Introduction to Data-Driven Dependency Parsing

Introductory Course, ESSLLI 2007

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Overview of the Course

- Dependency parsing (Joakim)
- Machine learning methods (Ryan)
- Transition-based models (Joakim)
- **Graph-based models** (Ryan)
- Loose ends (Joakim, Ryan):
  - Other approaches
  - Empirical results
  - Available software

Notation Reminder

- Sentence $x = w_0, w_1, \ldots, w_n$, with $w_0 = root$
- $L = \{l_1, \ldots, l_{|L|}\}$ set of permissible arc labels
- Let $G = (V, A)$ be a dependency graph for sentence x where:
  - $V = \{0, 1, \ldots, n\}$ is the vertex set
  - A is the arc set, i.e., $(i, j, k) \in A$ represents a dependency from w_i to w_j with label $l_k \in L$
- By the usual definition, G **is a tree**

Data-Driven Parsing

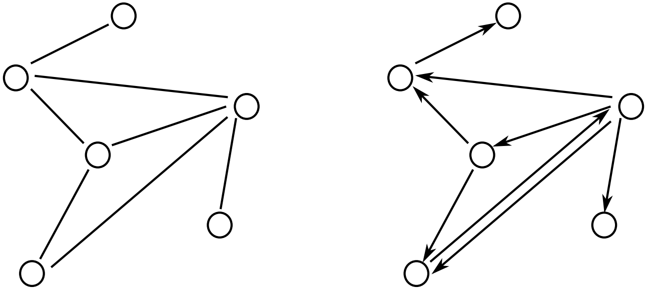
- Goal: Learn a good predictor of dependency graphs
- Input: x
- Output: dependency graph/tree G
- Last lecture:
  - Parameterize parsing by transitions
  - Learn to predict transitions given the input and a history
  - Predict new graphs using deterministic parsing algorithm
- This lecture:
  - Parameterize parsing by dependency arcs
  - Learn to predict entire graphs given the input
  - Predict new graphs using spanning tree algorithms

Lecture 4: Outline

- Graph theory refresher
- Arc-factored models (a.k.a. Edge-factored models)
  - Maximum spanning tree formulation
  - Projective and non-projective inference algorithms
  - Partition function and marginal algorithms – Matrix Tree Theorem
- Beyond Arc-factored Models
  - Vertical and horizontal markovization
  - Approximations

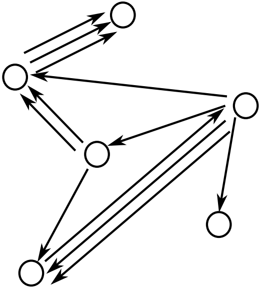
Some Graph Theory Reminders

- A graph $G = (V, A)$ is a set of verteces V and arcs $(i, j) \in A$, where $i, j \in V$
- Undirected graphs: $(i, j) \in A \Leftrightarrow (j, i) \in A$
- **Directed graphs (digraphs)**: $(i, j) \in A \nRightarrow (j, i) \in A$



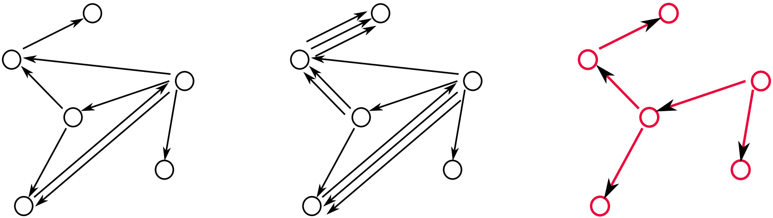
Multi-Digraphs

- A multi-digraph is a digraph where there can be multiple arcs between verteces
- $G = (V, A)$
- $(i, j, k) \in A$ represents the k^{th} arc from vertex i to vertex j



Directed Spanning Trees (a.k.a. Arborescence)

- A directed spanning tree of a (multi-)digraph $G = (V, A)$, is a subgraph $G' = (V', A')$ such that:
  - $V' = V$
  - $A' \subseteq A$, and $|A'| = |V'| - 1$
  - G' is a tree (acyclic)
- A spanning tree of the following (multi-)digraphs



