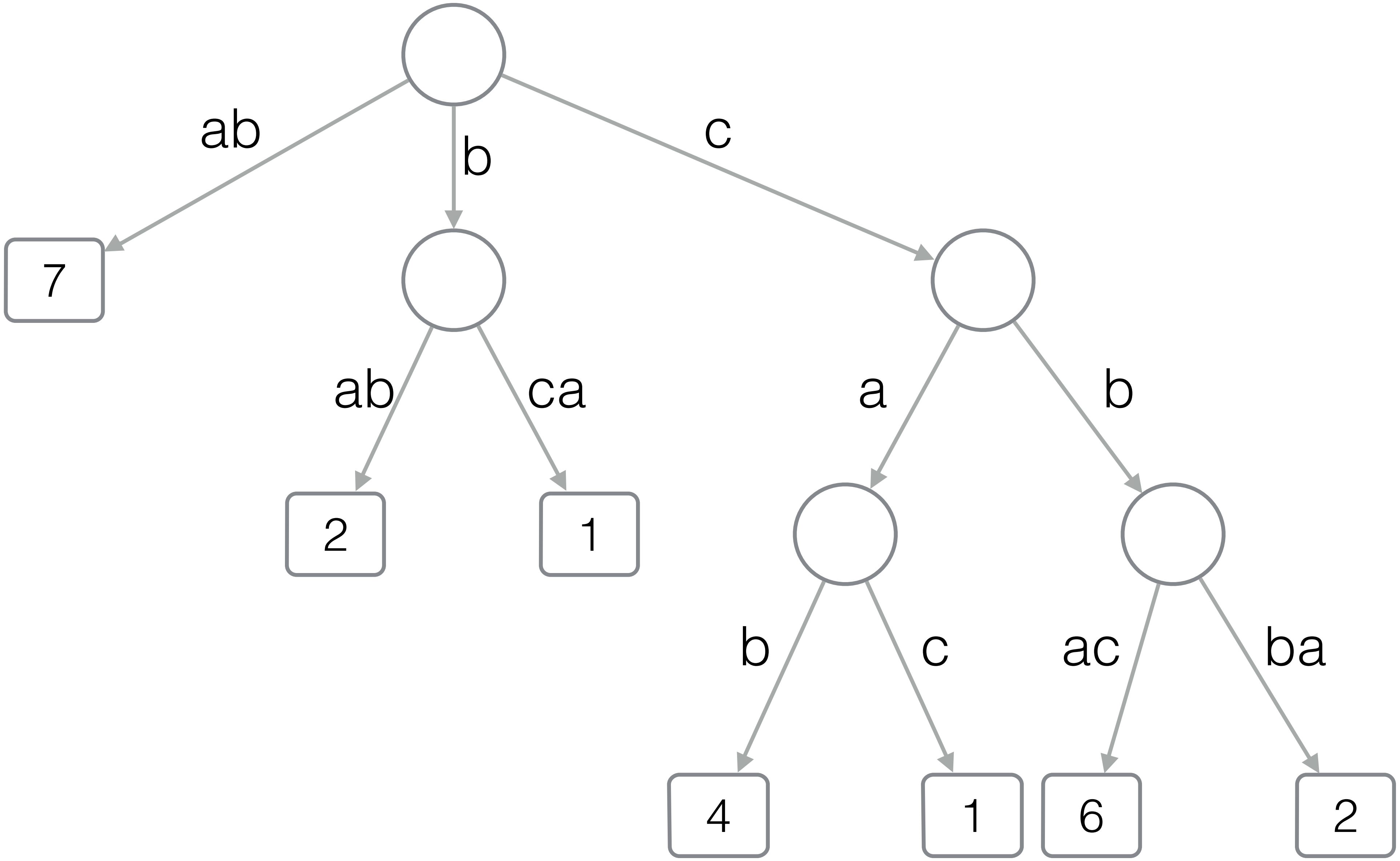
D = { ab (4), bab (2), bca (1), cab (2), cac (1), cbac (3), cbba (2) }

Trie

D = { ab (4), bab (2), bca (1), cab (2), cac (1), cbac (3), cbba (2) }

n = |D|, m total length of strings in D

Trie



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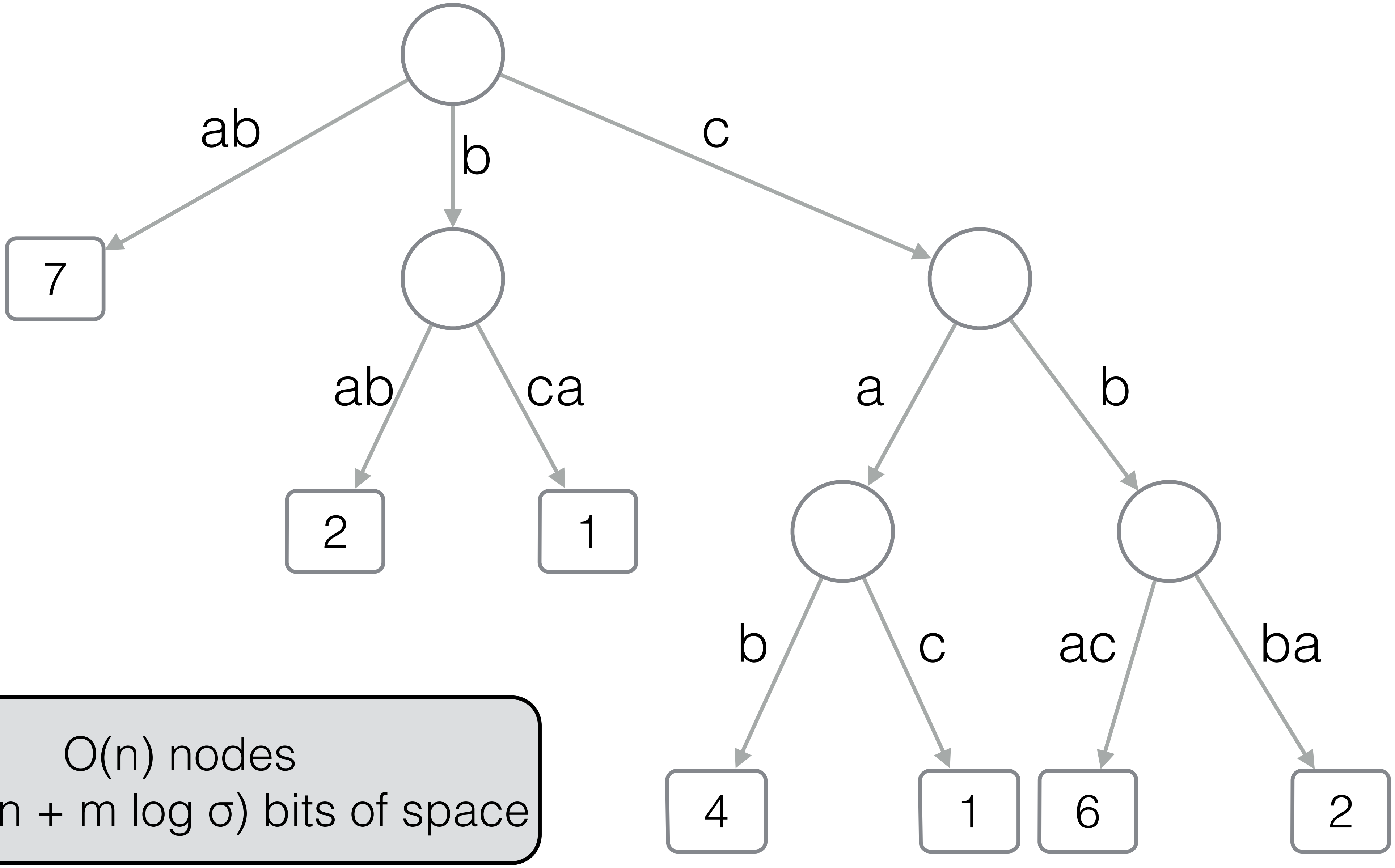
Trie

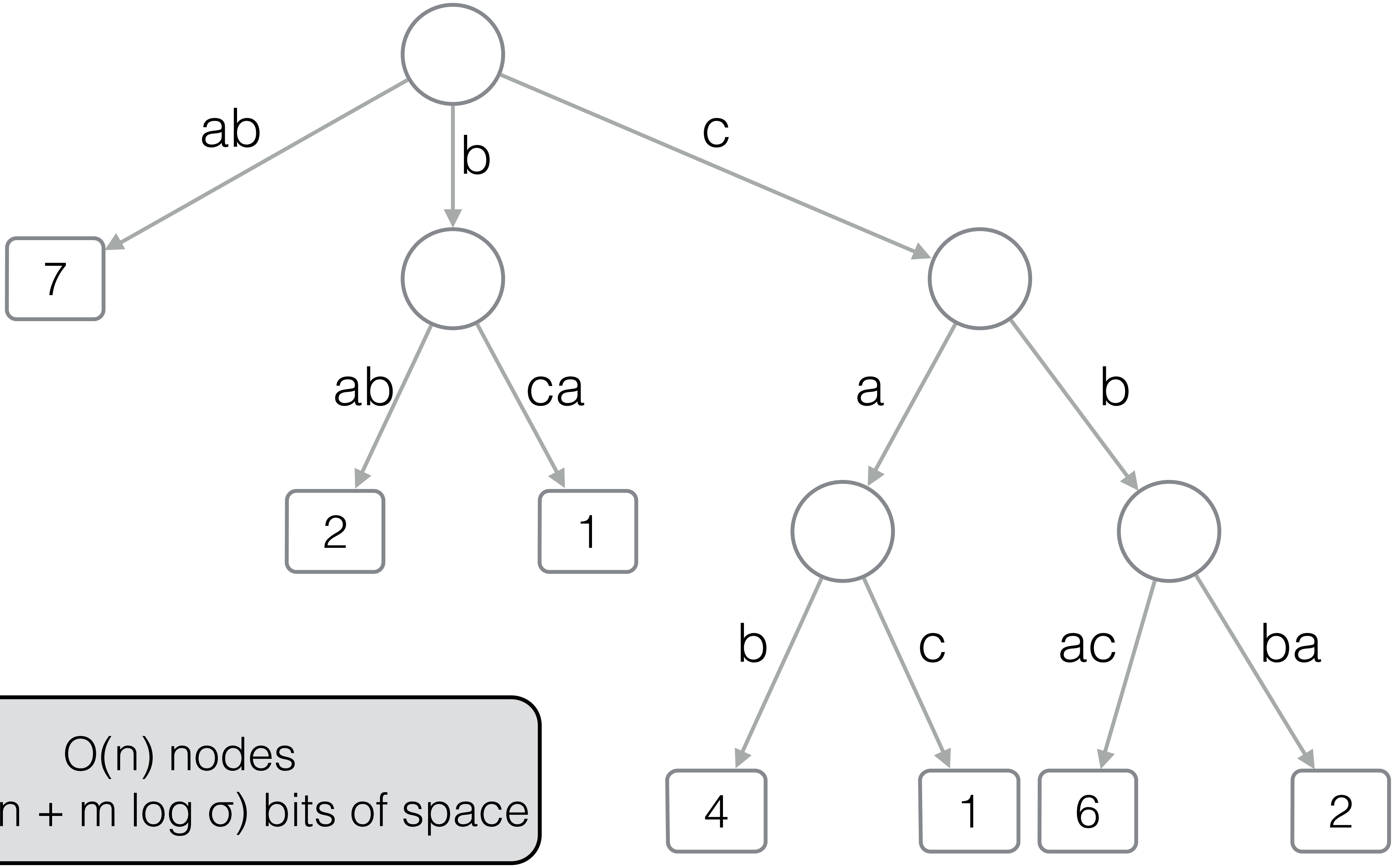


O(n) nodes
O(n log n + m log σ) bits of space

D = { ab (4), bab (2), bca (1), cab (2), cac (1), cbac (3), cbba (2) }

n = |D|, m total length of strings in D

Trie



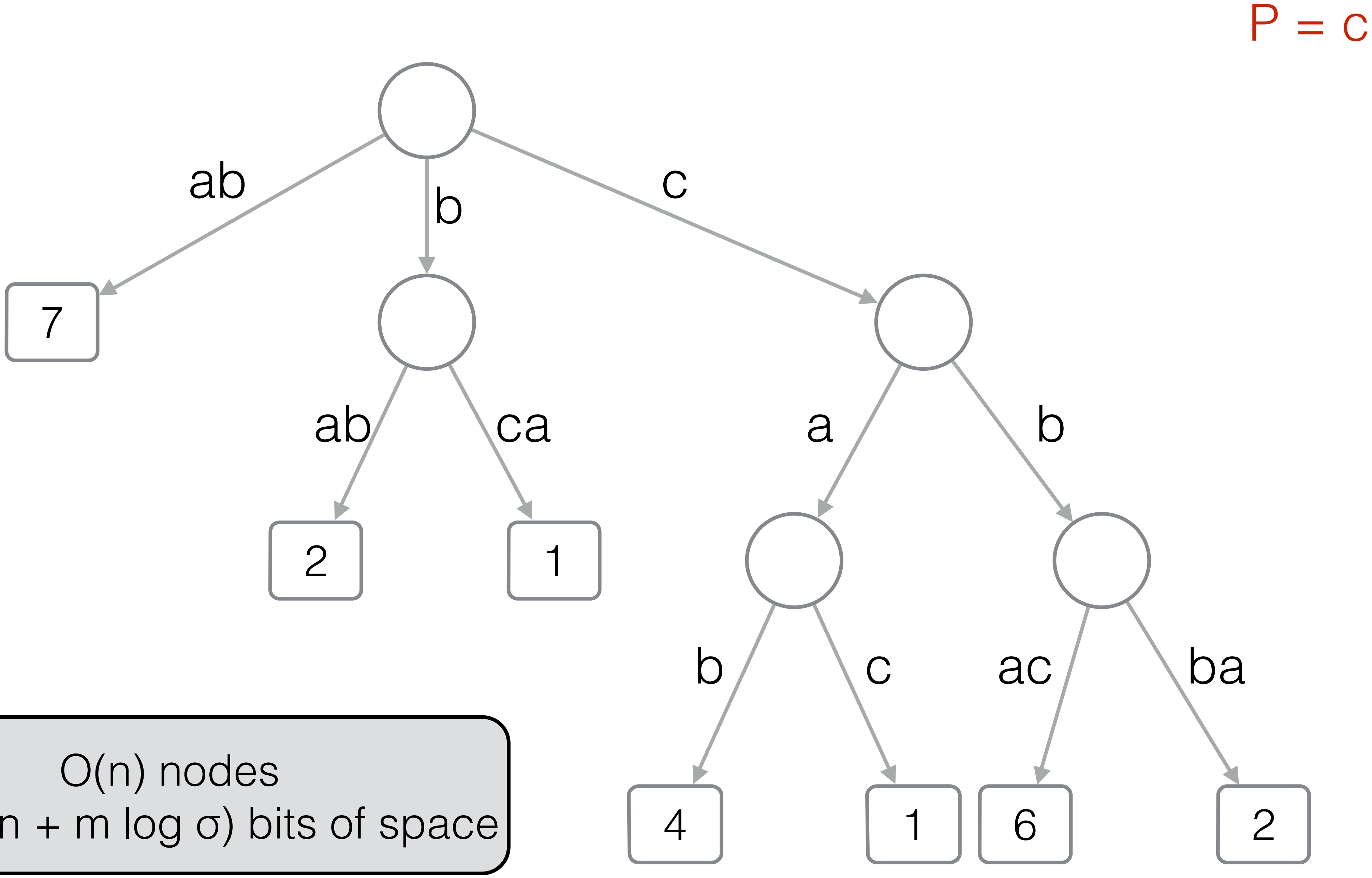
O(n) nodes
O(n log n + m log σ) bits of space

Find all the strings prefixed by any pattern P in O(|P|) time

D = { ab (4), bab (2), bca (1), cab (2), cac (1), cbac (3), cbba (2) }

n = |D|, m total length of strings in D

Trie



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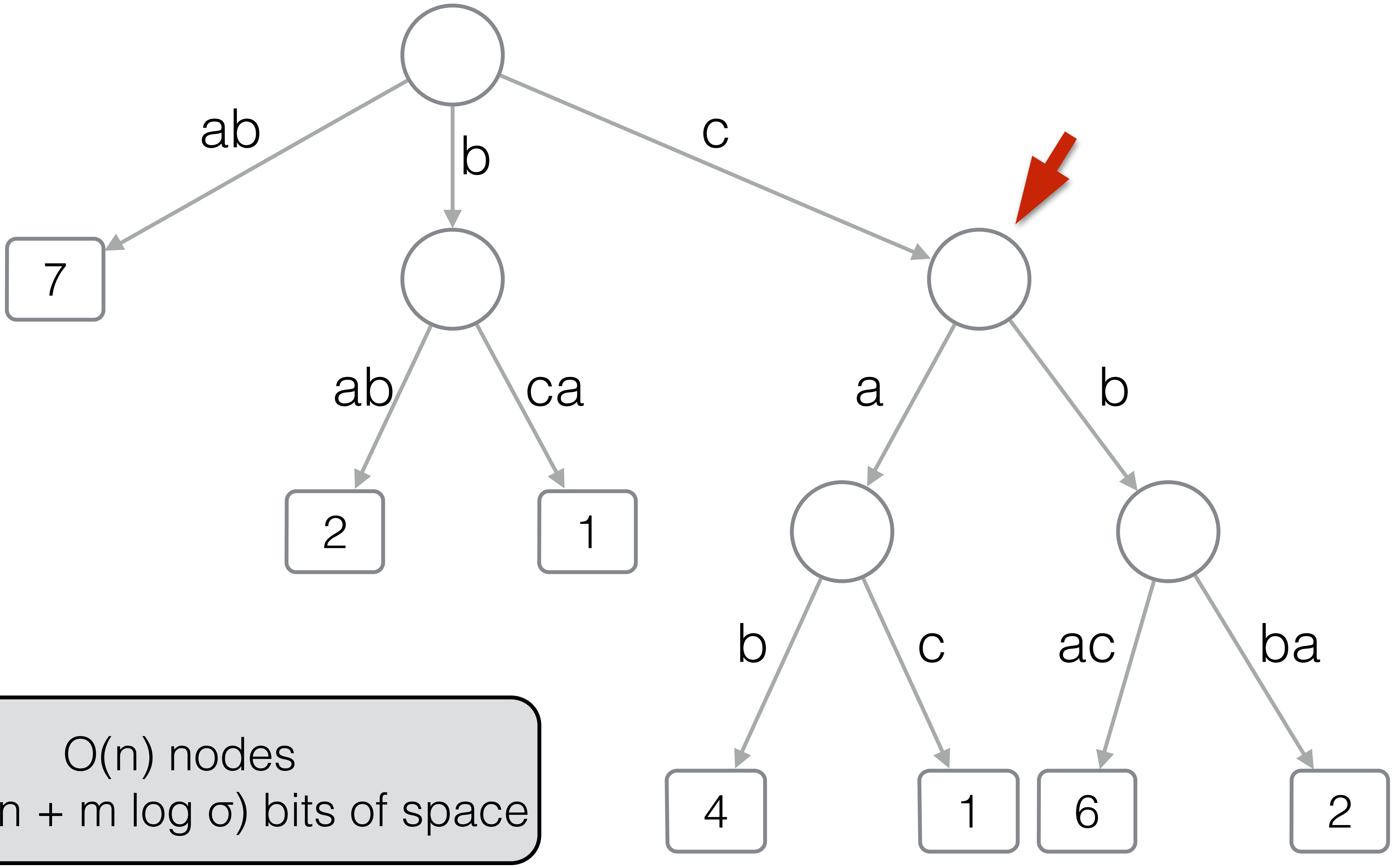
n = |D|, m total length of strings in D

Trie

P = c



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D = { ab (4), bab (2), bca (1), cab (2), cac (1), cbac (3), cbba (2) }

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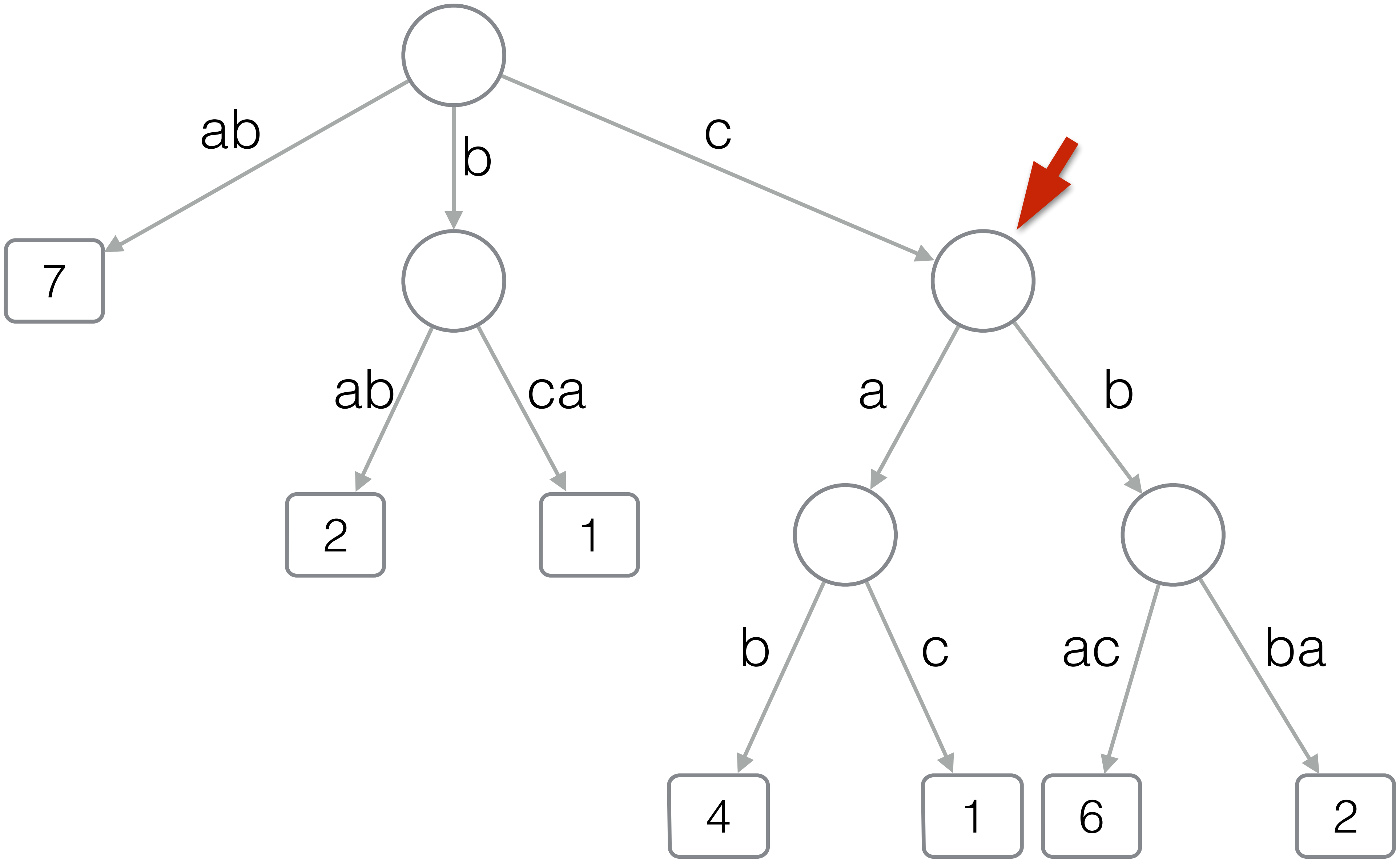
Finding Top-1

P = c



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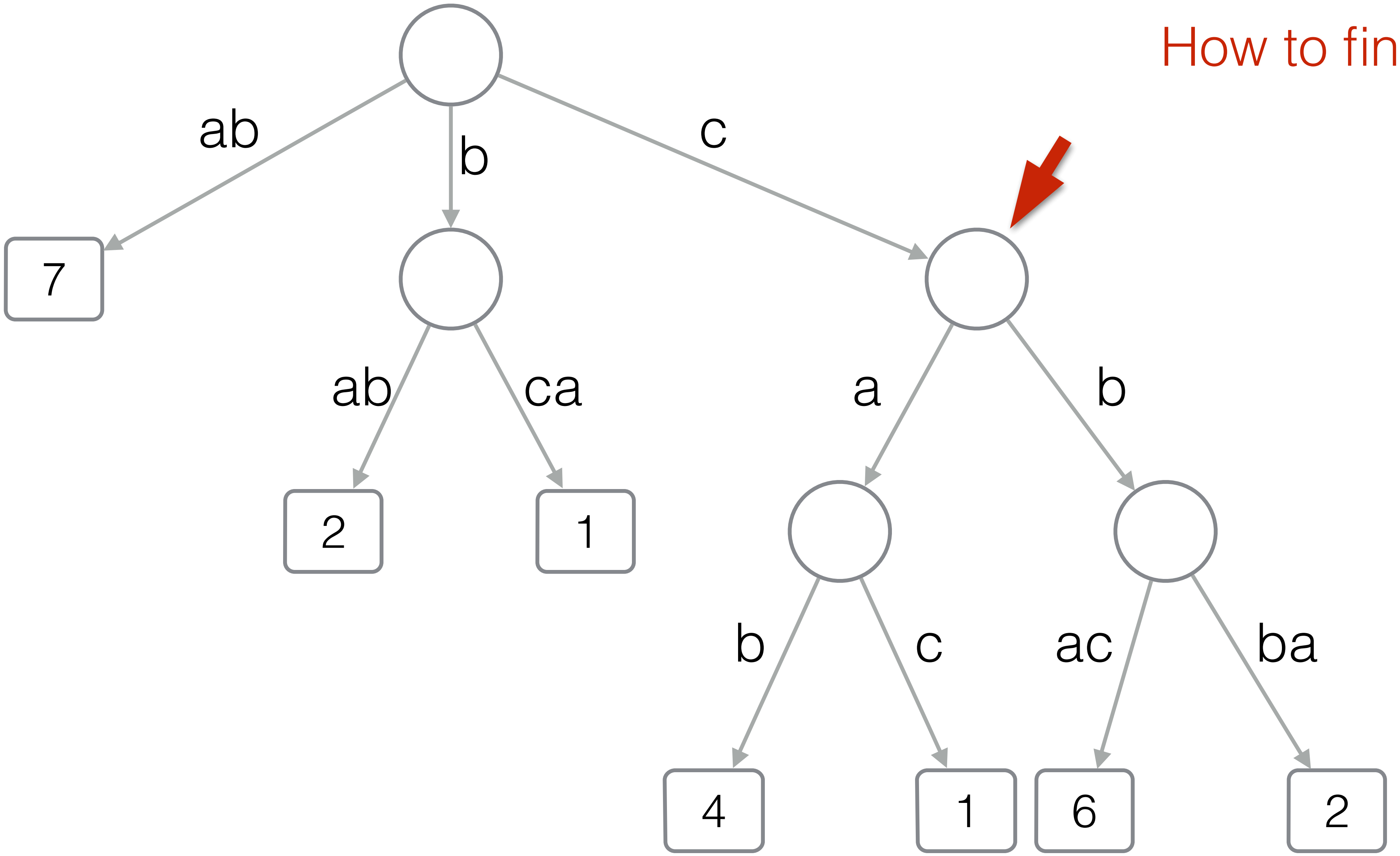
Finding Top-1



P = c

How to find Top-1?

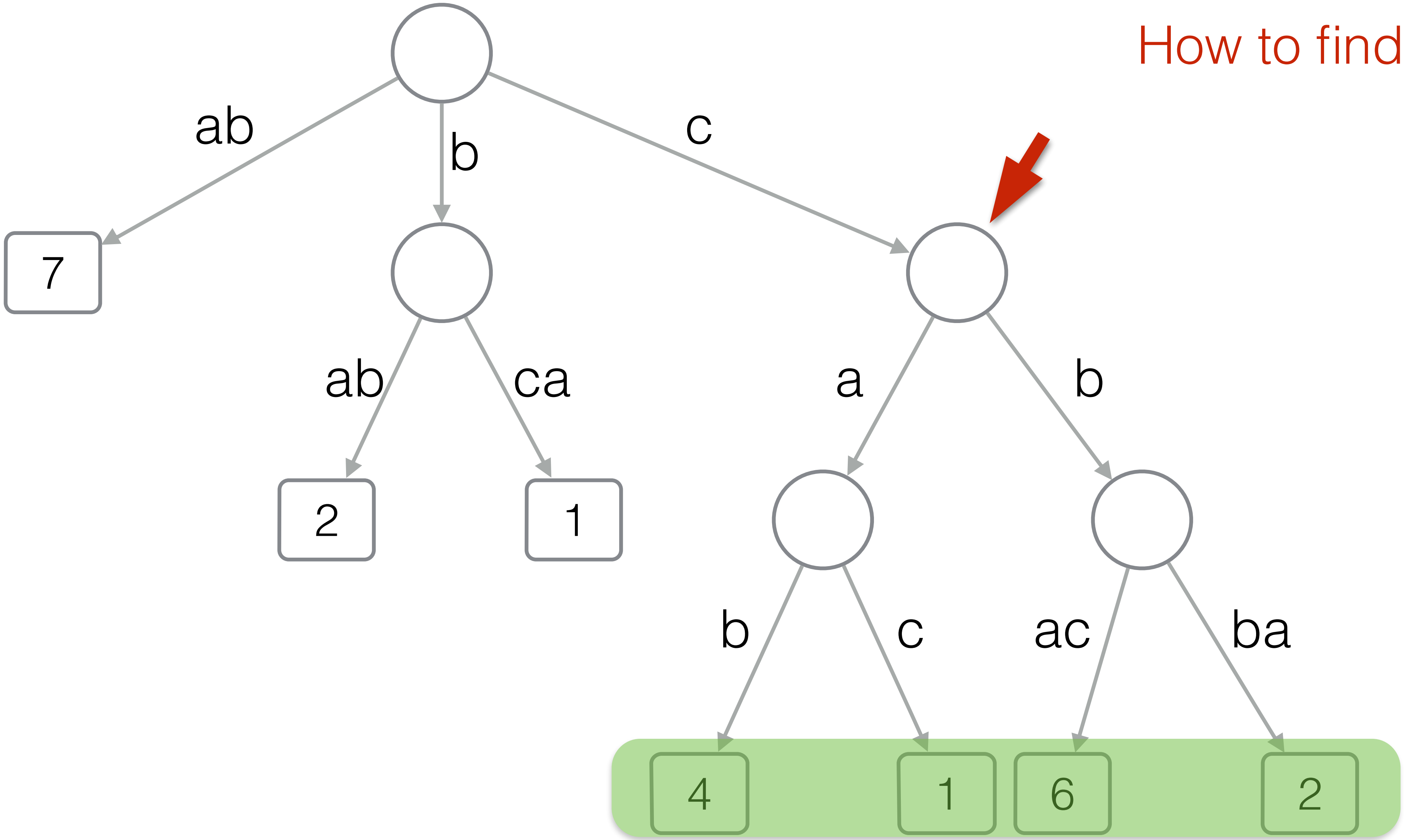
D = { ab (4), bab (2), bca (1), cab (2), cac (1), cbac (3), cbba (2) }

n = |D|, m total length of strings in D

Finding Top-1

P = c

How to find Top-1?



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Finding Top-1

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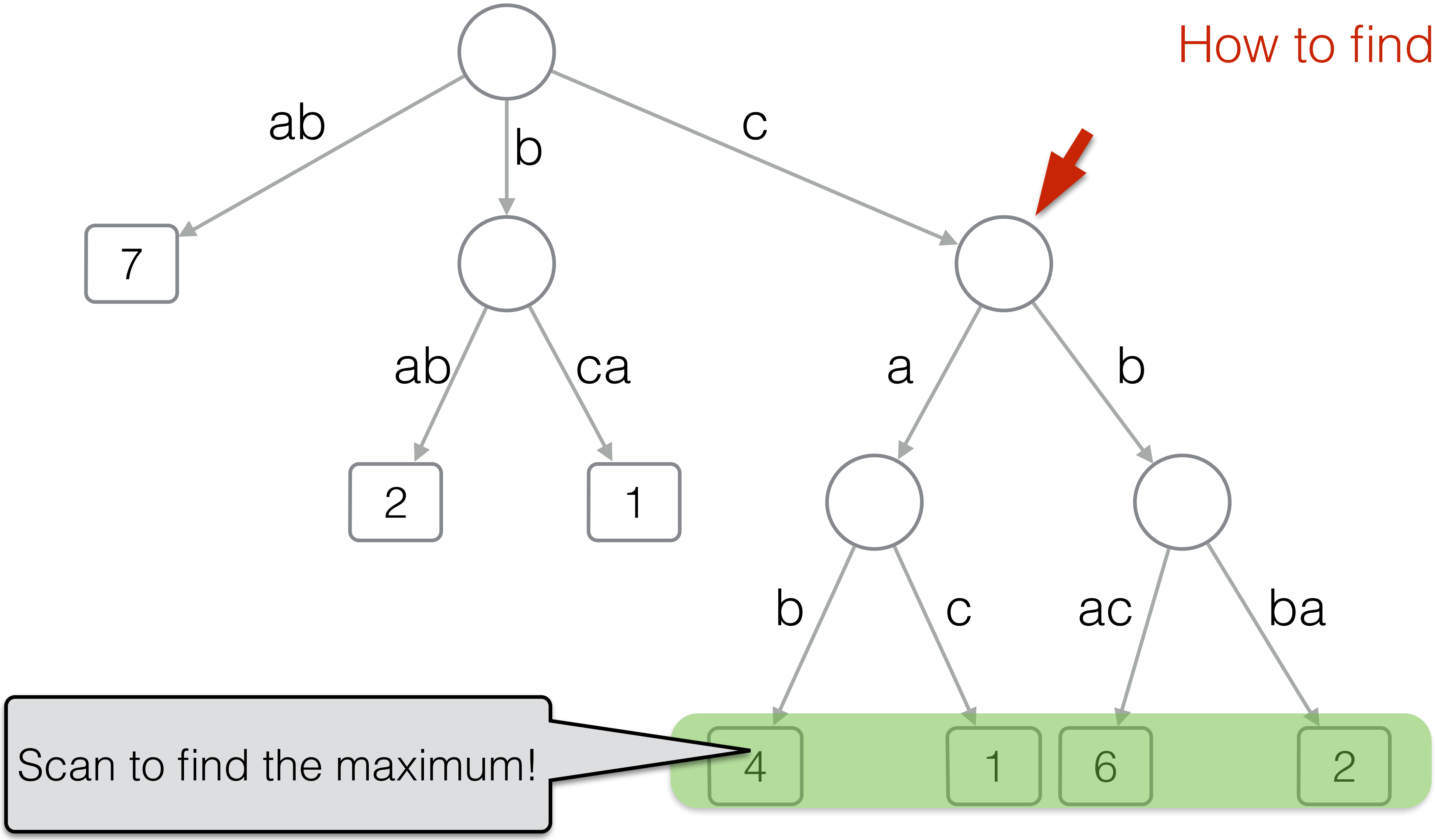
How to find Top-1?



Scan to find the maximum!

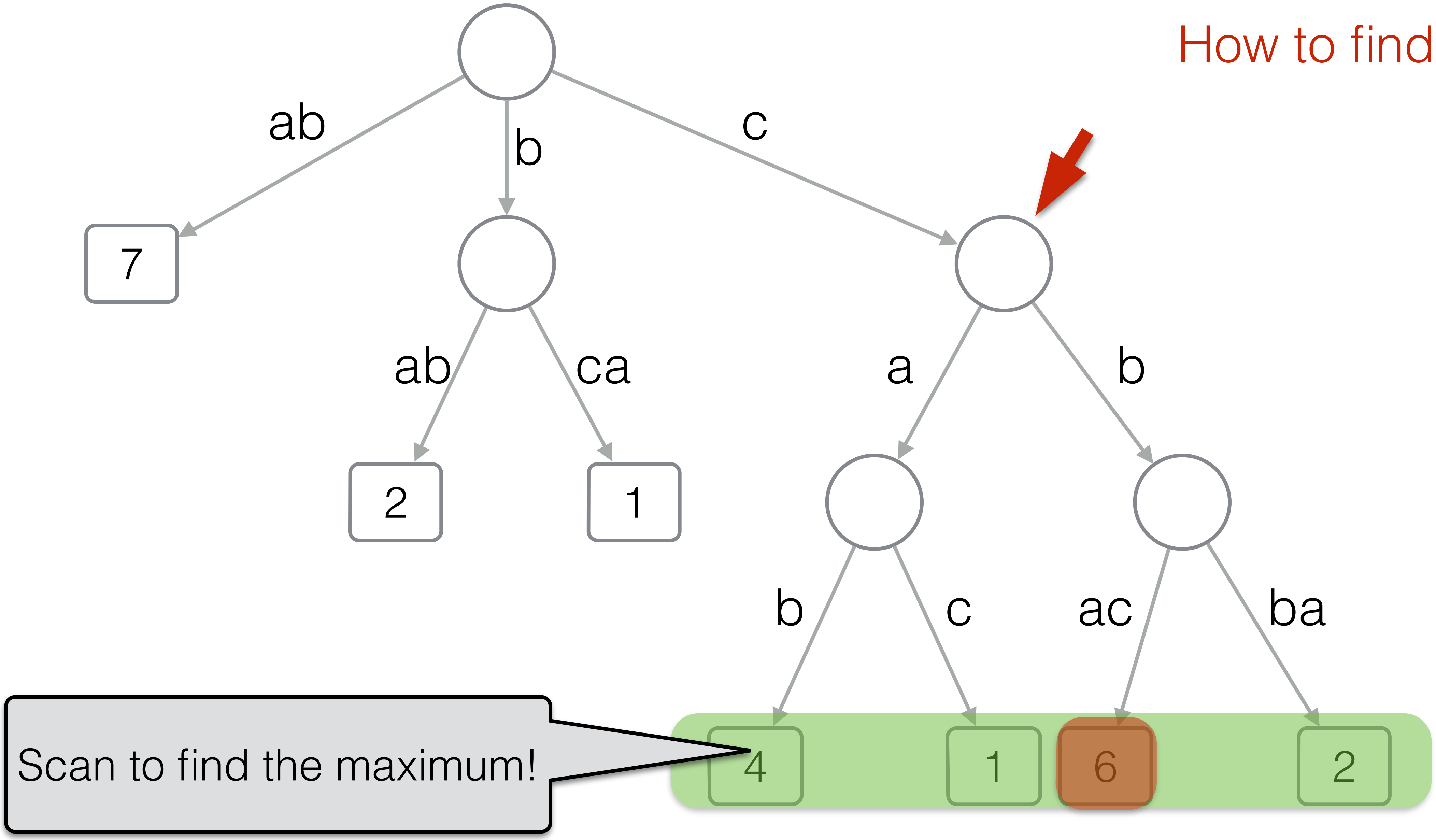
D = { ab (4), bab (2), bca (1), cab (2), cac (1), cbac (3), cbba (2) }

n = |D|, m total length of strings in D

Finding Top-1

P = c

How to find Top-1?