Weighted Directed Spanning Trees

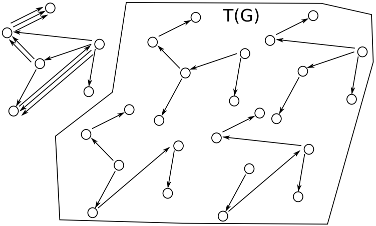
- Assume we have a weight function for each arc in a multi-digraph $G = (V, A)$
- Define $w_{ij}^{k} \geq 0$ to be the weight of $(i, j, k) \in A$ for a multi-digraph
- Define the weight of directed spanning tree G' of graph G as

$$w(G') = \prod_{(i,j,k) \in G'} w_{ij}^{k}$$

- Notation: $(i, j, k) \in G = (V, A) \Leftrightarrow$ the arc $(i, j, k) \in A$

Maximum Spanning Trees (MST) of (Multi-)Digraphs

- Let $T(G)$ be the set of all spanning trees for graph G



- The MST Problem: Find the spanning tree G' of the graph G that has highest weight

$$G' = \arg\max_{G' \in T(G)} w(G') = \arg\max_{G' \in T(G)} \prod_{(i,j,k) \in G'} w_{ij}^k$$

- Solutions … to come.

Arc-Factored Dependency Models

- Remember: Data-driven parsing parameterizes model and then learns parameters from data
- **Arc-factored model**
  - Assumes that the score / probability / **weight** of a dependency graph factors by its arcs

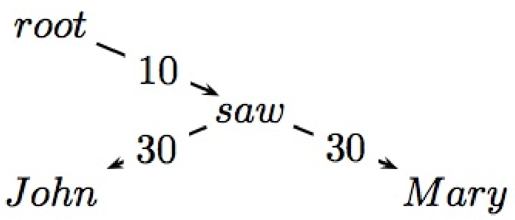
$$w(G) = \prod_{(i,j,k) \in G} w_{ij}^k$$

look familiar?

  - w_{ij}^k is the weight of creating a dependency from word w_i to w_j with label l_k
  - Thus there is an assumption that each dependency decision is independent
    - Strong assumption! Will address this later.

Arc-Factored Dependency Models Example

- Weight of dependency graph is $10 \times 30 \times 30 = 9000$



- In practice arc weights are much smaller

Important Concept G_x

- For input sentence $x = w_0, \ldots, w_n$, define $G_x = (V_x, A_x)$ as:
  - $V_x = \{0, 1, \ldots, n\}$
  - $A_x = \{(i, j, k) \mid \forall\ i, j \in V_x \text{ and } l_k \in L\}$
- Thus, G_x is complete multi-digraph over vertex set representing words

Theorem

Every valid dependency graph for sentence x is equivalent to a directed spanning tree for G_x that originates out of vertex 0

- Falls out of definitions of tree constrained dependency graphs and spanning trees
  - Both are spanning/connected (contain all words)
  - Both are trees

Three Important Problems

Theorem

Every valid dependency graph for sentence x is equivalent to a directed spanning tree for G_x that originates out of vertex 0

1. **Inference** $\equiv$ finding the MST of G_x

$$G = \underset{G \in T(G_x)}{\arg\max}\ w(G) = \underset{G \in T(G_x)}{\arg\max} \prod_{(i,j,k) \in G} w_{ij}^k$$

2. Defining w_{ij}^k and its **feature space**
3. **Learning** w_{ij}^k
   - Can use perceptron-based learning if we solve (1)

Inference - Getting Rid of Arc Labels

$$G = \arg\max_{G \in T(G_x)} w(G) = \arg\max_{G \in T(G_x)} \prod_{(i,j,k) \in G} w_{ij}^k$$

- Consider all the arcs between vertexes i and j
- Now, consider the arc (i, j, k) such that,

$$(i, j, k) = \arg\max_k w_{ij}^k$$

Theorem

The highest weighted dependency tree for sentence x must contain the arc (i, j, k) – (assuming no ties)

- Easy proof: if not, sub in (i, j, k) and get higher weighted tree

Inference - Getting Rid of Arc Labels

$$G = \underset{G \in T(G_x)}{\arg\max}\ w(G) = \underset{G \in T(G_x)}{\arg\max} \prod_{(i,j,k) \in G} w_{ij}^k$$

- Thus, we can reduce G_x from a multi-digraph to a simple digraph
- Just remove all arcs that do not satisfy

$$(i, j, k) = \underset{k}{\arg\max}\, w_{ij}^k$$

- Problem is now equal to the MST problem for digraphs

We will use the **Chu-Liu-Edmonds Algorithm**

[Chu and Liu 1965, Edmonds 1967]

Chu-Liu-Edmonds Algorithm

- Finds the MST originating out of a vertex of choice
- Assumes weight of tree is **sum** of arc weights
- No problem, we can use logarithms

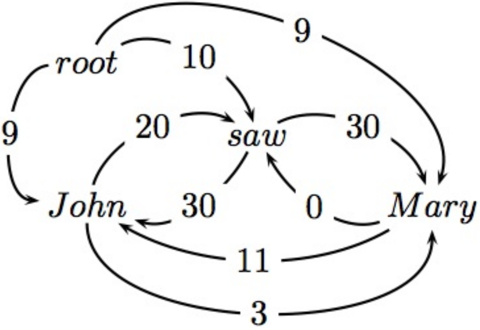
$$
\begin{aligned}
G &= \arg\max_{G \in T(G_x)} \prod_{(i,j,k) \in G} w_{ij}^k \\
&= \arg\max_{G \in T(G_x)} \log \prod_{(i,j,k) \in G} w_{ij}^k \\
&= \arg\max_{G \in T(G_x)} \sum_{(i,j,k) \in G} \log w_{ij}^k
\end{aligned}
$$

So if we let $w_{ij}^k = \log w_{ij}^k$, then we get

$$
G = \arg\max_{G \in T(G_x)} \sum_{(i,j,k) \in G} w_{ij}^k
$$

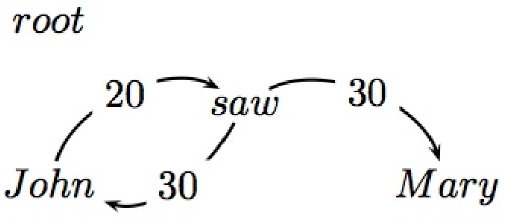
Chu-Liu-Edmonds

- x = root John saw Mary



Chu-Liu-Edmonds

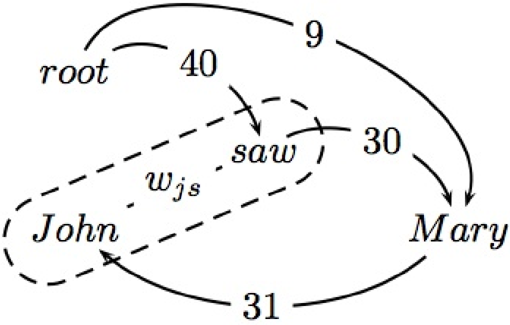
- Find highest scoring incoming arc for each vertex



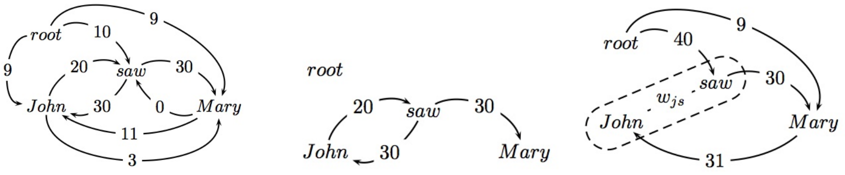
- If this is a tree, then we have found MST!!

Chu-Liu-Edmonds

- If not a tree, identify cycle and contract
- Recalculate arc weights into and out-of cycle

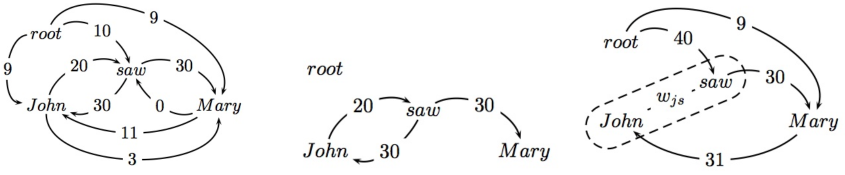


Chu-Liu-Edmonds



- Outgoing arc weights
  - Equal to the max of outgoing arc over all vertexes in cycle
  - e.g., John → Mary is 3 and saw → Mary is 30

Chu-Liu-Edmonds

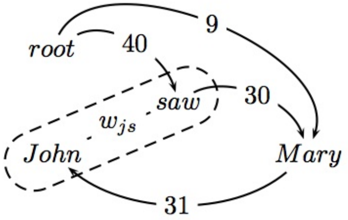


- Incoming arc weights
  - Equal to the weight of best spanning tree that includes head of incoming arc, and all nodes in cycle
  - root $\longrightarrow$ saw $\longrightarrow$ John is 40 (**)
  - root $\longrightarrow$ John $\longrightarrow$ saw is 29

Chu-Liu-Edmonds

Theorem

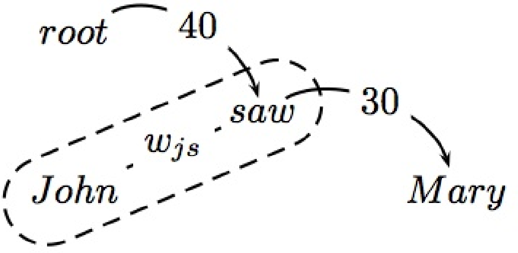
The weight of the MST of this contracted graph is equal to the weight of the MST for the original graph



- Therefore, recursively call algorithm on new graph

Chu-Liu-Edmonds

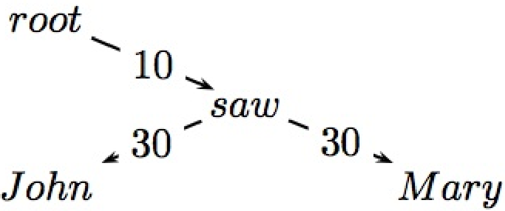
- This is a tree and the MST for the contracted graph!!