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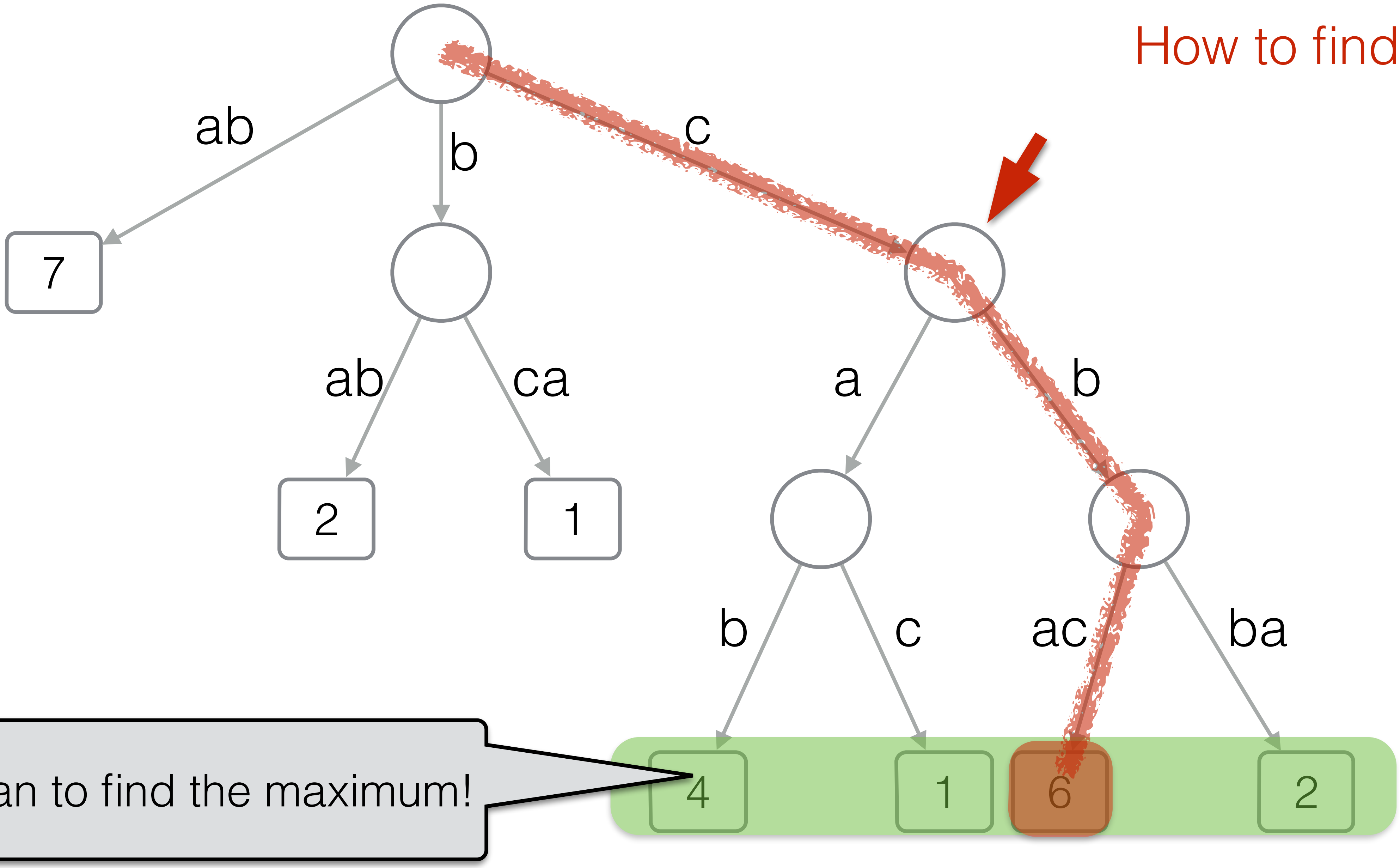

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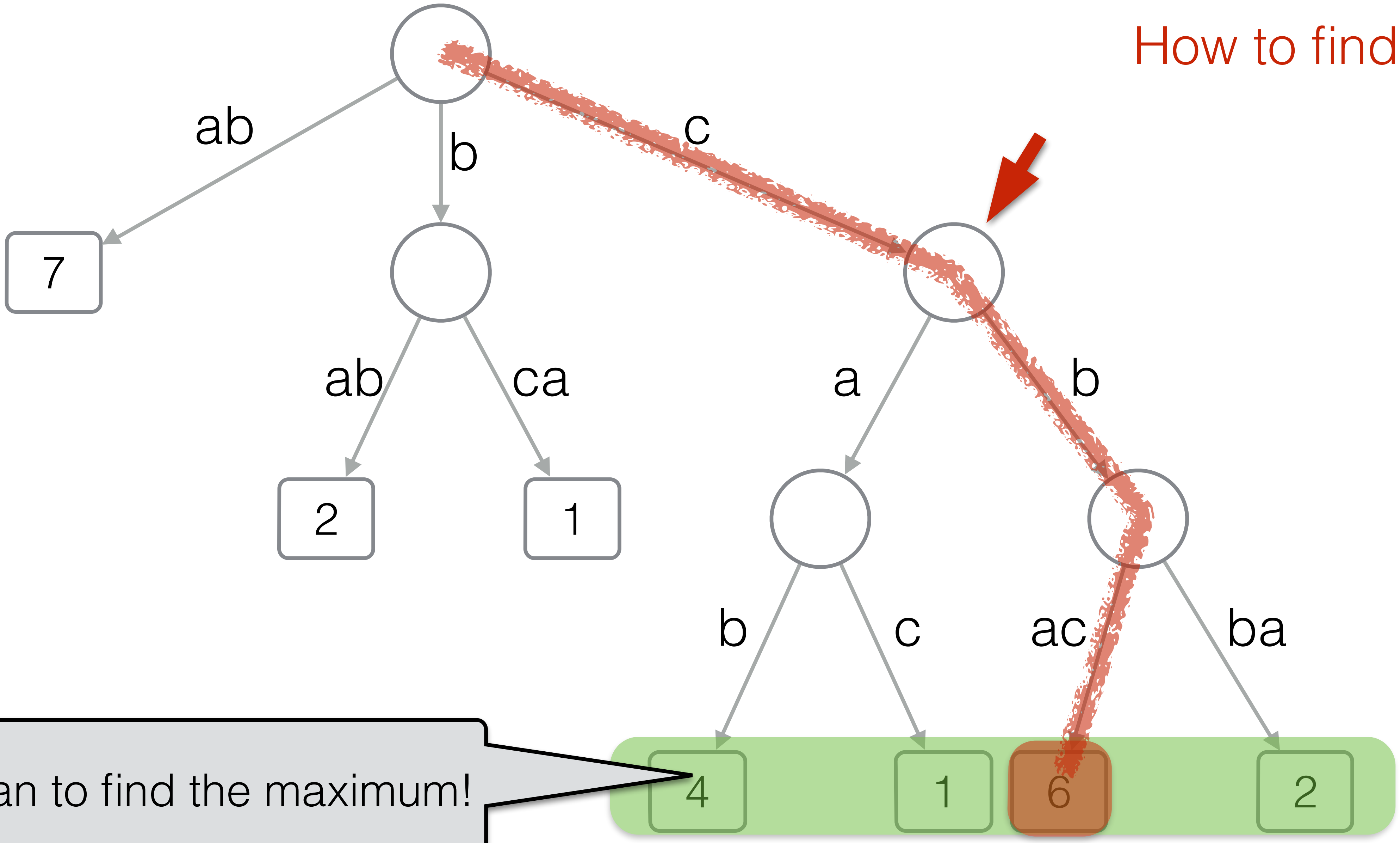


Scan to find the maximum!

O(n) query time :-(

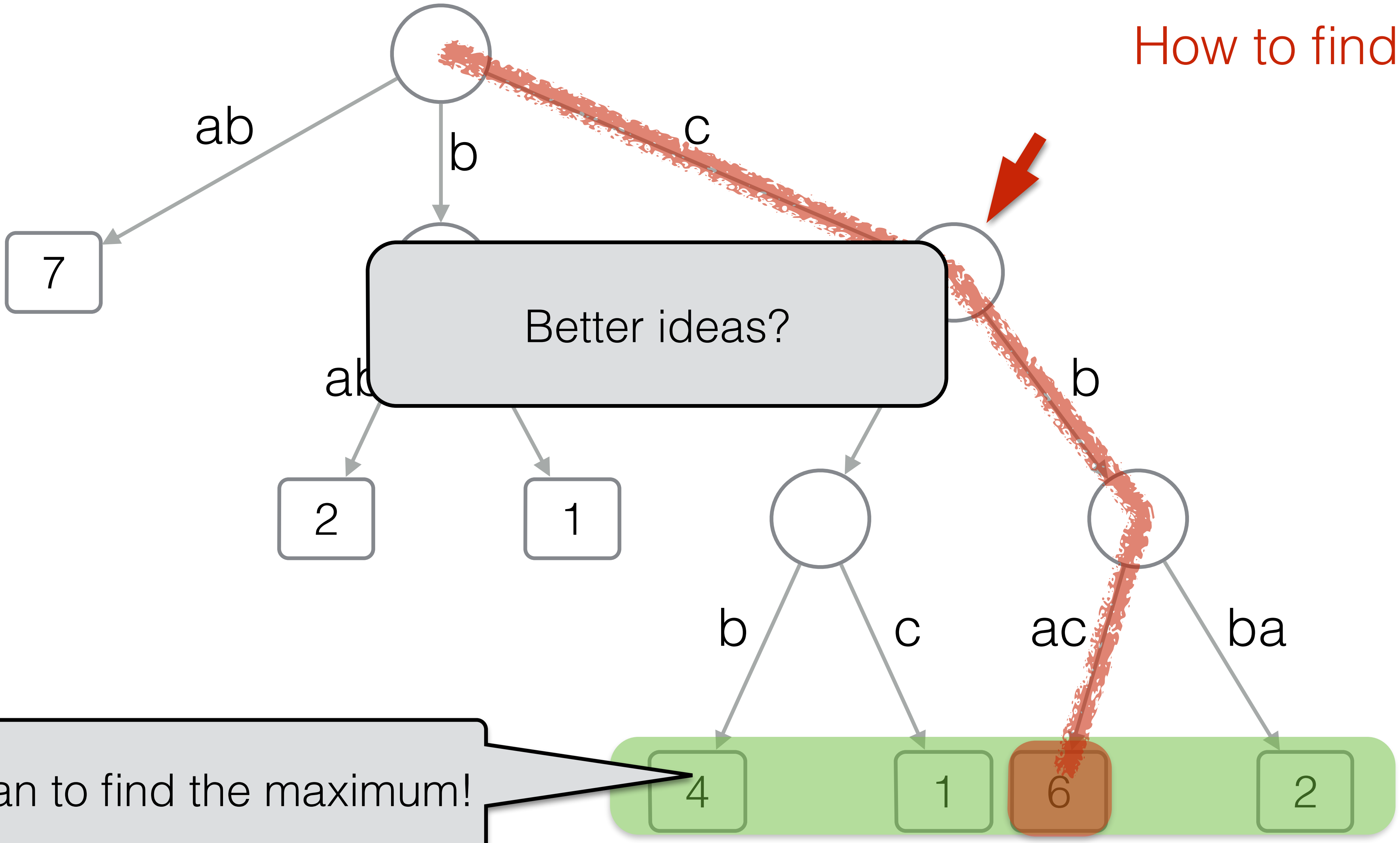
D = { ab (4), bab (2), bca (1), cab (2), cac (1), cbac (3), cbba (2) }

n = |D|, m total length of strings in D

Finding Top-1

P = c

How to find Top-1?



Better ideas?

Scan to find the maximum!

O(n) query time :-(

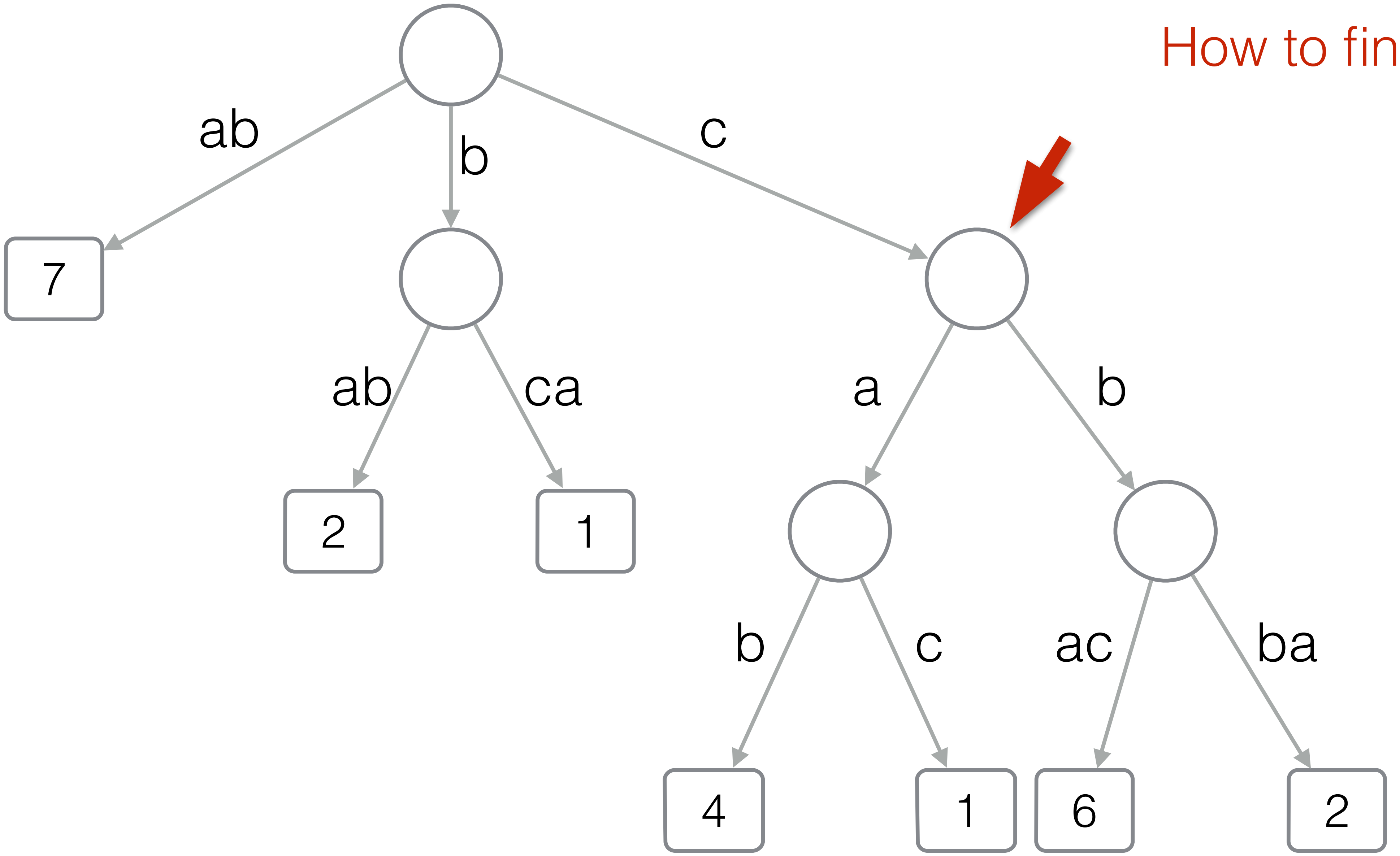
D = { ab (4), bab (2), bca (1), cab (2), cac (1), cbac (3), cbba (2) }

n = |D|, m total length of strings in D

Finding Top-1

P = c

How to find Top-1?



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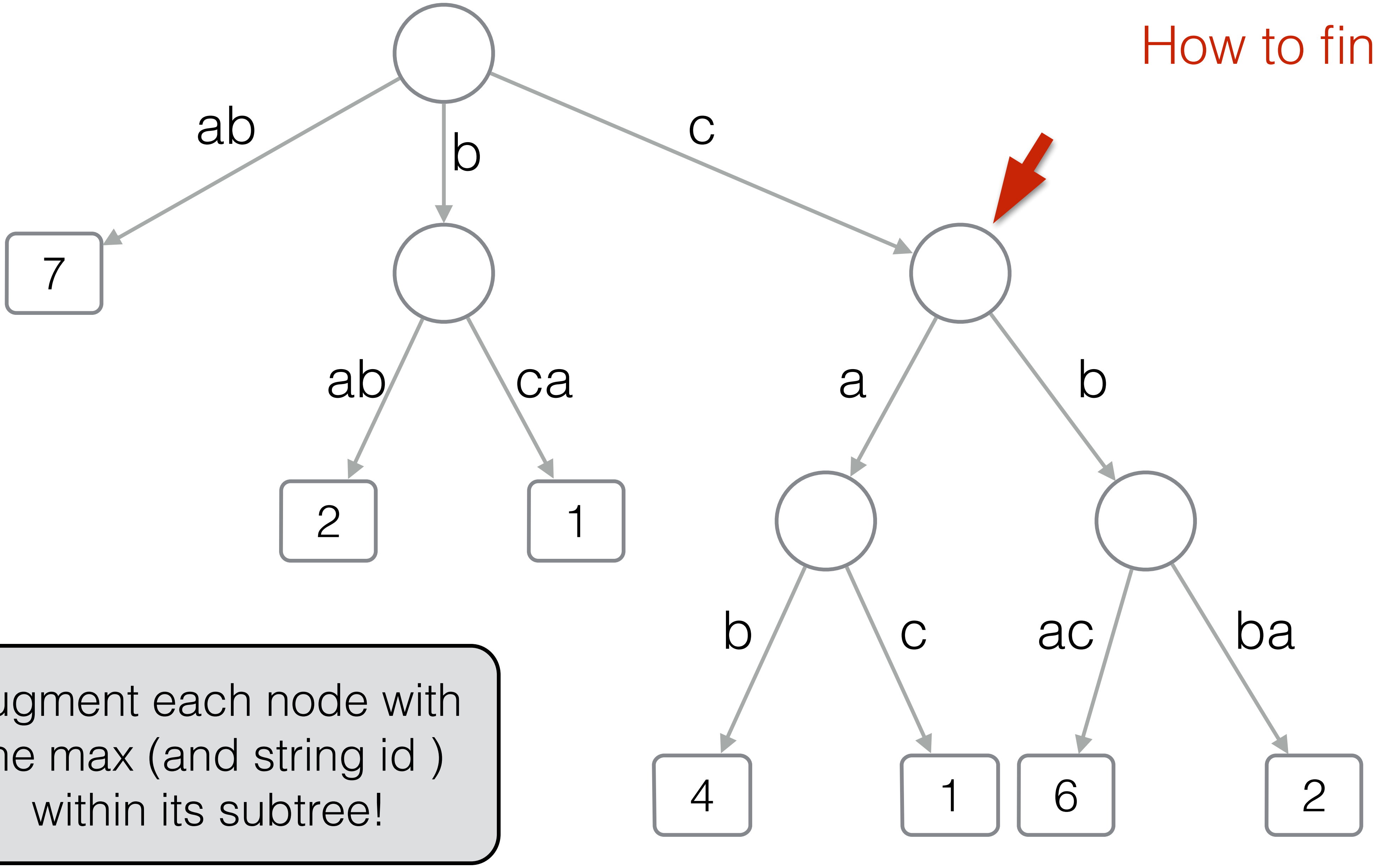
How to find Top-1?



Augment each node with the max (and string id) within its subtree!

D = { ab (4), bab (2), bca (1), cab (2), cac (1), cbac (3), cbba (2) }

n = |D|, m total length of strings in D

Finding Top-1

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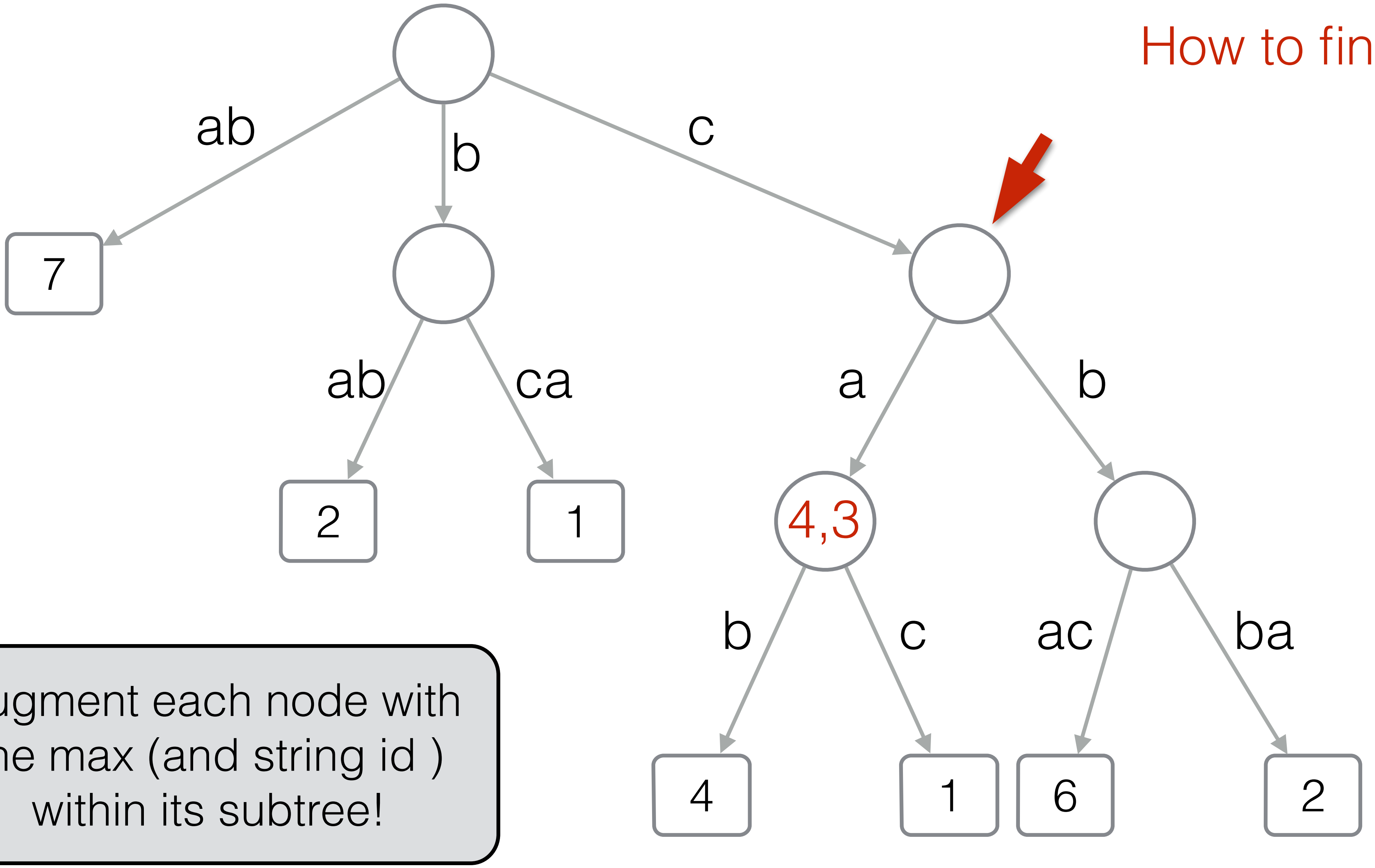
How to find Top-1?



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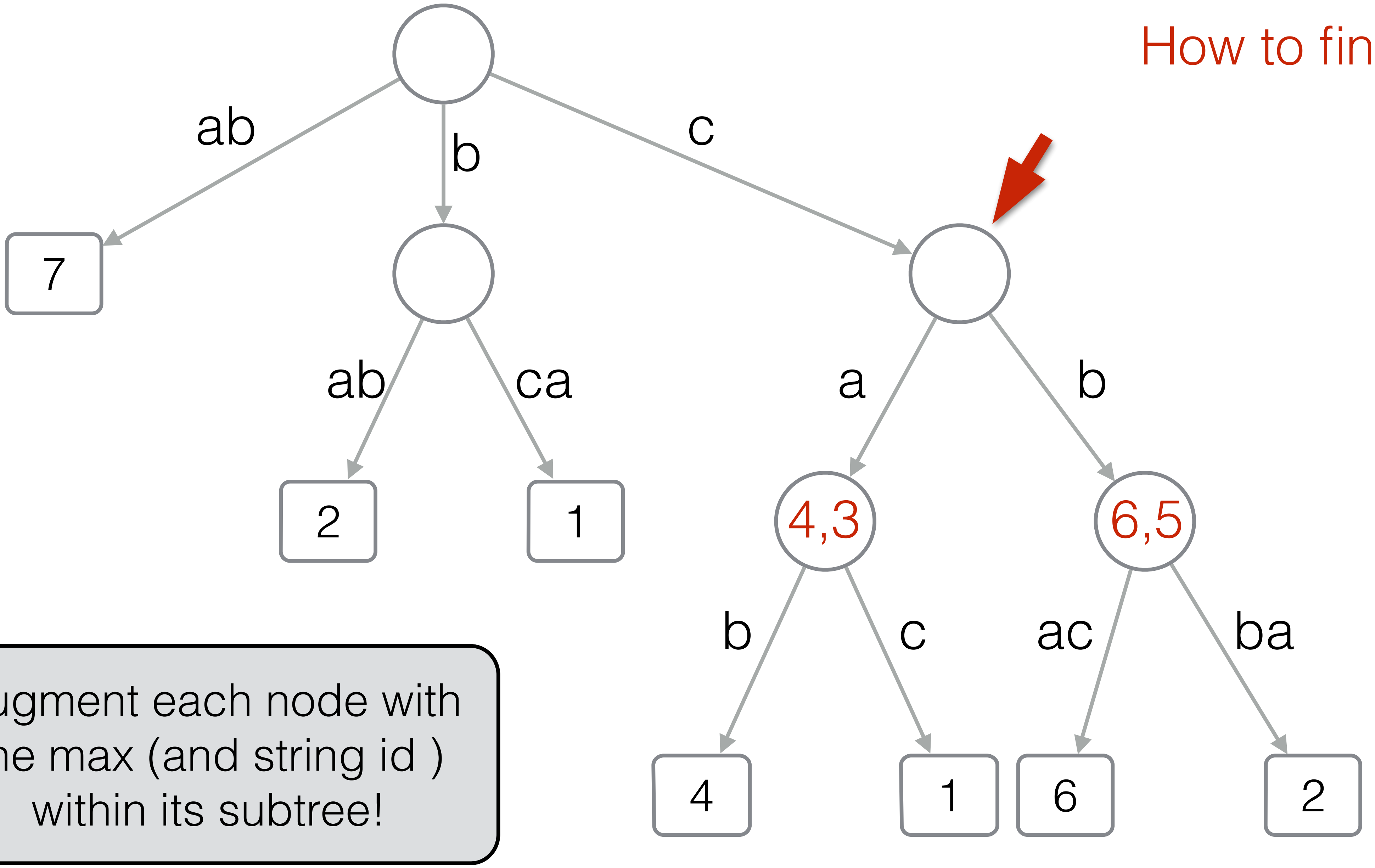

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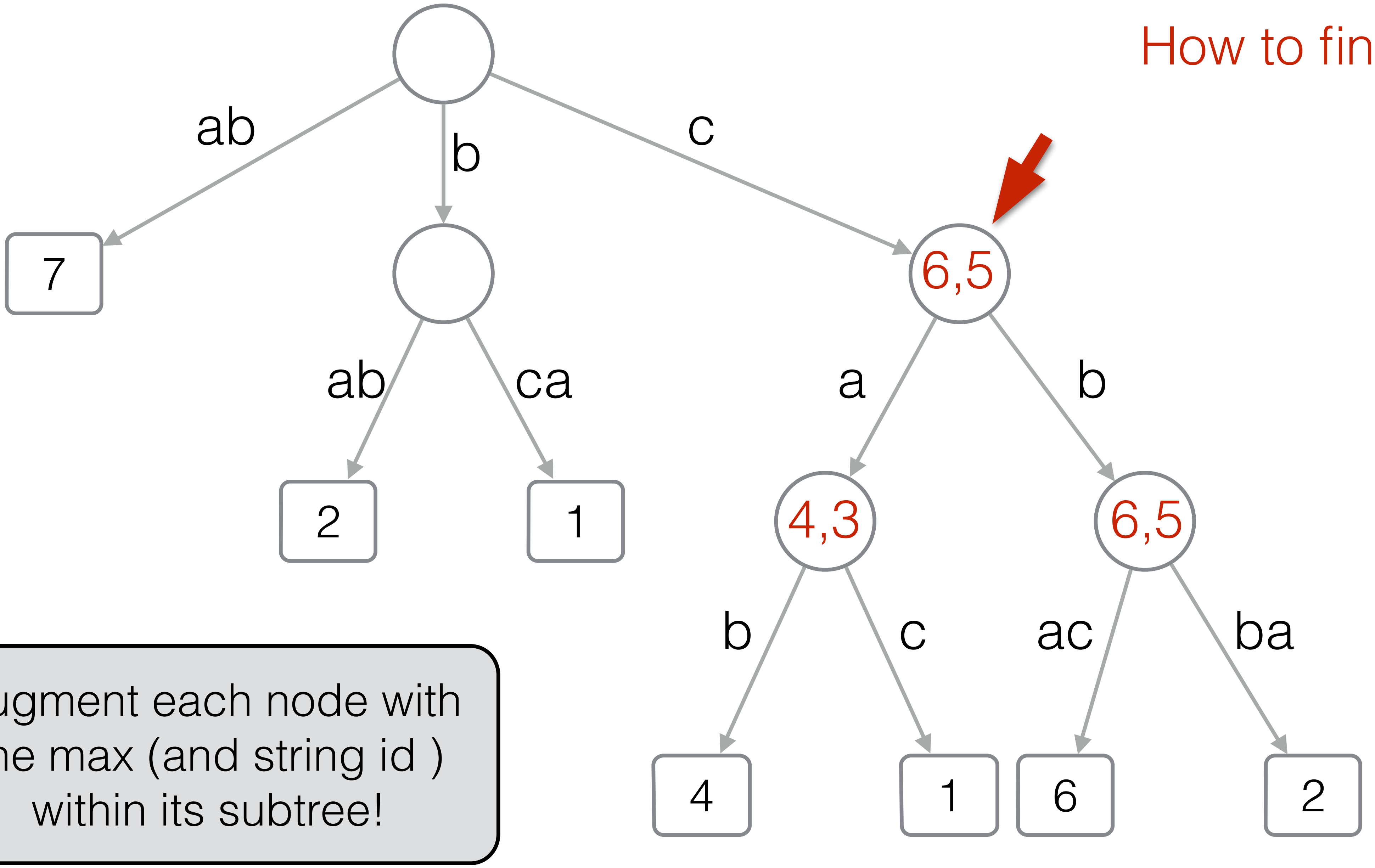

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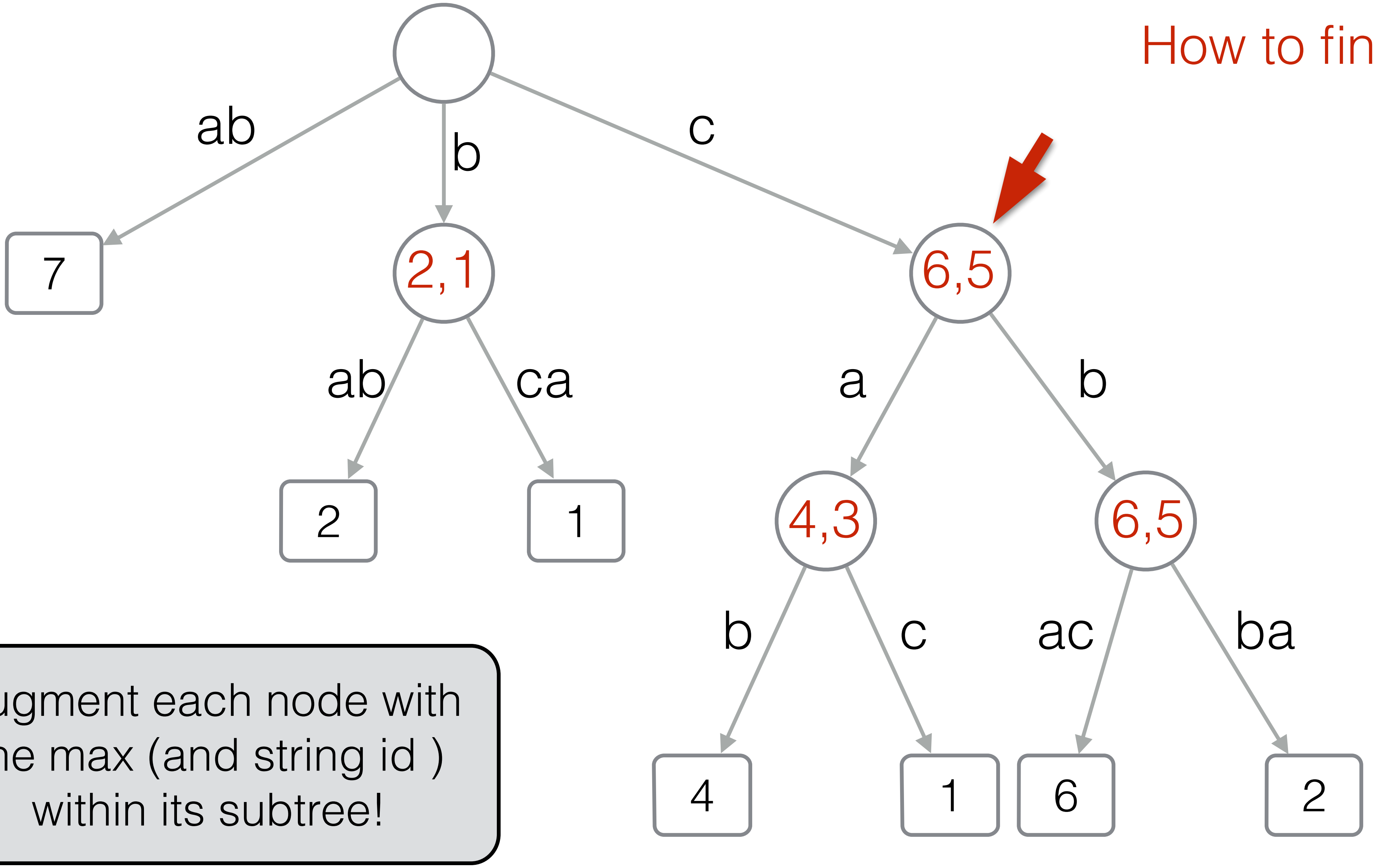