- Go back up recursive call and reconstruct final graph

Chu-Liu-Edmonds

- This is the MST!!



Chu-Liu-Edmonds Code

Chu-Liu-Edmonds(G_x, w)

Let $M = \{(i^*, j) : j \in V_x, i^* = \arg\max_{i'} w_{ij}\}$
Let $G_M = (V_x, M)$
If G_M has no cycles, then it is an MST: return G_M
Otherwise, find a cycle C in G_M
Let $< G_C, c, ma >=$ contract(G, C, w)
Let $G =$ Chu-Liu-Edmonds(G_C, w)
Find vertex $i \in C$ such that $(i', c) \in G$ and $ma(i', c) = i$
Find arc $(i'', i) \in C$
Find all arc $(c, i''') \in G$
$G = G \cup \{(ma(c, i'''), i''')\}_{\forall (c,i''') \in G} \cup C \cup \{(i', i)\} - \{(i'', i)\}$
Remove all vertices and arcs in G containing c
return G

- Reminder: $w_{ij} = \arg\max_k w_{ij}^k$

Chu-Liu-Edmonds Code (II)

contract$(G = (V, A), C, w)$

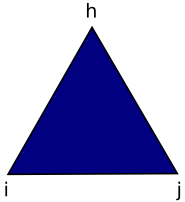
1. Let G_C be the subgraph of G excluding nodes in C
2. Add a node c to G_C representing cycle C
3. For $i \in V - C : \exists_{i' \in C}(i', i) \in A$
   - Add arc (c, i) to G_C with
     - $ma(c, i) = \arg\max_{i' \in C} score(i', i)$
     - $i' = ma(c, i)$
     - $score(c, i) = score(i', i)$
4. For $i \in V - C : \exists_{i' \in C}(i, i') \in A$
   - Add edge (i, c) to G_C with
     - $ma(i, c) = \arg\max_{i' \in C} [score(i, i') - score(a(i'), i')]$
     - $i' = ma(i, c)$
     - $score(i, c) = [score(i, i') - score(a(i'), i') + score(C)]$
       - where $a(v)$ is the predecessor of v in C
       - and $score(C) = \sum_{v \in C} score(a(v), v)$
5. return $< G_C, c, ma >$

Chu-Liu-Edmonds

- Naive implementation $O(n^3 + |L|n^2)$
  - Converting G_x to a digraph – $O(|L|n^2)$
  - Finding best arc – $O(n^2)$
  - Contracting cycles – $O(n^2)$
  - At most n recursive calls
- Better algorithms run in $O(|L|n^2)$ [Tarjan 1977]
- Chu-Liu-Edmonds searches all dependency graphs
  - Both projective and non-projective
  - Thus, it is an exact non-projective search algorithm!!!
- **What about the projective case?**

Arc-factored Projective Parsing

- Projective dependency structures are nested
- Can use CFG like parsing algorithms – chart parsing
- Each chart item (triangle) represents the weight of the best tree rooted at word h spanning all the words from i to j
  - Analog in CFG parsing: items represent best tree rooted at non-terminal NT spanning words i to j
- **Goal**: Find chart item rooted at 0 spanning 0 to n

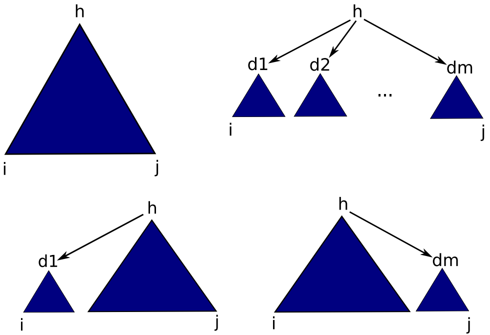


Base case

Length 1, $h = i = j$, has weight 1

Arc-factored Projective Parsing

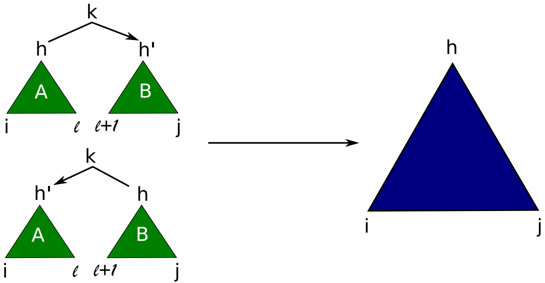
- All projective graphs can be written as the combination of two smaller adjacent graphs



- Inductive hypothesis – algorithm has calculated score of smaller items correctly (just like CKY)

Arc-factored Projective Parsing

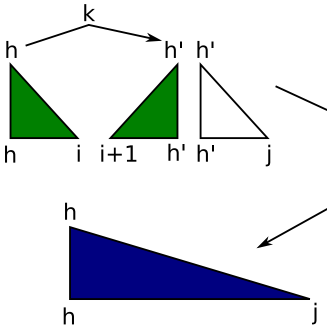
- Chart item filled in a bottom-up manner
  - First do all strings of length 1, then 2, etc. just like CKY



- Weight of new item: $\max_{l,j,k} \; w(A) \times w(B) \times w^{k}_{hh'}$
- Algorithm runs in $O(|L|n^5)$
- Use back-pointers to extract best parse (like CKY)

Arc-factored Projective Parsing

- $O(|L|n^5)$ is not that good
- [Eisner 1996] showed how this can be reduced to $O(|L|n^3)$
  - Key: split items so that sub-roots are always on periphery



Inference in Arc-Factored Models

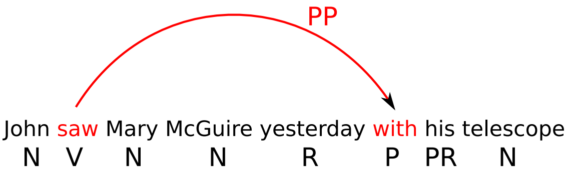
- Non-projective case
  - $O(|L|n^2)$ with the Chu-Liu-Edmonds MST algorithm
- Projective case
  - $O(|L|n^3)$ with the Eisner algorithm
- But we still haven't defined the form of w_{ij}^k
- Or how to learn these parameters

Arc weights as linear classifiers

$$w_{ij}^{k} = e^{\mathbf{w} \cdot \mathbf{f}(i,j,k)}$$

- Arc weights are a linear combination of features of the arc, **f**, and a corresponding weight vector **w**
- Raised to an exponent (simplifies some math ...)
- What arc features?
- [McDonald et al. 2005] discuss a number of binary features

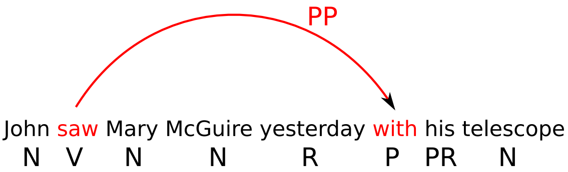
Arc Features: $\mathbf{f}(i, j, k)$



- Features from [McDonald et al. 2005]:
  - Identities of the words w_i and w_j and the label l_k

head=saw & dependent=with

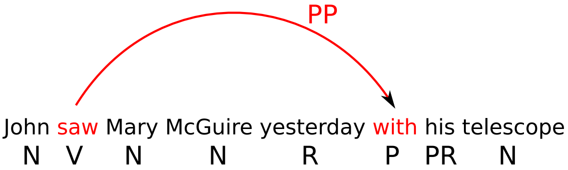
Arc Features: $\mathbf{f}(i, j, k)$



- Features from [McDonald et al. 2005]:
  - Part-of-speech tags of the words w_i and w_j and the label l_k

head-pos=Verb & dependent-pos=Preposition

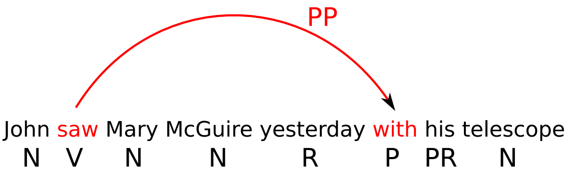
Arc Features: $\mathbf{f}(i, j, k)$



- Features from [McDonald et al. 2005]:
  - Part-of-speech of words surrounding and between w_i and w_j

inbetween-pos=Noun
inbetween-pos=Adverb
dependent-pos-right=Pronoun
head-pos-left=Noun
...