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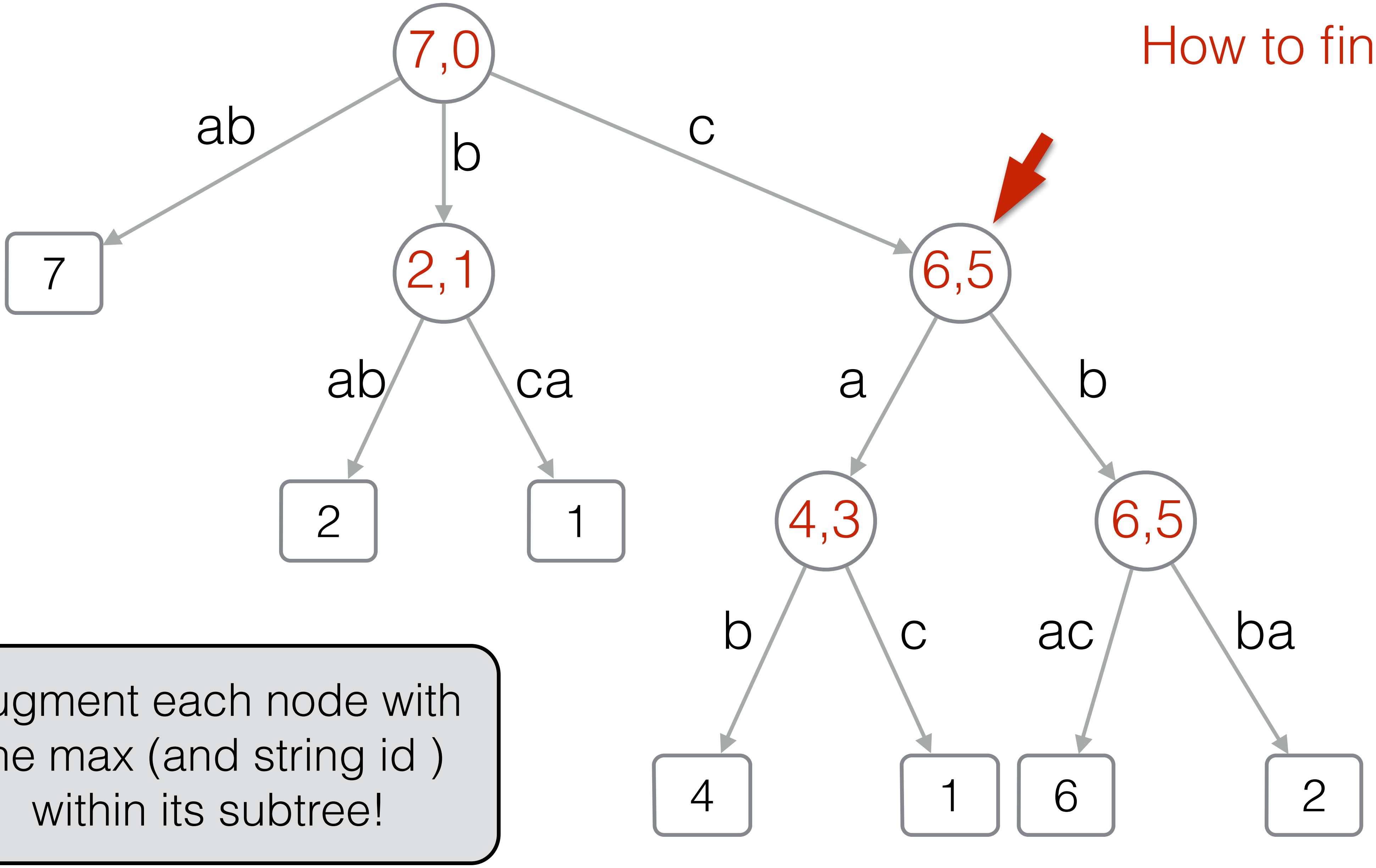

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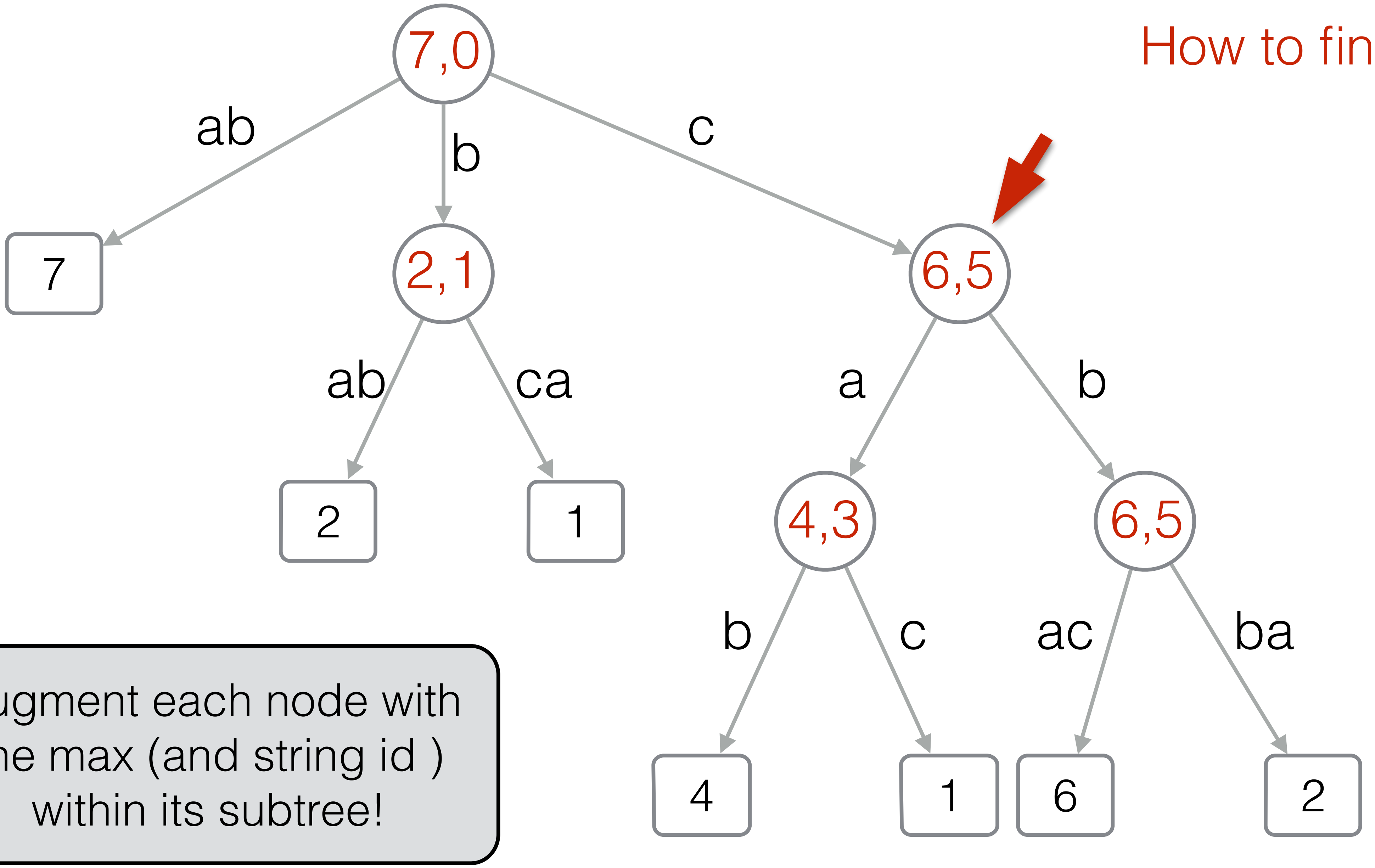

Augment each node with the max (and string id) within its subtree!

Preprocessing time: O(n)

Extra space: O(n log n) bits

Query time: O(1)

D (1), cab (2), cac (1), cbac (3), cbba (2) }

l length of strings in D

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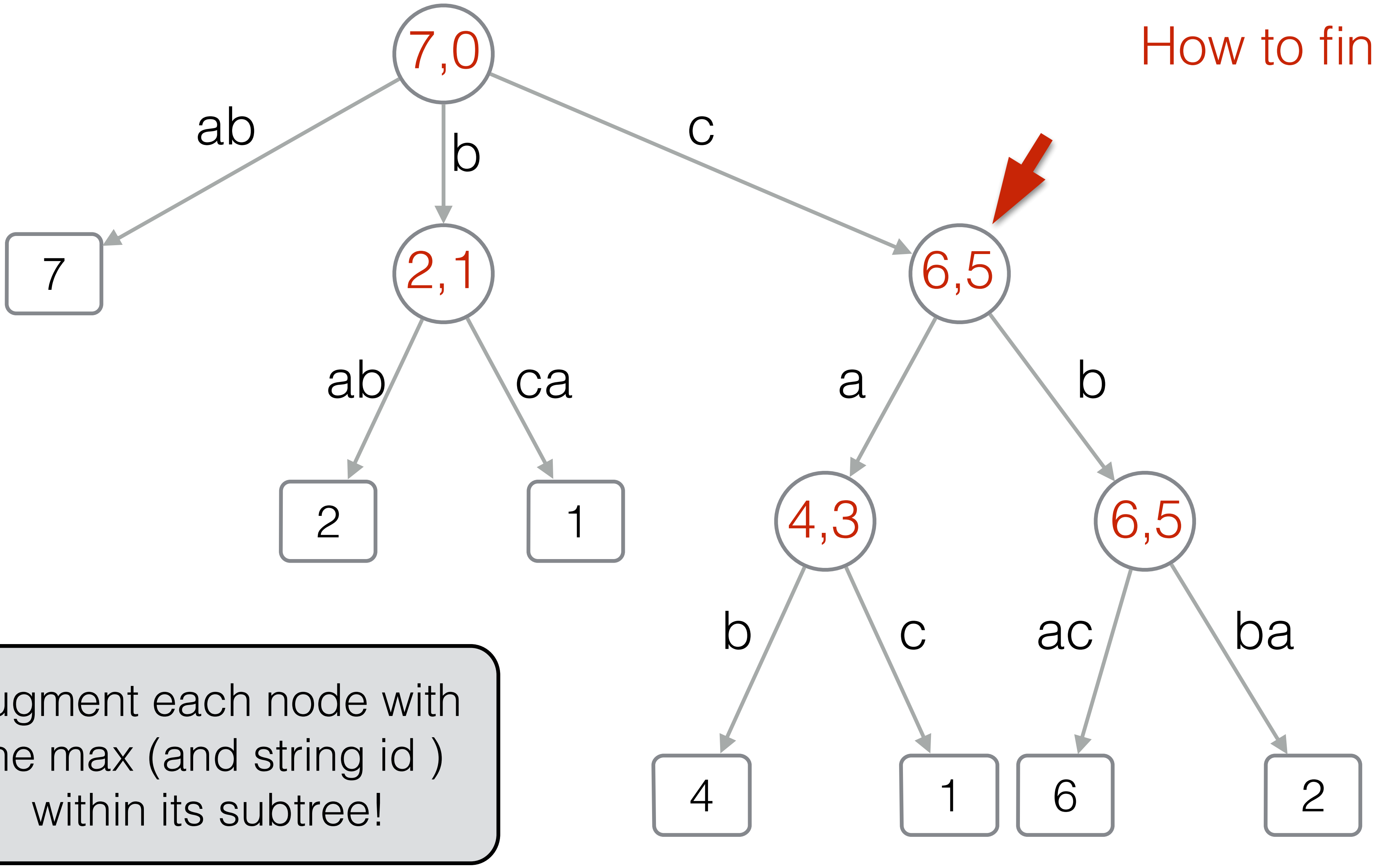
How to find Top-1?



Augment each node with the max (and string id) within its subtree!

Preprocessing time: O(n)

Extra space: O(n log n) bits

Query time: O(1)

Solving Top-k?

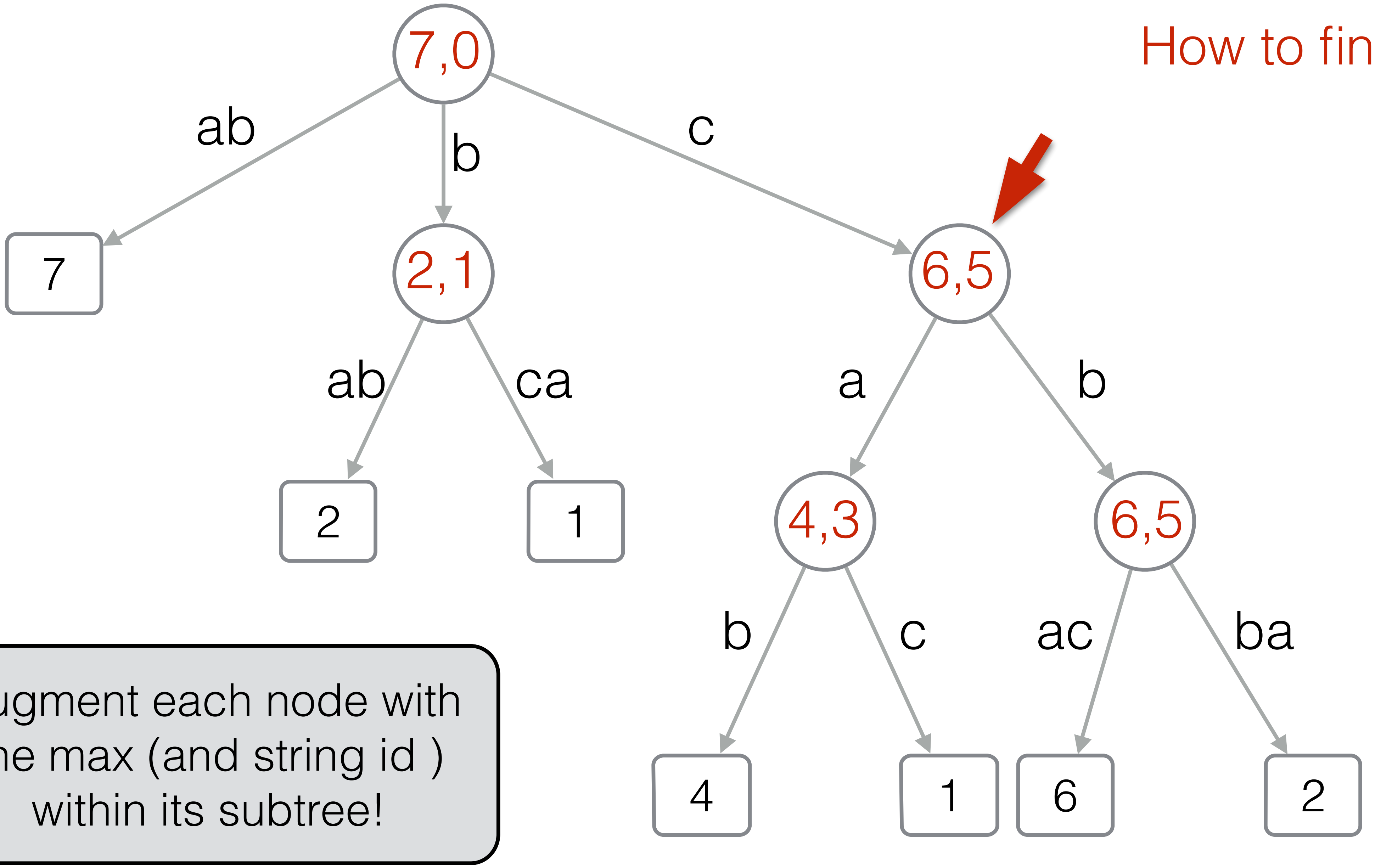
D (1), ca

l length of strings in D

Finding Top-1

P = c

How to find Top-1?



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Preprocessing time: O(n)

Extra space: O(n log n) bits

Query time: O(1)

D (1), ca

l length of strings in D

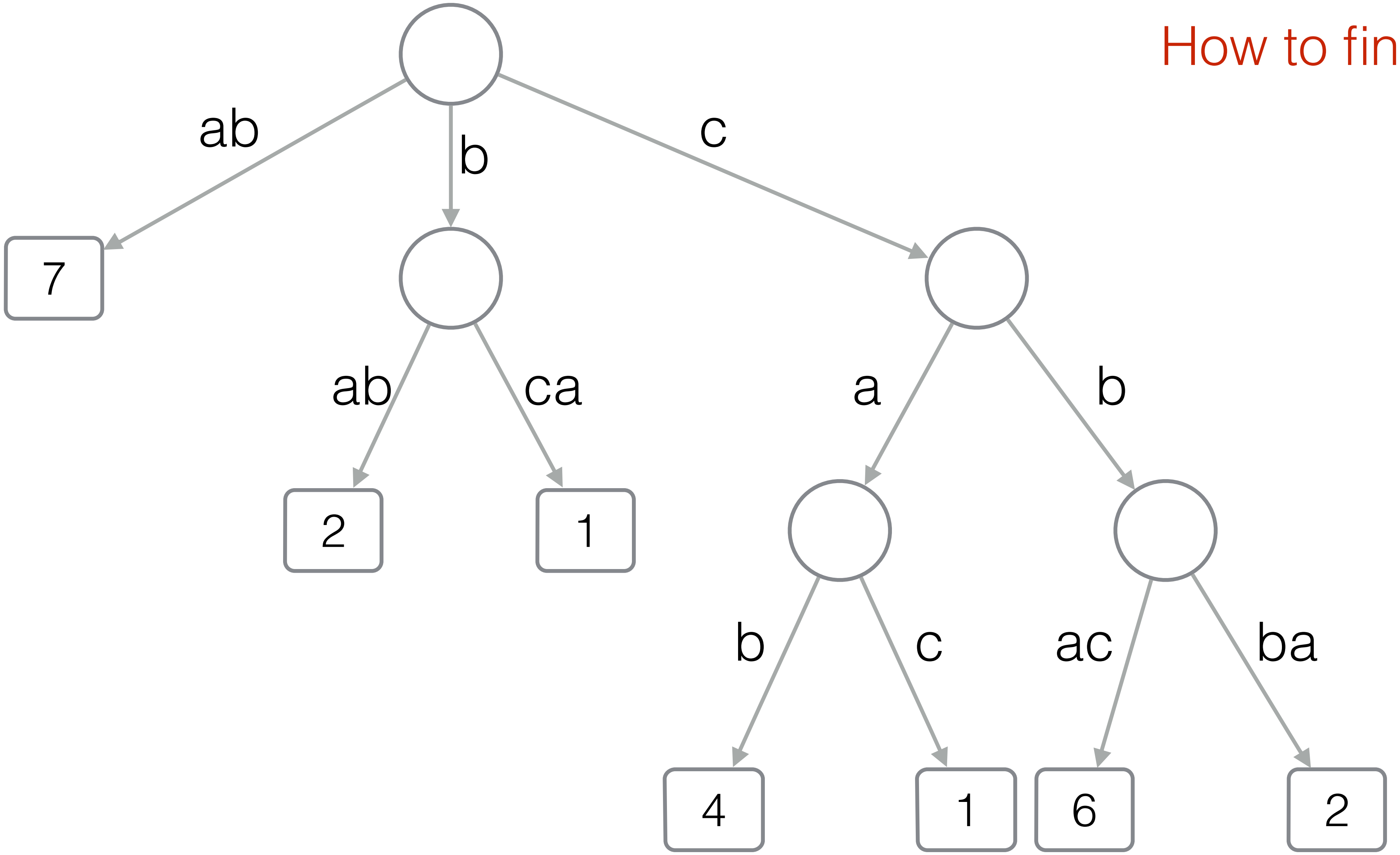
Solving Top-k?

- Extra space: O(k*n*log n) bits :-(
- You must know k at building time! :-(

Finding Top-1

P = c

How to find Top-1?



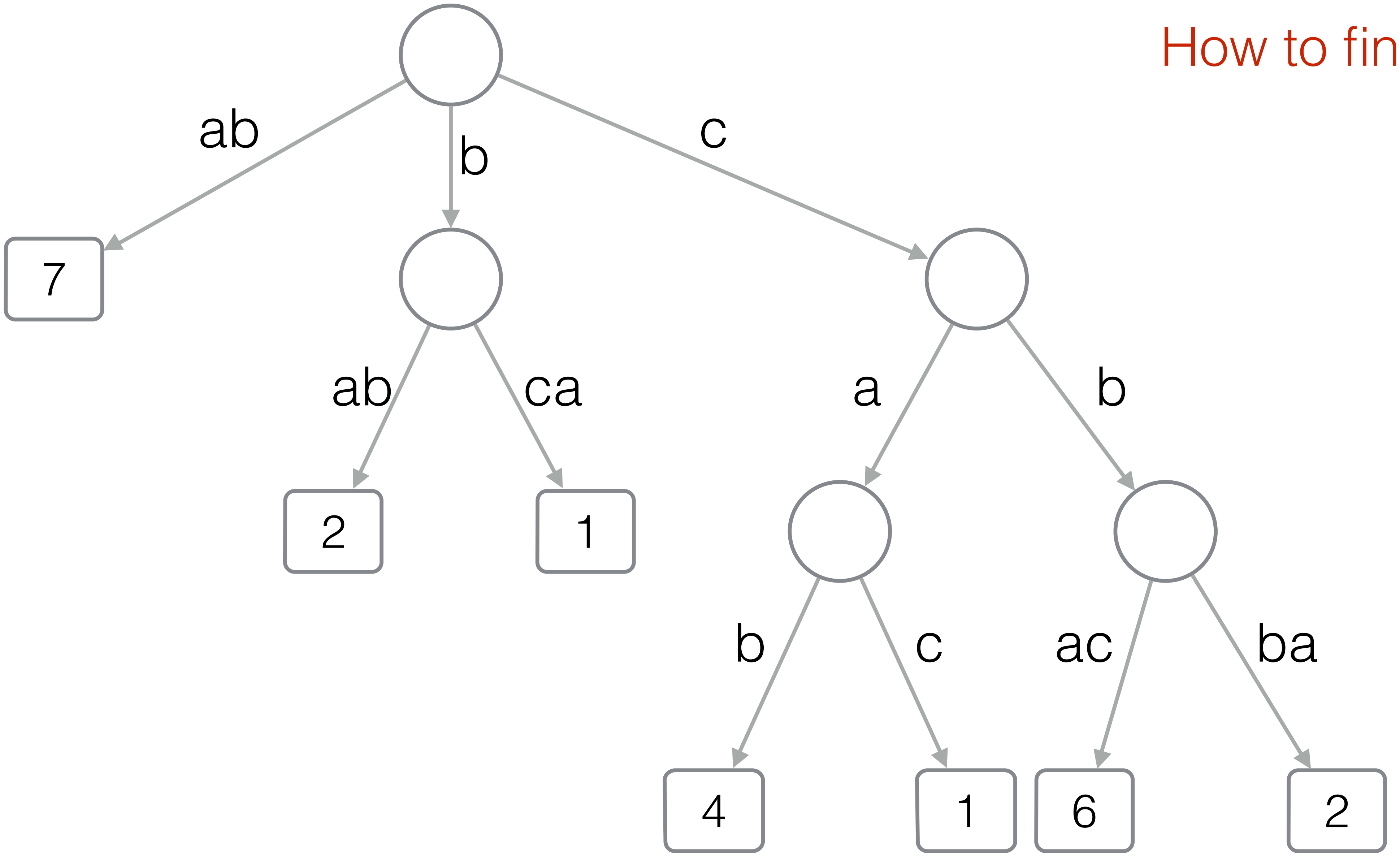
D = { ab (4), bab (2), bca (1), cab (2), cac (1), cbac (3), cbba (2) }

n = |D|, m total length of strings in D

Finding Top-1

P = c

How to find Top-1?



S							
	7	2	1	4	1	6	2

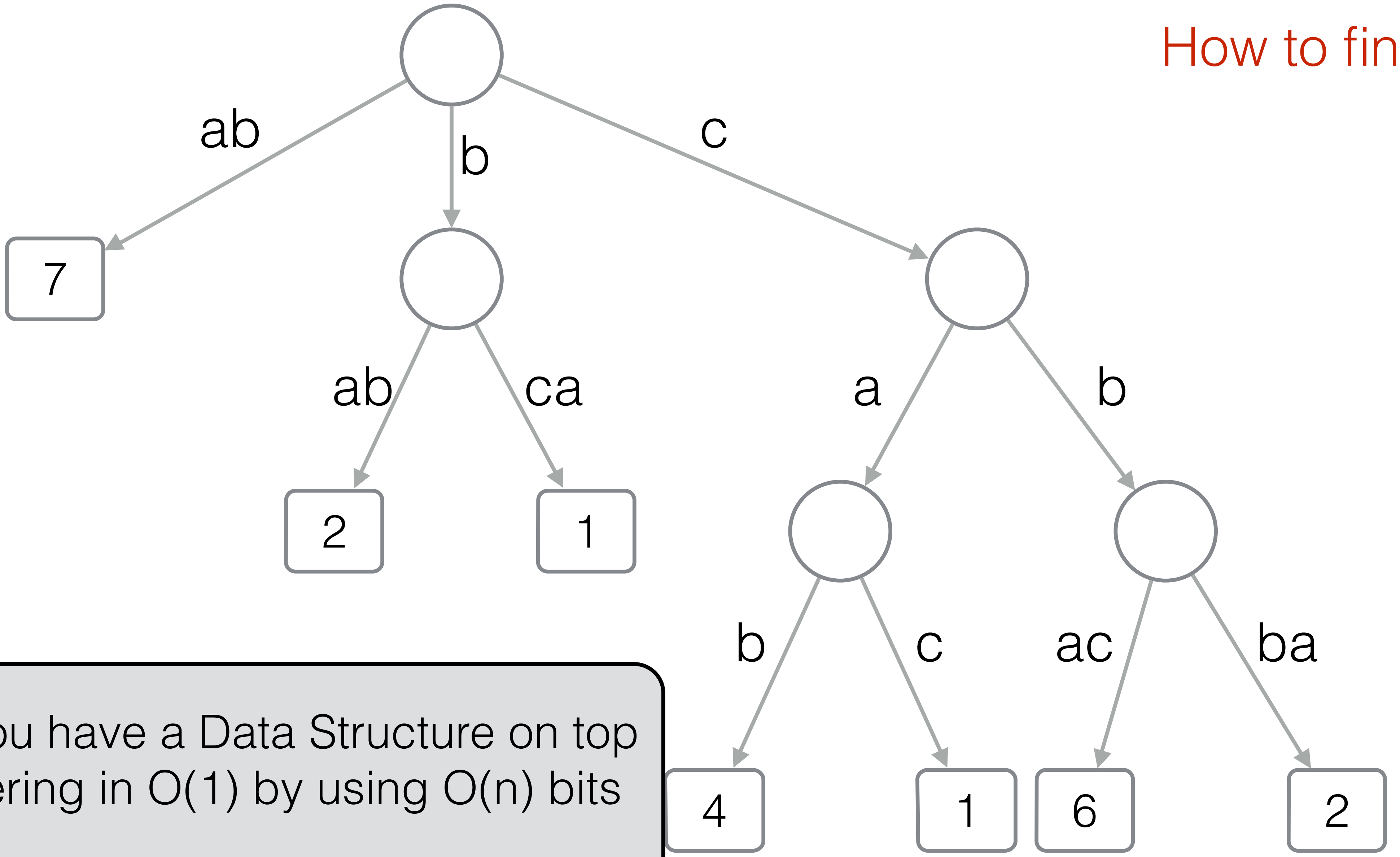
D = { ab (4), bab (2), bca (1), cab (2), cac (1), cbac (3), cbba (2) }

n = |D|, m total length of strings in D

Finding Top-1

P = c

How to find Top-1?



Assume you have a Data Structure on top of S answering in O(1) by using O(n) bits

RMQ(i,j) = position of the maximum in the range S[i,j]

S							
	7	2	1	4	1	6	2

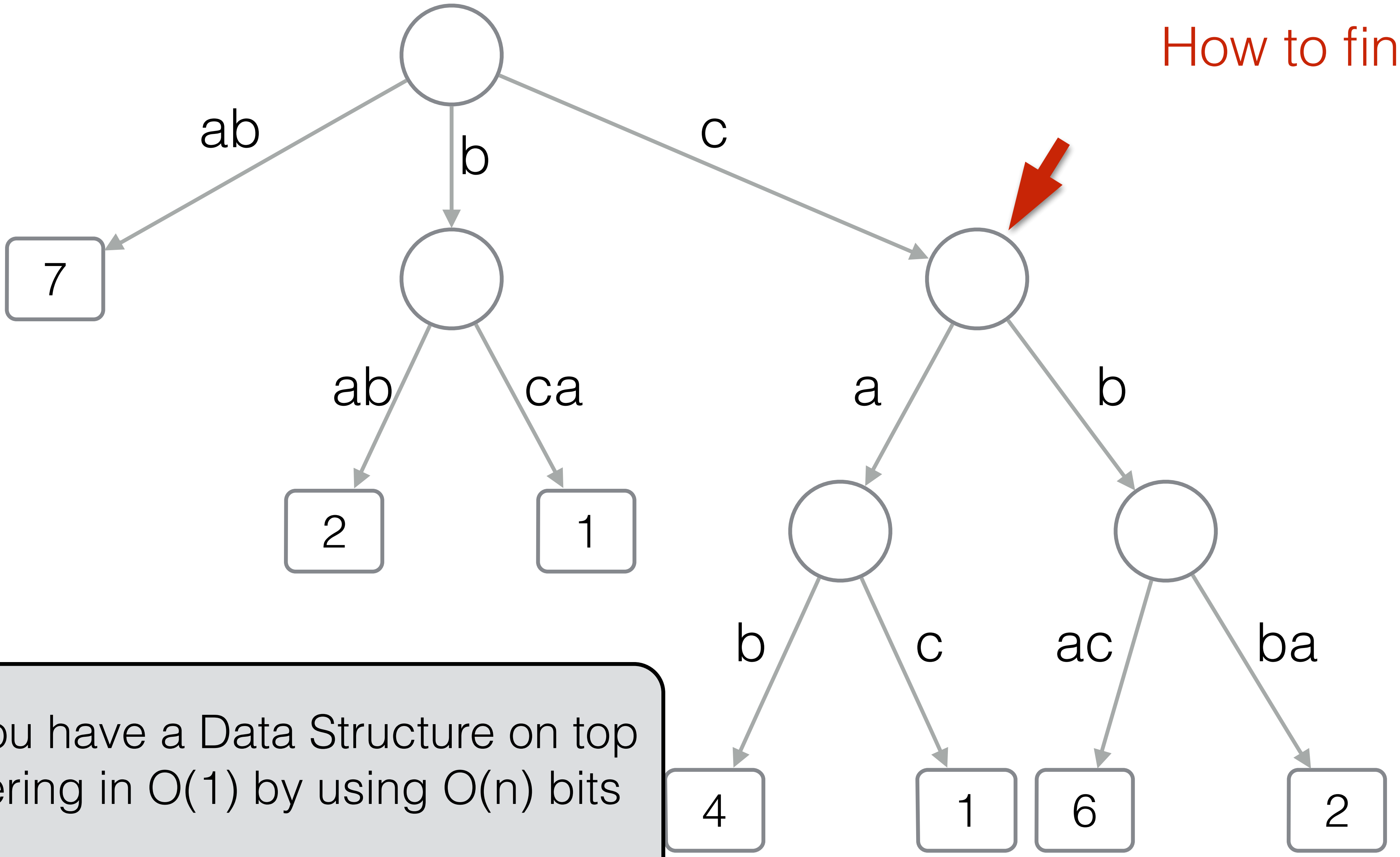
D = { ab (4), bab (2), bca (1), cab (2), cac (1), cbac (3), cbba (2) }

n = |D|, m total length of strings in D

Finding Top-1

P = c

How to find Top-1?



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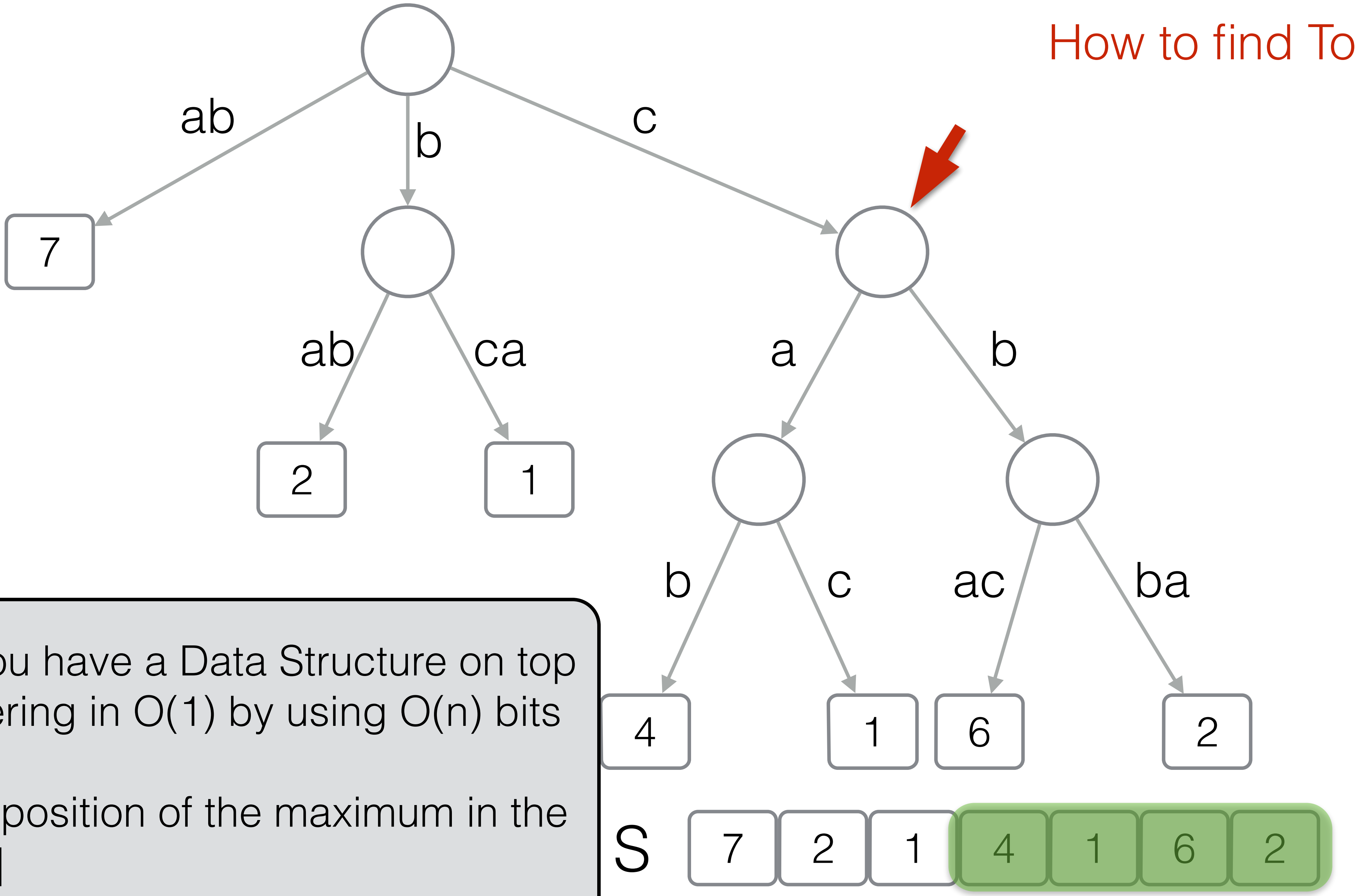
D = { ab (4), bab (2), bca (1), cab (2), cac (1), cbac (3), cbba (2) }

n = |D|, m total length of strings in D

Finding Top-1

P = c

How to find Top-1?



Assume you have a Data Structure on top of S answering in O(1) by using O(n) bits

RMQ(i,j) = position of the maximum in the range S[i,j]

S						
7	2	1	4	1	6	2

RMQ(3,6) = 5

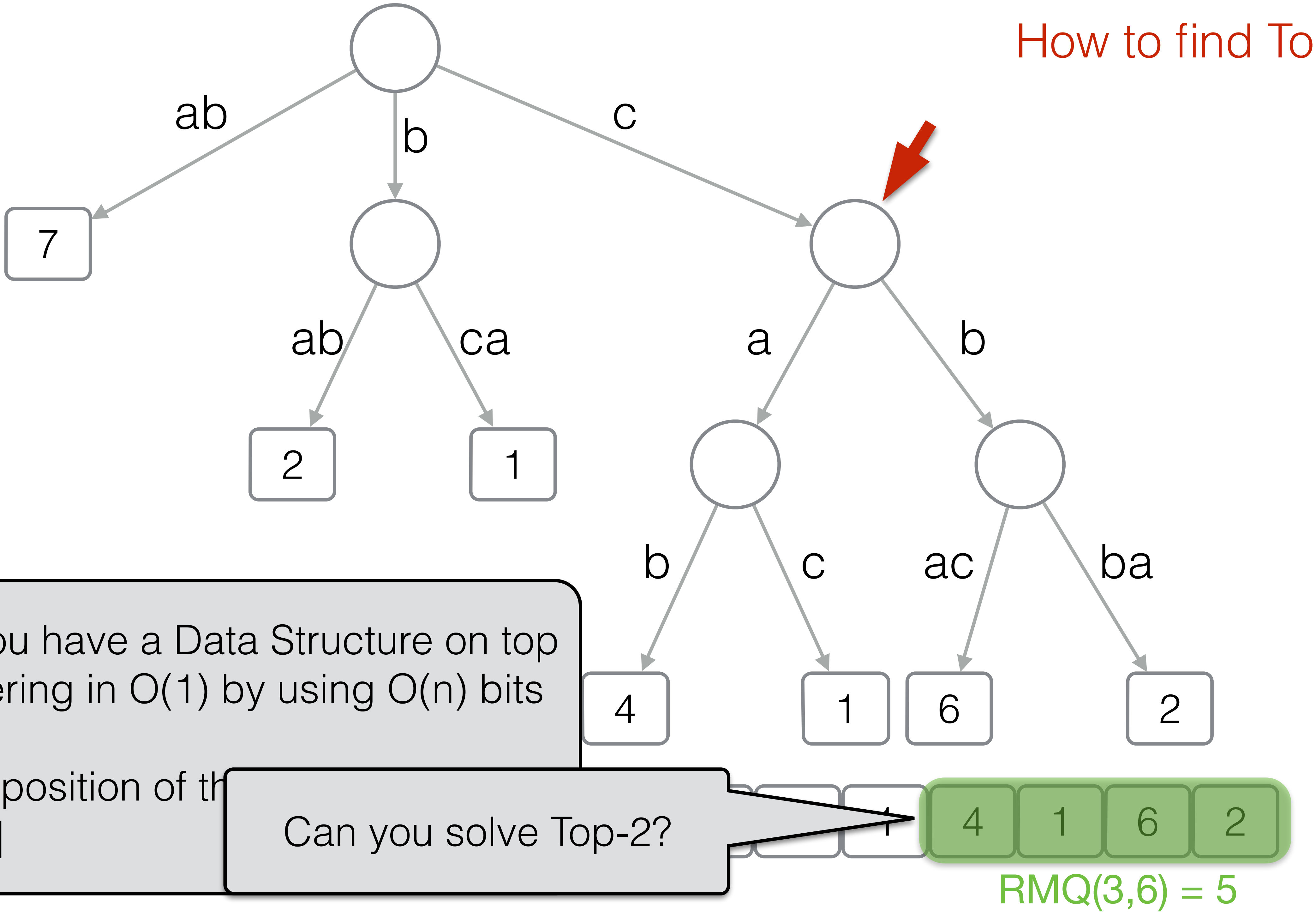
D = { ab (4), bab (2), bca (1), cab (2), cac (1), cbac (3), cbba (2) }

n = |D|, m total length of strings in D

Finding Top-1

P = c

How to find Top-1?