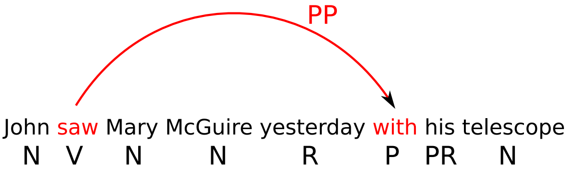
Arc Features: $\mathbf{f}(i, j, k)$



- Features from [McDonald et al. 2005]:
  - Number of words between w_i and w_j, and their orientation

arc-distance=3
arc-direction=right

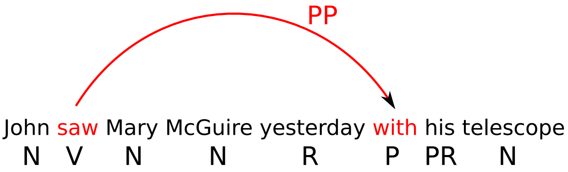
Arc Features: $\mathbf{f}(i, j, k)$



- Label features

arc-label=PP

Arc Features: $\mathbf{f}(i, j, k)$



- Combos of the above

head-pos=Verb & dependent-pos=Preposition & arc-label=PP

head-pos=Verb & dependent=with & arc-distance=3

...

- No limit: any feature over arc (i, j, k) or input x

Learning the parameters

- We can then re-write the inference problem

$$
\begin{aligned}
G &= \arg\max_{G \in T(G_x)} \prod_{(i,j,k)\in G} w_{ij}^{k} = \arg\max_{G \in T(G_x)} \prod_{(i,j,k)\in G} e^{\mathbf{w}\cdot\mathbf{f}(i,j,k)} \\
&= \arg\max_{G \in T(G_x)} \log \prod_{(i,j,k)\in G} e^{\mathbf{w}\cdot\mathbf{f}(i,j,k)} \\
&= \arg\max_{G \in T(G_x)} \sum_{(i,j,k)\in G} \mathbf{w}\cdot\mathbf{f}(i,j,k) \\
&= \arg\max_{G \in T(G_x)} \mathbf{w}\cdot \sum_{(i,j,k)\in G} \mathbf{f}(i,j,k) = \arg\max_{G \in T(G_x)} \mathbf{w}\cdot\mathbf{f}(G)
\end{aligned}
$$

- Which we can plug into online learning algorithms

Inference-based Learning

e.g., The Perceptron

Training data: $\mathcal{T} = \{(x_t, G_t)\}_{t=1}^{|\mathcal{T}|}$

1. $\mathbf{w}^{(0)} = 0; \; i = 0$
2. for $n : 1..N$
3. for $t : 1..T$
4. Let $G' = \arg\max_{G'} \mathbf{w}^{(i)} \cdot \mathbf{f}(G')$ (**)
5. if $G' \neq G_t$
6. $\mathbf{w}^{(i+1)} = \mathbf{w}^{(i)} + \mathbf{f}(G_t) - \mathbf{f}(G')$
7. $i = i + 1$
8. return $\mathbf{w}^i$

Other Important Problems

- K-best inference – $O(K \times |L|n^2)$ [Camerini et al. 1980]
- **Partition function**

$$Z_x = \sum_{G \in T(G_x)} w(G)$$

- **Arc expectations**

$$\langle i, j, k \rangle_x = \sum_{G \in T(G_x)} w(G) \times \mathbb{1}[(i, j, l) \in G]$$

- Important for some learning & inference frameworks
- Important for some applications

Partition Function: $Z_x = \sum_{G \in T(G_x)} w(G)$

- Lapacian Matrix Q for graph $G_x = (V_x, A_x)$

$$Q_{jj} = \sum_{i \neq j, (i,j,k) \in A_x} w_{ij}^k \quad \text{and} \quad Q_{ij} = \sum_{i \neq j, (i,j,k) \in A_x} -w_{ij}^k$$

- Cofactor Q^i is the matrix Q with the i^{th} row and column removed

The Matrix Tree Theorem [Tutte 1984]
The determinant of the cofactor Q^0 is equal to Z_x

- Thus $Z_x = |Q^0|$ – determinants can be calculated in $O(n^3)$
- Constructing Q takes $O(|L|n^2)$
- Therefore the whole process takes $O(n^3 + |L|n^2)$

Arc Expectations

$$\langle i, j, k \rangle_x = \sum_{G \in T(G_x)} w(G) \times \mathbb{1}[(i, j, k) \in A]$$

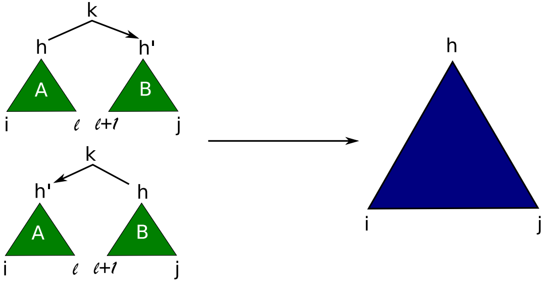
- Can easily be calculated, first reset some weights

$$w_{i'j}^{k'} = 0 \ \forall i' \neq i \text{ and } k' \neq k$$

- Now, $\langle i, j, k \rangle_x = Z_x$
- Why? All competing arc weights to zero, therefore every non-zero weighted graph must contain (i, j, k)
- Naively takes $O(n^5 + |L|n^2)$ to compute all expectations
- But can be calculated in $O(n^3 + |L|n^2)$ (see [McDonald and Satta 2007, Smith and Smith 2007, Koo et al. 2007])

Z_x for the Projective Case

- Just augment chart-parsing algorithm



- Weight of new item: $\sum_{l,j,k} \quad w(A) \times w(B) \times w^k_{hh'}$
- Weight of item rooted at 0 spanning 0 to n is equal to Z_x
- Also works for Eisner's algorithim – runtime $O(n^3 + |L|n^2)$

$\langle i, j, k \rangle_x$ for the Projective Case

- Can be calculated through Z_x, just like the non-projective case
- Can also be calculated using the inside-outside algorithm
- See [Paskin 2001] for more details

Why calculate Z_x and $\langle i, j, k \rangle_x$?

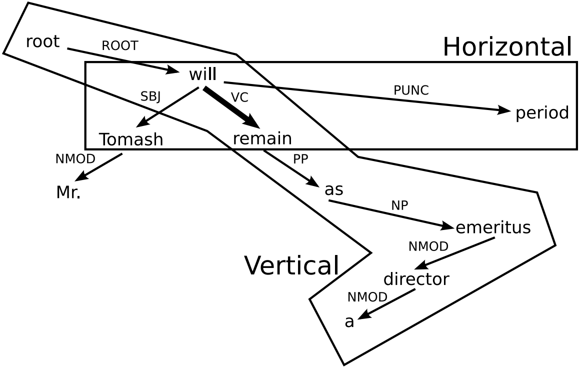
- Useful for many learning and inference problems
  - Min risk-decoding ($\langle i, j, k \rangle_x$)
  - Log-linear parsing models (Z_x and $\langle i, j, k \rangle_x$)
  - Syntactic language modeling (Z_x)
  - Unsupervised dependency parsing (Z_x and $\langle i, j, k \rangle_x$)
  - ...
- See [McDonald and Satta 2007] for more

Beyond Arc-factored Models

- Arc-factored models make strong independence assumptions
- Can we do better?
- Rest of lecture
  - NP-hardness of Markovization for non-projective parsing
  - But ... projective case has polynomial solutions!!
  - Approximate non-projective algorithms

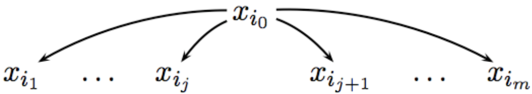
Vertical and Horizontal Markovization

- Dependency graphs weight factors over neighbouring arcs
- Vertical versus Horizontal neighbourhoods



N^{th} Order Horizontal Markov Factorization

- Assume the **unlabeled parsing** case (adding labels is easy)
- Weights factor over neighbourhoods of size N



- Normal (arc-factored = first-order)

$$\prod_{k=1}^{m} w_{i_0 i_k}$$

- Second-order – weights over pairs of adjacent (same side) arcs

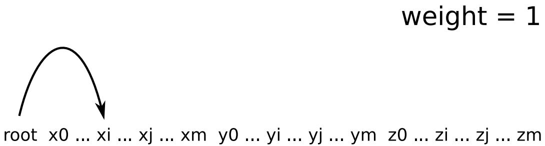
$$\prod_{k=1}^{j-1} w_{i_0 i_k i_{k+1}} \times w_{i_0 \cdot i_j} \times w_{i_0 \cdot i_{j+1}} \times \prod_{k=j+1}^{m-1} w_{i_0 i_k i_{k+1}}$$

Non-projective Horizontal Markovization

- Non-projective second-order parsing is NP-hard
  - Thus any order non-projective parsing is NP-hard
- **3-dimensional matching (3DM)**: Disjoint sets X, Y, Z each with m elements. A set $T \subseteq X \times Y \times Z$. Question – is there a subset $S \subseteq T$ such that $|S| = m$ and each $v \in X \cup Y \cup Z$ occurs in exactly one element of S
- **Reduction**: Define $G_x = (V_x, A_x)$ as a dense graph, where
  - $V_x = \{v \mid \forall, v \in X \cup Y \cup Z\} \cup \{0\}$
  - $w_{0x_ix_j} = 1, \forall x_i, x_j \in X$
  - $w_{x \cdot y} = 1, \forall x \in X, y \in Y$
  - $w_{x_iy_jz_k} = 1, \forall (x, y, z) \in T$
  - All other weights are 0