Assume you have a Data Structure on top of S answering in O(1) by using O(n) bits

RMQ(i,j) = position of th

range S[i,j]

Can you solve Top-2?

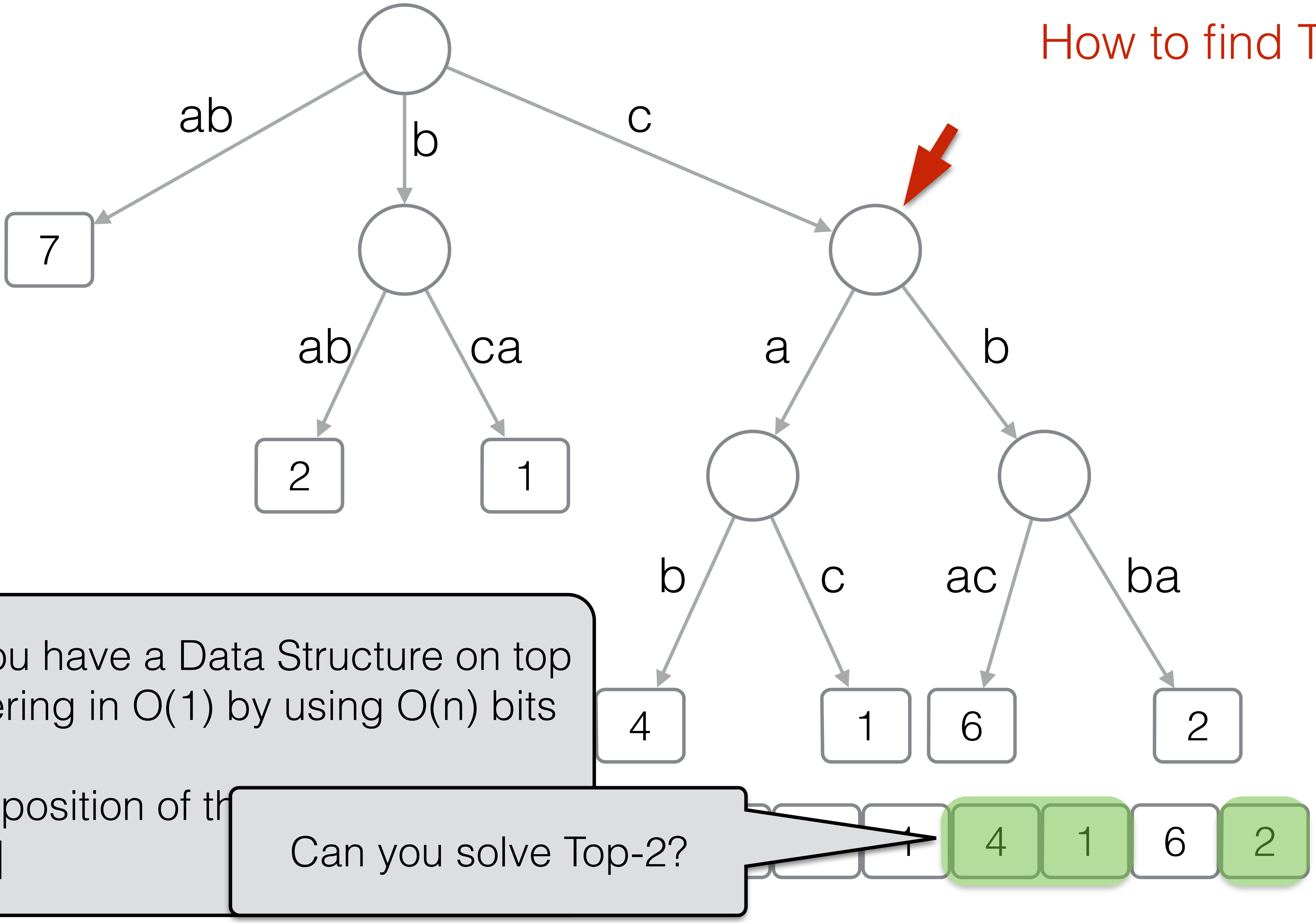
D = { ab (4), bab (2), bca (1), cab (2), cac (1), cbac (3), cbba (2) }

n = |D|, m total length of strings in D

Finding Top-1

P = c

How to find Top-1?



Assume you have a Data Structure on top of S answering in O(1) by using O(n) bits

RMQ(i,j) = position of th
range S[i,j]

Can you solve Top-2?

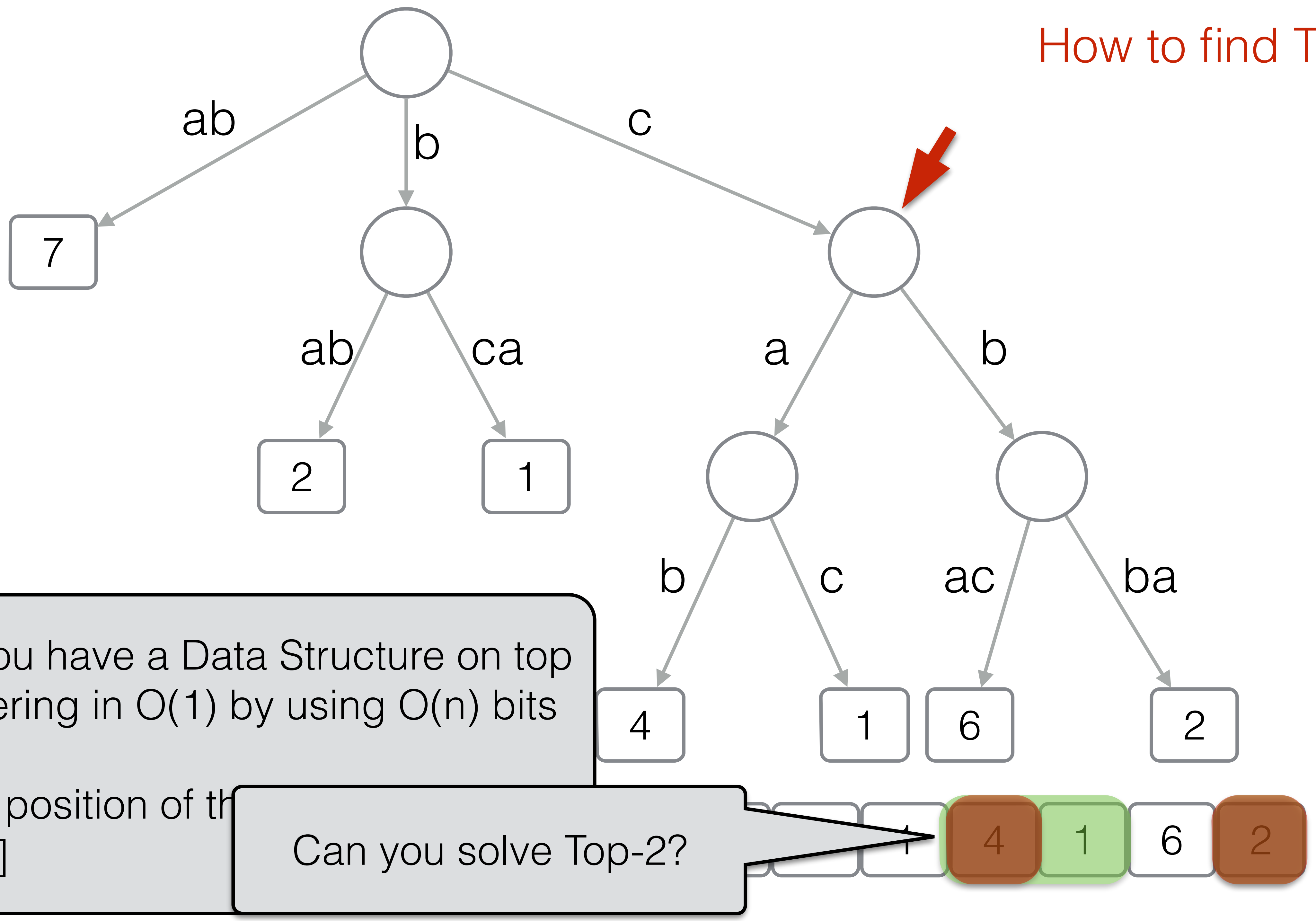
D = { ab (4), bab (2), bca (1), cab (2), cac (1), cbac (3), cbba (2) }

n = |D|, m total length of strings in D

Finding Top-1

P = c

How to find Top-1?



Assume you have a Data Structure on top of S answering in O(1) by using O(n) bits

RMQ(i,j) = position of th
range S[i,j]

Can you solve Top-2?

D = { ab (4), bab (2), bca (1), cab (2), cac (1), cbac (3), cbba (2) }

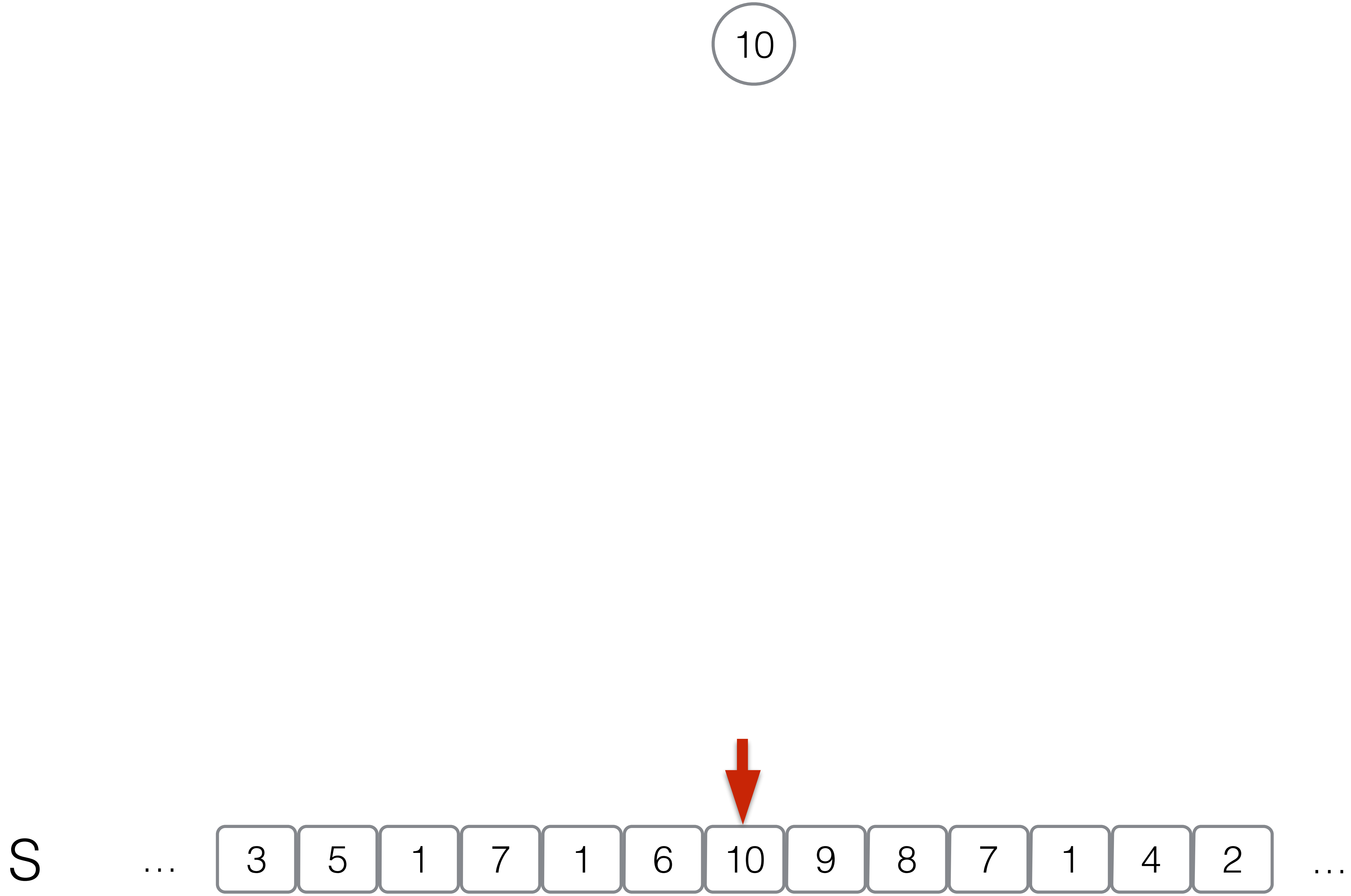
n = |D|, m total length of strings in D

Finding Top-k

Finding Top-k

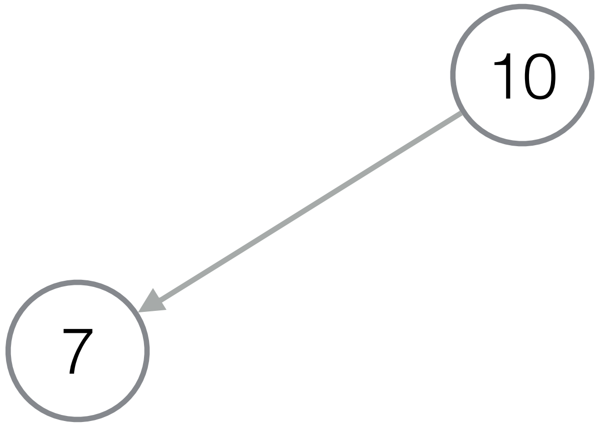
S	…	3	5	1	7	1	6	10	9	8	7	1	4	2	…

Finding Top-k



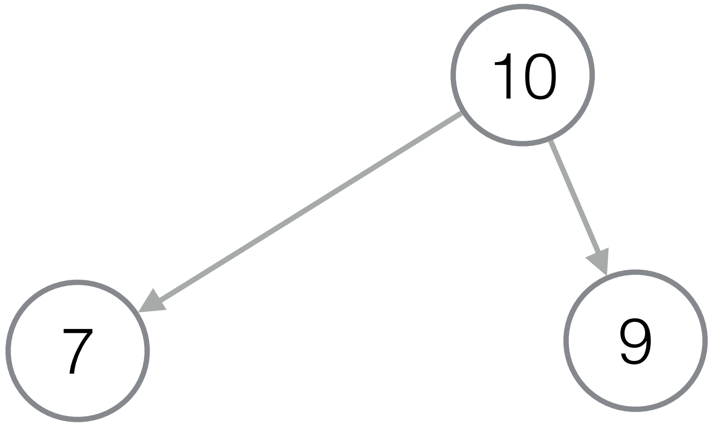
S … 3 5 1 7 1 6 10 9 8 7 1 4 2 …

Finding Top-k



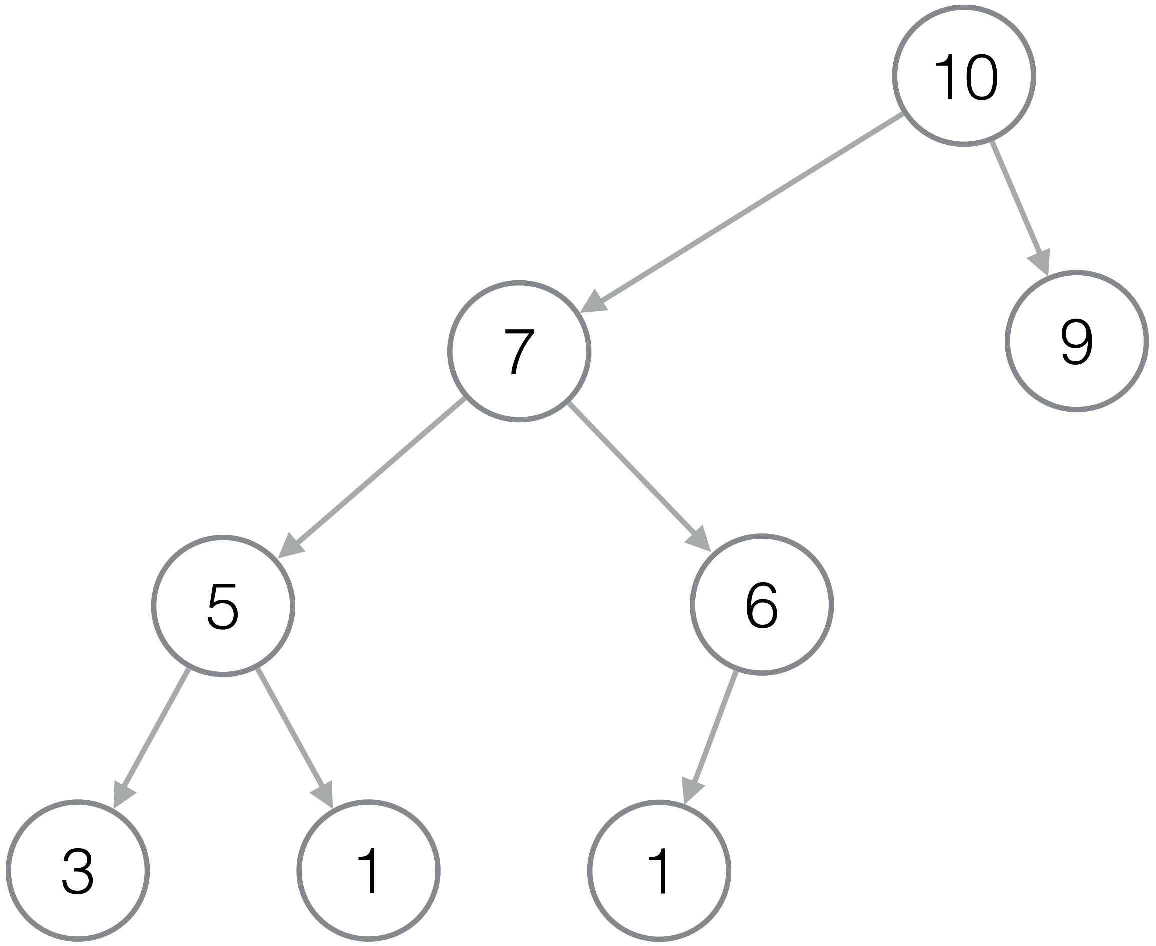
S	…	3	5	1	7	1	6	10	9	8	7	1	4	2	…

Finding Top-k



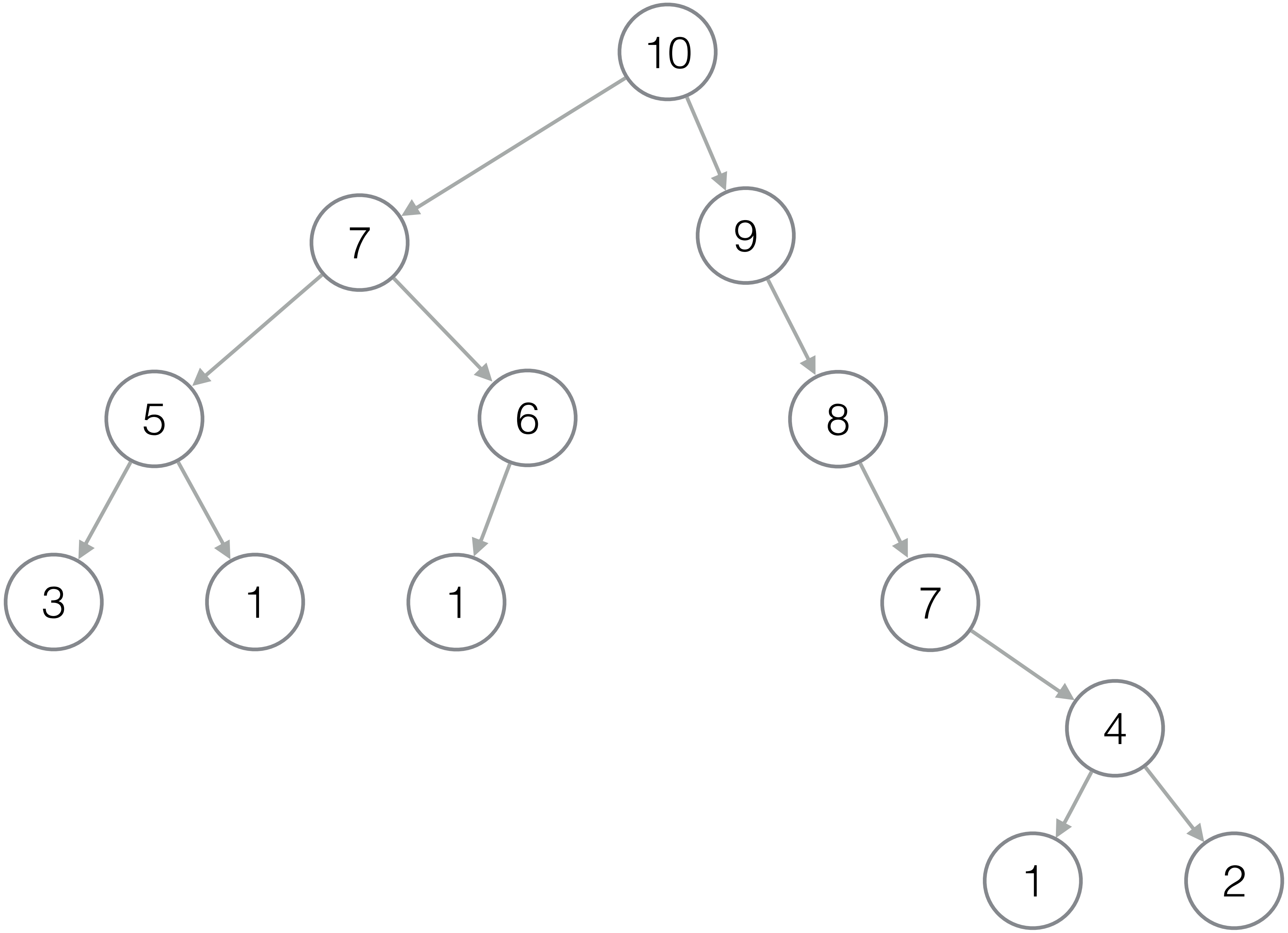
S …	3	5	1	7	1	6	10	9	8	7	1	4	2	…

Finding Top-k



S	…	3	5	1	7	1	6	10	9	8	7	1	4	2	…

Finding Top-k

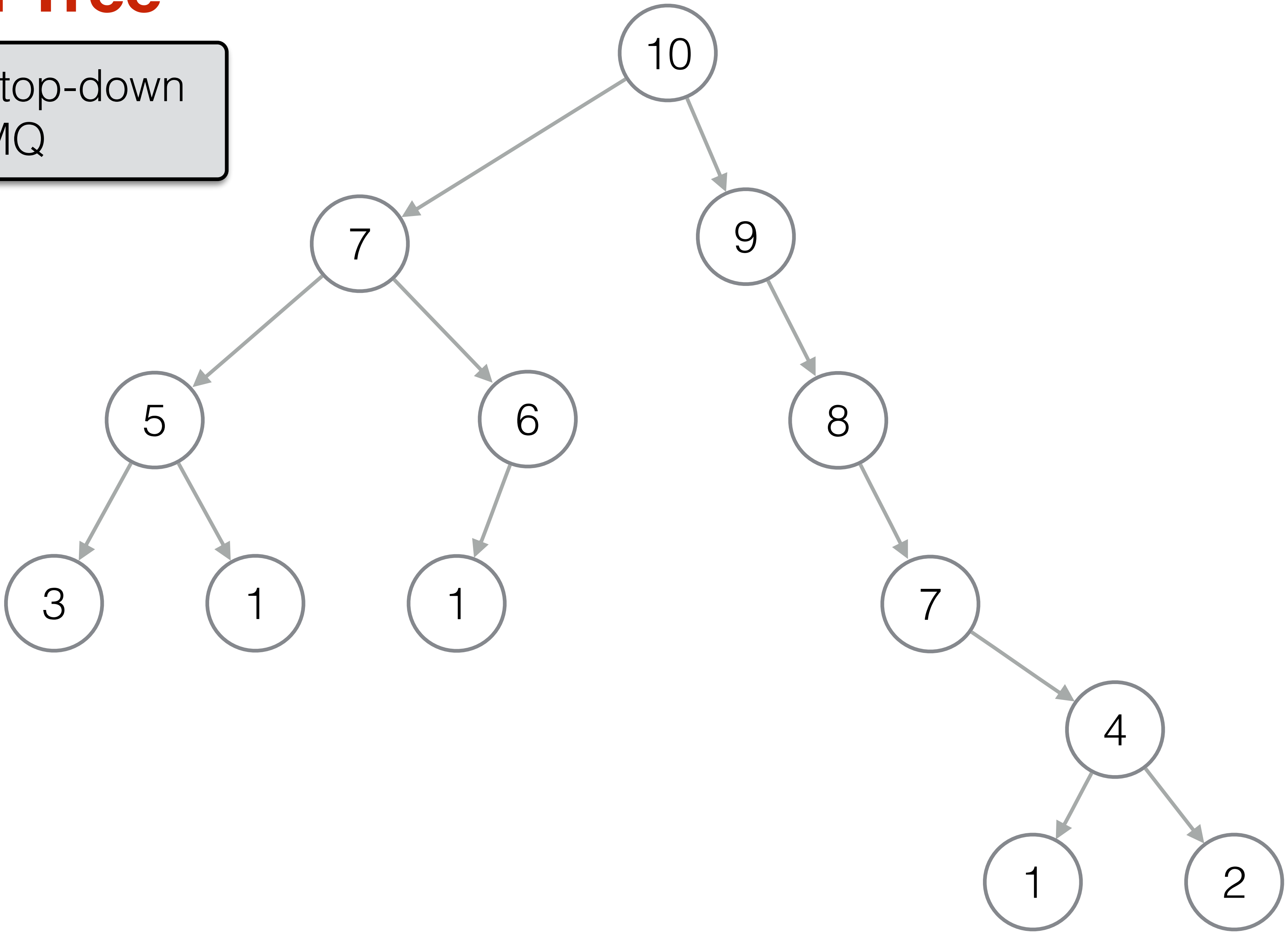


S …	3	5	1	7	1	6	10	9	8	7	1	4	2	…

Finding Top-k

Cartesian Tree

It can be built top-down with RMQ

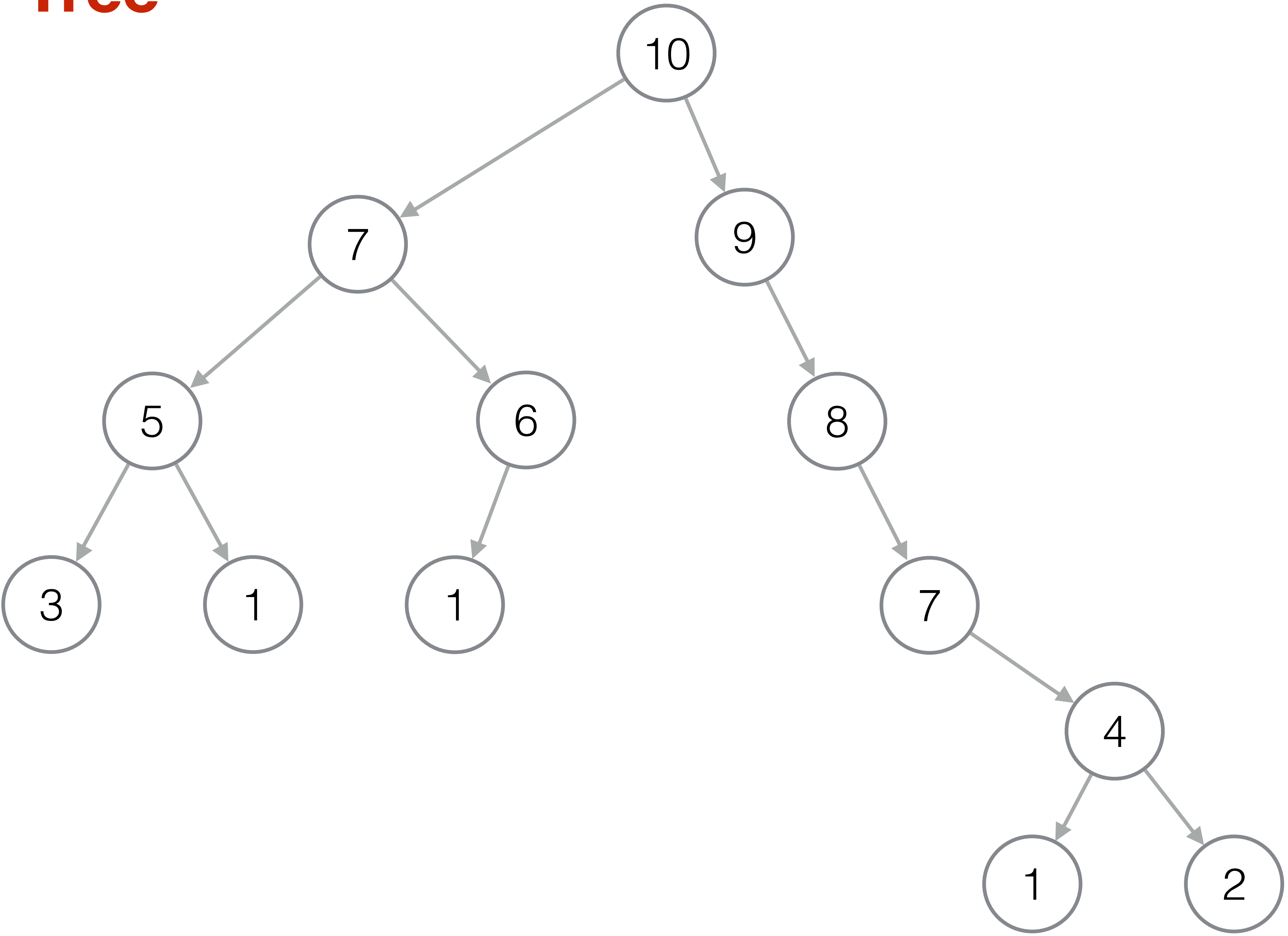


S	…	3	5	1	7	1	6	10	9	8	7	1	4	2	…

Finding Top-k

How to find Top-k?

Cartesian Tree



S …	3	5	1	7	1	6	10	9	8	7	1	4	2	…

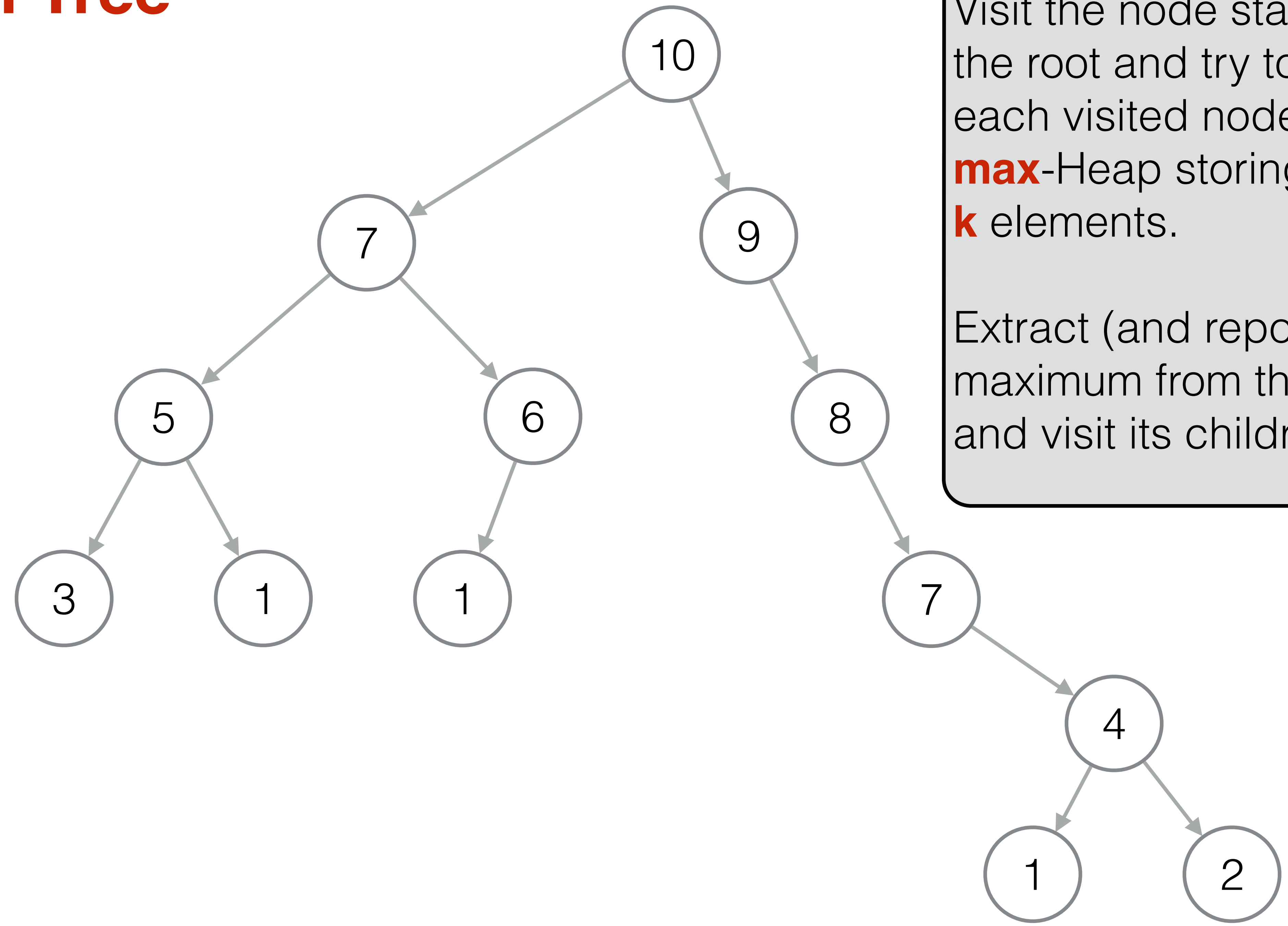
Finding Top-k

How to find Top-k?

Visit the node starting from the root and try to insert each visited node in a **max**-Heap storing at most **k** elements.

Extract (and report) the maximum from the heap and visit its children.