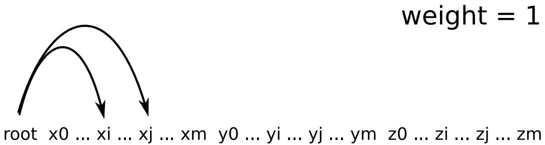
Non-projective Horizontal Markovization

- Non-projective second-order parsing is NP-hard
- Generate sentence from all $x \in X$, $y \in Y$ and $z \in Z$



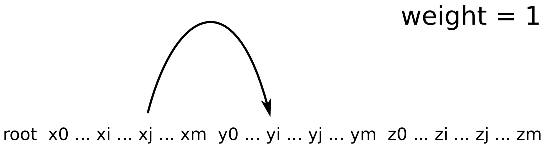
Non-projective Horizontal Markovization

- Non-projective second-order parsing is NP-hard



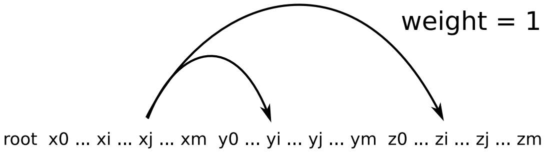
Non-projective Horizontal Markovization

- Non-projective second-order parsing is NP-hard



Non-projective Horizontal Markovization

- Non-projective second-order parsing is NP-hard



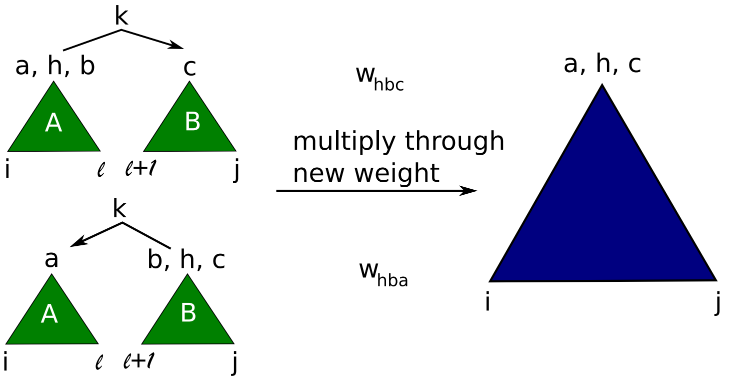
- All other arc weights are set to 0

Non-projective Horizontal Markovization

- **Theorem**: There is a 3DM iff there is a dependency graph of weight 1
- Proof:
  - All non-zero weight dependency graphs correspond to a 3DM
  - Every 3DM corresponds to a non-zero weight dependency graph
  - Therefore, there is a non-zero weight dependency graph iff then there is a 3DM
  - See [McDonald and Pereira 2006] for more

Projective Horizontal Markovization

- Can simply augment chart parsing algorithm
- Same for the Eisner algorithm – runtime still $O(|L|n^3)$



Approx Non-proj Horizontal Markovization

- Two properties:
  - Projective parsing is polynomial w/ horizontal Markovization
  - Most non-projective graphs are still primarily projective
- Use these facts to get an approximate non-projective algorithm
  - Find a high scoring projective parse
  - Iteratively modify to create a higher scoring non-projective parse
  - Post-process non-projectivity, which is related to pseudo-projective parsing

Approx Non-proj Horizontal Markovization

- **Algorithm**
  1. Let G be the highest weighted projective graph
  2. Find the arc $(i, j, k) \in G$, a node i' and label $l_{k'}$ such that
     - $G' = G \cup \{(i', j, k')\} - \{(i, j, k)\}$ is a valid graph (tree)
     - G' has highest weight of all possibly changes
  3. if $w(G') > w(G)$ then $G = G'$ and return to step 2
  4. Otherwise return G
- **Intuition**: Start with a high weighted graph and make local changes that increase the graphs weight until convergence
- Works well in practice [McDonald and Pereira 2006]

Vertical Markovization

- Also NP-hard for non-projective case [McDonald and Satta 2007]
  - Reduction again from 3DM
  - A little more complicated – relies on arc labels
- Projective case is again polynomial
  - Same method of augmenting the chart-parsing algorithm

Beyond Arc-Factorization

- For the non-projective case, increasing scope of weights (and as a result features) makes parsing intractable
- However, chart parsing nature of projective algorithms allows for simple augmentations
- Can approximate the non-projective case using the exact projective algorithms plus a post-process optimization
- Further reading:

  [McDonald and Pereira 2006, McDonald and Satta 2007]

Summary – Graph-based Methods

- Arc-factored models
  - Maximum spanning tree formulation
  - Projective and non-projective inference algorithms
  - Partition function and arc expectation algorithms – Matrix Tree Theorem
- Beyond Arc-factored Models
  - Vertical and horizontal markovization
  - Approximations

References and Further Reading

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