Cartesian Tree



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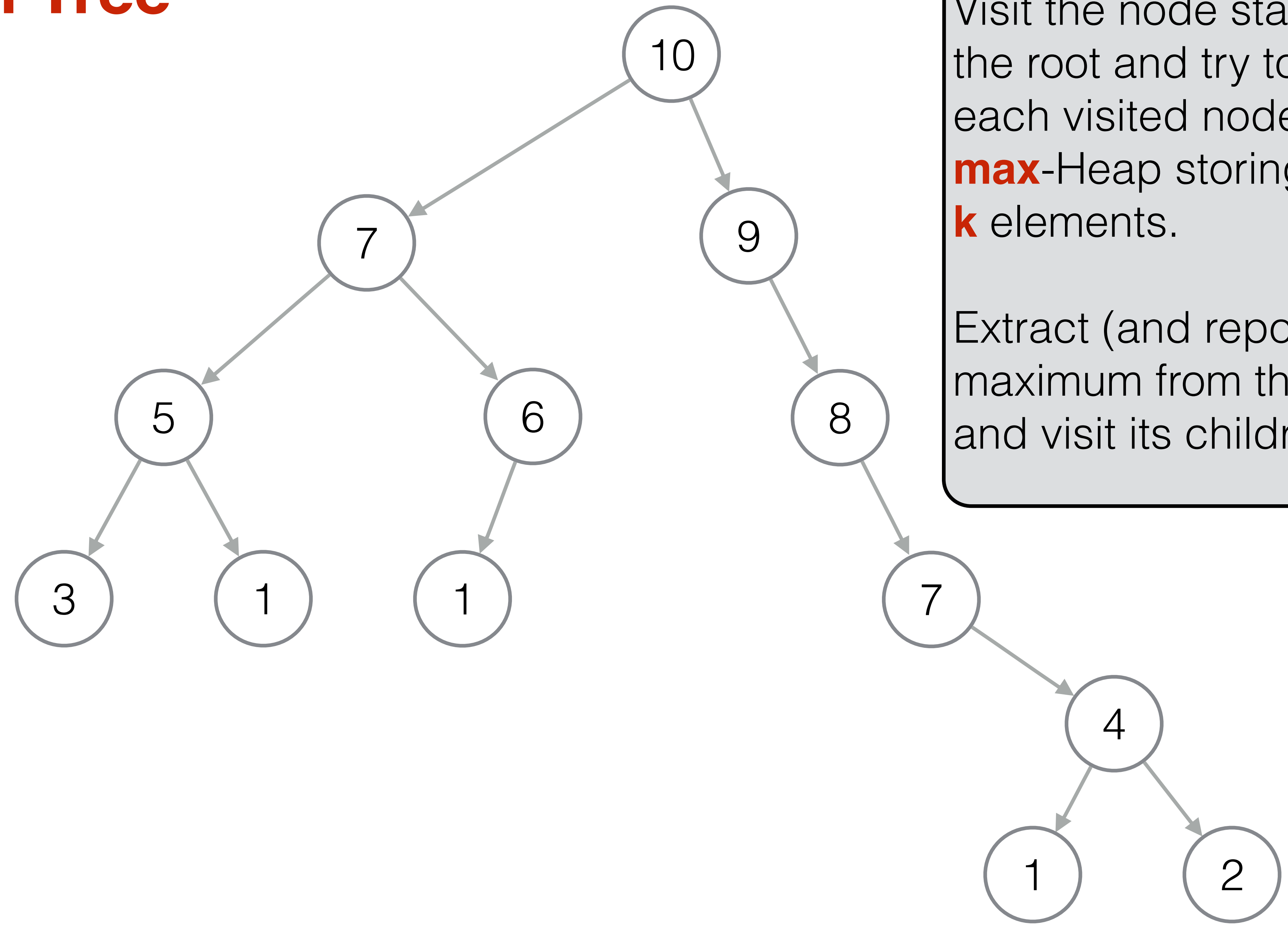

k=4

max-Heap

S	…	3	5	1	7	1	6	10	9	8	7	1	4	2	…

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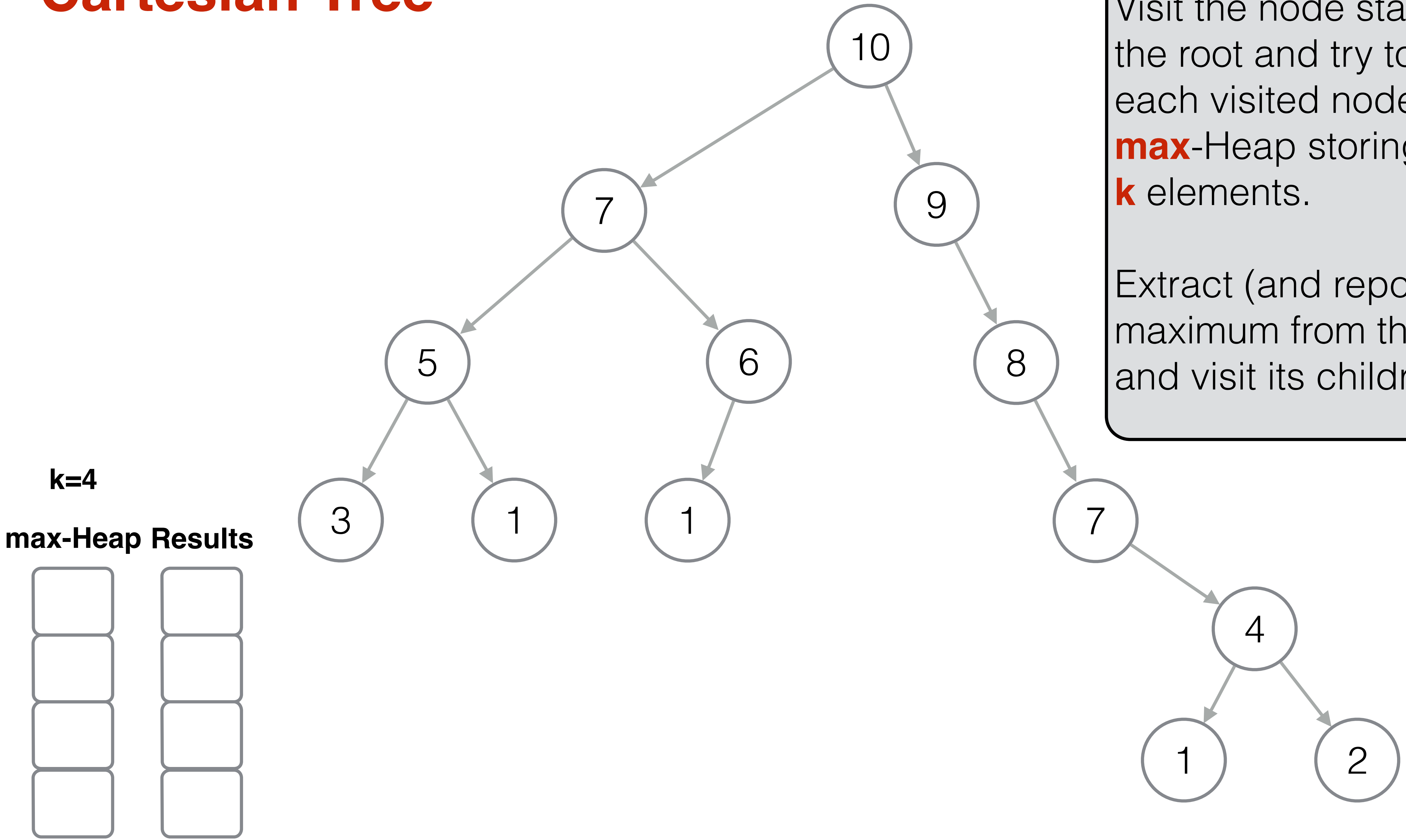

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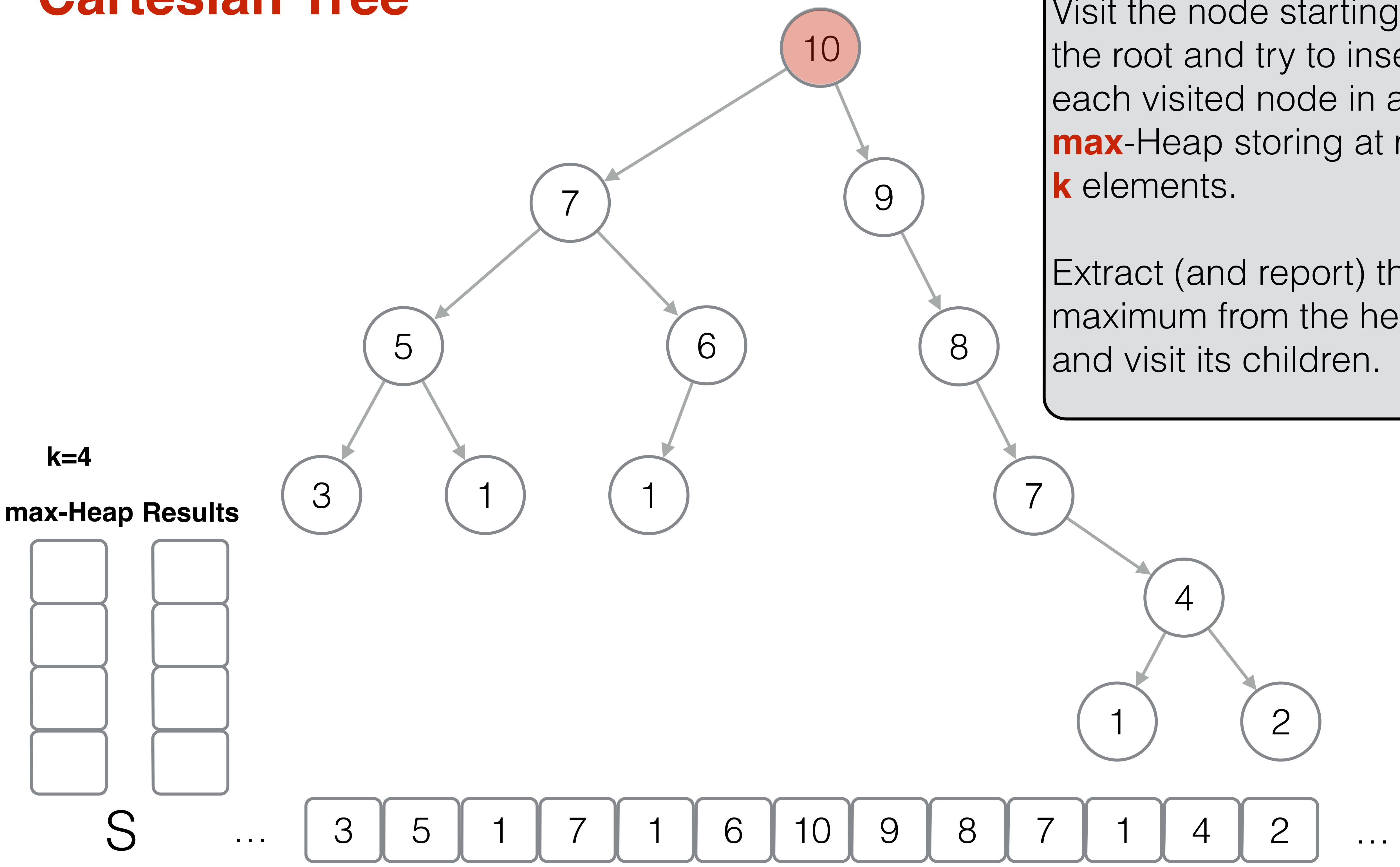

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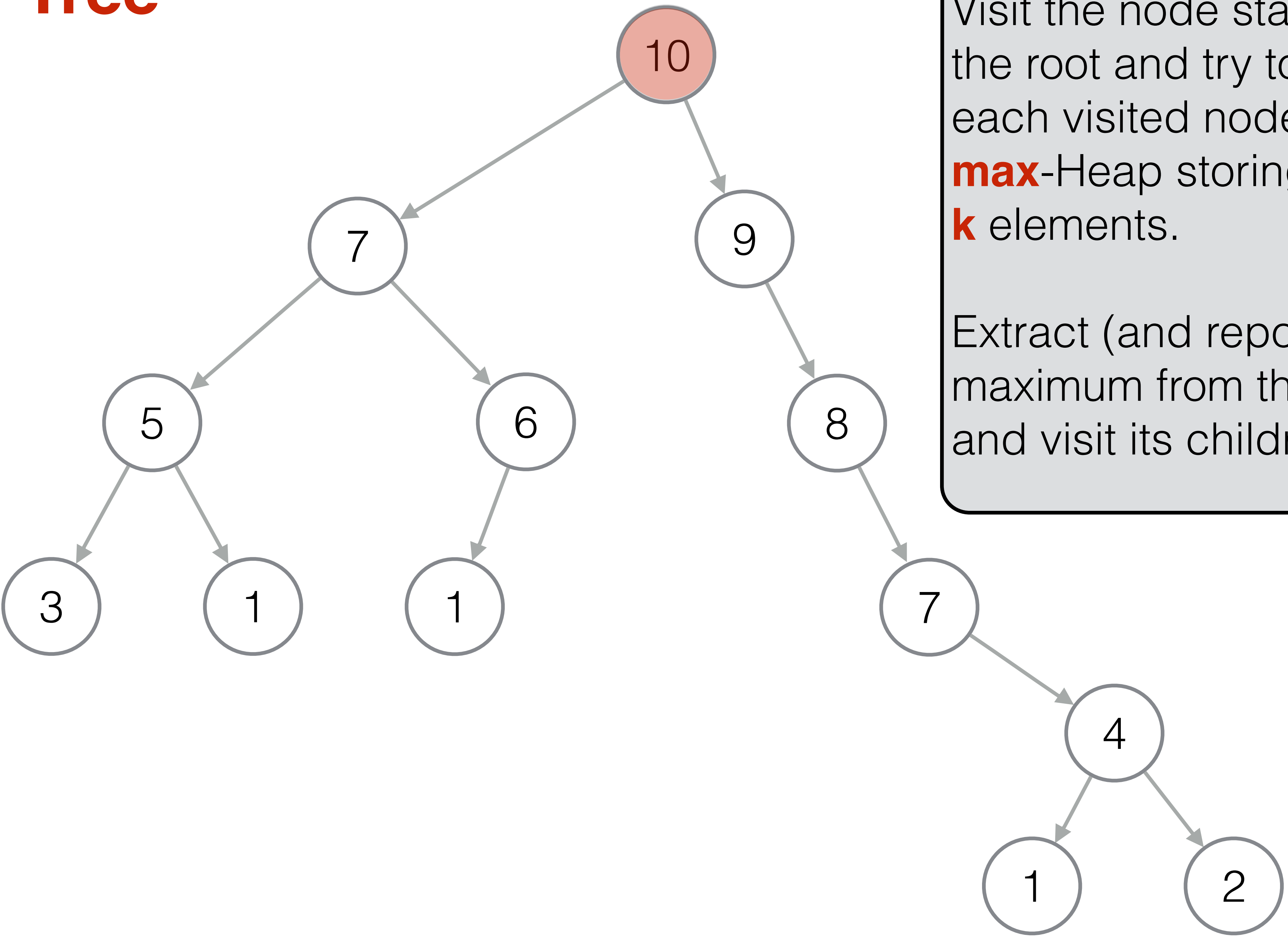


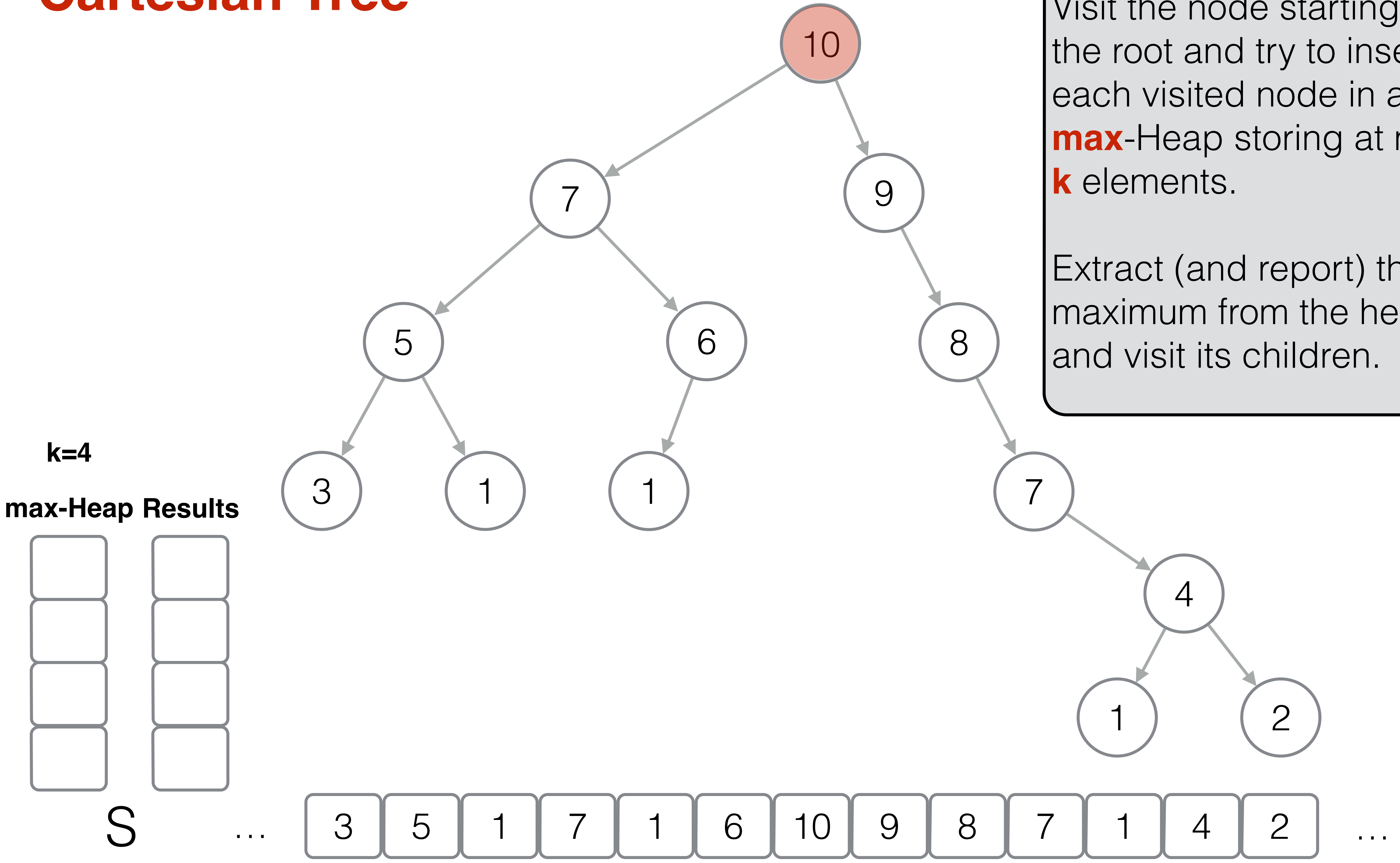
k=4

max-Heap	**Results**
10	

S …	3	5	1	7	1	6	10	9	8	7	1	4	2	…

Finding Top-k

Cartesian Tree



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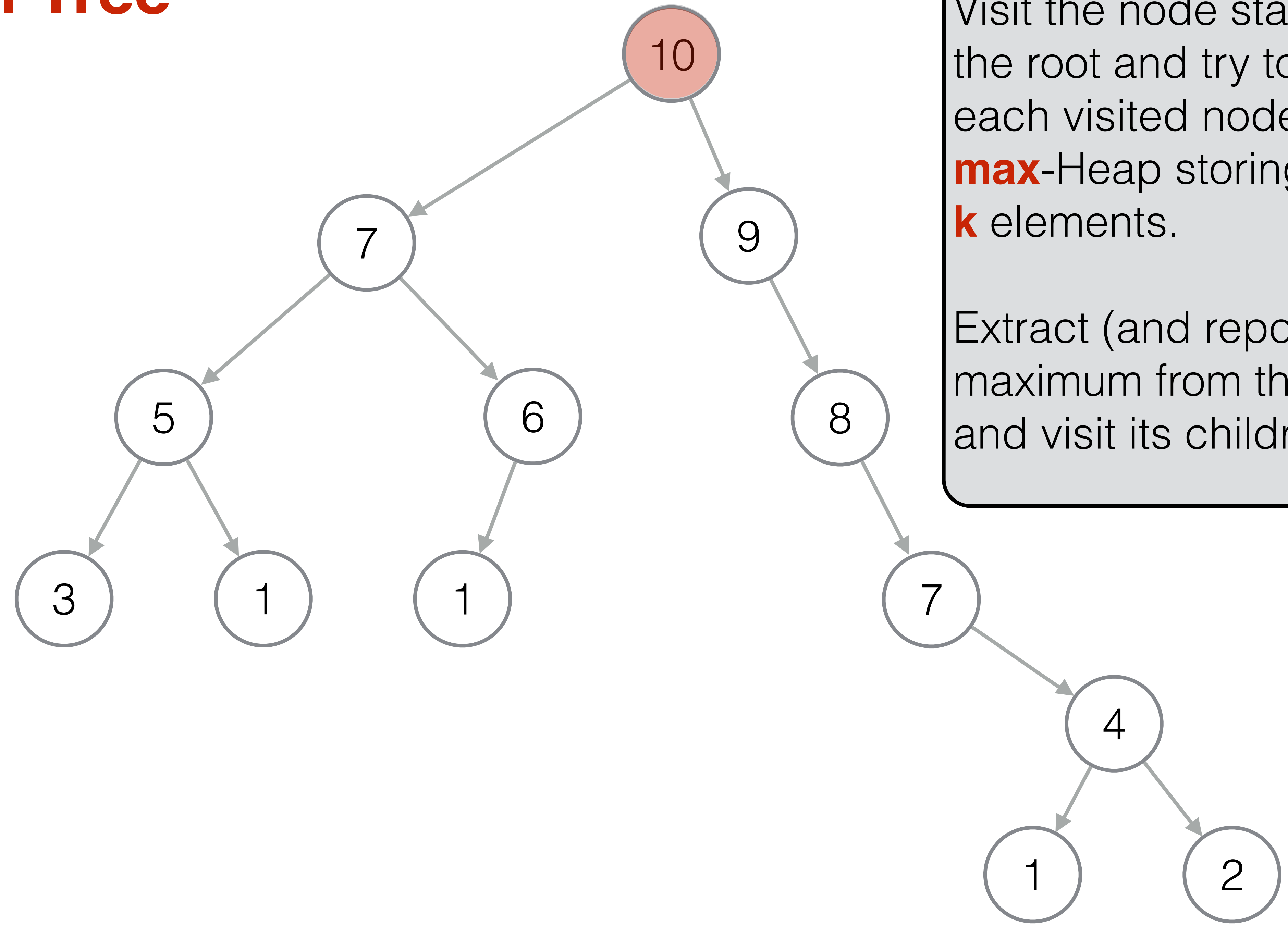


k=4

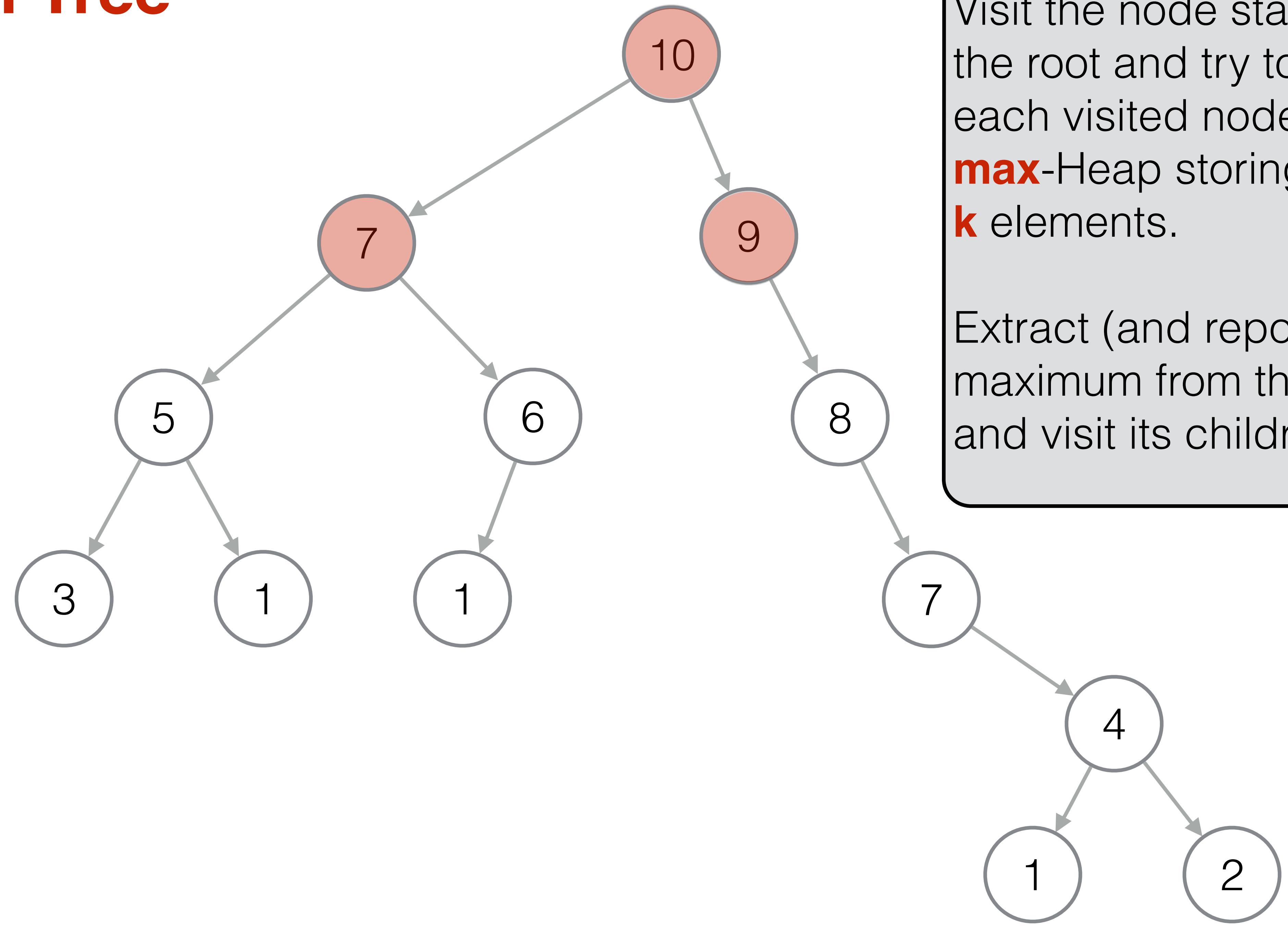
max-Heap	**Results**
	10

S	…	3	5	1	7	1	6	10	9	8	7	1	4	2	…

Finding Top-k

How to find Top-k?

Cartesian Tree



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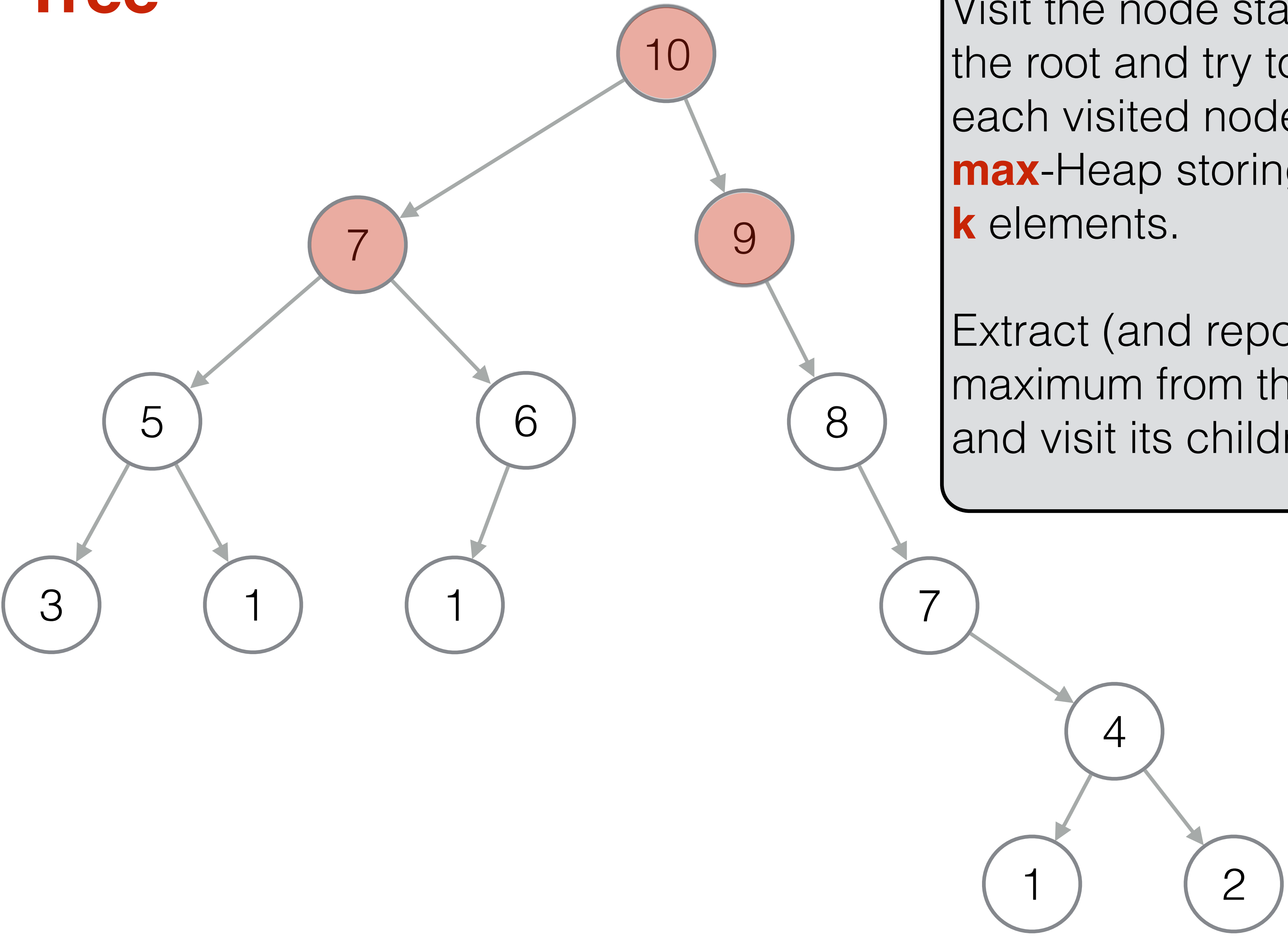
Cartesian Tree



k=4

max-Heap	**Results**
9	10
7	

S	…	3	5	1	7	1	6	10	9	8	7	1	4	2	…

Finding Top-k

Cartesian Tree

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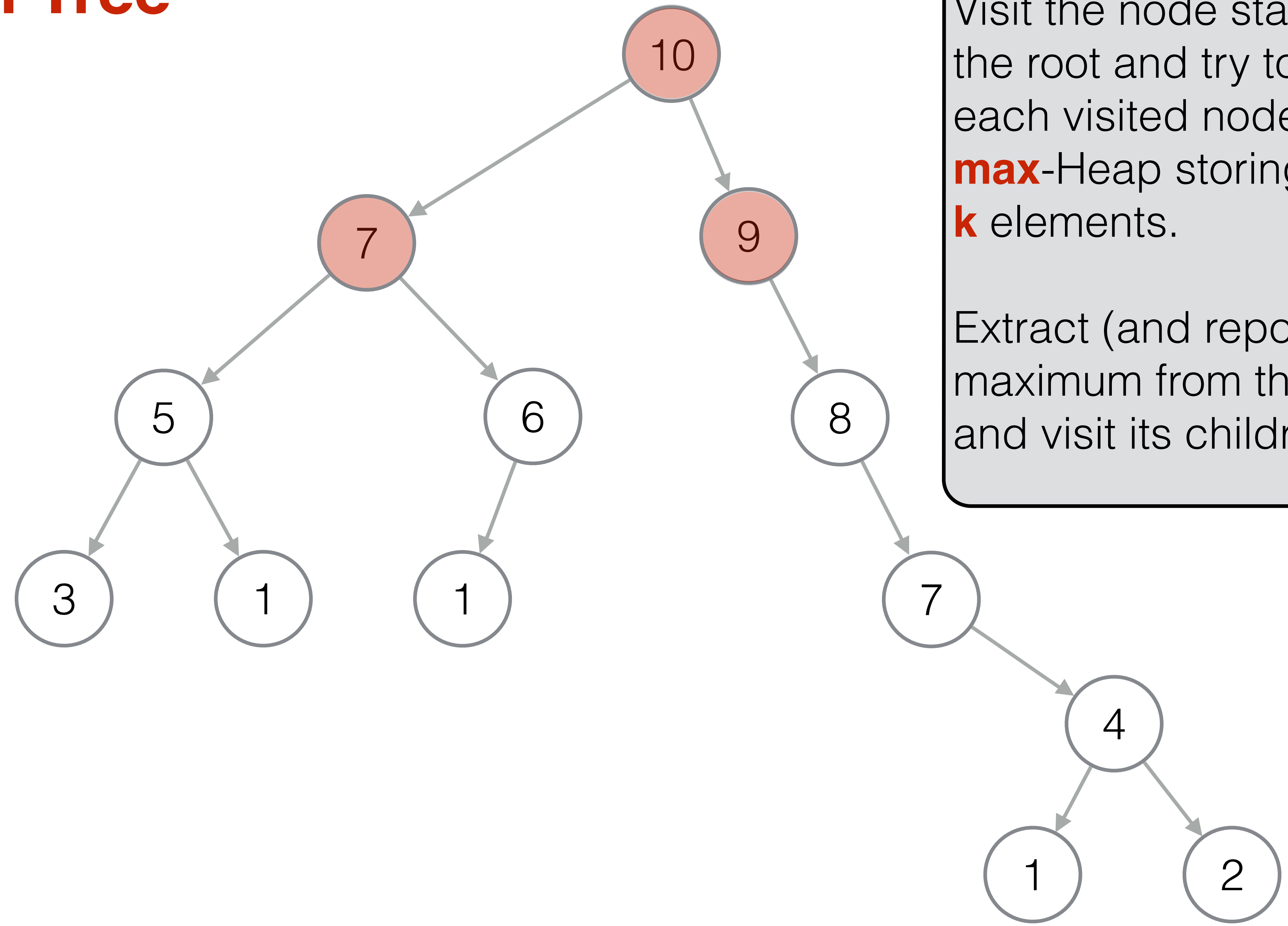

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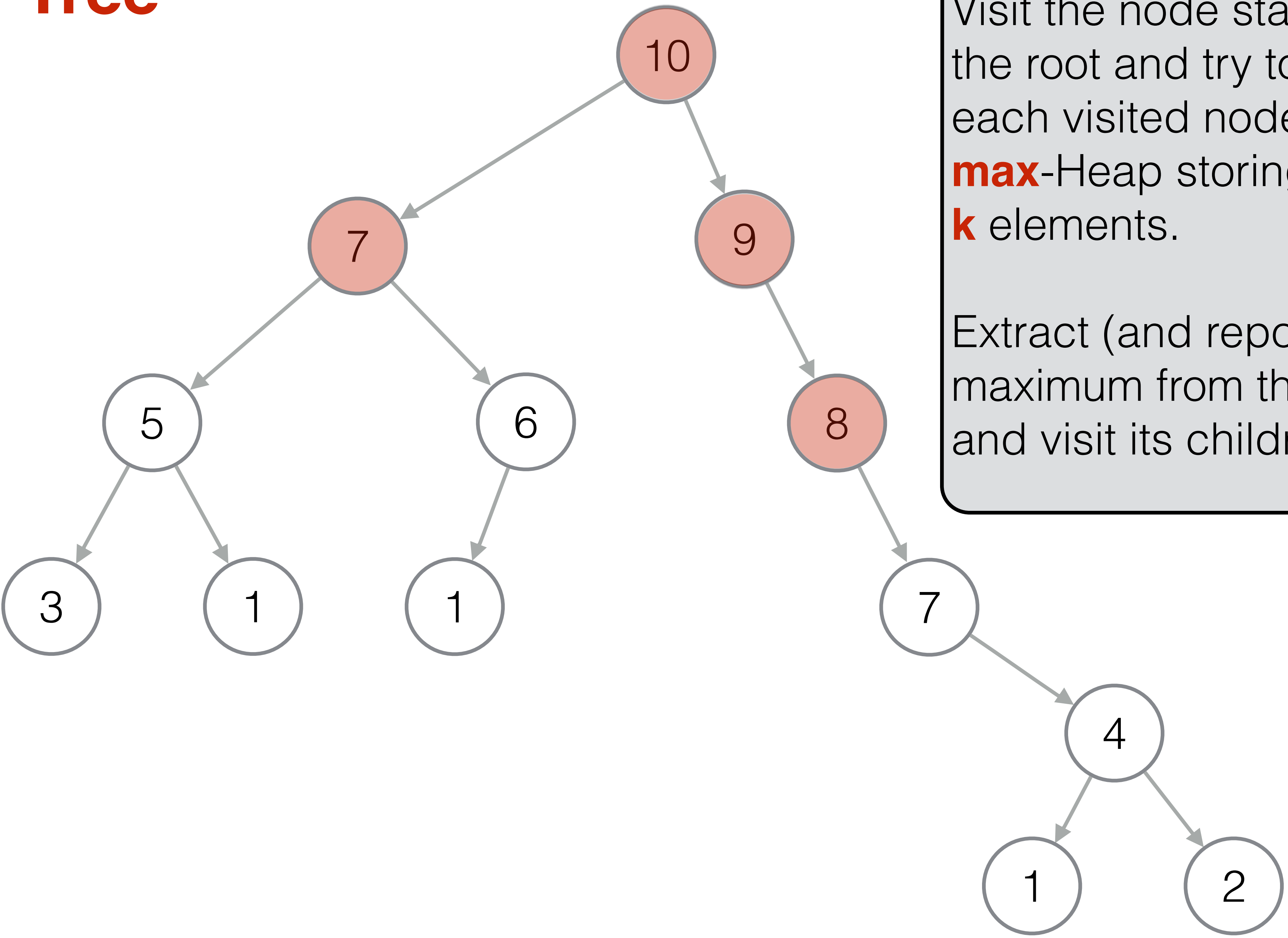

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	9

S

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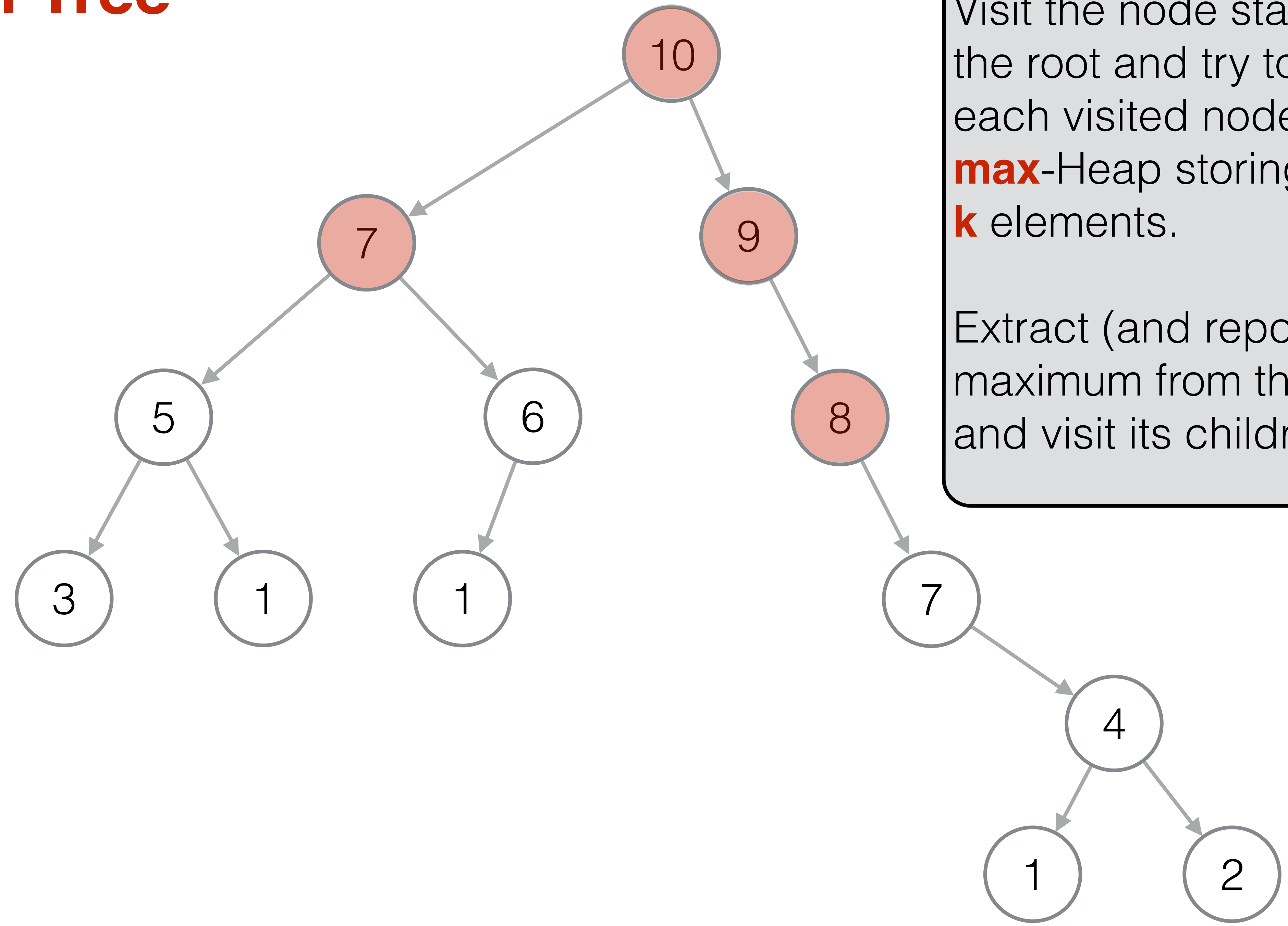
Finding Top-k

Cartesian Tree



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k=4

max-Heap	**Results**
8	10
7	9

S	…	3	5	1	7	1	6	10	9	8	7	1	4	2	…

Finding Top-k

How to find Top-k?

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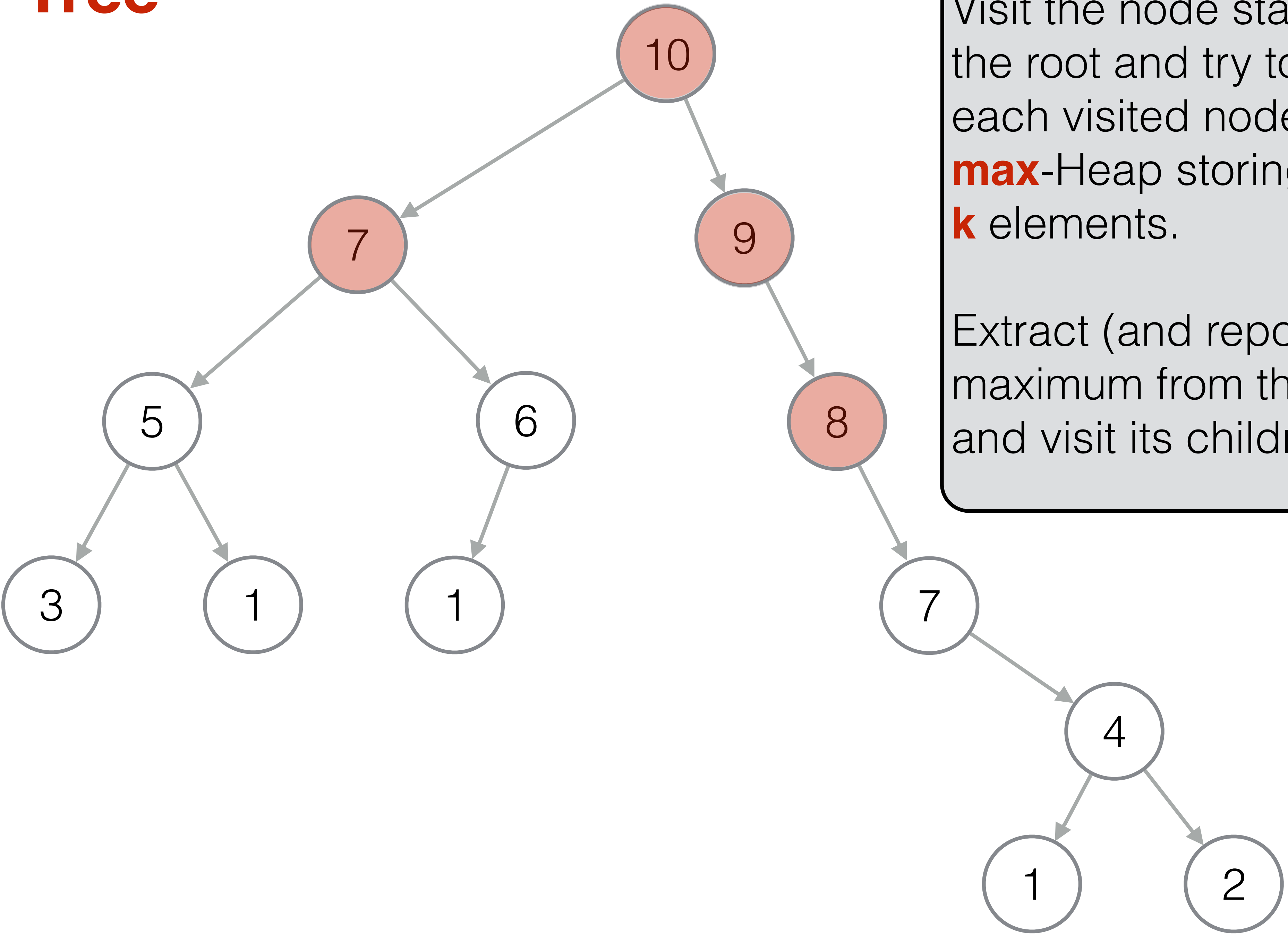
Cartesian Tree



k=4

max-Heap	**Results**
7	10
	9

S … 3 | 5 | 1 | 7 | 1 | 6 | 10 | 9 | 8 | 7 | 1 | 4 | 2 …

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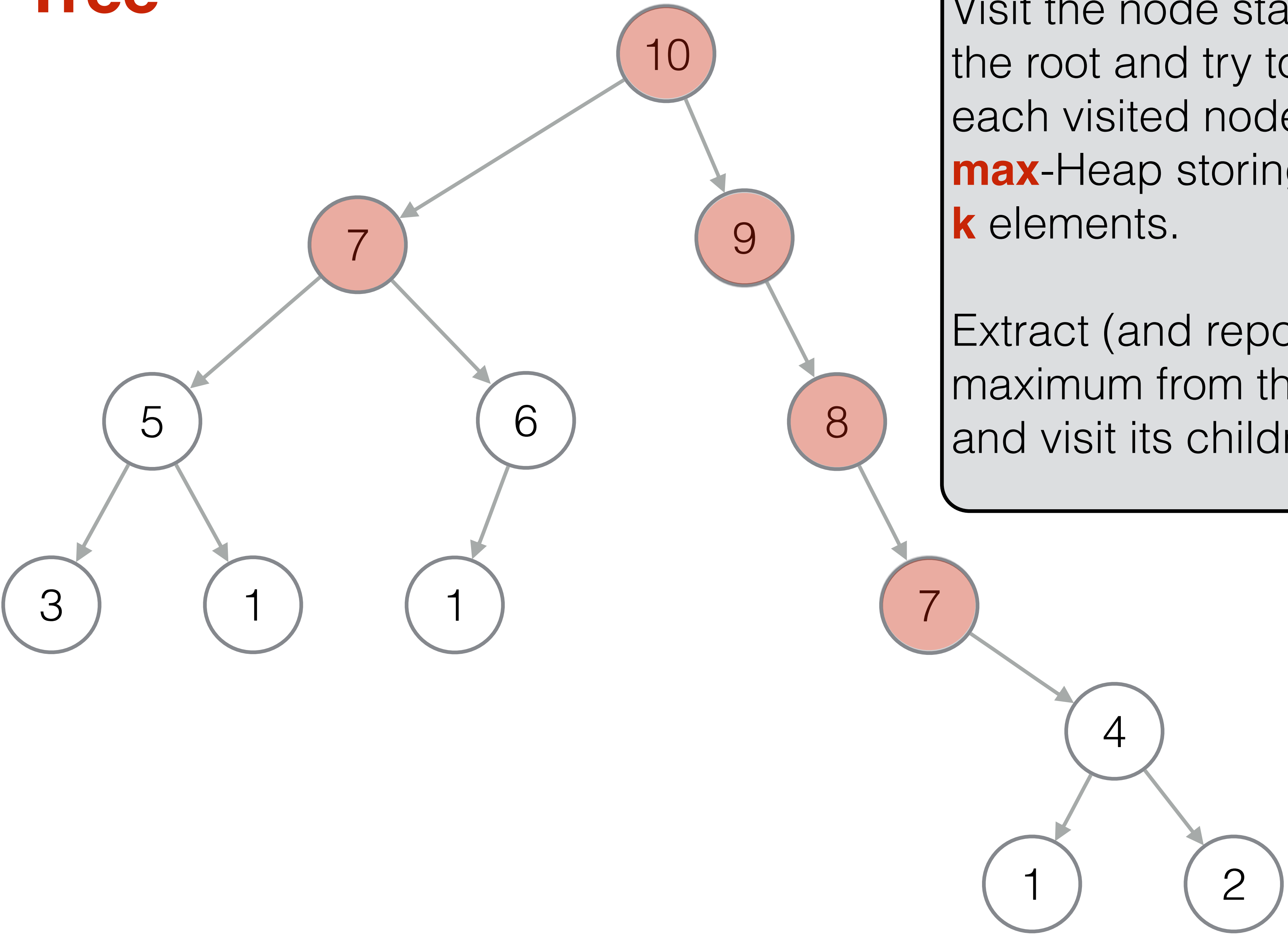

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	9
	8

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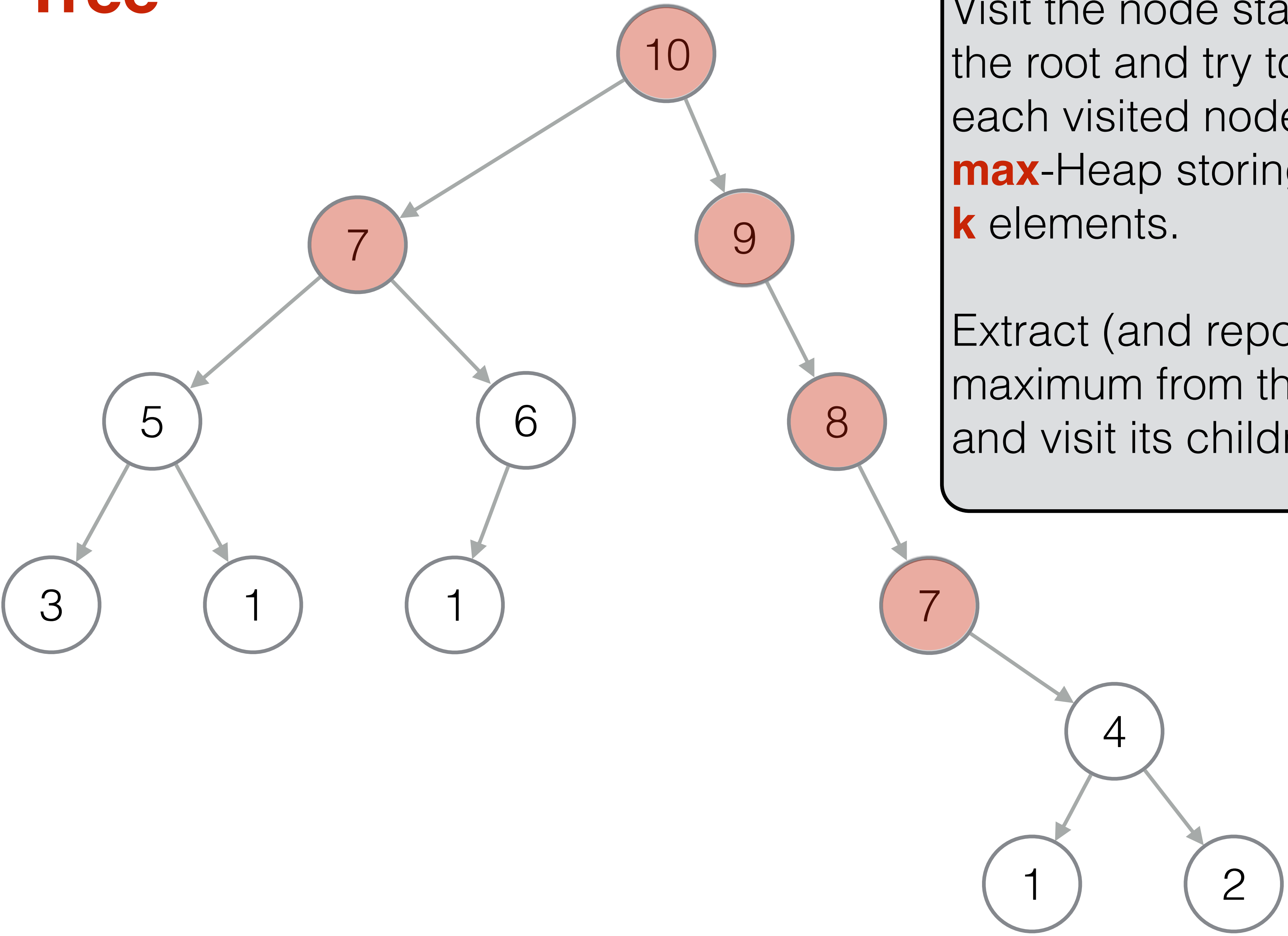


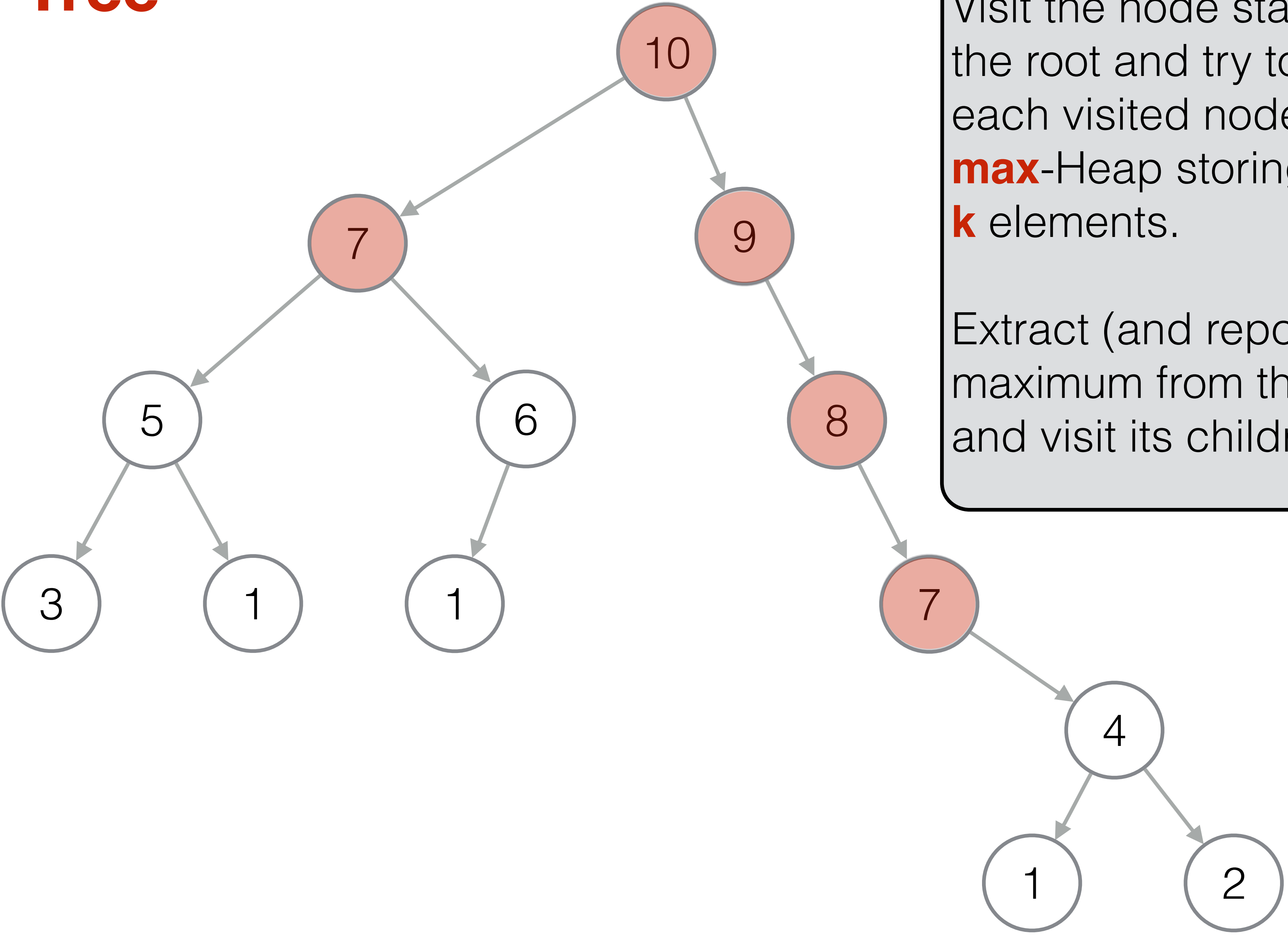
k=4

max-Heap	**Results**
7	10
7	9
	8

S	…	3	5	1	7	1	6	10	9	8	7	1	4	2	…

Finding Top-k

Cartesian Tree



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Extract (and report) the maximum from the heap and visit its children.

k=4

max-Heap	**Results**
7	10
	9
	8
	7

S	…	3	5	1	7	1	6	10	9	8	7	1	4	2	…

Finding Top-k

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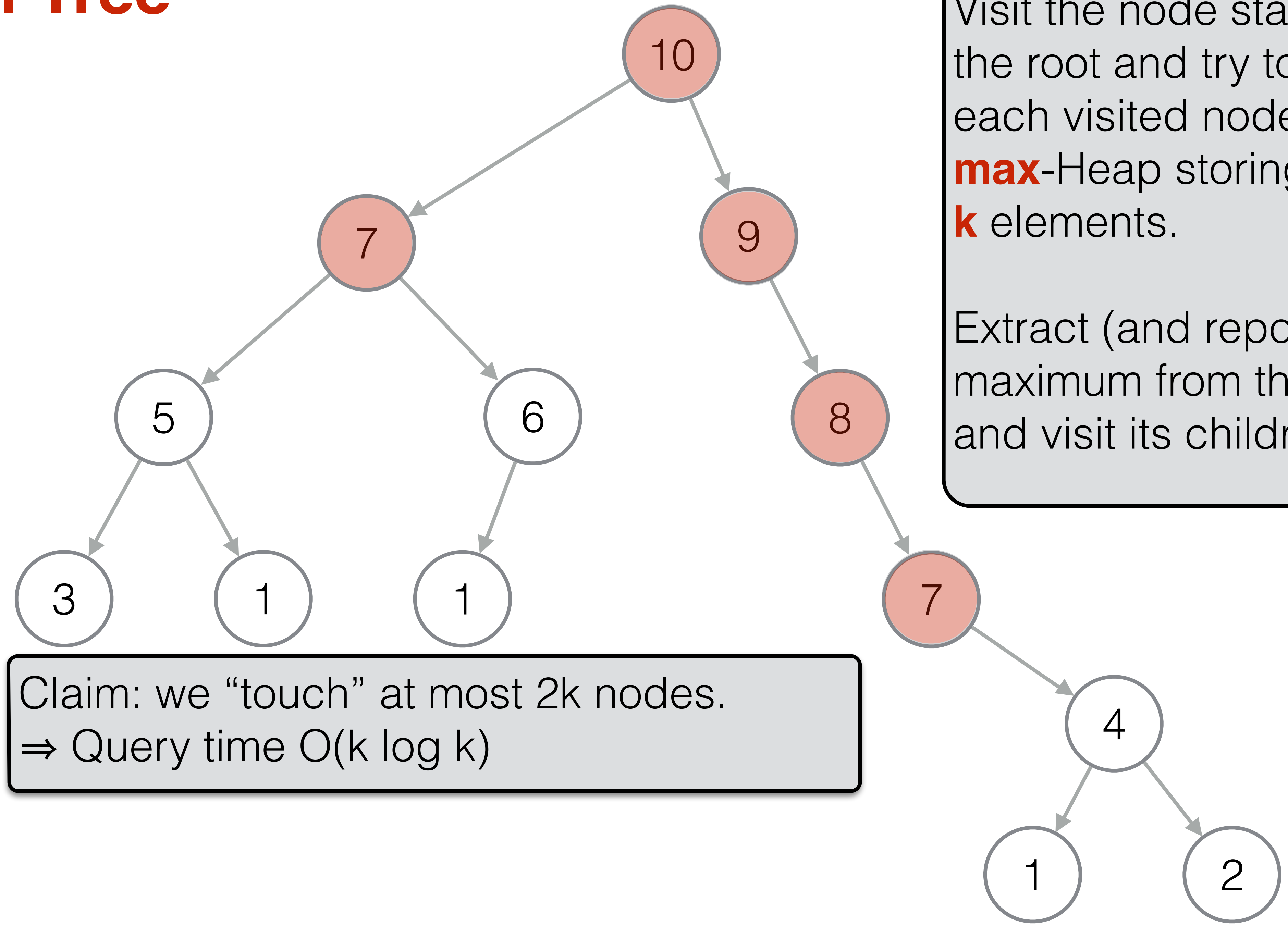
Cartesian Tree



Claim: we “touch” at most 2k nodes.
⇒ Query time O(k log k)

k=4

max-Heap	**Results**
7	10
	9
	8
	7

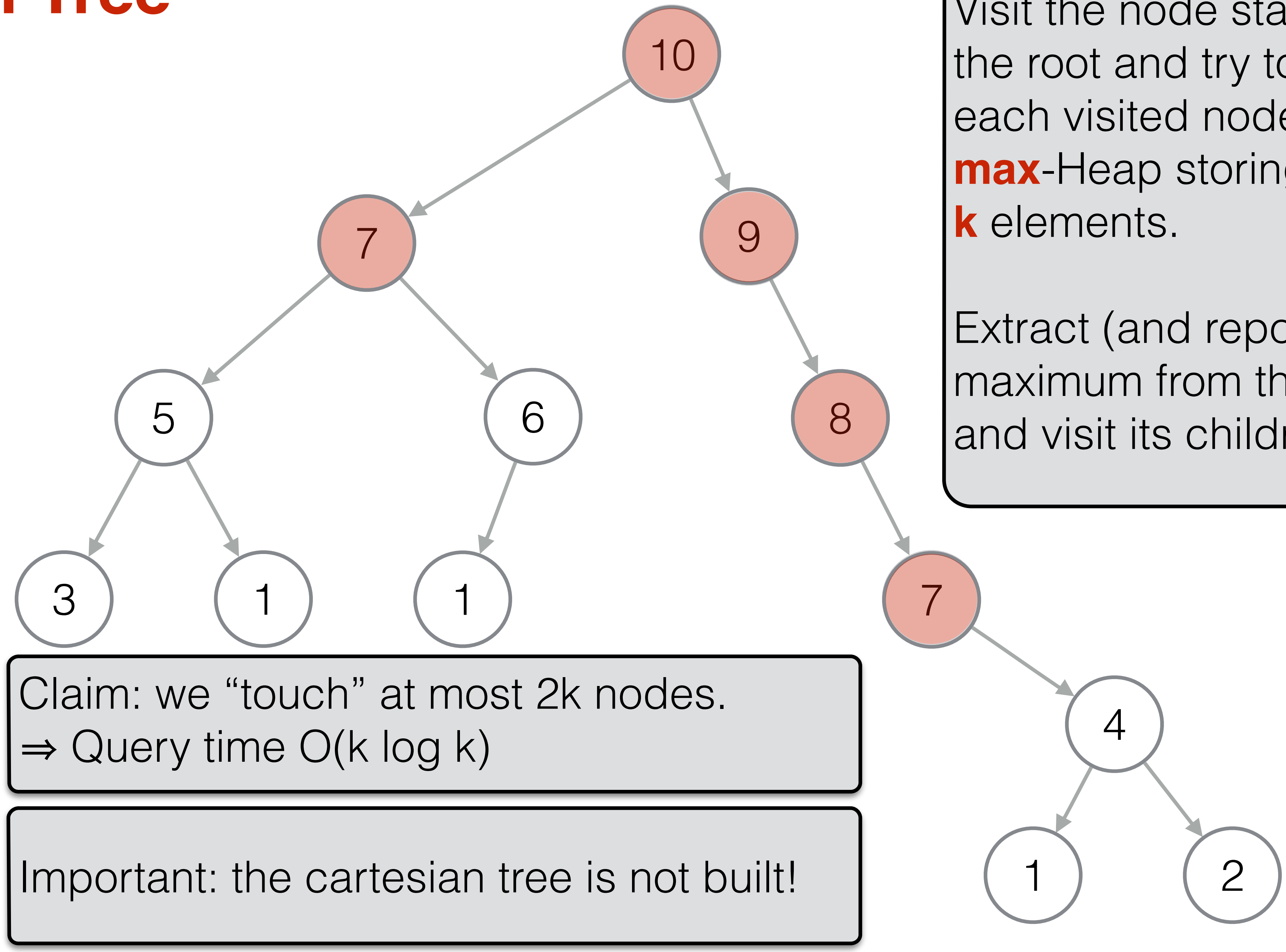
S

…	3	5	1	7	1	6	10	9	8	7	1	4	2	…

Finding Top-k

How to find Top-k?

Cartesian Tree



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Important: the cartesian tree is not built!

k=4

max-Heap	**Results**
7	10
	9
	8
	7

S	…	3	5	1	7	1	6	10	9	8	7	1	4	2	…

Finding Top-k

How to find Top-k?

Cartesian Tree

Visit the node starting from the root and try to insert each visited node in a **max**-Heap storing at most **k** elements.

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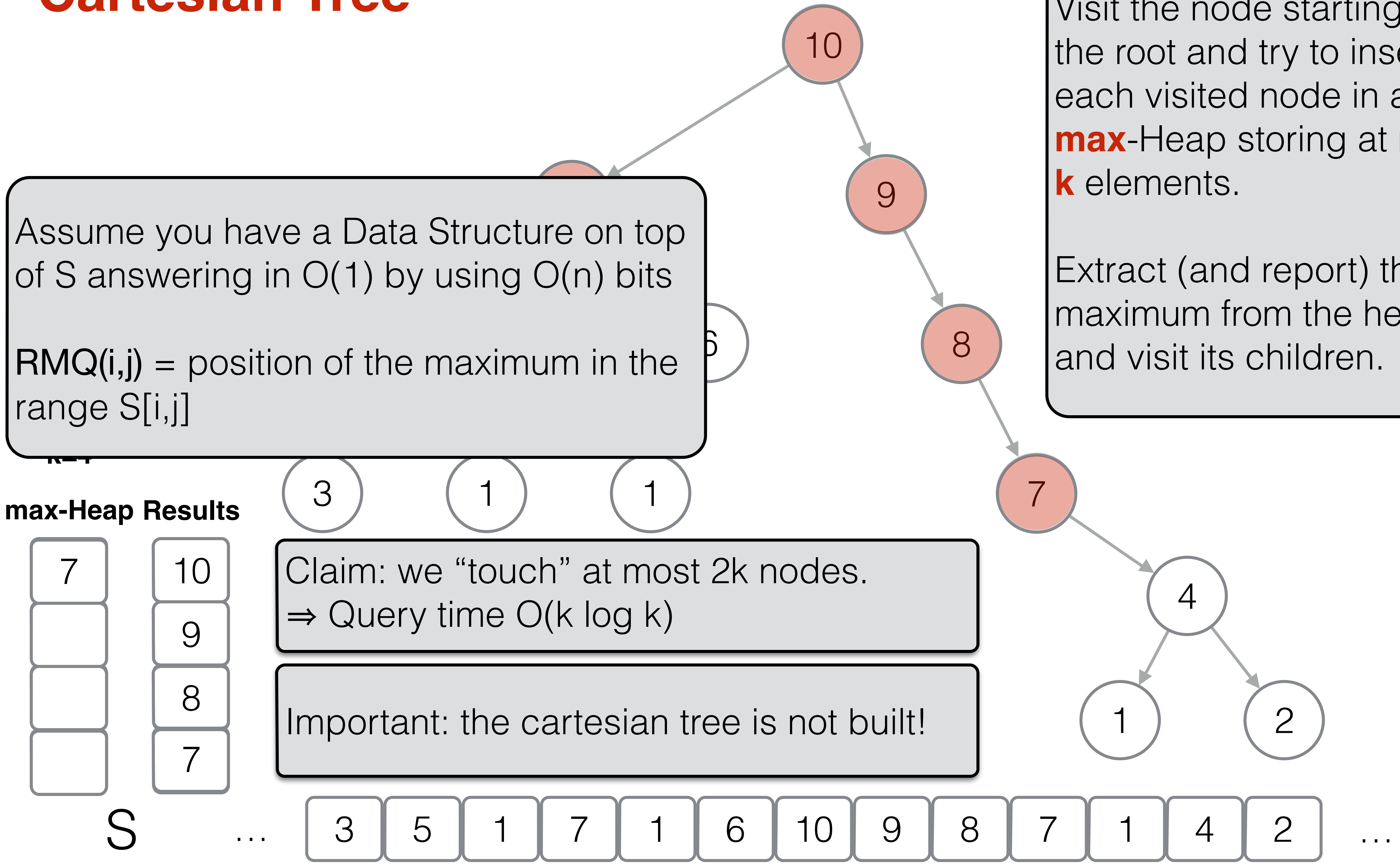

Assume you have a Data Structure on top of S answering in O(1) by using O(n) bits

RMQ(i,j) = position of the maximum in the range S[i,j]

Claim: we “touch” at most 2k nodes.
⇒ Query time O(k log k)

Important: the cartesian tree is not built!

max-Heap Results

max-Heap	Results
7	10
	9
	8
	7

S

…	3	5	1	7	1	6	10	9	8	7	1	4	2	…

Range Maximum Query (1)

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (1)

Space: $O(n^2 \log n)$ bits
Query time: $O(1)$

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (1)

Space: O(n^2 log n) bits
Query time: O(1)

Precompute the answer to any possible query.

There are O(n^2) distinct queries!

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (1)

Space: $O(n^2 \log n)$ bits
Query time: $O(1)$

$M[i,j] = RMQ(i,j)$

Precompute the answer to any possible query.

There are $O(n^2)$ distinct queries!

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (1)

Space: O(n^2 log n) bits
Query time: O(1)

$$M[i,j] = RMQ(i,j)$$

M

	0	1	2	3	4	5	6	7	8	9	10	11
0												
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												

Precompute the answer to any possible query.

There are O(n^2) distinct queries!

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (1)

Space: $O(n^2 \log n)$ bits
Query time: $O(1)$

Precompute the answer to any possible query.

There are $O(n^2)$ distinct queries!

$M[i,j] = RMQ(i,j)$

M	0	1	2	3	4	5	6	7	8	9	10	11
0												
1												
2						3						
3												
4												
5												
6												
7												
8												
9												
10												
11												

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: O(n log^2 n) bits
Query time: O(1)

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: O(n log² n) bits
Query time: O(1)

Maximum in a interval is the max between the maxima of any its subintervals

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: O(n $\log^2$ n) bits
Query time: O(1)

Maximum in a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

There are O(log n) possible intervals starting at any position i.

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum in a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

There are O(log n) possible intervals starting at any position i.

$$M[i,j] = RMQ(i,i+2^j)$$

M

	0	1	2	3	4
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

S

0	1	2	3	4	5	6	7	8	9	10	11
3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum in a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

There are $O(\log n)$ possible intervals starting at any position i.

$$M[i,j] = RMQ(i,i+2^j)$$

M	0	1	2	3	4
0					
1				?	
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

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M

	0	1	2	3	4
0					
1				?	
2					
3					
4					
5					
6					
7					
8					
9					
10					

$9=1+2^3$

S	0	1	2	3	4	5	6	7	8	9	10	11
	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum in a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

There are $O(\log n)$ possible intervals starting at any position i.

$$M[i,j] = RMQ(i, i+2^j)$$

M	0	1	2	3	4
0					
1				6	
2					
3					
4					
5					
6					
7					
8					
9					
10					

$9=1+2^3$

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum of a interval is the max between the maxima of any its subintervals

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$$M[i,j] = RMQ(i,i+2^j)$$

M

	0	1	2	3	4
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

S

0	1	2	3	4	5	6	7	8	9	10	11
3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: O(n $\log^2$ n) bits
Query time: O(1)

Maximum of a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

There are O(log n) possible intervals starting at any position i.

RMQ(1,7) =

$$M[i,j] = RMQ(i,i+2^j)$$

M	0	1	2	3	4
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum of a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

There are $O(\log n)$ possible intervals starting at any position i.

$$M[i,j] = \mathrm{RMQ}(i,i+2^j)$$

M	0	1	2	3	4
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

$\mathrm{RMQ}(1,7) = \mathrm{argmax}(S[M[1,1+2^2]], S[M[7-2^2,7]]) = 6$

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum of a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

There are $O(\log n)$ possible intervals starting at any position i.

$$M[i,j] = RMQ(i,i+2^j)$$

M	0	1	2	3	4
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

$RMQ(1,7) = \text{argmax}(S[M[1,1+2^2]], S[M[7-2^2,7]]) = 6$

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum of a interval is the max between the maxima of any its subintervals

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$$M[i,j] = RMQ(i,i+2^j)$$

M	0	1	2	3	4
0					
1			3		
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

$RMQ(1,7) = \text{argmax}(S[M[1,1+2^2]], S[M[7-2^2,7]]) = 6$

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum of a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

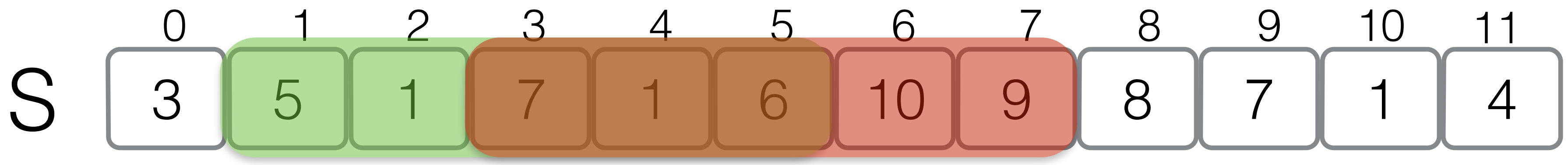
There are $O(\log n)$ possible intervals starting at any position i.

$$M[i,j] = RMQ(i,i+2^j)$$

M

	0	1	2	3	4
0					
1			3		
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

$RMQ(1,7) = \text{argmax}(S[M[1,1+2^2]], S[M[7-2^2,7]]) = 6$



	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum of a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

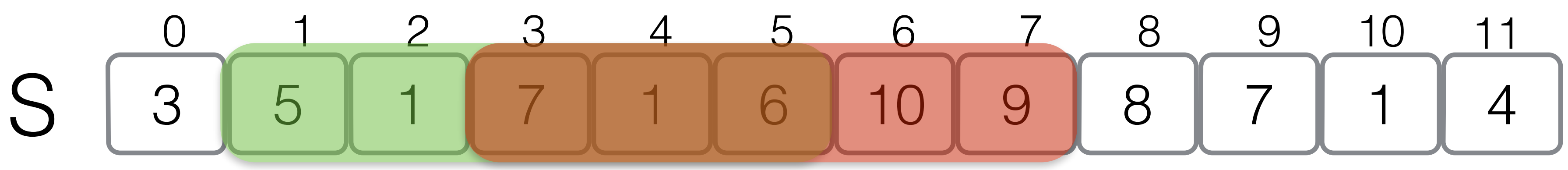
There are $O(\log n)$ possible intervals starting at any position i.

$$M[i,j] = RMQ(i,i+2^j)$$

M

	0	1	2	3	4
0					
1			3		
2					
3			6		
4					
5					
6					
7					
8					
9					
10					
11					

$$RMQ(1,7) = \text{argmax}(S[M[1,1+2^2]], S[M[7-2^2,7]]) = 6$$



	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum of a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

There are $O(\log n)$ possible intervals starting at any position i.

$$M[i,j] = RMQ(i,i+2^j)$$

M

	0	1	2	3	4
0					
1			3		
2					
3			6		
4					
5					
6					
7					
8					
9					
10					
11					

$$RMQ(1,7) = \text{argmax}(S[M[1,1+2^2]], S[M[7-2^2,7]]) = 6$$

$$RMQ(i,j) = \text{argmax}(S[M[i,i+2^{len}]], S[M[j-2^{len},j]])$$

where $len = \lfloor \log (j-i+1) \rfloor$

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (3)

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

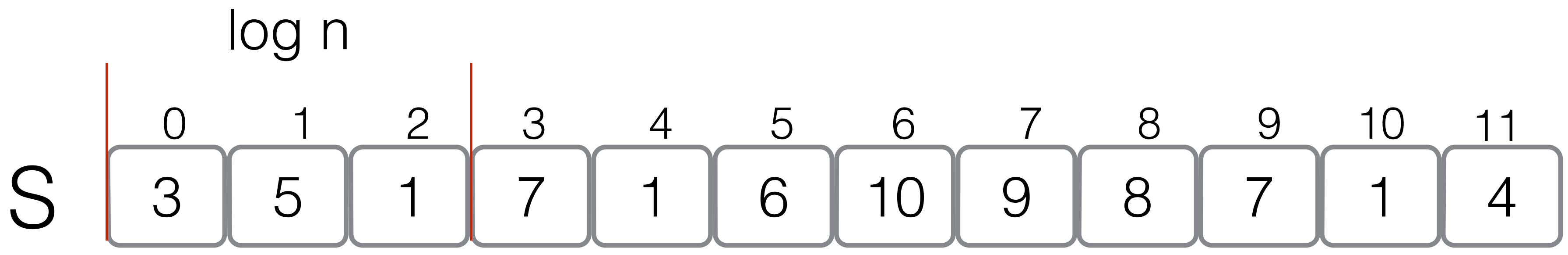
Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

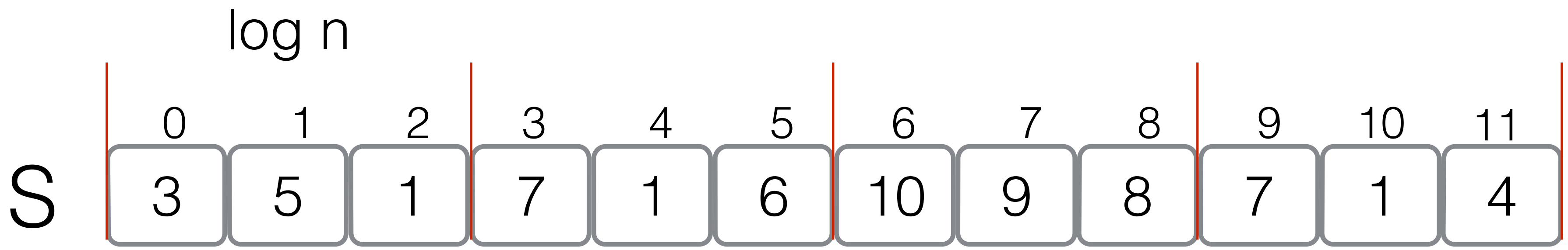


log n

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)



log n

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

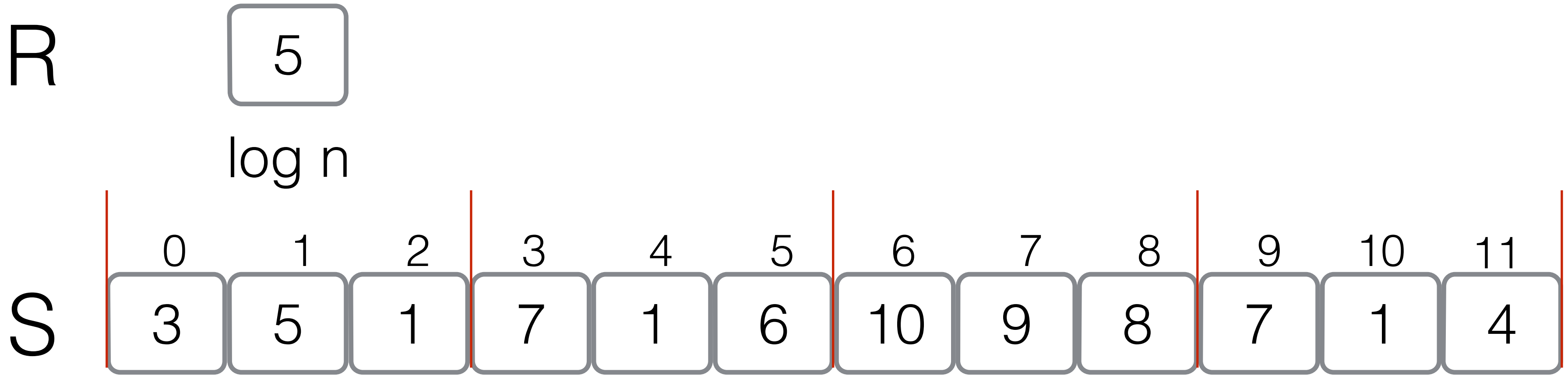
R

log n

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (3)

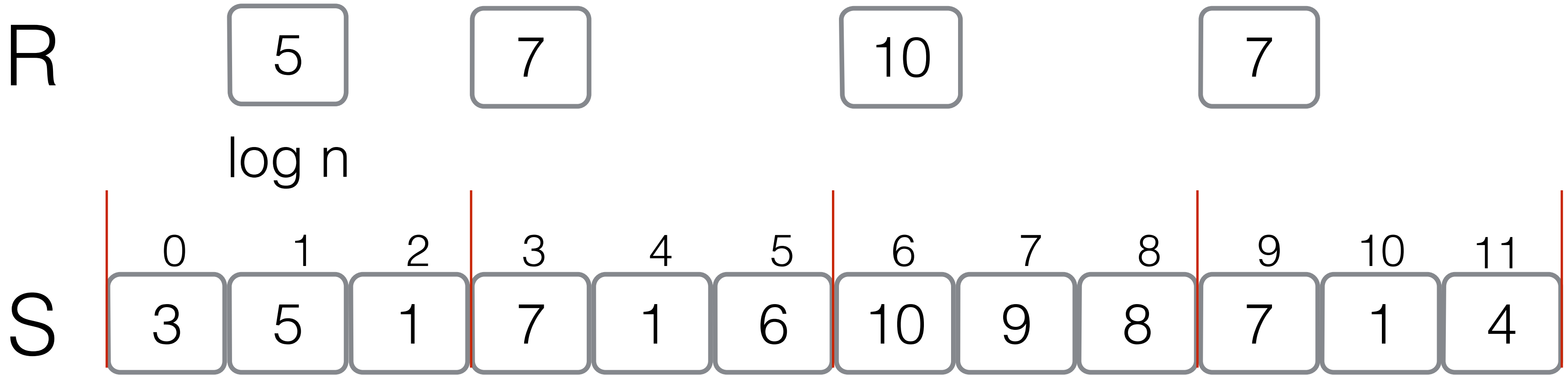
Space: O(n log n) bits
Query time: O(log n)



	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

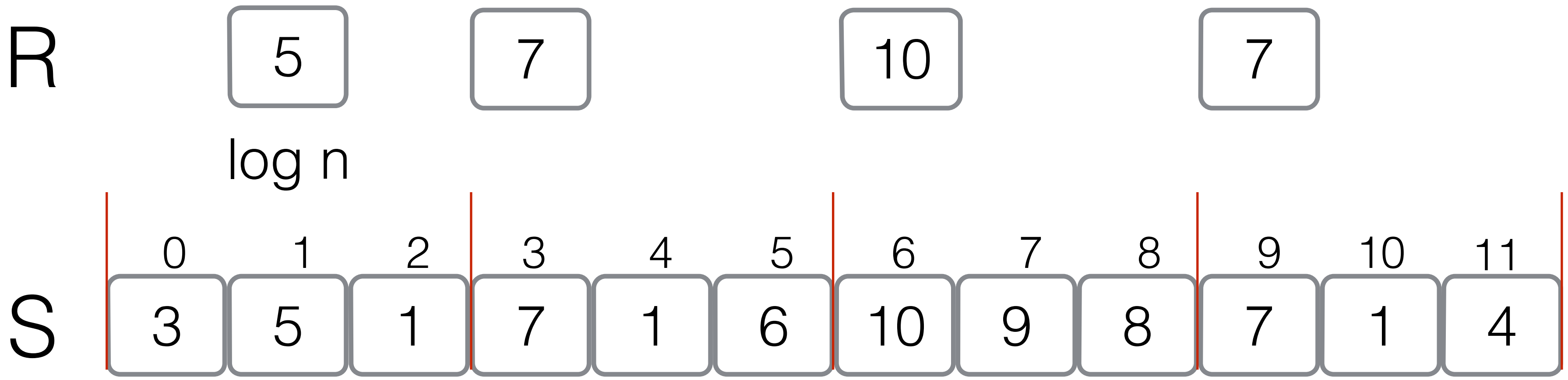


Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

Use the previous solution on R!

Space: ? bits
Query time: O(1)



R: 5 7 10 7

log n

0	1	2	3	4	5	6	7	8	9	10	11
3	5	1	7	1	6	10	9	8	7	1	4

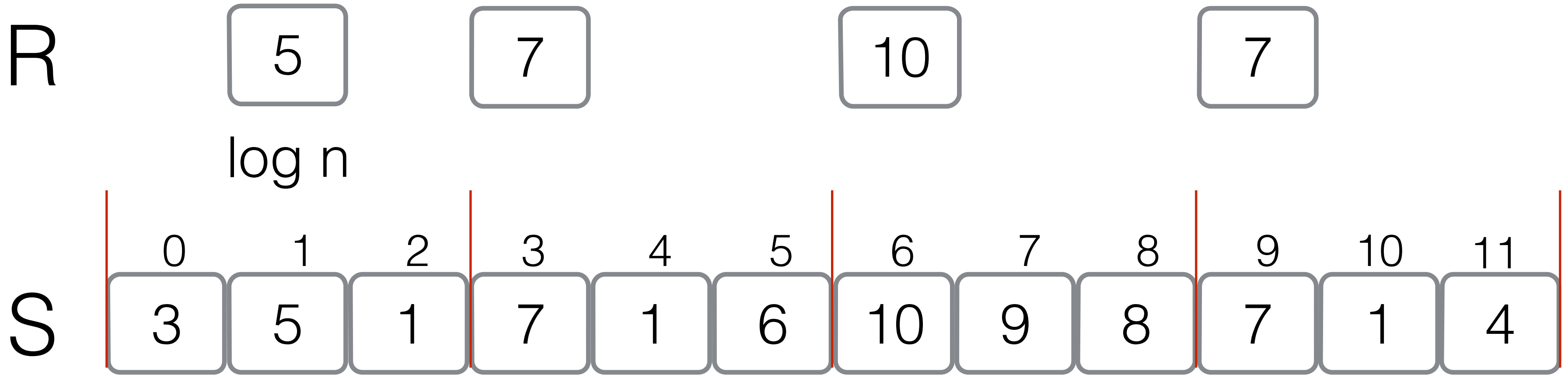
S

Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

Use the previous solution on R!

Space: O(n log n) bits
Query time: O(1)



	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

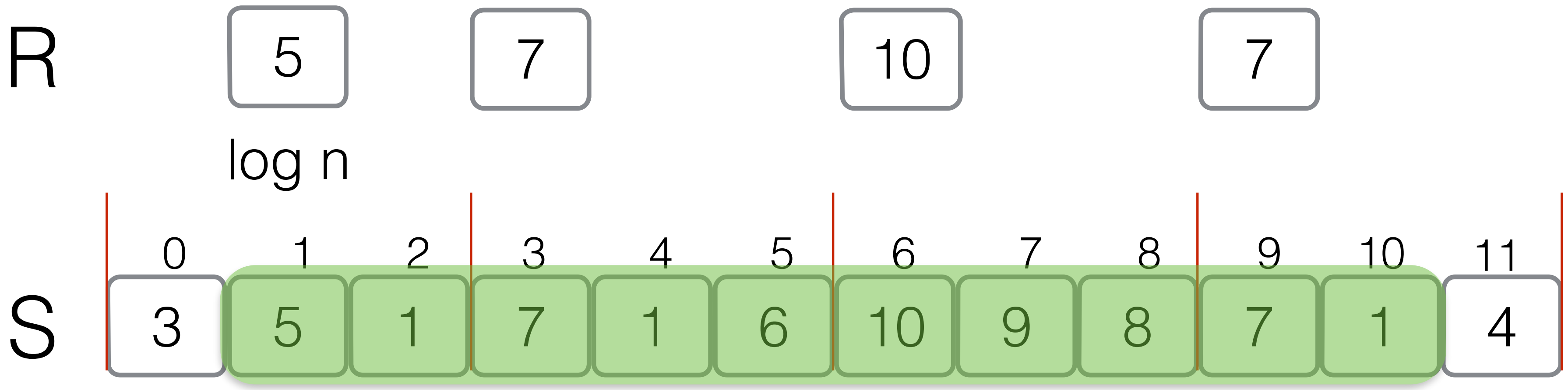
Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

Use the previous solution on R!

Space: O(n log n) bits
Query time: O(1)

RMQ(1,10) = ?



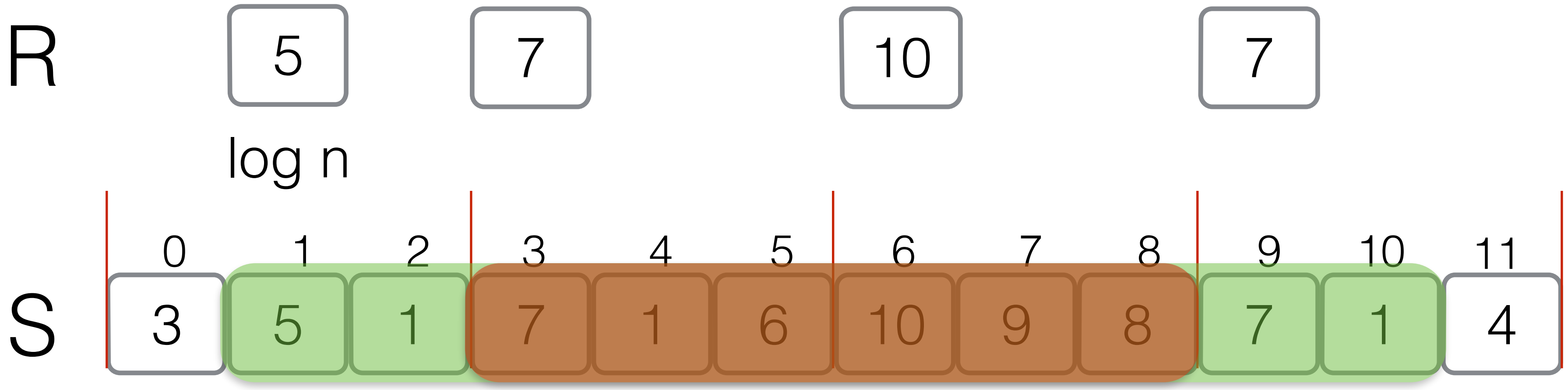
Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

Use the previous solution on R!

Space: O(n log n) bits
Query time: O(1)

RMQ(1,10) = ?



R: 5 7 10 7

log n

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

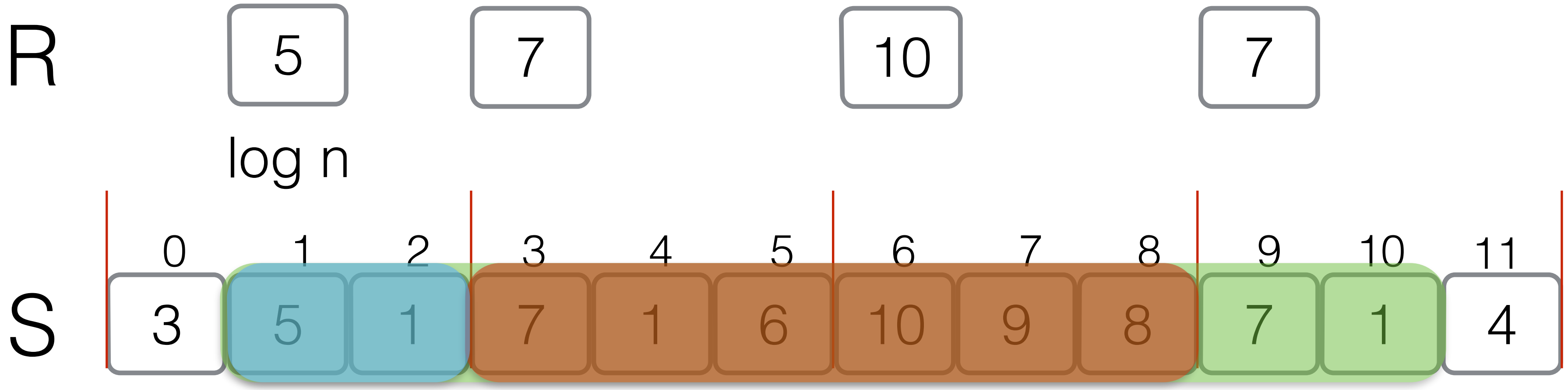
Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

Use the previous solution on R!

Space: O(n log n) bits
Query time: O(1)

RMQ(1,10) = ?



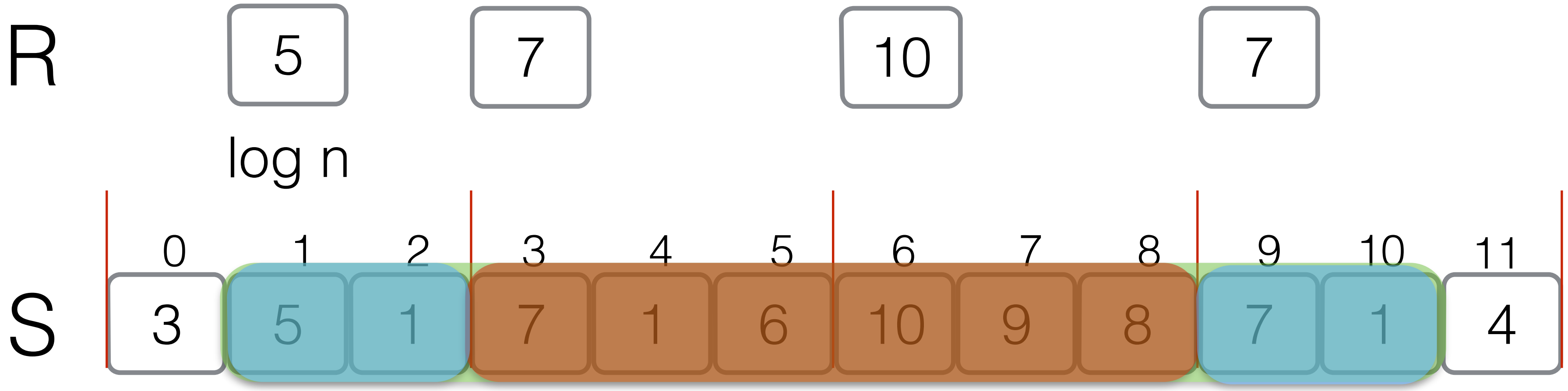
Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

Use the previous solution on R!

Space: O(n log n) bits
Query time: O(1)

RMQ(1,10) = ?



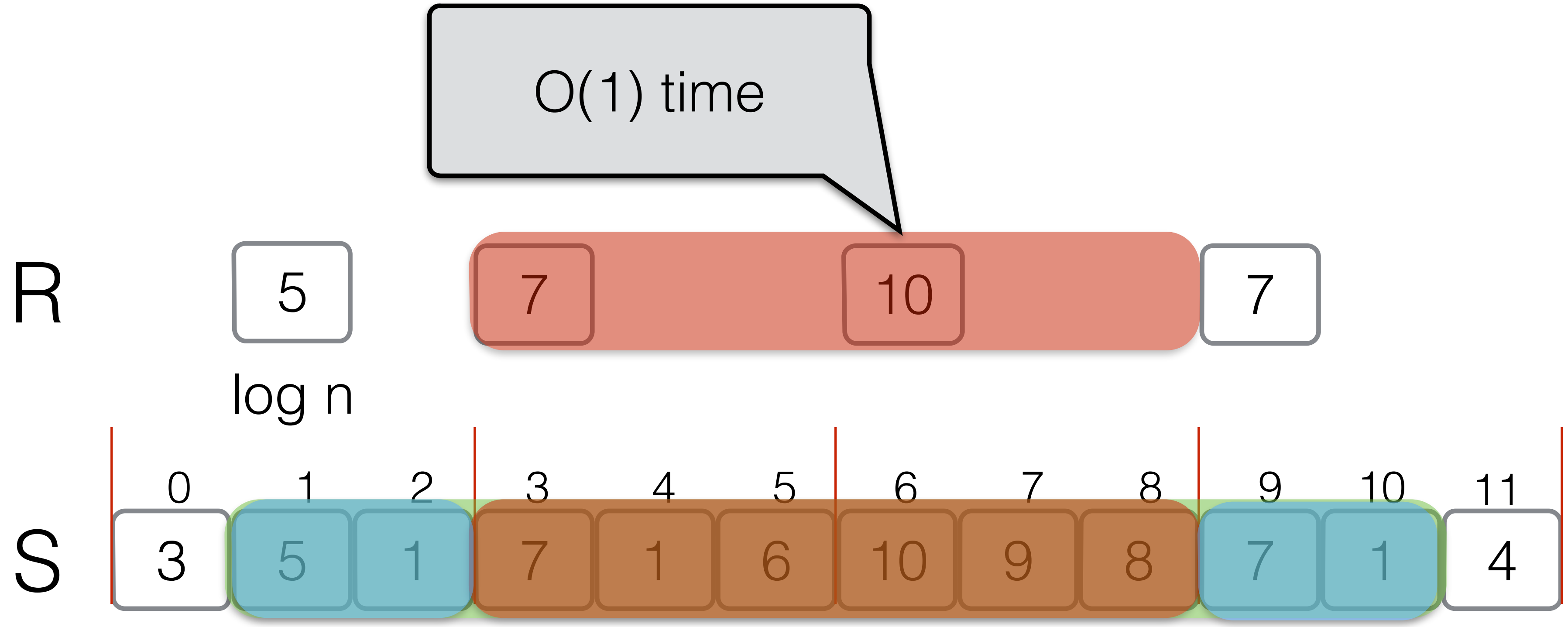
Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

Use the previous solution on R!

Space: O(n log n) bits
Query time: O(1)

RMQ(1,10) = ?



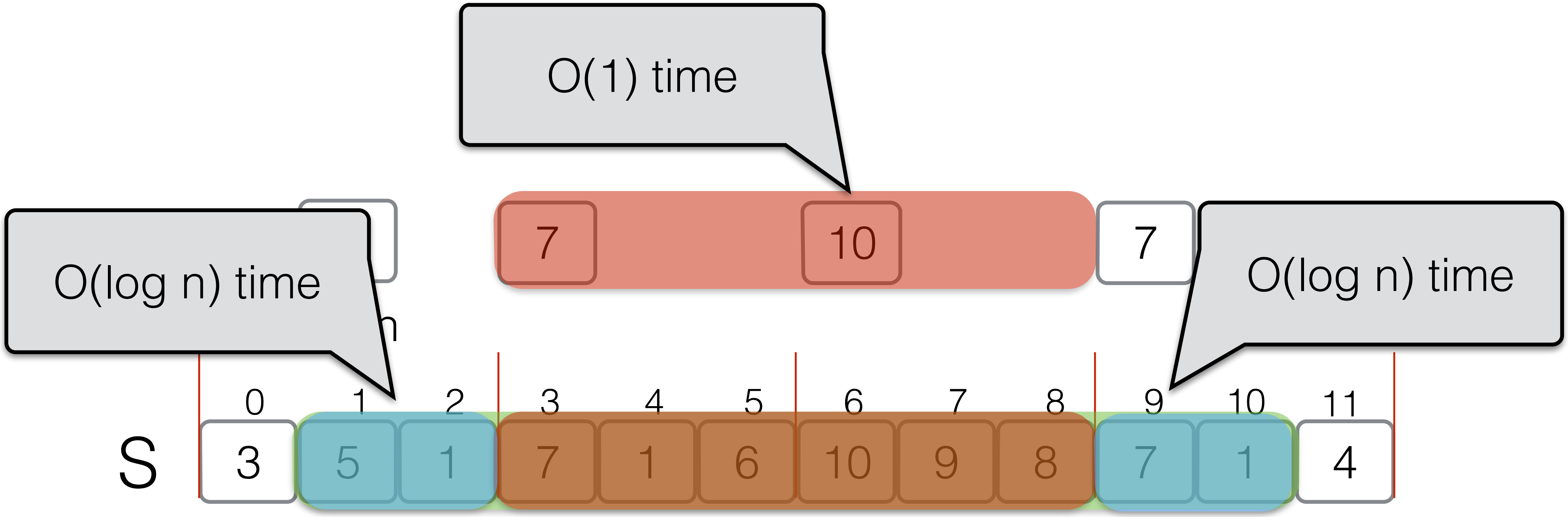
Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

Use the previous solution on R!

Space: O(n log n) bits
Query time: O(1)

RMQ(1,10) = ?



O(1) time

O(log n) time

O(log n) time

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (3)

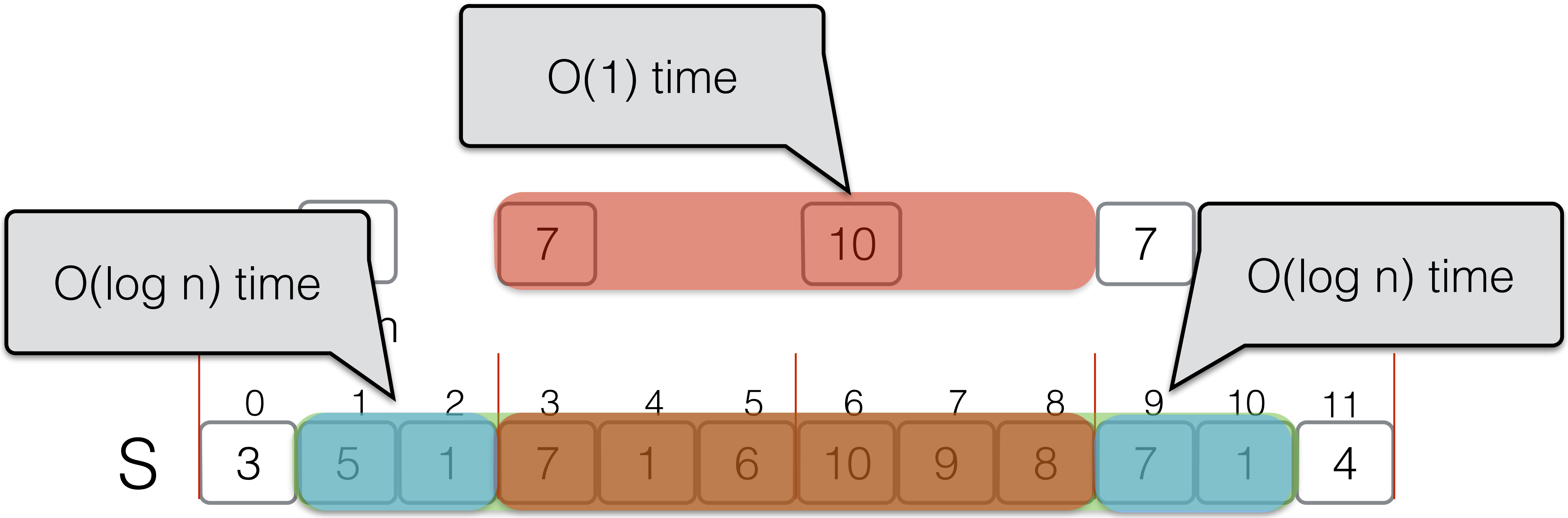
Space: O(n log n) bits
Query time: O(log n)

Space: O(n log n) bits
Query time: O(1)

Use the previous solution on R!

Space: O(n log n) bits
Query time: O(1)

RMQ(1,10) = ?



	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (3)

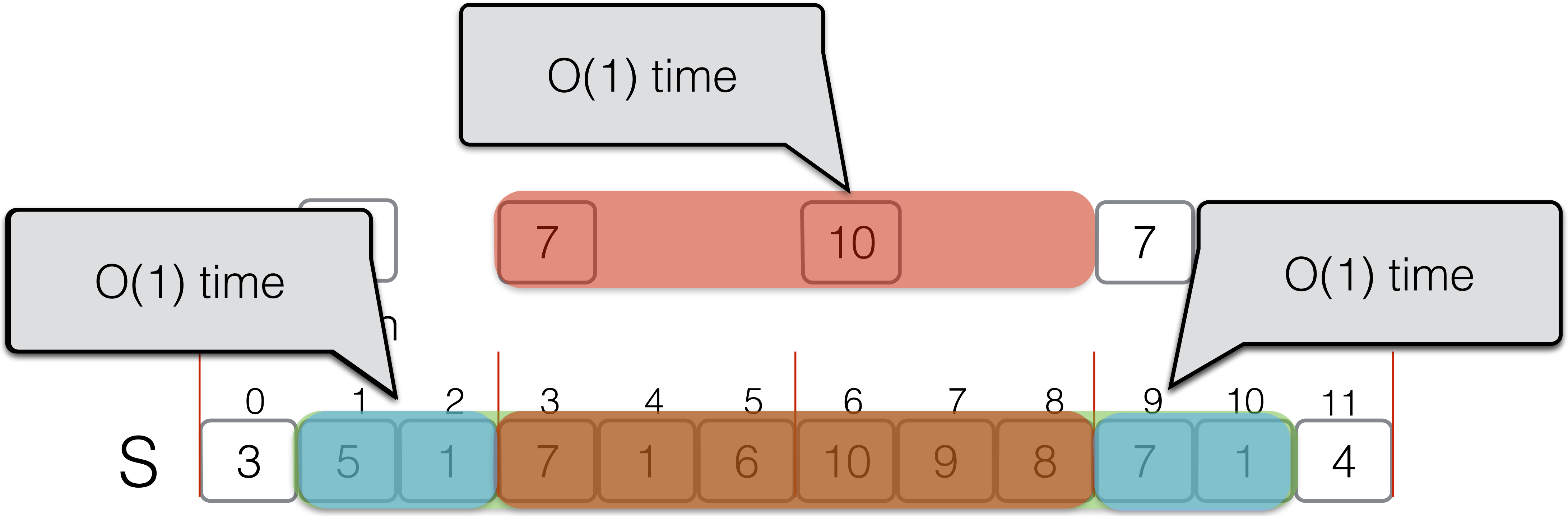
Space: O(n log n) bits
Query time: O(log n)

Space: O(n log n) bits
Query time: O(1)

Use the previous solution on R!

Space: O(n log n) bits
Query time: O(1)

RMQ(1,10) = ?



	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

O(1) time

O(1) time

O(1) time

Range Maximum Query (3)

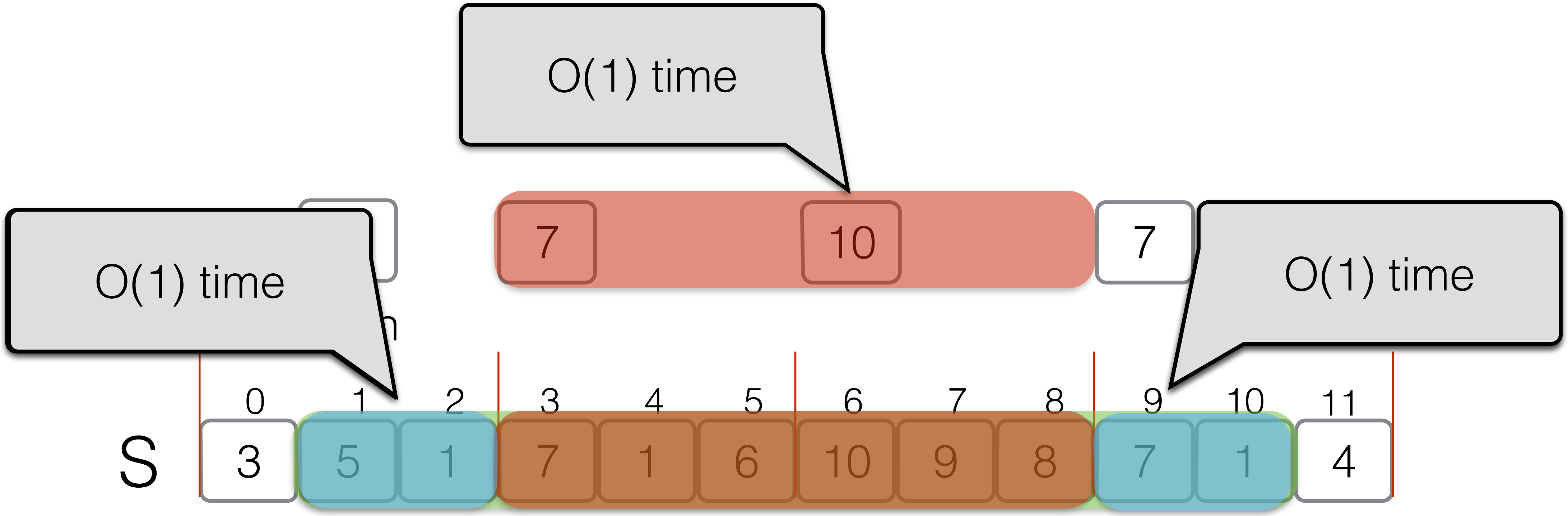
Space: O(n log n) bits
Query time: O(log n)

Space: O(n ~~log~~ n) bits
Query time: O(1)

Use the previous solution on R!

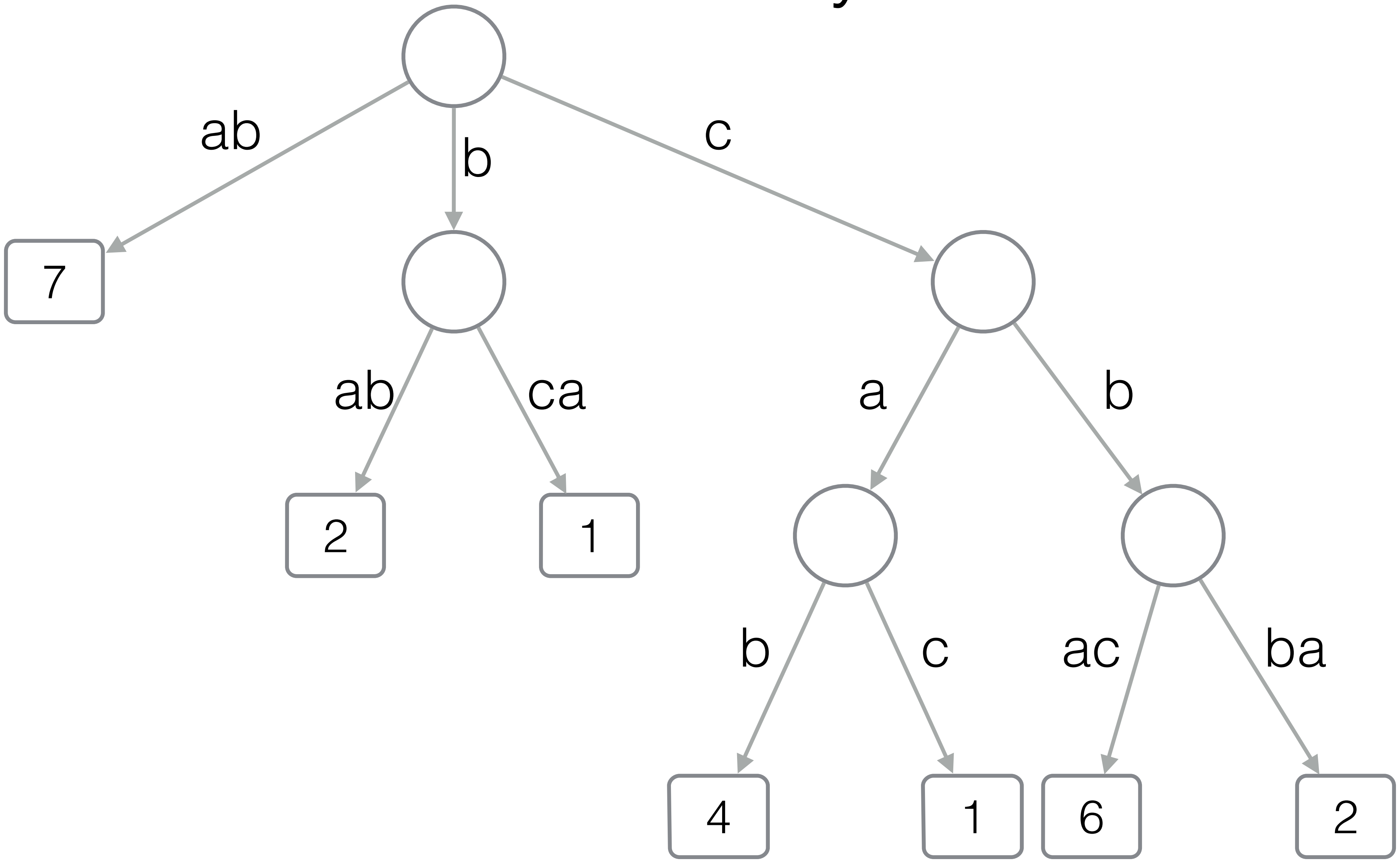
Space: O(n log n) bits
Query time: O(1)

RMQ(1,10) = ?



	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

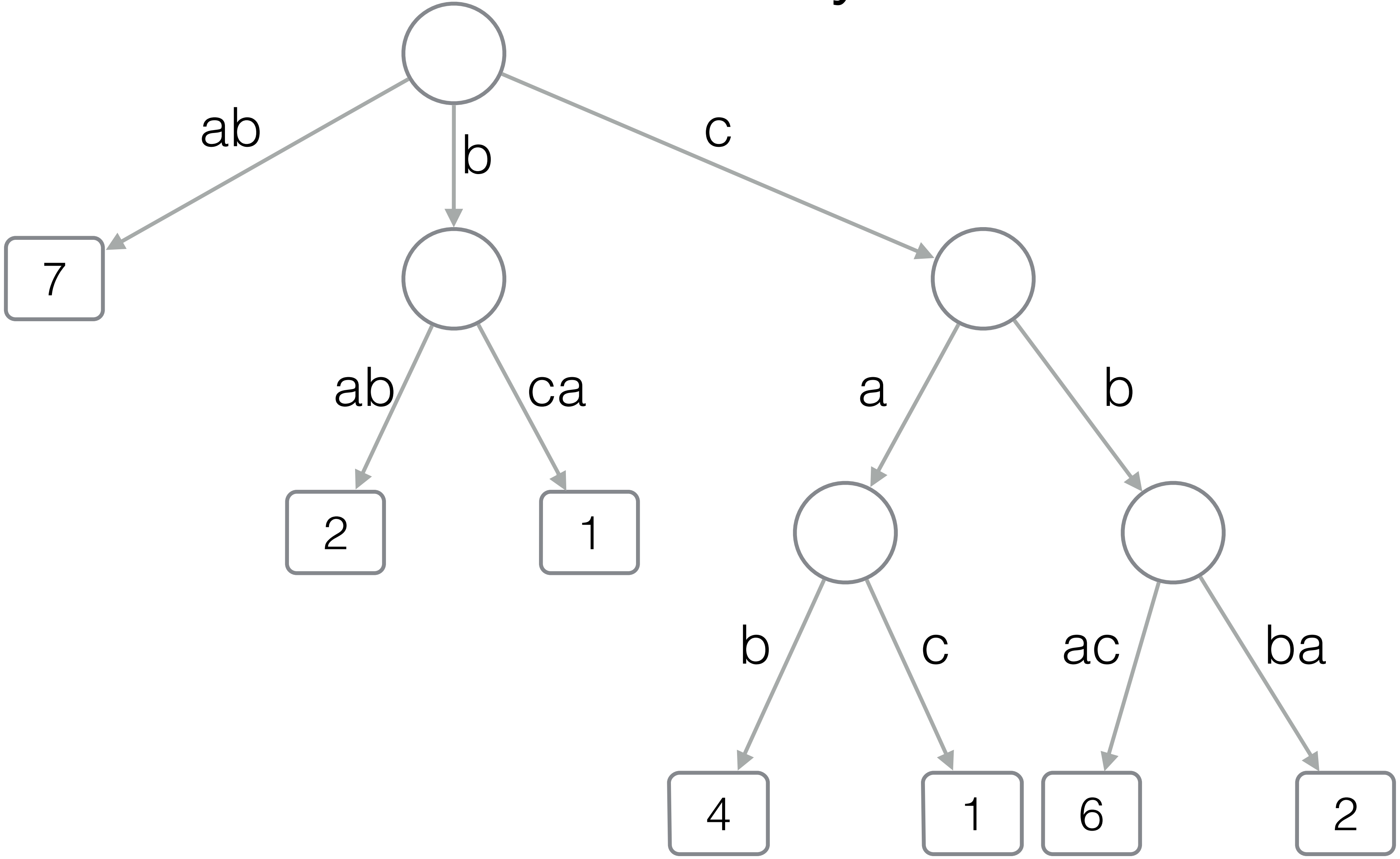
Summary



D = { ab (4), bab (2), bca (1), cab (2), cac (1), cbac (3), cbba (2) }

n = |D|, m total length of strings in D

Summary

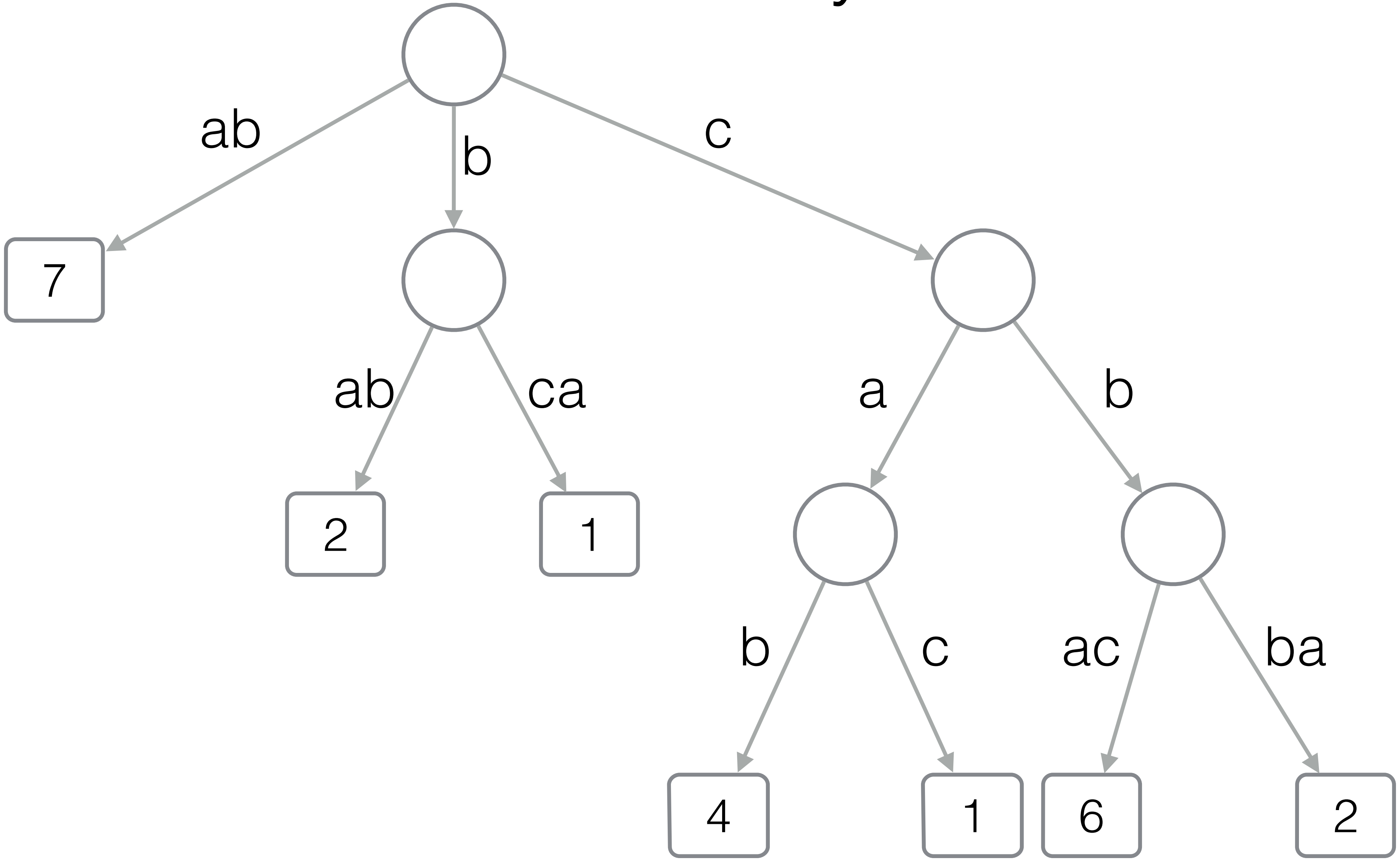


Find the node “prefixed” by P

D = { ab (4), bab (2), bca (1), cab (2), cac (1), cbac (3), cbba (2) }

n = |D|, m total length of strings in D

Summary



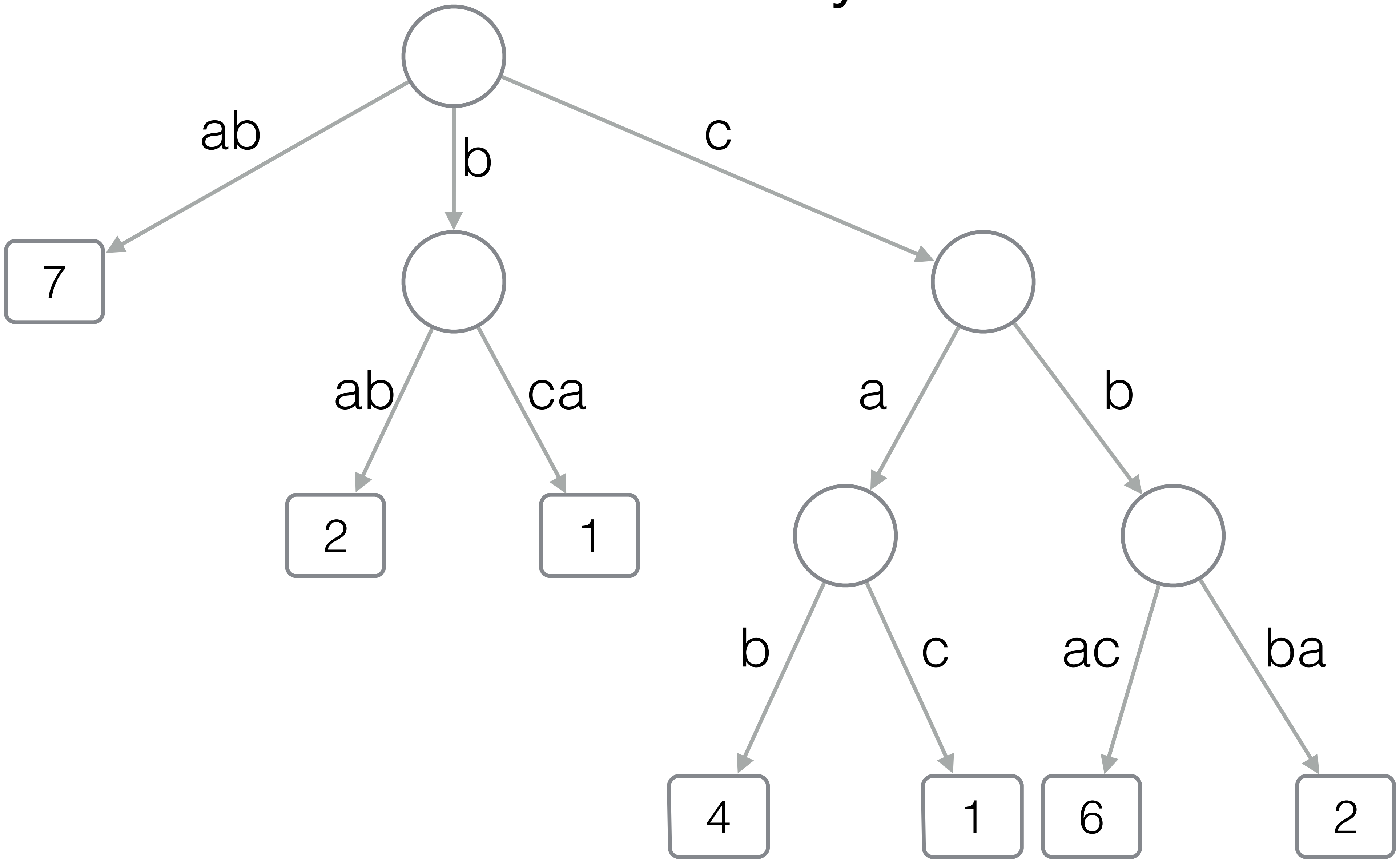
Find the node “prefixed” by P

O(|P|) time

D = { ab (4), bab (2), bca (1), cab (2), cac (1), cbac (3), cbba (2) }

n = |D|, m total length of strings in D

Summary



Find the node "prefixed" by P

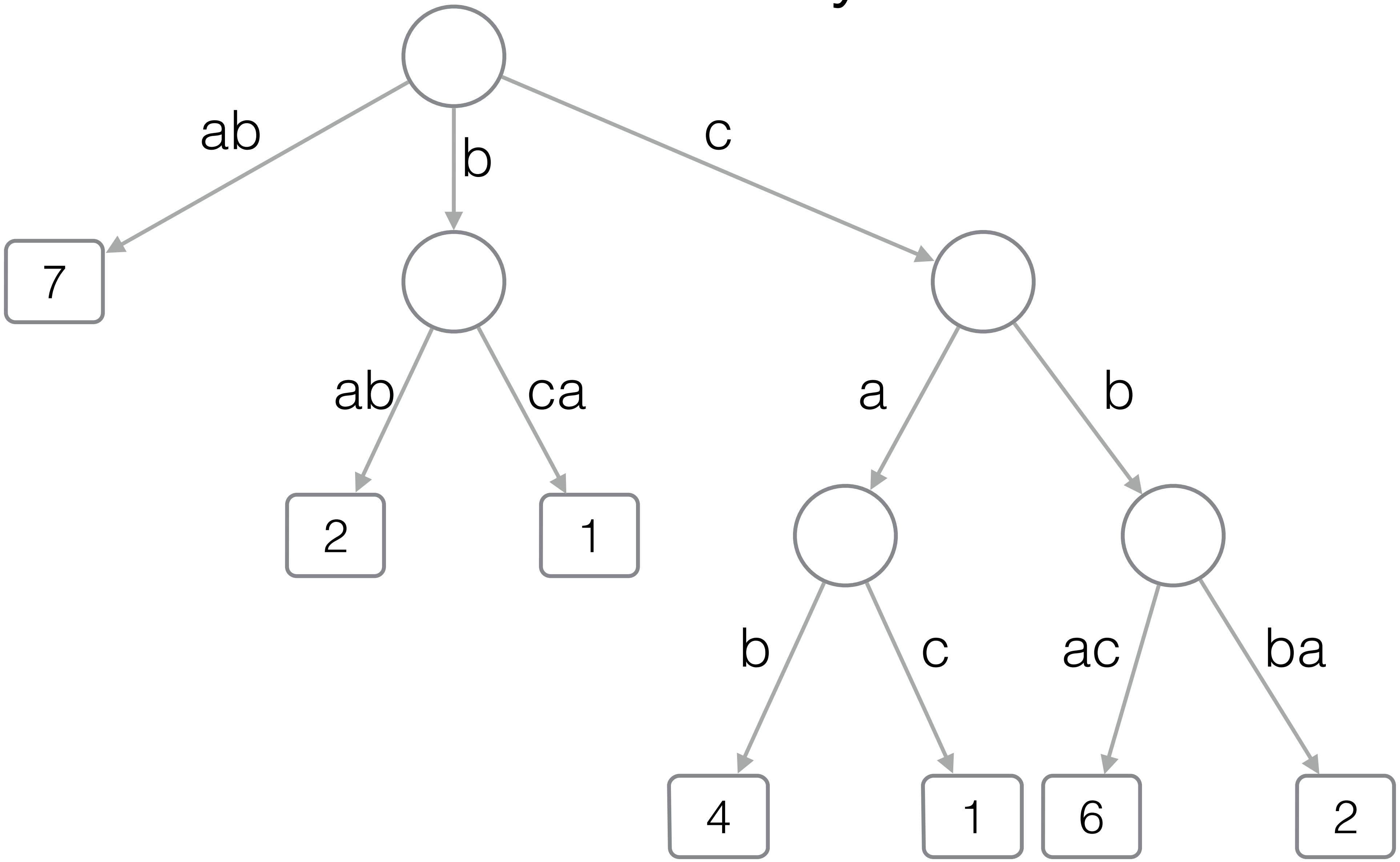
O(|P|) time

O(n log n + m log σ) bits

D = { ab (4), bab (2), bca (1), cab (2), cac (1), cbac (3), cbba (2) }

n = |D|, m total length of strings in D

Summary



Find the node “prefixed” by P

O(|P|) time

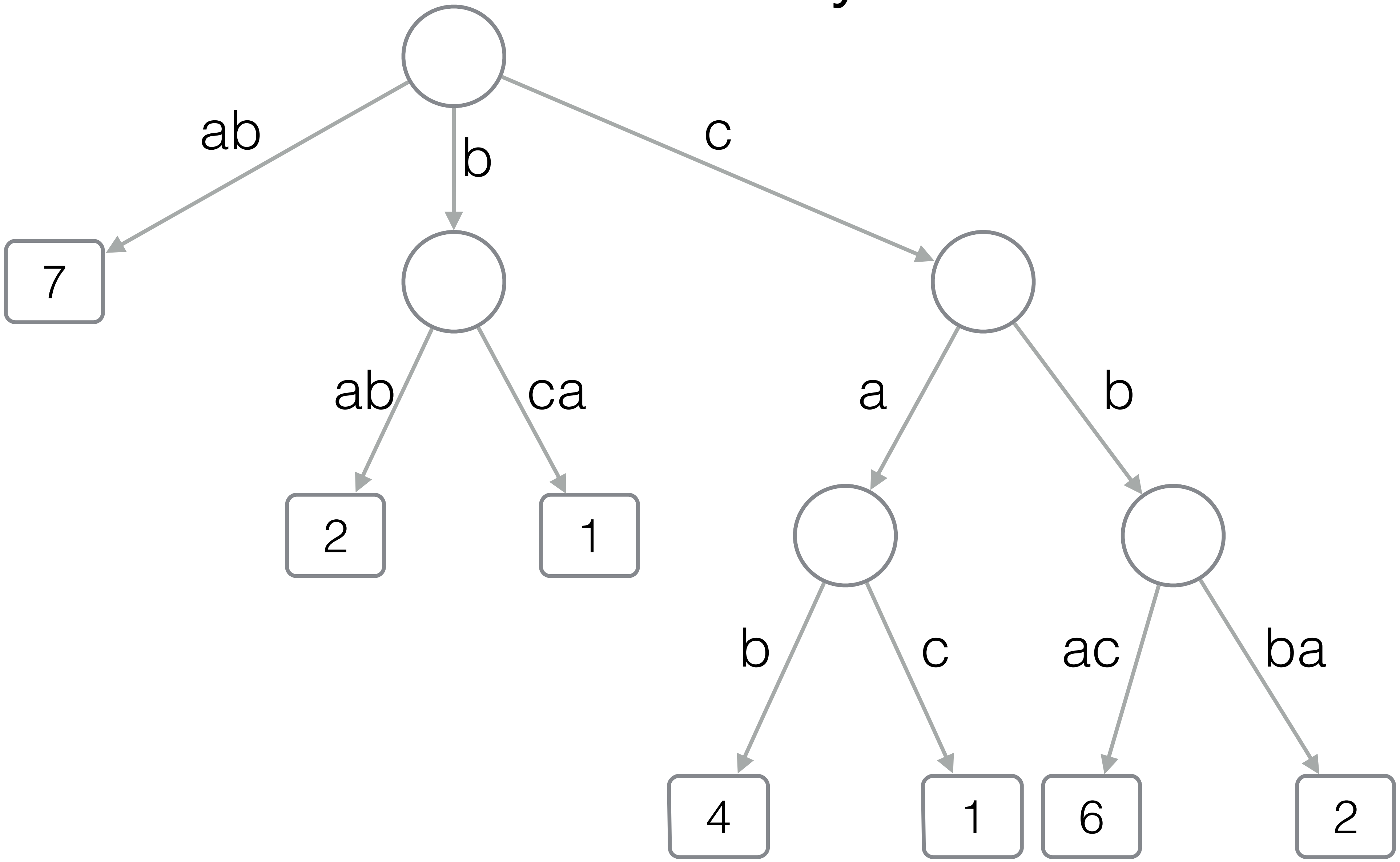
O(n log n + m log σ) bits

Compute the top-k strings

a (1), cab (2), cac (1), cbac (3), cbba (2) }

n = |D|, m total length of strings in D

Summary



Find the node “prefixed” by P

O(|P|) time

O(n log n + m log σ) bits

Compute the top-k strings

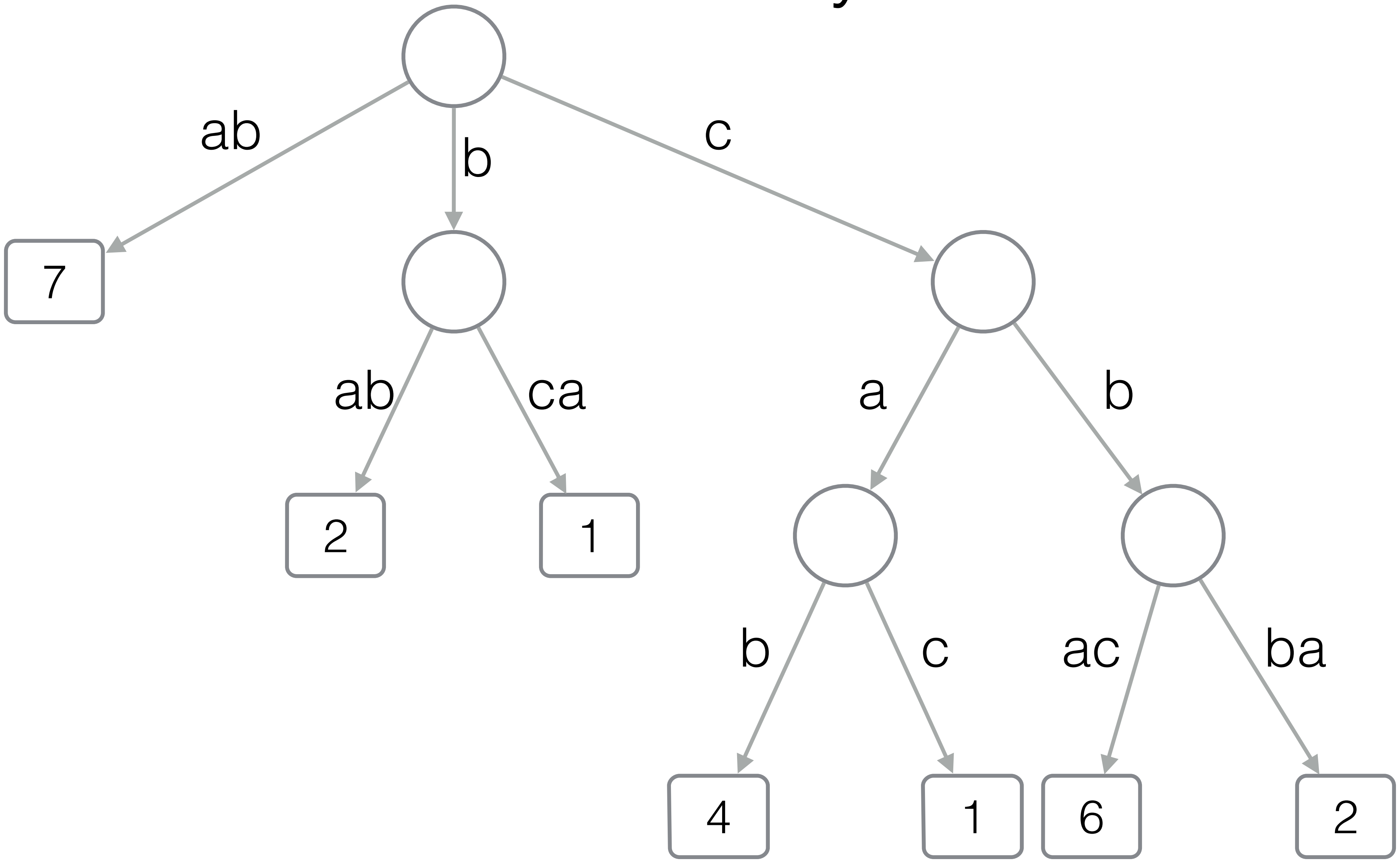
a

O(k log k) time

cbac (3), cbba (2) }

n = |D|, m total length of strings in D

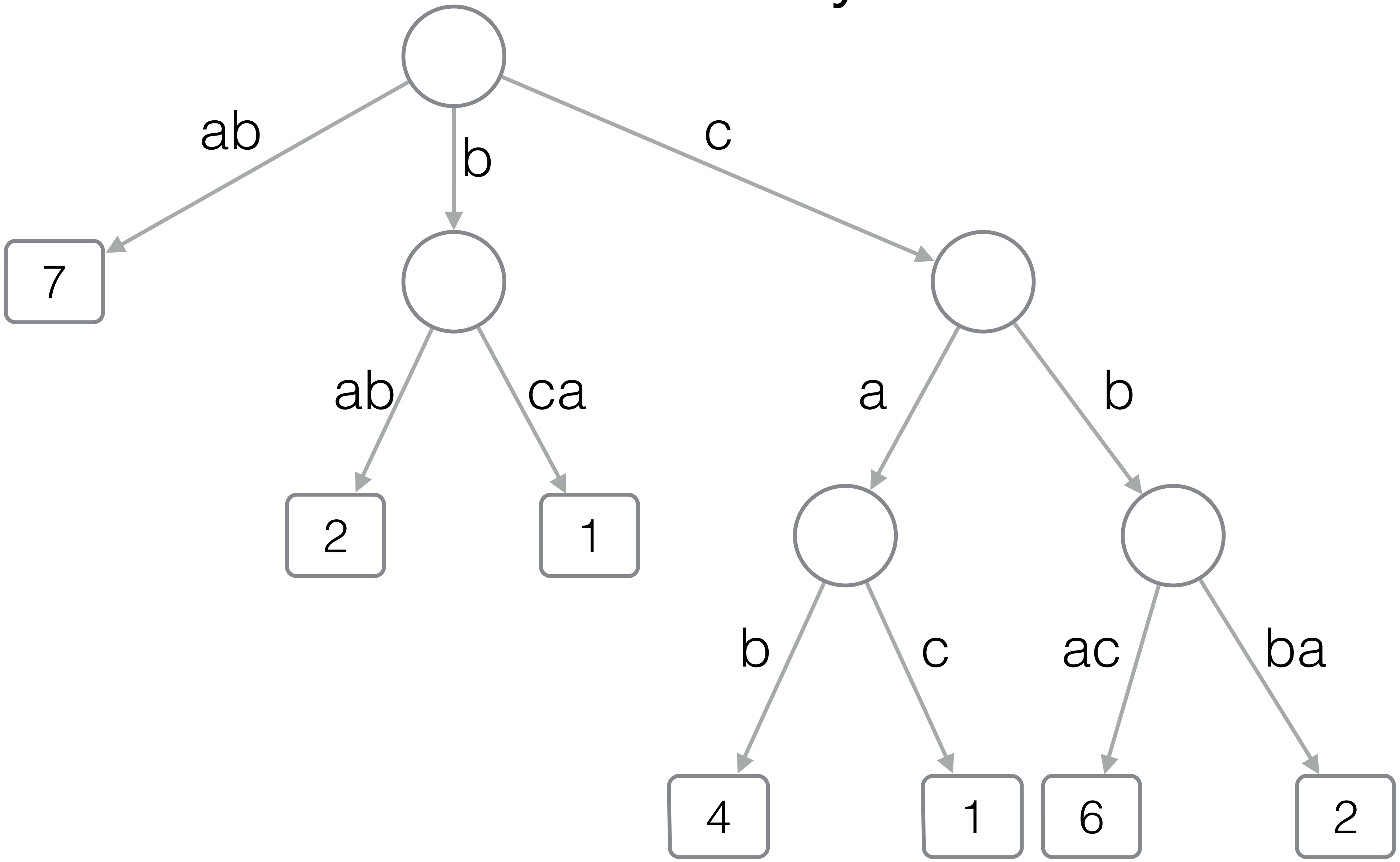
Summary



Find the node “prefixed” by P	O(|P|) time	O(n log n + m log σ) bits
Compute the top-k strings	O(k log k) time	O(n) bits

n = |D|, m total length of strings in D

Summary



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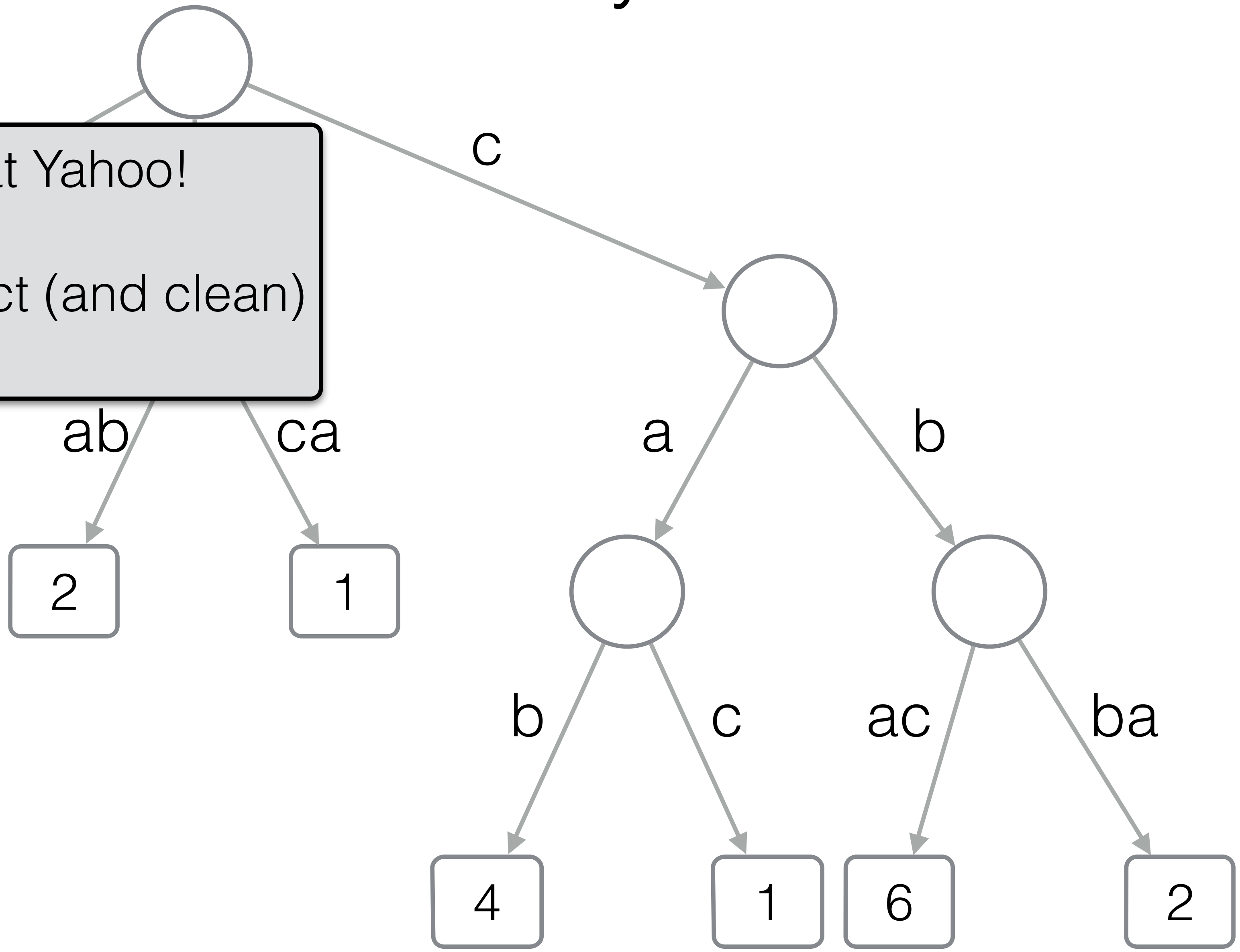

3 months query log at Yahoo!

≈600 million of distinct (and clean) queries

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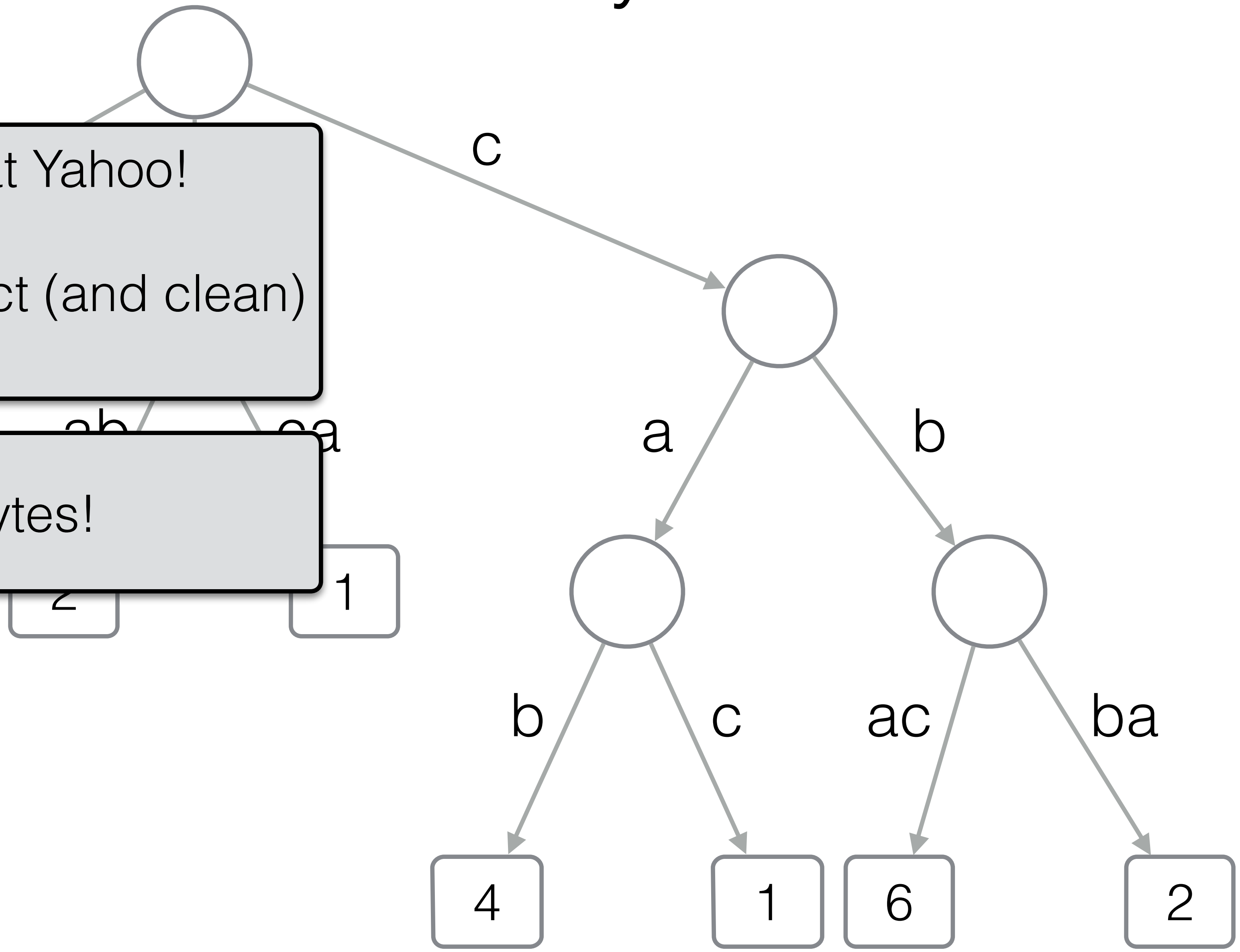

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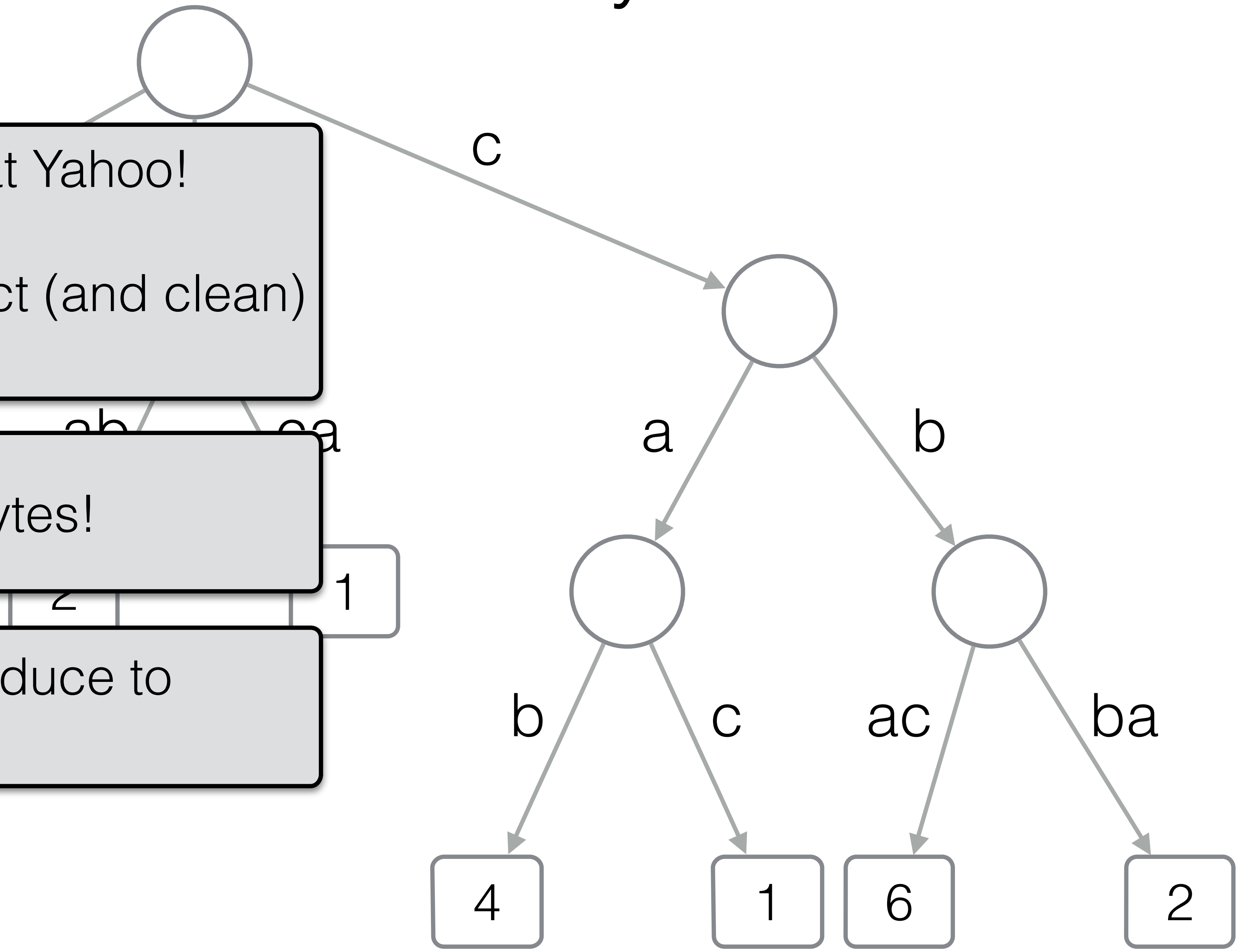

3 months query log at Yahoo!

≈600 million of distinct (and clean) queries

Trie requires ≈50 Gbytes!

We will see how to reduce to ≈5 Gbytes!

Find the node “prefixed” by P	O(|P|) time	O(n log n + m log σ) bits
Compute the top-k strings	O(k log k) time	O(n) bits

n = |D|, m total length of strings in D